

# Virtual Laboratory Design for Pendulum-Cart Control System

Feni Isdaryani and Trihastuti Agustinah

Department of Electrical Engineering

Institut Teknologi Sepuluh Nopember

Surabaya, Indonesia

feni14@mhs.ee.its.ac.id; trihastuti@ee.its.ac.id

**Abstract**— This paper describes the design of virtual laboratory using MATLAB for pendulum-cart system. The control system is designed using fuzzy control method. The aims of this virtual laboratory are to help user to do an experiment and learn the dynamic of the pendulum-cart system. The control system designed is applied to the virtual laboratory. The physical form of pendulum-cart system in this virtual laboratory is visualized with 3D graphics simulation and interactive interface. The results of virtual laboratory simulation show that 3D graphics, control system, and the interface can be well integrated.

**Keywords**—component; Virtual Laboratory, pendulum-cart system, fuzzy control.

## I. INTRODUCTION

The development of computer technology provides various advantages in improving the quality of education. Various learning methods has been produced along with the development of computer technology, where the learning methods can simplify and facilitate the process of learning and teaching. Lately, in the field of control systems, learning method using virtual laboratory become popular, because it can allow users to learn and test a system with interactive interface. Besides, virtual laboratory has several advantages over conventional learning in the real laboratory, such as low cost, maintenance free, and experiment can be done at anytime and from anywhere.

Educational tool such as virtual laboratory has been developed [1]-[4]. These researches use various types of plant as control objects, e.g. process-level, two-wheeled inverted robot, gantry cranes, and DC motor. Virtual laboratory has been developed not only in the field of engineering course, but also in other fields. Several classical control methods have been proposed for control of the plant, can be found in [1]-[3]. The multiple software is used in [3] to create the virtual laboratory. It is required high programming level and other tool to communicate with different software, e.g. computation engine software and graphical interface.

In this paper, a virtual laboratory is designed for pendulum-cart control system. In contrast to [1]-[2], that used classical control methods for the inverted pendulum, this virtual laboratory control system is designed using intelligent control,

i.e. fuzzy control. The proposed controller consists of two controllers: fuzzy swing-up controller and fuzzy stabilization controller. The control objective is to bring the pendulum from pendant position and stabilize the pendulum in the inverted position. Mamdani type fuzzy controller is used to swing the pendulum from pendant position to upright position, and Takagi-Sugeno (TS) fuzzy model is used to stabilize the pendulum-cart system. The aims of this virtual laboratory are to help user to do an experiment and learn the pendulum-cart system using fuzzy control method.

In virtual laboratory, the physical form of the system is visualized in the simulation, so that students can be more familiar with the system before performing experiments on actual plant [5]. For creating this virtual laboratory Matlab software is selected because it's simple programming to create interface, 3D graphic, and controller. It does not require any other communication tools to generate the programs. Moreover, 3D graphics pendulum-cart animation with interactive interface provide a venue for testing and demonstrating various control methods for the pendulum-cart system.

## II. VIRTUAL LABORATORY OVERVIEW

Virtual laboratory is an interactive environment for creating and conducting simulated experiments [5]. In this research, virtual laboratory is used as a tool in learning process, such as learn the characteristic of the system, and to do an experiment with various fuzzy control. This virtual laboratory is operated using a computer and can perform simulations of system as if the users doing the actual experiments in the laboratory.

The design of virtual laboratory consists of two parts: virtual reality model of pendulum-cart system and interface design. Virtual reality model is a dynamic 3D representation of pendulum-cart system. In this paper, virtual laboratory interface design is conducted on MATLAB version R2011b. The interface consists of several components such as Push Button, Check Box, Slider, Pop-up Menu and Edit Text.

