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A psychological perspective on augmented reality in the mathematics classroom

Keith R. Bujak [a,*], Iulian Radu [b], Richard Catrambone [a], Blair MacIntyre [b], Ruby Zheng [b,1], Gary Golubski [c]

[a] Georgia Institute of Technology, School of Psychology, 654 Cherry Street, Atlanta, GA 30332-0170, , USA
[b] Georgia Institute of Technology, College of Computing, 801 Atlantic Drive, Atlanta, GA 30332-0280, USA
[c] United States Army, 1129 Plymouth Rock Way, Greenwood, IN 46142, USA

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ABSTRACT

Physical objects and virtual information are used as teaching aids in classrooms everywhere, and until recently, merging these two worlds has been difficult at best. Augmented reality offers the combination of physical and virtual, drawing on the strengths of each. We consider this technology in the realm of the mathematics classroom, and offer theoretical underpinnings for understanding the benefits and limitations of AR learning experiences. The paper presents a framework for understanding AR learning from three perspectives: physical, cognitive, and contextual. On the physical dimension, we argue that physical manipulation affords natural interactions, thus encouraging the creation of embodied representations for educational concepts. On the cognitive dimension, we discuss how spatiotemporal alignment of information through AR experiences can aid student's symbolic understanding by scaffolding the progression of learning, resulting in improved understanding of abstract concepts. Finally, on the contextual dimension, we argue that AR creates possibilities for collaborative learning around virtual content and in non-traditional environments, ultimately facilitating personally meaningful experiences. In the process of discussing these dimensions, we discuss examples from existing AR applications and provide guidelines for future AR learning experiences, while considering the pragmatic and technological concerns facing the widespread implementation of augmented reality inside and outside the classroom.

1. Introduction

Augmented reality (AR) is just starting to scratch the surface in educational applications. We believe this technology has great potential for educational outcomes. However, in order to take full advantage of this technology, we must understand the psychological factors that influence AR designs. In this paper we analyze the literature along three dimensions – physical, cognitive, and contextual – and we use these as lenses for analyzing the potential benefits and design considerations of AR technology as applied to learning. In the process we provide illustrations from existing AR systems, and generate guidelines for AR application designers. We believe this knowledge will be beneficial for educators interested in understanding the potential of AR as a learning technology, and to technology designers interested in pursuing educational applications.

We focus on mathematics education, specifically math manipulatives, to guide our analysis. We selected this domain given the long history of using physical objects in the mathematics classroom. We consider both physical and virtual manipulatives, gleaning insights from the research regarding the benefits of each, in order to understand the potential of future AR learning experiences. For example, physical objects afford certain actions based on their shapes and offer immediate kinesthetic feedback, while virtual objects can include additional

* Corresponding author. Tel.: +1 404 314 1060; fax: +1 404 894 8905.
E-mail addresses: bujak@gatech.edu (K.R. Bujak), iulian@cc.gatech.edu (I. Radu), rc7@prism.gatech.edu (R. Catrambone), blair@cc.gatech.edu (B. MacIntyre), zhengsan@gmail.com (R. Zheng), golubskig@gmail.com (G. Golubski).
[1] Current address: 328 W 83rd St. Apt. 5d, New York, NY 10024.

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instructional information built right into them, such as allowing or disallowing certain configurations of the objects. AR has the potential to leverage the strengths of each, given its combination of the physical and the virtual in the same space. Because there are not many AR applications for math learning, we use examples from other AR learning domains. Although we are focusing on math education, we believe our analysis is applicable to AR learning applications in a variety of other domains.

Researchers define a math manipulative as "an object that can be handled by an individual in a sensory manner during which conscious and unconscious mathematical thinking will be fostered" (Swan & Marshall, 2010, p. 14). Manipulative objects have been used since ancient times. Babylonians used counting boards, precursors to the abacus, as early as 300 BCE (Boggan, Harper, & Whitmire, 2010). Recently in the 19th and early 20th centuries, Froebel and Montessori designed manipulative objects for children, with the aim of teaching mathematics concepts. In contemporary school classrooms, a variety of physical manipulatives are now frequently used in Kindergarten and elementary grades (Swan & Marshall, 2010), and, more recently, the proliferation of technology in schools has given rise to the development of computer-based "virtual" manipulatives.

Physical manipulatives encompass an array of objects such as analog clocks, balance scales, coins, dice, and spinners. Each manipulative might be suitable for teaching multiple mathematics concepts. For instance, Unifix cubes can be used to teach students about repeating patterns, counting, number composition and decomposition, addition, subtraction, multiplication, base 10 representation, and fractions, among others (Glenn & Carpenter, 2007, p. 260). These types of objects are useful for teaching children about abstract concepts by using concrete items that they are likely to be familiar with.

