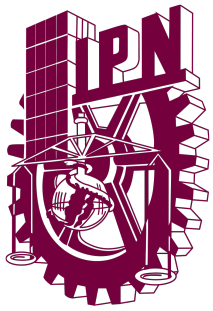

Instituto Politécnico Nacional

Escuela Superior de Cómputo

Ecuaciones simbólicas de los ECA para la regla 22 y 54

Unidad de aprendizaje: Computing Selected Topics

Grupo: 3CM19

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Índice

1. Introducción

En este documento se harán los cálculos correspondientes para hallar las ecuaciones simbólicas de un autómata celular elemental para la regla 22 y la regla 54 mediante el uso de la ecuación recursiva siguiente.

$$R_{ij}^{(k)} = R_{ij}^{(k-1)} + R_{ik}^{(k-1)}(R_{kk}^{(k-1)})^* R_{kj}^{(k-1)} \tag{1}$$

2. Ecuaciones simbólicas regla 22

Para poder encontrar una ecuación que describa a nuestro autómata necesitaremos el diagrama de 'de Bruijn' de nuestro ECA para la regla 22, el cual es el que se representa en la figura 1.

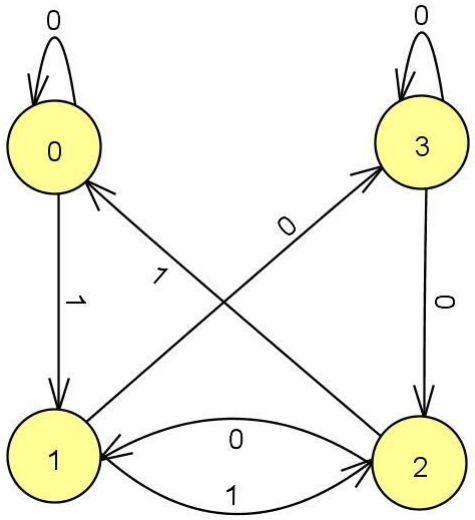

Figura 1: Diagrama de 'de Bruijn' para ECA regla 22

2.1. Cálculos para k=0

$$R_{00}^{(0)} = \varepsilon + 0; R_{01}^{(0)} = 1; R_{02}^{(0)} = \emptyset; R_{03}^{(0)} = \emptyset$$
$$R_{10}^{(0)} = \emptyset; R_{11}^{(0)} = \varepsilon; R_{12}^{(0)} = 1; R_{13}^{(0)} = 0$$
$$R_{20}^{(0)} = 1; R_{21}^{(0)} = 0; R_{22}^{(0)} = \varepsilon; R_{23}^{(0)} = \emptyset$$
$$R_{30}^{(0)} = \emptyset; R_{31}^{(0)} = \emptyset; R_{32}^{(0)} = 0; R_{33}^{(0)} = \varepsilon + 0$$

2.2. Cálculos para k=1

$$R_{00}^{(1)} = R_{00}^{0} + R_{01}^{0}(R_{11}^{0})^* R_{10}^{0} = (\varepsilon + 0) + (1)(\varepsilon)^*(\emptyset) = \varepsilon + 0$$
$$R_{01}^{(1)} = R_{01}^{0} + R_{01}^{0}(R_{11}^{0})^* R_{11}^{0} = (1) + (1)(\varepsilon)^*(\varepsilon) = 1$$
$$R_{02}^{(1)} = R_{02}^{0} + R_{01}^{0}(R_{11}^{0})^* R_{12}^{0} = \emptyset + (1)(\varepsilon)^*(1) = 11$$