### A. Virtual Reality Toolbox

Virtual Reality Toolbox is a MATLAB facility to create and visualize dynamic system in the form of 3D graphics or virtual world. Virtual Reality Toolbox has complementary components such as Virtual Reality Modeling Language

(VRML) Editing Tools and VRML viewer [6]. VRML Editing Tools is used to create a virtual world using VRML editor or text editor. While VRML viewer is used to view the virtual world that have been created using VRML editor. Connecting virtual world with MATLAB Simulink can be done by using Simulink 3D Animation toolbox. The interface of the virtual reality toolbox can be seen in Figure 1.

VR model is designed in 3D World Editor. It is created by building a physical model of pendulum-cart system in 3D form. Physical model of the pendulum-cart system consists of a cart with four wheels that mounted on both sides, two pairs of pendulum, and rail. It is can be seen in Figure 2. Furthermore, the 3D model is made correspond with the physical model of pendulum-cart system. VR modeling is designed by forming a hierarchical tree structure objects (nodes) that consists of the parent object and child object. The position and orientation of child objects are specified relative to the parent object. Every node in tree represents some functionality of the 3D scene.

### B. GUIDE MATLAB

Graphical User Interface Development Editor (GUIDE) or commonly called the GUI builder, is a facility in MATLAB to create a GUI with graphic objects such as, button, text box, slider, and menu. In MATLAB, GUI can be made with two steps, first is create the GUI on the MATLAB GUIDE, and then create the action of each component through the programming [7].

The interface of the GUIDE can be seen in Figure 3. Main menu in GUIDE consists of Layout Editor, Component Palette, Menu Editor, Alignment Tool, M-file Editor, Property Inspector and Running Button.

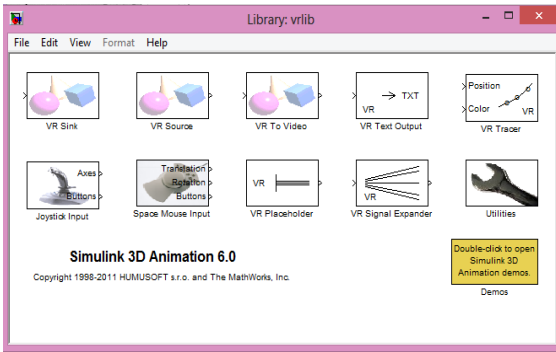


Fig. 1. Virtual Reality Toolbox



Fig. 2. Pendulum-Cart System

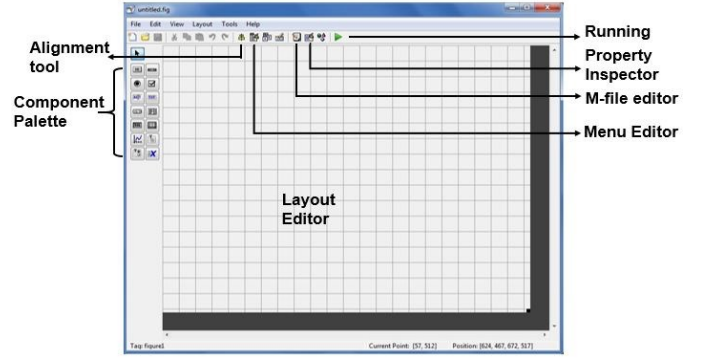


Fig. 3. GUIDE Interface

Creating interface at GUIDE, can be done by drag and drop the desired components in Component Palette to the Layout Editor. The GUI programming can be written in the M-file Editor menu on the GUIDE menu bar. Then, it will pop up a script that contains a Callback for each component. Callbacks are functions that execute in response to user-generated events, such as a mouse click. On each component that included in the GUIDE, it automatically add a new Callback. Add the code of desired command to the Callbacks to perform the desired functions of components.

Virtual laboratory interfaces must be designed to be user friendly and interactive, such as provide information about the plant/controller and also users can change parameters that available on interface. The interface is designed using several components such as Push Button, Pop-up Menu, Slider, and Edit Text. Push Button is used to view the Simulink diagram, change the model parameters or control parameters, to view the control action scope, and to start/stop the simulation. To select types of controllers for pendulum-cart system can be used Pop-up Menu. Component Edit Text is used to set the stop time for MATLAB Simulink.