Virtual manipulatives are digital interactive experiences that depict mathematical concepts. Virtual manipulatives have been defined as "interactive, Web-based visual representations of a dynamic object that presents opportunities for constructing mathematical knowledge" (Moyer, Bolyard, & Spikell, 2002, p. 373). We adhere to this definition, although we also include locally executed software in addition to Web-based experiences. There exist a wide variety of virtual mathematics manipulatives. Some replicate physical manipulatives, extending them with digital information that might facilitate student learning, for instance the virtual manipulative replicates a physical Base 10 Blocks manipulative, and the system automatically computes the number that the manipulative represents (Utah State University, 1999a). Others allow students to explore concepts that would be difficult to explore in the physical world, for instance students can investigate differences in shapes of cross-sectional slices taken from 3D geometric shapes (Utah State University, 1999b).

There are many technologies that currently allow people to interact in ways that go beyond the traditional mouse and keyboard. Many of these systems are reported as motivational when experienced by children. There is much interest in exploring the potential of mixed-reality for children's entertainment, as can be seen in popular products such as Sony PlayStation Eye, Nintendo Wii and Microsoft Kinect. Many studies with children frequently report that children have fun when using their body to control the game, and this might occur even when the game is difficult to play. The emergence of AR technology gives rise to the possibility of new kinds of educational manipulatives that combine physical objects and virtual information. AR can leverage the concreteness of physical manipulatives yet provide the flexibility of virtual manipulatives. In the following sections, we will analyze manipulatives through several perspectives from the fields of education and psychology. We will describe the benefits and drawbacks of physical, virtual, and AR manipulatives, while suggesting future applications of AR manipulatives that draw strengths from both the physical and the virtual.

The manuscript is organized into three main sections in addition to a concluding section that provides a summary and addresses some pragmatic limitations. AR manipulatives will be considered along three dimensions: physical, cognitive, and contextual. In each of these sections, we draw upon existing knowledge in areas of physical and virtual manipulatives, and discuss these in relation to theories of children's learning. We make the argument that physical and virtual manipulates each bring unique benefits to the learning experience, and that augmented reality technology has the potential to not only unite the benefits of the two types of manipulatives, but also to place the education in relevant social and environmental contexts, leading to novel opportunities for learning experiences.

2. Physical dimension

Along the first dimension, physical, we assert that physical interaction with objects supports two learning goals, the first being pragmatically oriented and the second being cognitively oriented. Infants begin developing motor skills from birth, as seen by their rapid learning of eye movement control and hand reaching and grasping. Feedback is inherent in the experience of physical actions; if a reach and grasp action is initiated but it results in the infant failing to obtain the desired object, it becomes apparent that a second and modified attempt must be made. Motor movements are refined through this process, and the child learns to manipulate the physical environment. In this section we first argue that educational experiences are aided when leveraging people's natural interaction in the physical world, as opposed to the abstract interactions afforded by mice, keyboards, and graphical user interfaces. Furthermore, we argue that retention of knowledge can be further increased when abstract educational concepts are related to physical spaces and actions.

2.1. Intuitive interactions

Typically when a student accesses educational content using a computer, she must hold knowledge regarding computer-based interactions. At the very least, she must be able to use a keyboard and mouse; additionally, she might need to be familiar with a variety of interaction techniques (such as moving windows, accessing menus, etc.). Because the student must know how to apply these interactions, there is a learning cost while the student trains to use a virtual manipulative application. There is a cognitive load imposed while the student interacts with the application (Sweller, 2010). Cognitive load occurs when activities use the resources of working memory, decreasing the potential for learning. The use of "natural" interfaces are believed to reduce *extraneous* cognitive load – studies show that cognitive load in AR environments can be less than when learning in a computer-based environment (Tang, Owen, Biocca, & Mou, 2003, p. 73). Extraneous load is defined as mental workload generated by cognitive activities that are not directly related to the learning goal. In the case of a mathematics virtual manipulative, the learning goal might be counting and number operations; however, using a mouse, keyboard, and GUI are extraneous to that goal, and thus is likely to reduce the overall learning effects of the experience.

Augmented reality can lower the barrier to entry for students engaging virtual content, as it makes use of natural interactions that allow more – and potentially younger – students to engage with educational content. The ease of interacting within AR-based experiences can invite teachers to bring virtual educational experiences into the early school years. AR technology allows students to interact with the educational content by leveraging what they already know about interacting with the physical world. They can move around to change perspective, move closer/farther to change scale, they can select virtual objects by pointing to them, they can reach out to touch and move objects, etc. Because AR permits these natural interactions, there is a reduction in the knowledge and skills required of users, increasing the transparency of the interface between student and educational content. The user still needs to learn interactions specific to the application itself, but the operations of basic navigation and object manipulation are likely to be intuitive to most users. In a study of an AR storytelling system where children controlled story characters through movement of paddles (Hornecker & Dünser, 2009), the authors observed that children were not only comfortable with controlling the characters through simple movements, but that children also expected the characters to react to complex interactions found in the real world (such as jumping when flicked, or colliding when bumped), which were not programmed into the AR system's repertoire of interactions.