$$R_{03}^{(1)} = R_{03}^{0} + R_{01}^{0}(R_{11}^{0})^{*}R_{13}^{0} = \emptyset + (1)(\varepsilon)^{*}(0) = 10$$
$$R_{10}^{(1)} = R_{10}^{0} + R_{11}^{0}(R_{11}^{0})^{*}R_{10}^{0} = \emptyset + (\varepsilon)(\varepsilon)^{*}(\emptyset) = \emptyset$$
$$R_{11}^{(1)} = R_{11}^{0} + R_{11}^{0}(R_{11}^{0})^{*}R_{11}^{0} = (\varepsilon) + (\varepsilon)(\varepsilon)^{*}(\varepsilon) = \varepsilon$$
$$R_{12}^{(1)} = R_{12}^{0} + R_{11}^{0}(R_{11}^{0})^{*}R_{12}^{0} = (1) + (\varepsilon)(\varepsilon)^{*}(1) = 1$$
$$R_{13}^{(1)} = R_{13}^{0} + R_{11}^{0}(R_{11}^{0})^{*}R_{13}^{0} = (0) + (\varepsilon)(\varepsilon)^{*}(0) = 0$$
$$R_{20}^{(1)} = R_{20}^{0} + R_{21}^{0}(R_{11}^{0})^{*}R_{10}^{0} = 1 + (0)(\varepsilon)^{*}(\emptyset) = 1$$
$$R_{21}^{(1)} = R_{21}^{0} + R_{21}^{0}(R_{11}^{0})^{*}R_{11}^{0} = (0) + (0)(\varepsilon)^{*}(\varepsilon) = 0$$
$$R_{22}^{(1)} = R_{22}^{0} + R_{21}^{0}(R_{11}^{0})^{*}R_{12}^{0} = (\varepsilon) + (0)(\varepsilon)^{*}(1) = \varepsilon + 01$$
$$R_{23}^{(1)} = R_{23}^{0} + R_{21}^{0}(R_{11}^{0})^{*}R_{13}^{0} = \emptyset + (0)(\varepsilon)^{*}(0) = 00$$
$$R_{30}^{(1)} = R_{30}^{0} + R_{31}^{0}(R_{11}^{0})^{*}R_{10}^{0} = \emptyset + (\emptyset)(\varepsilon)^{*}(\emptyset) = \emptyset$$
$$R_{31}^{(1)} = R_{31}^{0} + R_{31}^{0}(R_{11}^{0})^{*}R_{11}^{0} = \emptyset + (\emptyset)(\varepsilon)^{*}(\varepsilon) = \emptyset$$
$$R_{32}^{(1)} = R_{32}^{0} + R_{31}^{0}(R_{11}^{0})^{*}R_{12}^{0} = 0 + (\emptyset)(\varepsilon)^{*}(1) = 0$$
$$R_{33}^{(1)} = R_{33}^{0} + R_{31}^{0}(R_{11}^{0})^{*}R_{13}^{0} = (\varepsilon + 0) + (\emptyset)(\varepsilon)^{*}(0) = \varepsilon + 0$$