### III. CONTROL SYSTEM OVERVIEW

The system that will be used in this virtual laboratory is a pendulum-cart control system, where the pendulum-cart system consists of two moving parts that is pendulum and cart. Cart can only move horizontally along the rail with a limited path length, while the pendulum can swing in accordance to the axis of rotation that located on the side of the cart. To swinging and balancing pendulum, cart is moved to the left or to the right rail.

The pendulum-cart system has single input and multiple output. The input is control signal, and the output are cart position ( $x_1$ ), pendulum angular position ( $x_2$ ), cart velocity ( $x_3$ ), and pendulum angular velocity ( $x_4$ ), where state space equation presented in (1-4) [8]:

$$\dot{x}_1 = x_3 \quad (1)$$

$$\dot{x}_2 = x_4 \quad (2)$$

$$\dot{x}_3 = \frac{a(u - T_c - \mu x_4^2 \sin x_2) + l \cos x_2 (\mu g \sin x_2 - f_p x_4)}{J + \mu l \sin^2 x_2} \quad (3)$$

$$\dot{x}_4 = \frac{l \cos x_2 (u - T_c - \mu x_4^2 \sin x_2) + \mu g \sin x_2 - f_p x_4}{J + \mu l \sin^2 x_2} \quad (4)$$

with:

$$a = l^2 + \frac{J}{m_c + m_p}; \quad \mu = (m_c + m_p)l$$

Mass of pendulum denoted as  $m_p$ , mass of cart as  $m_c$ , gravity as  $g$ , inertia as  $J$ , distance between axis of rotation to the center of mass system as  $l$ . Control forces applied to the cart as  $u$ , the friction between cart and rail is  $T_c$ , and  $f_p$  is a constant friction of pendulum. The pendulum-cart system parameters that used in this paper can be seen in Table 1.

In order to pendulum-cart system can be controlled, then two controllers are designed; the fuzzy swing-up controller and the stabilization controller. For stabilization, the controller is created using T-S fuzzy model, where the rules of the controller are created using the parallel distributed compensation (PDC) [9] with pole placement. The control scheme can be seen in Figure 4.

#### A. Fuzzy Swing-up Controller

Fuzzy swing-up controller for pendulum-cart system is intended to swing the pendulum and raise it from hanging position towards inverted position. In pendulum-cart system there are two objects that can be controlled, the cart and the pendulum. By moving the cart to the right, pendulum will swing to the left, and vice versa. This controller has two inputs, cart position ( $x_1$ ) and cart velocity ( $x_3$ ), while the output is control action ( $u$ ).

The premise variable  $x_1$  is divided into five membership functions (as shown in Figure 5), i.e. *Negative Big* (NB), *Negative Small* (NS), *Zero* (Z), *Positive Small* (PS), and *Positive Big* (PB). The premise variable  $x_3$  is divided into two membership functions that are *Positive* (P) and *Negative* (N) as shown in Figure 6. Whereas the control action ( $u$ ) has ten membership functions as shown in Figure 7. The rule base for fuzzy swing-up controller is described in Table 2.

TABLE 1. General Parameter of Pendulum-Cart System

Parameter	Value
$m_c$ (Kg)	1.12
$m_p$ (Kg)	0.12
$l$ (m)	0.0167903
$J$ (Kg.m <sup>2</sup> )	0.0135735
$f_p$ (Kg.m <sup>2</sup> /s)	0.000107

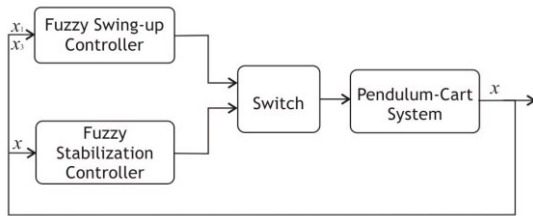


Fig. 4. Control Scheme for Pendulum-Cart System

Table 2. The rule base of fuzzy swing-up controller

		Premise $x_1$				
		NB	NS	Z	PS	PB
Premise $x_3$	N	PVB	NVS	NS	NM	NB
	P	PB	PM	PS	PVS	NVB