There is evidence to suggest that AR learning environments support learning outcomes through the use of natural interactions. One side of this research has been in the investigation of epistemic actions. When performing a task, people take actions in order to explore the task domain. For instance, when solving a jigsaw puzzle, people will pick up a piece and move it around while trying to fit it in different spots, in a trial-and-error fashion. Research studying children's activities while solving physical or PC-based puzzles show that children perform more epistemic actions and perform tasks faster with physical puzzles (Antle, Droumeva, & Ha, 2009). It is possible that this difference is due to the children's familiarity with manipulating physical objects. Thus, it is possible that such actions are less likely to occur with virtual manipulatives, and more likely employed by children interacting with physical manipulatives, yielding more exploration of the learning content. Another side of this research is in the domain of embodied cognition. Educational experiences can leverage learners' existing embodied knowledge, by appealing to bodily motions corresponding to certain abstract concepts. For instance, researchers have demonstrated how children intuitively understand musical properties as bodily motions (Antle, Droumeva, & Corness, 2008). This research indicates that technologies such as AR can lead to richer learning experiences by coupling educational content to physical motion.

Physical movements in AR can help students learn spatial content. Existing AR systems enable learners to easily explore 3D spaces by simply moving their body to change perspective. Some AR systems, that connect virtual objects to physical objects, also allow learners to rearrange the mixed-reality space through physical manipulation. Researchers reported one study where learners used either AR or a PC-desktop interface to view geographical landscapes (Shelton & Hedley, 2004). Learners in the AR condition showed better memory for the observed spaces. The authors found that the learners' amount of physical interaction corresponded to increased differences in scores on pre/post tests when investigating knowledge of earth–sun relationships during yearly cycles. One potential reason for this finding is that when students interact with educational content through AR, they have more control over the way information is being delivered. Student preferences can vary; some would prefer to learn through looking at a stable image while others would prefer to learn by moving around objects. AR allows students to have control over how they examine the content, leading to improved learning of spatial content.

Learning can also be improved by spatially anchoring virtual content to physical locations and objects. Research in spatial learning has found that memory retrieval and learning is aided when information is associated with physical locations. For instance, the ancient mnemonic device of a 'memory walk'–whereby a person intending to memorize several pieces of information associates information with features of a space (Yates, 1966) – leads to better memory recall as the learner imagines navigating the space. Research shows that properly connecting abstract concepts with physical objects can also help support memory and understanding of symbolic representations (Tversky, 2001). AR manipulatives can leverage the same process to aid learning, by aligning information to objects and locations in the student's environment.

2.2. Physical action encoding

Researchers have found that physical actions can be beneficial for recall of information. Children are better at recalling facts and relationships about a story when they read sentences and then act the story using physical props compared to children who do not perform the actions (Glenberg, Brown, & Levin, 2007). Researchers demonstrated the potential of AR technology to create embodied representations; students in their study had better memory for physically interactive story content compared to non-interactive content (Hornecker & Dünser, 2009). The researchers hypothesized that the learning in these cases occurs due to proprioceptive encoding of the information. Research also indicates that children internalize and later simulate physical experiences to solve problems. For example, after learning about inequalities using a physical balance, children reproduced the balance when solving problems on a test sheet (Suh & Moyer, 2007). This research suggests that physical actions are beneficial for strengthening the memory for learning content.

Embodied cognition research demonstrates that people form metaphorical associations between physical activities and conceptual abstractions (Lakoff & Johnson, 1980, p. 256). An example from the domain of mathematics is the association between the concept of "addition" with the physical activity of "putting things together" (Lakoff & Núñez, 2000, p. 512). Researchers demonstrated how mixed-reality technology can be used to create embodied representations of the mathematical concept of ratios (Abrahamson & Trninic, 2011). Constructing AR experiences in which children must perform physical activities enacting abstract concepts might thus be beneficial because they can help the creation of such embodied knowledge. Physical manipulatives are useful for prompting learners to engage in physical movements that are necessary for developing robust memories, but physical manipulatives are limited in their ability to display mathematical concepts and to guide students through an educational experience. Virtual manipulatives, in comparison, can be easily accompanied by pedagogical information on or around the manipulatives. Current software for virtual manipulatives, based in PC and virtual-reality environments, deliver content through visual and auditory modalities. However, in these virtual environments there is a lack of natural motor feedback because the user is not interacting directly with objects in the physical world.

AR technology can aid the creation of embodied metaphors, by combining physical and virtual manipulatives into experiences where students use physical objects augmented with virtual information. With AR, the manipulative can provide real-time information in response to students' physical motions that might lead to better encoding and internalization. AR manipulatives can facilitate the creation of embodied metaphors inspired by physical manipulatives, or new kinds of metaphors otherwise difficult to convey through concrete physical objects. With current AR technology, applications can trigger actions in response to physical changes in rotation of physical objects, changes in spatial relationships between objects, or movements of physical objects on paths. Examples of these actions include dropping an item when a paddle is tilted, increasing the force on an item when a physical object is brought closer, and selecting an item when a physical object circles it. Future improvements in vision processing and gesture recognition will increase the repertoire of actions, such as allowing detection of fast motions such as throwing objects, or allowing detection of complex and subtle movements such as non-verbal gestures.