2.3. Cálculos para k=2

$$R_{00}^{(2)} = R_{00}^{1} + R_{02}^{1}(R_{22}^{1})^{*}R_{20}^{1} = (\varepsilon + 0) + (11)(\varepsilon + 01)^{*}(1) = \varepsilon + 0 + 11(01)^{*}1$$
$$R_{01}^{(2)} = R_{01}^{1} + R_{02}^{1}(R_{22}^{1})^{*}R_{21}^{1} = (1) + (11)(\varepsilon + 01)^{*}(0) = 1 + 11(01)^{*}0$$
$$R_{02}^{(2)} = R_{02}^{1} + R_{02}^{1}(R_{22}^{1})^{*}R_{22}^{1} = (11) + (11)(\varepsilon + 01)^{*}(\varepsilon + 01) = 11(01)^{*}$$
$$R_{03}^{(2)} = R_{03}^{1} + R_{02}^{1}(R_{22}^{1})^{*}R_{23}^{1} = (10) + (11)(\varepsilon + 01)^{*}(00) = 1(0 + 1(01)^{*}00)$$
$$R_{10}^{(2)} = R_{10}^{1} + R_{12}^{1}(R_{22}^{1})^{*}R_{20}^{1} = \emptyset + (1)(\varepsilon + 01)^{*}(1) = 1(01)^{*}1$$
$$R_{11}^{(2)} = R_{11}^{1} + R_{12}^{1}(R_{22}^{1})^{*}R_{21}^{1} = (\varepsilon) + (1)(\varepsilon + 01)^{*}(0) = \varepsilon + 1(01)^{*}0$$
$$R_{12}^{(2)} = R_{12}^{1} + R_{12}^{1}(R_{22}^{1})^{*}R_{22}^{1} = (1) + (1)(\varepsilon + 01)^{*}(\varepsilon + 01) = 1(01)^{*}$$
$$R_{13}^{(2)} = R_{13}^{1} + R_{12}^{1}(R_{22}^{1})^{*}R_{23}^{1} = 0 + (1)(\varepsilon + 01)^{*}(00) = 0 + 1(01)^{*}00$$
$$R_{20}^{(2)} = R_{20}^{1} + R_{22}^{1}(R_{22}^{1})^{*}R_{20}^{1} = 1 + (\varepsilon + 01)(\varepsilon + 01)^{*}(1) = (01)^{*}1$$
$$R_{21}^{(2)} = R_{21}^{1} + R_{22}^{1}(R_{22}^{1})^{*}R_{21}^{1} = 0 + (\varepsilon + 01)(\varepsilon + 01)^{*}(0) = (01)^{*}0$$
$$R_{22}^{(2)} = R_{22}^{1} + R_{22}^{1}(R_{22}^{1})^{*}R_{22}^{1} = (\varepsilon + 01) + (\varepsilon + 01)(\varepsilon + 01)^{*}(\varepsilon + 01) = (01)^{*}$$
$$R_{23}^{(2)} = R_{23}^{1} + R_{22}^{1}(R_{22}^{1})^{*}R_{23}^{1} = 00 + (\varepsilon + 01)(\varepsilon + 01)^{*}(00) = (01)^{*}00$$
$$R_{30}^{(2)} = R_{30}^{1} + R_{32}^{1}(R_{22}^{1})^{*}R_{20}^{1} = \emptyset + (0)(\varepsilon + 01)^{*}(1) = 0(01)^{*}1$$
$$R_{31}^{(2)} = R_{31}^{1} + R_{32}^{1}(R_{22}^{1})^{*}R_{21}^{1} = \emptyset + (0)(\varepsilon + 01)^{*}(0) = 0(01)^{*}0$$
$$R_{32}^{(2)} = R_{32}^{1} + R_{32}^{1}(R_{22}^{1})^{*}R_{22}^{1} = 0 + (0)(\varepsilon + 01)^{*}(\varepsilon + 01) = 0(01)^{*}$$
$$R_{33}^{(2)} = R_{33}^{1} + R_{32}^{1}(R_{22}^{1})^{*}R_{23}^{1} = (\varepsilon + 0) + (0)(\varepsilon + 01)^{*}(00) = \varepsilon + 0 + 0(01)^{*}00$$

2.4. Cálculos para k=3

$$R_{00}^{(3)} = R_{00}^2 + R_{03}^2(R_{33}^2)^* R_{30}^2 = (\varepsilon + 0 + 11(01)^*1) + (1(0 + 1(01)^*00))(\varepsilon + 0 + 0(01)^*00)^*(0(01)^*1) = \varepsilon + 0 + 1(1 + (0 + 1(01)^*00)(0 + 0(01)^*00)^*0)(01)^*1$$

$$R_{01}^{(3)} = R_{01}^2 + R_{03}^2(R_{33}^2)^* R_{31}^2 = (1 + 11(01)^*0) + (1(0 + 1(01)^*00))(\varepsilon + 0 + 0(01)^*00)^*(0(01)^*0) = 1 + 1(1 + (0 + 1(01)^*00)(0 + 0(01)^*00)^*0)(01)^*0$$

$$R_{02}^{(3)} = R_{02}^2 + R_{03}^2(R_{33}^2)^* R_{32}^2 = (11(01)^*) + (1(0 + 1(01)^*00))(\varepsilon + 0 + 0(01)^*00)^*(0(01)^*) = 1(1 + (0 + 1(01)^*00)(0 + 0(01)^*00)^*0)(01)^*$$

$$R_{03}^{(3)} = R_{03}^2 + R_{03}^2(R_{33}^2)^* R_{33}^2 = (1(0+1(01)^*00)) + (1(0+1(01)^*00))(\varepsilon+0+0(01)^*00)^*(\varepsilon+0+0(01)^*00) = 1(0 + 1(01)^*00)(0 + 0(01)^*00)^*$$

$$R_{10}^{(3)} = R_{10}^2 + R_{13}^2(R_{33}^2)^* R_{30}^2 = (1(01)^*1) + (0 + 1(01)^*00)(\varepsilon + 0 + 0(01)^*00)^*(0(01)^*1) = (1 + (0 + 1(01)^*00)(0 + 0(01)^*00)^*0)(01)^*1$$