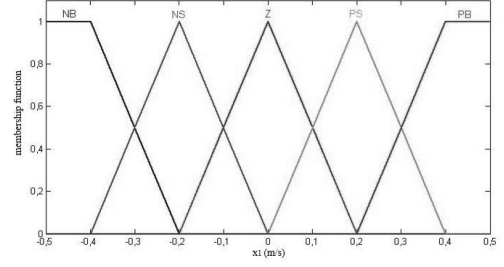


Fig. 5. Cart position ( $x_1$ ) membership functions

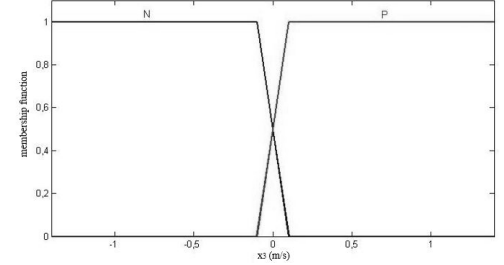


Fig. 6. Cart velocity ( $x_3$ ) membership functions

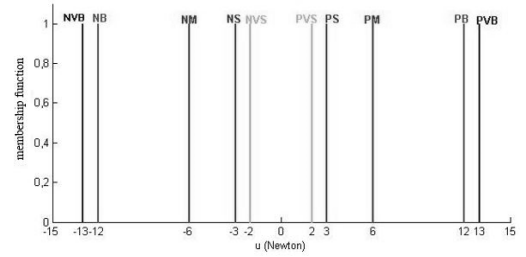


Fig. 7. Control action ( $u$ ) membership functions

#### B. Fuzzy Stabilization Controller

The controller for stabilization pendulum-cart system is designed using the T-S fuzzy model. The fuzzy model represent the dynamics of nonlinear system where the consequent part of each rule is represented as a linear model. Overall system model is achieved by blending fuzzy of several linear models.

The proposed T-S fuzzy model has two rules, and one premise variable. The premise variable is pendulum angle ( $x_2$ ). The rules of plant can be written as:

Rule 1 for plant:

**If**  $x_2(t)$  is  $M_1$  (about 0 rad)

**Then**  $\dot{x}(t) = A_1 x(t) + B_1 u(t)$

$y(t) = C_1 x(t)$

Rule 2 for plant:

**If**  $x_2(t)$  is  $M_2$  ( $\pm 0.2$  rad)

**Then**  $\dot{x}(t) = A_2 x(t) + B_2 u(t)$

$y(t) = C_2 x(t)$

According to the concept of PDC [9], every rule of plant is compensated by the corresponding rule of the controller. The rules of the controller can be written as follows:

Rule 1 for controller:

**if**  $x_2(t)$  is  $M_1$  (about 0 rad)

**Then**  $u(t) = -K_1 x(t)$

Rule 2 for controller:

**if**  $x_2(t)$  is  $M_2$  ( $\pm 0.2$  rad)

**Then**  $u(t) = -K_2 x(t)$

The pole placement technique [10] is used to obtain the state feedback gain of the controller rules. Because the pendulum-cart system consists of four states, it is required four-poles. By determining the desired poles for each rule, the state feedback gain can be obtained.

#### IV. SIMULATION RESULTS

The simulation results of this research is intended to test the virtual laboratories and virtual word with an interface that has been created in the GUI, and to test the control system that has been created in the Simulink.

##### A. Virtual World Simulation

Virtual World Simulation is done by running virtual world file (with extension \*.wrl) that has been designed. Figure 8, 9, and 10 shows the VR model of pendulum-cart system when viewed from front, side, and above, respectively.