Complex physical control might not always be desired because the complexity of performing the input might outweigh the learning benefits. For instance, manipulating a chemical molecule in 3D space through physical motions might be more difficult and tedious than manipulating a symbolic representation of the molecule. Furthermore, enabling users to use natural interactions assumes that their skills such as motor manipulation, attention, and spatial cognition, are developed enough for the interaction. This issue applies especially to children. Children's skills such as hand-eye coordination, multiple-hand coordination, and fine motor skills continue to develop through middle and late childhood. Children might be unable to intercept moving items with their hands, or to move in indirect motions, such as when the AR system mimics a mirror (Hornecker & Dünser, 2009). Coordination between hands might be undeveloped, thus children should not be required to move two objects concurrently, such as holding a handheld phone while gesturing with a paddle. Finally, they might not be able to perform fine motions such as placing objects at a specific location/angle, or following a path without error. These limitations influence the ability of learners to experience AR-based educational content.

2.3. Summary

In this section, we have discussed how the physical affordances of augmented-reality experiences can benefit student learning, specifically focusing on natural interaction and embodied representations. Natural interaction reduces the extraneous cognitive load involved in engaging with virtual educational content, allowing children to use their knowledge from the real world to interact with the experience. Natural interaction lowers the barrier of entry for students to use the system, and gives children more control over their learning. Physical control of the learning experience encourages epistemic actions and the formation of embodied representations, and can lead to improved understanding of spatial concepts, and better recall of the learning content.

To nurture these learning effects, AR experiences should strive to leverage natural interactions so users can intuitively transfer their knowledge of the real world to the use of the system. AR experiences should allow users to enact the learning concepts, not simply observe them. Furthermore, the use of embodied metaphors is encouraged in the design of interactions, whereby the physical interactions are directly related to the educational concepts. Associating content to spatial locations strengthens understanding of abstract content. Additionally, AR experiences are beneficial when learners are allowed freedom to explore the content from different spatial perspectives, and when they can control the pace of their own learning. Finally, AR designers should be wary of the capabilities and limitations of young students, as their developmental stage will limit their ability to physically interact with the technology.

3. Cognitive dimension

In elementary math classes, children must learn abstract mathematical concepts such as numeric operations, as well as the notations used to represent those concepts. While learning mathematics from interacting with manipulatives, children must understand relationships between the manipulative objects and mathematical concepts (for instance, the student needs to understand that a Cuisenaire rod represents a number), and between the manipulatives and their specific mathematical notation (for instance, a Cuisenaire rod five blocks long should be understood as the same as the number "5"). Mathematics involves a large amount of abstract information that must be understood by children. In later school years, abstract information is conveyed through symbolic notations, but in early years it is difficult to use symbols because children are not familiar with mathematical notation; thus physical objects are intended to serve as bridge to the world of symbols. However, there are inherent limitations with the use of physical objects because the connections to abstract concepts might not be obvious to learners.

Gaining an understanding of symbolic relationships can be difficult for young children playing with physical manipulatives. The concrete representation of a manipulative might distract from the learning tasks (M. C. Brown, McNeil, & Glenberg, 2009). Learners might miss the mathematical concept represented by a manipulative because they are distracted by the physical features of the manipulative, especially if manipulative is highly representative or contains many salient features (Uttal, Scudder Judy, & Kathyrn, 1997). Another difficulty noted by researchers is that there is a lack of alignment between manipulatives and their notational representations (M. C. Brown et al., 2009). When students use physical manipulatives they do not immediately see the mathematical notation represented by the manipulative. Furthermore, a time lag between exposures to different representations can cause difficulty in understanding symbolic relationships (Uttal et al., 1997). In the following sections we will argue that learning is improved when the educational experience presents information at spatially- and temporally-appropriate locations. Additionally, we argue that AR experiences are especially suited to manipulating the learner's perception of reality, potentially causing improved learning of symbolic relationships and understanding of invisible phenomena.

3.1. Spatial and temporal contiguity

Studies have shown that student learning is improved when related pieces of information are presented spatially or temporally close to each other (Ginns, 2006; Sweller, 2010). Generally, a learning environment should focus on invoking cognitive processes that are germane to the learning activity, and reduce extraneous tasks that will increase the cognitive load. Extraneous cognitive load can be induced when presenting instructional information at a location other than where the learner is directing their attention, or at a time when the learner is not thinking about related content. Such occurrences divide the learner's attention, requiring that the learner mentally connect disjoint pieces of information, thus increasing extraneous workload and decreasing working memory capacity for the task at hand. Integrating the information with the learner's active workspace can increase learning effects. For example, researchers found that when students learn about an electric circuit, they learn better when a circuit diagram and its properties are shown in one display, rather than on two separate displays where one display is used for the circuit diagram and another display for the circuit properties (Kester, Kirschner, & van Merriënboer, 2005). This principle also applies when students process step-by-step instructions in the learning task. For example, when learning procedural knowledge related to performing a multi-step process, it is beneficial for students to see instructions integrated with the materials being manipulated (Tang et al., 2003, p. 73).