$$R_{11}^{(3)} = R_{11}^2 + R_{13}^2(R_{33}^2)^* R_{31}^2 = (\varepsilon + 1(01)^*0) + (0 + 1(01)^*00)(\varepsilon + 0 + 0(01)^*00)^*(0(01)^*0) = \varepsilon + (1 + (0 + 1(01)^*00)(0 + 0(01)^*00)^*0)(01)^*0$$

$$R_{12}^{(3)} = R_{12}^2 + R_{13}^2(R_{33}^2)^* R_{32}^2 = (1(01)^*) + (0 + 1(01)^*00)(\varepsilon + 0 + 0(01)^*00)^*(0(01)^*) = (1 + (0 + 1(01)^*00)(0 + 0(01)^*00)^*0)(01)^*$$

$$R_{13}^{(3)} = R_{13}^2 + R_{13}^2(R_{33}^2)^* R_{33}^2 = (0 + 1(01)^*00) + (0 + 1(01)^*00)(\varepsilon + 0 + 0(01)^*00)^*(\varepsilon + 0 + 0(01)^*00) = (0 + 1(01)^*00)(0 + 0(01)^*00)^*$$

$$R_{20}^{(3)} = R_{20}^2 + R_{23}^2(R_{33}^2)^* R_{30}^2 = ((01)^*1) + ((01)^*00)(\varepsilon + 0 + 0(01)^*00)^*(0(01)^*1) = (01)^*(1 + 00(0 + 0(01)^*00)^*0(01)^*1)$$

$$R_{21}^{(3)} = R_{21}^2 + R_{23}^2(R_{33}^2)^* R_{31}^2 = ((01)^*0) + ((01)^*00)(\varepsilon + 0 + 0(01)^*00)^*(0(01)^*0) = (01)^*(0 + 00(0 + 0(01)^*00)^*0(01)^*0)$$

$$R_{22}^{(3)} = R_{22}^2 + R_{23}^2(R_{33}^2)^* R_{32}^2 = ((01)^*) + ((01)^*00)(\varepsilon + 0 + 0(01)^*00)^*(0(01)^*) = (01)^* + (01)^*00(0 + 0(01)^*00)^*0(01)^*$$

$$R_{23}^{(3)} = R_{23}^2 + R_{23}^2 (R_{33}^2)^* R_{33}^2 = ((01)^*00) + ((01)^*00)(\varepsilon + 0 + 0(01)^*00)^*(\varepsilon + 0 + 0(01)^*00) = (01)^*00(0 + 0(01)^*00)^*)$$

$$R_{30}^{(3)} = R_{30}^2 + R_{33}^2 (R_{33}^2)^* R_{30}^2 = (0(01)^*1) + (\varepsilon + 0 + 0(01)^*00)(\varepsilon + 0 + 0(01)^*00)^*(0(01)^*1) = (0 + 0(01)^*00)^*0(01)^*1$$

$$R_{31}^{(3)} = R_{31}^2 + R_{33}^2 (R_{33}^2)^* R_{31}^2 = (0(01)^*0) + (\varepsilon + 0 + 0(01)^*00)(\varepsilon + 0 + 0(01)^*00)^*(0(01)^*0) = (0 + 0(01)^*00)^*0(01)^*0$$

$$R_{32}^{(3)} = R_{32}^2 + R_{33}^2 (R_{33}^2)^* R_{32}^2 = (0(01)^*) + (\varepsilon + 0 + 0(01)^*00)(\varepsilon + 0 + 0(01)^*00)^*(0(01)^*) = (0 + 0(01)^*00)^*0(01)^*$$

$$R_{33}^{(3)} = R_{33}^2 + R_{33}^2 (R_{33}^2)^* R_{33}^2 = (\varepsilon + 0 + 0(01)^*00) + (\varepsilon + 0 + 0(01)^*00)(\varepsilon + 0 + 0(01)^*00)^*(\varepsilon + 0 + 0(01)^*00) = (0 + 0(01)^*00)^*$$

3. Ecuaciones simbólicas regla 54

Para poder encontrar una ecuación que describa a nuestro autómata necesitaremos el diagrama de 'de Bruijn' de nuestro ECA para la regla 54, el cual es el que se representa en la figura 2.