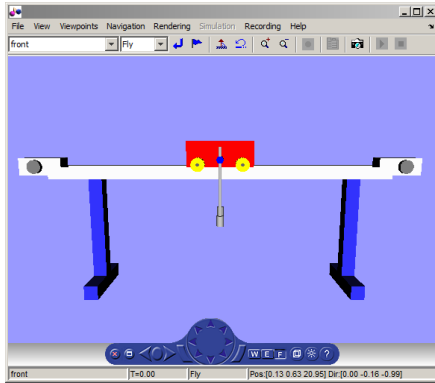


Fig. 8. VR model front-view

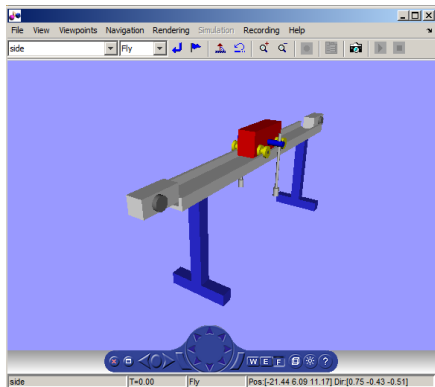


Fig. 9. VR model side-view

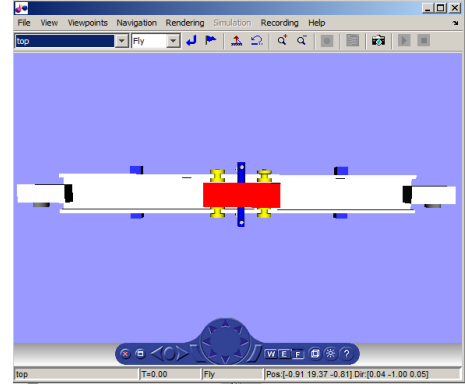


Fig. 10. VR model above-view

##### B. Virtual Laboratory Simulation

Interface simulation is done by running the GUIDE m-file that has been designed. The interface on the virtual laboratory can be seen in Figure 11, consists of five panels that is Pendulum Cart System, Controller, Parameters, View Response, and Open Buttons. The dynamic motion of a pendulum-cart system is displayed by 3D graphics scheme on the Pendulum-Cart System panel.

Controller panel is consists of two menu that is used to select the desired controller and choose the desired view point pendulum-cart system. This virtual laboratory offers three different controllers, where the first is proposed controller, the second controller refers to [11], and the third refers to the controller in [12].

Users can change the parameter values of pendulum-cart system by moving the slider that has been provided in the parameter panel. Moreover, users can add disturbance and change the initial condition values of the desired pendulum angle. In this panel there are also a start, stop, and exit button.

Graph response of the system can be viewed by pressing the buttons on the View Response panel. The response button consist of cart position, pendulum angular position, cart velocity, pendulum angular velocity and control signal. The graph responses also has a zoom feature.

Open Button panel consists of four push button that is used to show the information about the Simulink model, fuzzy swing-up control, T-S fuzzy control for stabilization, and the values of the state feedback gain that are used in the control system. As can be seen in Figure 12, if the State Feedback Gain button is pressed, it will display a new window that contain information about the state feedback gain values are used in the selected controller.

Virtual laboratory simulation test is done by testing every button and menu. Figure 13 show a screenshot when running a virtual laboratory simulation, where the pendulum rod toward the inverted position.