Spatial and temporal contiguity is more difficult to achieve with physical manipulatives than with virtual manipulatives. A common problem observed by researchers has been the fact that with physical manipulatives, students cannot easily see the instructional information represented by manipulative (M. C. Brown et al., 2009). Virtual manipulatives can easily provide a display of information spatially close to the concrete representation of the manipulative, and update this symbolic representation in real-time as students interact with the manipulative (Moyer et al., 2002). Augmented reality manipulatives can provide the same level of spatial and temporal contiguity as virtual manipulatives, and they can provide this for learning situations that involve physical objects. Information can be tied to physical objects and locations, and updated as the learner progresses through the task. The spatially- and temporally-aligned information can indicate the use and manipulation of physical manipulates, as well as their symbolic representation. For the novice learner, this continuity of information will reduce the extraneous cognitive load – for instance, searching for instructions in a place other than where the physical manipulatives are located – and allow the learner to focus on the task at hand.

Some current AR systems facilitate the observation and manipulation of information to which students would otherwise not have access if they were using physical objects only. Researchers developed a system for visualizing a human body's internal organs (Nischelwitzer, Lenz, Searle, & Holzinger, 2007). The system allows the user to spread apart and re-connect organs, enabling spatial visualization of the body's internal organization. Although such a system would be possible to construct using physical materials, it would be more difficult for learners to achieve the same manipulations compared to the AR system. Other researchers presented a system for simulating plant growth (Theng, Mei-Ling, Liu, & Cheok, 2007), whereby students manipulate environmental factors such as light and water, while watching the speeded-up growth and adaptation of a virtual plant. This system could be replicated using a real plant; however, the learner would need to dedicate much more time and care in order to achieve the same effect as observed through the AR system. In these examples, virtual representations are used for conveying concepts that are difficult or impossible to achieve using physical objects.

3.2. Abstract-physical encoding

Most AR systems leverage spatiotemporal contiguity by overlaying virtual information relevant to physical objects and spaces (Azuma, 1997; Azuma et al., 2001). Applications of spatiotemporal contiguity can be observed in systems such as the word-learning application (Chen, Su, Lee, & Wu, 2007). In this system, the children must show a Chinese glyph to the computer, and the computer overlays the image of the glyph. The system aligns the image with the physical glyph, and provides feedback to the user as to whether their choice was correct or incorrect. Another similar AR system allows children to visualize animals when the correct symbol is selected (Campos & Pessanha, 2011). In the above examples, students used the AR system to visualize the meaning of paper-based symbols. Other experiences have been demonstrated where physical objects are more integral to the learning experience. In one AR system, users are guided through steps for assembling LEGO-like structures, while the AR view shows users the next step in the physical assembly task (Theng et al., 2007). When learning to assemble 2D and 3D structures, AR has been shown to improve skill transfer and decrease assembly times when compared to learning from a completed structure and replicating it (Pathomaree & Charoenseang, 2005). Such AR systems are successful because they display information relevant to the user at the appropriate time and at the appropriate location.

Augmented reality manipulatives can alleviate the issues associated with symbolic understanding of physical manipulatives. Similar to virtual manipulatives, AR manipulatives can help students see relationships between concrete objects and their symbolic representation by showing the representations in the same view. The AR learning systems described above for teaching children about symbolic relationships (Campos & Pessanha, 2011; Chen et al., 2007) operate by simply matching physical card containing the symbol, with their graphical representation. This coupling potentially helps learners understand what the graphical symbols mean. We suggest that AR technology can bridge the gap between physical manipulatives and their symbolic representation by morphing the physical object into its representation – for instance, shape-shifting the physical Cuisenaire rod into its numerical representation. This visual effect can help students understand the relationship between a physical manipulative and its symbolic representation because it conceptually indicates that the physical manipulative "is" its symbolic representation. The experience can be scaffolded, such that the emphasis on symbolic content is increased or decreased as the learner acquires knowledge. Furthermore, augmented reality has the potential to convert abstract information into concrete representations, for example, representing pollution as physical objects in the learner's environment. In an AR system, learners might even use physical gestures to interact with these reifications. However, it is worth noting that although such physical reifications have potential learning benefits, they might confuse the learner due to their concrete appearance as objects in the real world. Learners need to understand that such reifications are merely representations of phenomena that are inherently invisible. This topic can be a challenge when designing for young children, as elementary school children think concretely and rely on perception to drive their judgments (Flavell, Miller, & Miller, 1993).