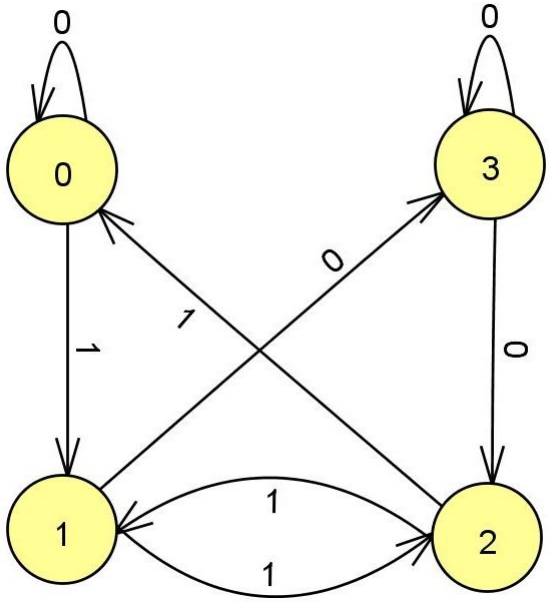

Figura 2: Diagrama de 'de Bruijn' para uso de la ecuación 1 para ECA regla 54

3.1. Cálculos para k=0

$$R_{00}^{(0)} = \varepsilon + 0; R_{01}^{(0)} = 1; R_{02}^{(0)} = \emptyset; R_{03}^{(0)} = \emptyset$$
$$R_{10}^{(0)} = \emptyset; R_{11}^{(0)} = \varepsilon; R_{12}^{(0)} = 1; R_{13}^{(0)} = 0$$
$$R_{20}^{(0)} = 1; R_{21}^{(0)} = 1; R_{22}^{(0)} = \varepsilon; R_{23}^{(0)} = \emptyset$$
$$R_{30}^{(0)} = \emptyset; R_{31}^{(0)} = \emptyset; R_{32}^{(0)} = 0; R_{33}^{(0)} = \varepsilon + 0$$

3.2. Cálculos para k=1

$$R_{00}^{(1)} = R_{00}^{0} + R_{01}^{0}(R_{11}^{0})^{*}R_{10}^{0} = (\varepsilon + 0) + (1)(\varepsilon)^{*}(\emptyset) = \varepsilon + 0$$
$$R_{01}^{(1)} = R_{01}^{0} + R_{01}^{0}(R_{11}^{0})^{*}R_{11}^{0} = (1) + (1)(\varepsilon)^{*}(\varepsilon) = 1$$
$$R_{02}^{(1)} = R_{02}^{0} + R_{01}^{0}(R_{11}^{0})^{*}R_{12}^{0} = \emptyset + (1)(\varepsilon)^{*}(1) = 11$$
$$R_{03}^{(1)} = R_{03}^{0} + R_{01}^{0}(R_{11}^{0})^{*}R_{13}^{0} = \emptyset + (1)(\varepsilon)^{*}(0) = 10$$
$$R_{10}^{(1)} = R_{10}^{0} + R_{11}^{0}(R_{11}^{0})^{*}R_{10}^{0} = \emptyset + (\varepsilon)(\varepsilon)^{*}(\emptyset) = \emptyset$$
$$R_{11}^{(1)} = R_{11}^{0} + R_{11}^{0}(R_{11}^{0})^{*}R_{11}^{0} = (\varepsilon) + (\varepsilon)(\varepsilon)^{*}(\varepsilon) = \varepsilon$$
$$R_{12}^{(1)} = R_{12}^{0} + R_{11}^{0}(R_{11}^{0})^{*}R_{12}^{0} = (1) + (\varepsilon)(\varepsilon)^{*}(1) = 1$$
$$R_{13}^{(1)} = R_{13}^{0} + R_{11}^{0}(R_{11}^{0})^{*}R_{13}^{0} = (0) + (\varepsilon)(\varepsilon)^{*}(0) = 0$$
$$R_{20}^{(1)} = R_{20}^{0} + R_{21}^{0}(R_{11}^{0})^{*}R_{10}^{0} = 1 + (1)(\varepsilon)^{*}(\emptyset) = 1$$
$$R_{21}^{(1)} = R_{21}^{0} + R_{21}^{0}(R_{11}^{0})^{*}R_{11}^{0} = (1) + (1)(\varepsilon)^{*}(\varepsilon) = 1$$
$$R_{22}^{(1)} = R_{22}^{0} + R_{21}^{0}(R_{11}^{0})^{*}R_{12}^{0} = (\varepsilon) + (1)(\varepsilon)^{*}(1) = \varepsilon + 11$$
$$R_{23}^{(1)} = R_{23}^{0} + R_{21}^{0}(R_{11}^{0})^{*}R_{13}^{0} = \emptyset + (1)(\varepsilon)^{*}(0) = 10$$
$$R_{30}^{(1)} = R_{30}^{0} + R_{31}^{0}(R_{11}^{0})^{*}R_{10}^{0} = \emptyset + (\emptyset)(\varepsilon)^{*}(\emptyset) = \emptyset$$
$$R_{31}^{(1)} = R_{31}^{0} + R_{31}^{0}(R_{11}^{0})^{*}R_{11}^{0} = \emptyset + (\emptyset)(\varepsilon)^{*}(\varepsilon) = \emptyset$$
$$R_{32}^{(1)} = R_{32}^{0} + R_{31}^{0}(R_{11}^{0})^{*}R_{12}^{0} = 0 + (\emptyset)(\varepsilon)^{*}(1) = 0$$
$$R_{33}^{(1)} = R_{33}^{0} + R_{31}^{0}(R_{11}^{0})^{*}R_{13}^{0} = (\varepsilon + 0) + (\emptyset)(\varepsilon)^{*}(0) = \varepsilon + 0$$