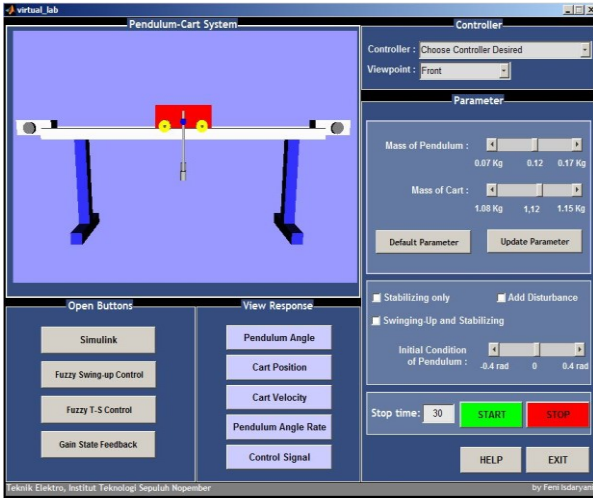


Fig. 11. Virtual Laboratory Interface

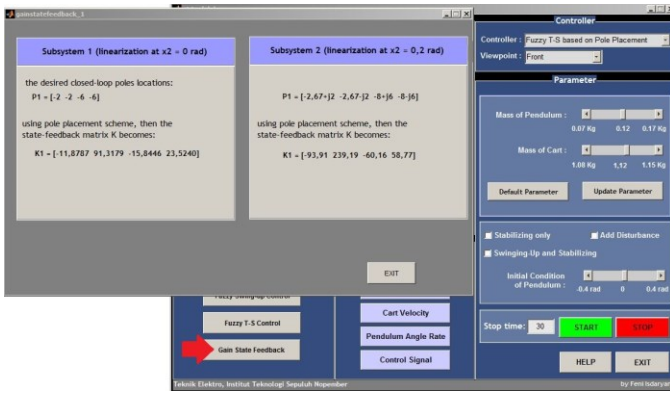


Fig. 12. Window of Gain State Feedback button menu

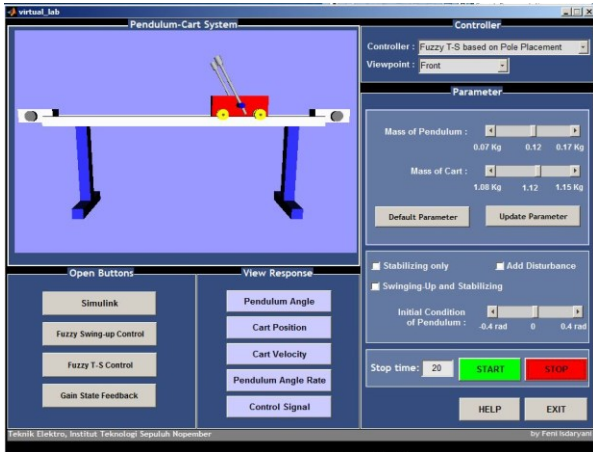


Fig. 13. Virtual Laboratory Component Testing

### C. Control System Simulation

Simulation results of the proposed control system can be seen by pressing the buttons on the View Response interface panel. Figure 14 consists of interface and graphics response. There are three responses shown in Figure 14, seen from the upper left that is the response of the pendulum position, cart position, and control signal applied to the pendulum-cart system. For the clearer picture, the cart position and pendulum angle responses are depicted in Figure 15 and 16, respectively.

The nonlinear system in (1-4) is linearized using two operating points. The state space equation of the system can be written as

$$\dot{x}(t) = A_i x(t) + B_i u(t), \quad i = 1, 2$$

with the matrices  $A_i$  and  $B_i$  are obtained by linearization for two operating points, i.e.  $x_2 = 0$  rad, and  $x_2 = 0.2$  rad. The linear matrices of the system are as below

$$A_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0.2524 & 0 & 0 \\ 0 & 15.0319 & 0 & -0.0079 \end{bmatrix}; B_1 = \begin{bmatrix} 0 \\ 0 \\ 0.8272 \\ 1.2370 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0.2298 & 0 & 0 \\ 0 & 14.6544 & 0 & -0.0079 \end{bmatrix}; B_2 = \begin{bmatrix} 0 \\ 0 \\ 0.8263 \\ 1.2086 \end{bmatrix}$$