3.3. Summary

We have argued that learning through physical manipulatives is a challenging task due to the difficulty of understanding symbolic links between the physical objects and abstract concepts. The construct of spatiotemporal contiguity indicates that aligning information in time and space can help learners connect disjoint pieces of information, and augmented-reality experiences are suitable to leverage this contiguity for physical contexts. Augmented reality can present information associated with physical objects and locations, leading to improved learning of symbolic associations, and improved performance on students following physical instructions. Furthermore, as augmented reality allows students to experience interactive 3D simulations, leading to deeper insights on phenomena that might otherwise be difficult to explore.

Augmented reality experiences are educationally effective when they align information to the student's attention. AR experiences can monitor a student's tasks and respond by presenting contextually relevant information at the appropriate level of scaffolding, which is aligned with the physical objects that students are attending to. For children who cannot yet think symbolically, AR can also transform one representation into another, such as morphing a Cuisenaire rod into a number, bypassing the need for unassisted symbolic thinking. For children who might not easily think about symbolic representations of objects, an AR system can simplify the visual appearance of objects. This can reduce the amount of visual richness or clutter and therefore potentially lead children to focus on the abstract meanings of the objects. Augmented reality experiences can also encourage understanding of invisible phenomena by converting abstract information into concrete objects, or visualizing phenomena that are otherwise infeasible for students to access.

4. Contextual dimension

Disappearing are the days in which learning is an individual process limited to the confines of the classroom. Learning occurs in the context of other people and in the context of the real world. Students learning in such contexts gain experience with not only learning, but also with understanding how classroom concepts apply to problem solving in real world situations. In such contexts, students gain a deeper appreciation for learning, by relating the learning content to their own experiences. We argue that AR can lead to improve learning experiences by allowing students to easily collaborate around virtual content, to access contextually relevant content, and to engage with personally-relevant content.

4.1. Micro-scale interactions

Collaboration can be a facilitator to learning because it enables students to engage with other learners as well as the educational content at the same time. This allows for deeper learning as students consider different perspectives and direct each other to study different aspects of the educational content (Chi, 2009). As students need to communicate their thoughts, they must think about their knowledge and how to match it to what others know. Communicating ideas to others has the potential to lead to meta-cognitive skills of determining one's own learning and tackling problems that will enhance it (Bransford, Brown, & Cocking, 2000). Various factors come into play in an effective collaboration. Non-verbal behaviors, such as gestures, body language, and eye gaze, have a purpose in communication as well as in directing attention (Billinghurst, Kato, & Poupyrev, 2001). Physical objects often play an important part in collaboration due to their affordances, semantic meaning, or spatial relationship to other objects.

Physical manipulatives in a collaborative setting have been shown to be effective in teaching dimensional analysis. For instance, researchers found that students in such an environment were engaged in active learning and were able to proceed at their own pace and in the manner that they preferred (Saitta, Gittings, & Geiger, 2011). This approach, however, was not successful in eliciting better performance with the more complex problems assigned to the students. It is possible that, in this particular case, students' ability to solve more complex problems was limited by the affordances of the physical manipulative, and potentially, a virtual manipulative would have been helpful. Though collaboration is possible with virtual manipulatives, some of the benefits of student–student interactions are lost when students collaborate around a virtual environment. If students collaborate by looking at a computer screen, they must switch between looking at the screen and looking at the other persons (Shelton & Hedley, 2004). Using such a system is also not suitable for large groups of students, and it also makes it difficult for a student to have individual control over the virtual content. On the other hand, if students collaborate in a virtual world (such as in a virtual-reality environment), then more people can collaborate around the same educational content; however, in such virtual environments communication through non-verbal cues is cumbersome or nonexistent (Billinghurst et al., 2001).

Augmented reality can take the best of both scenarios: students can see the virtual content and each other in the same space, thus the collaboration activity can take advantage of non-verbal cues as well as the affordances of physical objects. Many people can collaborate around the same educational content, each having an individual perspective and control over their experience. People collaborating in a shared space using AR have been shown to exhibit similar behaviors as in face-to-face collaboration (Kiyokawa, Iwasa, Takemura, & Yokoya, 1998). With mathematics concepts in a collaborative setting, the virtual environment can generally support higher-level communication about the curriculum topics, while the physical environment supports learning through mirroring manipulation of the objects (Evans, Feenstra, Ryon, & McNeill, 2011). At the present moment, not many AR learning systems are specifically designed for multi-person collaborations. Currently, the typical model in children's collaborative AR experiences is to have one person controlling the AR experience while the remaining group of students observes the experience on a shared display. For example, researchers projected an AR display onto a classroom wall and one student chose the physical manipulatives while the class watched and offered suggestions (Pasqualotti & dal Sasso Freitas, 2002). Other collaborative AR systems enable multiple children to interact with the experience that is displayed on a shared screen such as a PC monitor or TV (Hornecker & Dünser, 2009; Theng et al., 2007). One system enabled learners to experience their own perspective and interactions with the AR world through individual HMD or handheld displays (Kaufmann & Schmalstieg, 2003). In each of these examples, students are not only interacting with the educational content, they are also interacting with each other. Students gain the experiences of learning about the material and explaining the content to each other, which can lead to improved comprehension and retention of knowledge.