3.3. Cálculos para k=2

$$R_{00}^{(2)} = R_{00}^{1} + R_{02}^{1}(R_{22}^{1})^{*}R_{20}^{1} = (\varepsilon + 0) + (11)(\varepsilon + 11)^{*}(1) = \varepsilon + 0 + 11(11)^{*}1$$
$$R_{01}^{(2)} = R_{01}^{1} + R_{02}^{1}(R_{22}^{1})^{*}R_{21}^{1} = (1) + (11)(\varepsilon + 11)^{*}(1) = 1 + 11(11)^{*}1$$
$$R_{02}^{(2)} = R_{02}^{1} + R_{02}^{1}(R_{22}^{1})^{*}R_{22}^{1} = (11) + (11)(\varepsilon + 11)^{*}(\varepsilon + 11) = 11(11)^{*}$$
$$R_{03}^{(2)} = R_{03}^{1} + R_{02}^{1}(R_{22}^{1})^{*}R_{23}^{1} = (10) + (11)(\varepsilon + 11)^{*}(10) = 1(0 + 1(11)^{*}10)$$
$$R_{10}^{(2)} = R_{10}^{1} + R_{12}^{1}(R_{22}^{1})^{*}R_{20}^{1} = \emptyset + (1)(\varepsilon + 11)^{*}(1) = 1(11)^{*}1$$
$$R_{11}^{(2)} = R_{11}^{1} + R_{12}^{1}(R_{22}^{1})^{*}R_{21}^{1} = (\varepsilon) + (1)(\varepsilon + 11)^{*}(1) = \varepsilon + 1(11)^{*}1$$
$$R_{12}^{(2)} = R_{12}^{1} + R_{12}^{1}(R_{22}^{1})^{*}R_{22}^{1} = (1) + (1)(\varepsilon + 11)^{*}(\varepsilon + 11) = 1(11)^{*}$$
$$R_{13}^{(2)} = R_{13}^{1} + R_{12}^{1}(R_{22}^{1})^{*}R_{23}^{1} = 0 + (1)(\varepsilon + 11)^{*}(10) = 0 + 1(11)^{*}10$$
$$R_{20}^{(2)} = R_{20}^{1} + R_{22}^{1}(R_{22}^{1})^{*}R_{20}^{1} = 1 + (\varepsilon + 11)(\varepsilon + 11)^{*}(1) = (11)^{*}1$$
$$R_{21}^{(2)} = R_{21}^{1} + R_{22}^{1}(R_{22}^{1})^{*}R_{21}^{1} = 1 + (\varepsilon + 11)(\varepsilon + 11)^{*}(1) = (11)^{*}1$$
$$R_{22}^{(2)} = R_{22}^{1} + R_{22}^{1}(R_{22}^{1})^{*}R_{22}^{1} = (\varepsilon + 11) + (\varepsilon + 11)(\varepsilon + 11)^{*}(\varepsilon + 11) = (11)^{*}$$

$$R_{23}^{(2)} = R_{23}^{1} + R_{22}^{1}(R_{22}^{1})^{*}R_{23}^{1} = 10 + (\varepsilon + 11)(\varepsilon + 11)^{*}(10) = (11)^{*}10$$
$$R_{30}^{(2)} = R_{30}^{1} + R_{32}^{1}(R_{22}^{1})^{*}R_{20}^{1} = \emptyset + (0)(\varepsilon + 11)^{*}(1) = 0(11)^{*}1$$
$$R_{31}^{(2)} = R_{31}^{1} + R_{32}^{1}(R_{22}^{1})^{*}R_{21}^{1} = \emptyset + (0)(\varepsilon + 11)^{*}(1) = 0(11)^{*}1$$
$$R_{32}^{(2)} = R_{32}^{1} + R_{32}^{1}(R_{22}^{1})^{*}R_{22}^{1} = 0 + (0)(\varepsilon + 11)^{*}(\varepsilon + 11) = 0(11)^{*}$$
$$R_{33}^{(2)} = R_{33}^{1} + R_{32}^{1}(R_{22}^{1})^{*}R_{23}^{1} = (\varepsilon + 0) + (0)(\varepsilon + 11)^{*}(10) = \varepsilon + 0 + 0(11)^{*}10$$

3.4. Cálculos para k=3

$$R_{00}^{(3)} = R_{00}^{2} + R_{03}^{2}(R_{33}^{2})^{*}R_{30}^{2} = (\varepsilon + 0 + 11(11)^{*}1) + (1(0 + 1(11)^{*}10))(\varepsilon + 0 + 0(11)^{*}10)^{*}(0(11)^{*}1) = \varepsilon + 0 + 1(1 + (0 + 1(11)^{*}10)(0 + 0(11)^{*}10)^{*}0)(11)^{*}1$$

$$R_{01}^{(3)} = R_{01}^{2} + R_{03}^{2}(R_{33}^{2})^{*}R_{31}^{2} = (1 + 11(11)^{*}1) + (1(0 + 1(11)^{*}10))(\varepsilon + 0 + 0(11)^{*}10)^{*}(0(11)^{*}1) = 1 + 1(1 + (0 + 1(11)^{*}10)(0 + 0(11)^{*}10)^{*}0)(11)^{*}1$$

$$R_{02}^{(3)} = R_{02}^{2} + R_{03}^{2}(R_{33}^{2})^{*}R_{32}^{2} = (11(11)^{*}) + (1(0 + 1(11)^{*}10))(\varepsilon + 0 + 0(11)^{*}10)^{*}(0(11)^{*}) = 1(1 + (0 + 1(11)^{*}10)(0 + 0(11)^{*}10)^{*}0)(11)^{*}$$

$$R_{03}^{(3)} = R_{03}^{2} + R_{03}^{2}(R_{33}^{2})^{*}R_{33}^{2} = (1(0 + 1(11)^{*}10)) + (1(0 + 1(11)^{*}10))(\varepsilon + 0 + 0(11)^{*}10)^{*}(\varepsilon + 0 + 0(11)^{*}10) = 1(0 + 1(11)^{*}10)(0 + 0(11)^{*}10)^{*}$$

$$R_{10}^{(3)} = R_{10}^{2} + R_{13}^{2}(R_{33}^{2})^{*}R_{30}^{2} = (1(11)^{*}1) + (0 + 1(11)^{*}10)(\varepsilon + 0 + 0(11)^{*}10)^{*}(0(11)^{*}1) = (1 + (0 + 1(11)^{*}10)(0 + 0(11)^{*}10)^{*}0)(11)^{*}1$$

$$R_{11}^{(3)} = R_{11}^{2} + R_{13}^{2}(R_{33}^{2})^{*}R_{31}^{2} = (\varepsilon + 1(11)^{*}1) + (0 + 1(11)^{*}10)(\varepsilon + 0 + 0(11)^{*}10)^{*}(0(11)^{*}1) = \varepsilon + (1 + (0 + 1(11)^{*}10)(0 + 0(11)^{*}10)^{*}0)(11)^{*}1$$