4.2. Macro-scale interactions

Learning is a process that occurs throughout the learner's life, not limited to classroom experiences. Despite this, it can often be difficult to access learning materials outside of school hours. In typical classrooms, the materials for physical manipulatives are held in storage bins and supply rooms, and starting and stopping learning activities requires some effort because, at the beginning of manipulative-assisted activities, teachers must distribute the materials, then collect and restock them at the end. Virtual manipulatives are more attractive to use in classrooms because they do not require a lengthy setup and cleanup time (Moyer et al., 2002). In order for a student to use a virtual manipulative, a computer program is started. Once the student is done interacting, the computer program is exited. Furthermore, unlike physical manipulatives, many students can observe and manipulate the same manipulative through the computer. Currently, virtual content can be accessed through computerized devices such as desktops, laptops, smart phones or specialized kiosks. One of the greatest benefits of AR to education is increasing the access to virtual educational content in environments that go beyond the walls of the classroom. AR manipulatives can permit students to have learning experiences in situations where virtual manipulatives might be infeasible – for instance, the experience might start as a student points the AR manipulative at an object on the playground. Highly portable devices such as mobile phones can allow students to access educational content in environments when traditional computers are not readily available, and provide children with the ability to link the learning content to their specific environmental context.

Situating learning in relevant contexts can help to improve learning outcomes. Situated cognition is an attempt to bridge the gap between knowing and doing. "Learning from dictionaries, like any method that tries to teach abstract concepts independently of authentic situations, overlooks the way understanding is developed through situated use" (J. S. Brown, Collins, & Duguid, 1989). The idea of situated cognition is embodied in the idea of authentic activity, which is defined by the culture in which learning occurs. Activities that are meaningful to a society take place in and are defined by the culture surrounding their application. Learning in schools is often defined by the culture of the school while ignoring the culture in which a learner might want to use that knowledge. Situated cognition aims to bridge the gap between the culture in which learning occurs and the culture in which the knowledge will be applied.

Some AR systems permit users to connect to information relevant to their context (Agarwal, Belhumeur, & Feiner, 2006; Wikitude, 2011; Yelp Inc, 2011). The AR Electronic Field Guide, for instance, permitted users to identify trees by pointing the phone camera at a leaf (Agarwal et al., 2006). Yelp and Wikitude applications permitted users to point their camera at their surroundings and see what stores or landmarks exist in the vicinity. With an AR manipulative that relies on physical objects, its accessibility depends on the type of physical objects involved in the experience. A manipulative that uses large physical blocks and runs on expensive handheld devices might not be able to be used at a student's home in a low-income community. In contrast, an AR manipulative that uses playing cards and runs off a web-camera might be accessible to a greater variety of children. As AR becomes more popular, and the ownership of smart phones expands, we can expect to see more and more learners using AR technology outside the classroom. Potentially, the semantic web or web-of-things will allow more contextually driven information to be accessible (Bizer, Heath, & Berners-Lee, 2009), allowing for AR applications that easily adapt to a user's environment. Although there is technology to place educational content in the world, there are many questions that remain open. For instance: What information can be learned in an authentic environment? How does learning in these environments compare to more traditional classrooms? What will motivate students to learn when grades are less relevant and students are learning beyond the watchful gaze of a teacher?

4.3. Personal relevance

Students' affective attitudes toward learning experiences can facilitate or impede learning (Davies & Brember, 2001). Positive attitudes can motivate students to engage and spend time with the learning environment. When students interact with computers, positive attitudes can motivate students to quickly master technical skills, yet negative attitudes such as anxiety can cause difficulty with such skill acquisition (Teo & Noyes, 2008). Developing manipulatives that leverage these positive interactions with computers, yet limit the anxiety of the experience, are crucial to student learning. Physical manipulatives are emotionally effective because their toy-like forms are familiar and easy to use by children. Furthermore, virtual manipulatives bypass a common motivational problem in which some students at certain grade levels perceive physical manipulatives as toys for younger kids. The perceived sophistication of virtual manipulatives can alleviate this issue, and might draw children to want to engage and explore them (Moyer et al., 2002). Furthermore, unlike their physical counterparts, virtual manipulatives lend themselves more easily to modification and personalization by students – for instance, students can paint the virtual manipulative or augment it with the student's own photos.

Augmented reality can aid learning due to a combination of its relevance of the learner's physical world and the customizability of the virtual world. Children have described AR technology as being "magic" (Billinghurst et al., 2001). The experience is magical because the reality around the user can be believably transformed into something out of a fantasy book. In opposition to computer games and pure virtual-reality, AR does not separate the user from his reality but instead uses it and realistically transforms it. This effect can cause a high degree of surprise and curiosity in users. Students' motivation to engage with manipulatives can be amplified. AR can appeal to students' interests through experiences that integrate personally meaningful objects, such as allowing students to measure one's own body or involve one's own toys. Furthermore, similar to virtual manipulatives, the elements of an AR manipulative can be permanently modified for the duration of the experience to relate to the individual learner. The emotional state of the student can make the experience more memorable, and can help memory encoding and learning (Fredrickson, 1998). Currently there is wide interest from marketing departments in using this "magical" technology to create memorable experiences for users. However, once people become accustomed to the technology, research will be able to observe whether the emotional effects of the technology are sustainable in longer-term learning experiences.