$$R_{12}^{(3)} = R_{12}^{2} + R_{13}^{2}(R_{33}^{2})^{*}R_{32}^{2} = (1(11)^{*}) + (0 + 1(11)^{*}10)(\varepsilon + 0 + 0(11)^{*}10)^{*}(0(11)^{*}) = (1 + (0 + 1(11)^{*}10)(0 + 0(11)^{*}10)^{*}0)(11)^{*}$$

$$R_{13}^{(3)} = R_{13}^{2} + R_{13}^{2}(R_{33}^{2})^{*}R_{33}^{2} = (0 + 1(11)^{*}10) + (0 + 1(11)^{*}10)(\varepsilon + 0 + 0(11)^{*}10)^{*}(\varepsilon + 0 + 0(11)^{*}10) = (0 + 1(11)^{*}10)(0 + 0(11)^{*}10)^{*}$$

$$R_{20}^{(3)} = R_{20}^{2} + R_{23}^{2}(R_{33}^{2})^{*}R_{30}^{2} = ((11)^{*}1) + ((11)^{*}10)(\varepsilon + 0 + 0(11)^{*}10)^{*}(0(11)^{*}1) = (11)^{*}(1 + 10(0 + 0(11)^{*}10)^{*}0(11)^{*}1)$$

$$R_{21}^{(3)} = R_{21}^2 + R_{23}^2(R_{33}^2)^* R_{31}^2 = ((11)^*1) + ((11)^*10)(\varepsilon + 0 + 0(11)^*10)^*(0(11)^*1) = (11)^*(1 + 10(0 + 0(11)^*10)^*0(11)^*1)$$

$$R_{22}^{(3)} = R_{22}^2 + R_{23}^2(R_{33}^2)^* R_{32}^2 = ((11)^*) + ((11)^*10)(\varepsilon + 0 + 0(11)^*10)^*(0(11)^*) = (11)^* + (11)^*10(0 + 0(11)^*10)^*0(11)^*$$

$$R_{23}^{(3)} = R_{23}^2 + R_{23}^2(R_{33}^2)^* R_{33}^2 = ((11)^*10) + ((11)^*10)(\varepsilon + 0 + 0(11)^*10)^*(\varepsilon + 0 + 0(11)^*10) = (11)^*10(0 + 0(11)^*10)^*)$$

$$R_{30}^{(3)} = R_{30}^2 + R_{33}^2(R_{33}^2)^* R_{30}^2 = (0(11)^*1) + (\varepsilon + 0 + 0(11)^*10)(\varepsilon + 0 + 0(11)^*10)^*(0(11)^*1) = (0 + 0(11)^*10)^*0(11)^*1$$

$$R_{31}^{(3)} = R_{31}^2 + R_{33}^2(R_{33}^2)^* R_{31}^2 = (0(11)^*1) + (\varepsilon + 0 + 0(11)^*10)(\varepsilon + 0 + 0(11)^*10)^*(0(11)^*1) = (0 + 0(11)^*10)^*0(11)^*1$$

$$R_{32}^{(3)} = R_{32}^2 + R_{33}^2(R_{33}^2)^* R_{32}^2 = (0(11)^*) + (\varepsilon + 0 + 0(11)^*10)(\varepsilon + 0 + 0(11)^*10)^*(0(11)^*) = (0 + 0(11)^*10)^*0(11)^*$$

$$R_{33}^{(3)} = R_{33}^2 + R_{33}^2(R_{33}^2)^* R_{33}^2 = (\varepsilon + 0 + 0(11)^*10) + (\varepsilon + 0 + 0(11)^*10)(\varepsilon + 0 + 0(11)^*10)^*(\varepsilon + 0 + 0(11)^*10) = (0 + 0(11)^*10)^*$$

4. Conclusiones

Como conclusión para este reporte se escriben las ecuaciones que describen a los autómatas celulares elementales para la regla 22 y 54.

Ecuación para la regla 22:

$$(0 + 1(01)^*00)(0 + 0(01)^*00)^*$$

Ecuación para la regla 54:

$$(0 + 1(11)^*10)(0 + 0(11)^*10)^*$$

Referencias

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