4.4. Summary

We argue that learning environments are enhanced by the presence of three general factors: collaboration, contextual relevance, and personal relevance. One benefit of AR experiences is they combine face-to-face collaboration with access to virtual learning content, permitting learners have their own perspective and control over the virtual content, while maintaining visual contact with their collaborators. Furthermore, AR experiences leverage situated cognition, by allowing the student to connect to the virtual educational content by simply pointing a camera at their environment, whether inside or outside the classroom. This ease of access is highly beneficial to students because contextually relevant information can be procured to satisfy the student's interest. Finally, student motivation can be increased through AR technology because AR experiences seemingly change student perceptions of reality, and provide learning experiences that are associated with personally-relevant content. To leverage the benefit of collaboration, AR experiences must not only allow students to view virtual content and other people in the same mixed-reality space, but the experience should allow students to independently control the content, as well as have personalized views of the content.

There are some design issues to consider with developing collaborative AR experiences. For example, if collaborators use HMD devices, occlusion might be an issue because virtual content might overlap with that of other collaborators; or, if collaborators use handheld devices, they will have a limited view into the shared virtual space, and they will have difficulty knowing what others are seeing. Because learners will want to discuss virtual content, it is important for the virtual space to be anchored to landmarks in the real space, such that they can be used as reference points. Furthermore, young children experiencing such shared virtual spaces might have difficulties collaborating because they might not easily visualize other children's point of view. AR experiences should also strive to leverage a user's context to create contextually relevant learning experiences that can be accessed virtually anywhere. In recent years, mobile devices have become a commodity for most people – smart phones such as the iPhone, Blackberry and Android are becoming gateways for AR technology to reach the general public, and innovations such as the semantic web-of-things will make it easier to build contextually-intelligent applications. Although there are still technical limits due to lack of processing power from these small devices, it is expected that learning with portable AR applications will be harnessed in the near future. Finally, AR applications should strive to generate motivational and personally-relevant learning experiences that take advantage of students' inherent motivation to experience the "magic" of AR, and which allow users to customize the content according to their preferences.

5. Conclusion and future work

We propose three dimensions along which to consider physical and virtual manipulatives inside and outside of the classroom. Physical objects afford more natural interactions compared to traditional computer input devices (e.g., keyboards, mice), potentially resulting in memory encoding that is strengthened through motor actions. Additional information can be presented with these physical objects in the virtual space, helping students make connections between the physical and abstract. Collaborative learning can take place in the presence of other learners within the learner's world, generating a more motivating and personalized learning experience. Although technology will evolve, we suggest augmented reality is a technology that will help bring together the benefits of both physical and virtual learning experiences.

We have highlighted the potential benefits and limitations of using AR to deliver learning experiences, by presenting an analysis based on psychological constructs, and by comparing AR applications to physical and virtual manipulatives. We envision a classroom in which children effortlessly interact with physical learning materials without specialized training. Gone are the superfluous lessons in teaching students how to *use* the computer before they start *learning* with the computer. Instructional information is embedded in the classroom space, tied directly to the relevant objects both spatially and temporally. Learners are focused on the task at hand. As they engage with the content, information seamlessly morphs from one representation to another, creating strong links between various concepts. Children physically manipulate the objects and representations, building strong connections through acting out various concepts. Students are captivated by the dynamic relationship between the real and virtual, the concrete and the abstract. We see students deeply engaged with these learning processes not just by themselves, but also with others. They interact naturally, taking advantage of social cues and eye contact as they learn from each other. Students are not limited by the confines of the classroom space because the playground, the street, and the home become fodder for augmented educational experiences.

We have proposed a three-tiered framework for understanding how to design AR learning experiences, but there are other concerns that need to be addressed in order to fully realize AR in the classroom. Teachers will be challenged by learning curves of adapting to a new technology in the fast-paved environment of the classroom. Schools and districts will need to commit to making the investment. Furthermore, computing power must increase to allow for more photorealistic rendering and complex natural interactions. Researchers will need to better understand how to design specific experiences to teach specific topics, while understanding the capabilities and limitations of learners of varying ages and skills. These and other pragmatic concerns must be addressed to create a robust learning technology.

Although AR shows great promise for extending the resources used for educating our students, there is much research to be done. Researchers must more specifically address the usefulness of AR from a psychological perspective. Specifically, a theory is needed to describe and predict learning outcomes in the space where the real and virtual combine. This domain is ripe for understanding not only the advantage of learning from the real and learning from the virtual, but how these two manifestations of learning materials combine and give rise to educationally beneficial experiences.

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