The simulation is done by using initial condition of pendulum angle is  $\pi$  radians (pendant position) with the feedback gain, given by:

$$K_1 = [-11.9 \quad 91.3 \quad -15.8 \quad 23.5]$$

$$K_2 = [-93.9 \quad 239.2 \quad 60.2 \quad 58.7]$$

As result, to swing the pendulum toward the inverted position, the cart moves 2 times to the right and 1 times to the left. Cart deviation when swing-up is -0.392 m to 0.392 m. At the time of 2.33 seconds, the cart is around 0.2 m and moving toward the midpoint of the rails about 4 seconds. The pendulum reaches the inverted position after swinging two times. Pendulum angle can reach 0.3 rad at the time of 2.33 seconds and steady at 0 rad (inverted position) after 2.75 seconds.

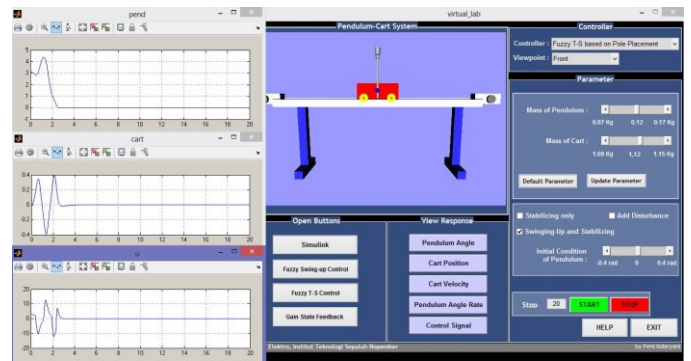


Fig. 14. Screenshot of the virtual laboratory interface and control system responses

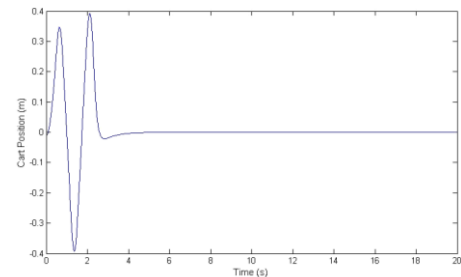


Fig. 15. The cart position of the pendulum-cart system



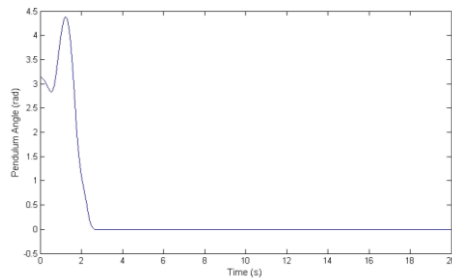


Fig. 16. The pendulum angle of the pendulum-cart system

Figure 15 and 16 show that the cart arrived to its desired position, i.e. midpoint of rail, and pendulum angle is at the inverted position. It can be concluded that the proposed controller meet the designed objective.

## V. CONCLUSION

After conducting the design and simulation of this research, virtual laboratory design for fuzzy control of pendulum-cart system, can be seen that the simulation results for virtual laboratory show that 3D graphics, control system, and the interface can be well integrated. The result of pendulum-cart control system simulation are shown in 3D graphic display. The proposed controller based on fuzzy control method has good performance to stabilize the pendulum-cart system.

For further research development, the authors suggest that this virtual laboratory design can be integrated with the internet, and can be used for tracking control problem for pendulum-cart system.

## REFERENCES

- [1] J. L. Lima, J. C. Gonçalves, P.G. Costa, and A. P. Moreira, "Interactive educational software for dynamic system control," *International Conference on Engineering Education, ICEE*, 2007.
- [2] R. Murillo Gracia, F. Wornle, B. G. Stewart, and D. K. Harrison, "Real-time remote network control of an inverted pendulum using ST-RTL", *IEEE Frontiers in Education Conference*, 2002.
- [3] A. Villar-Zafra, S. Zarza- Sánchez, J. A. Lázaro-Villal, and R. M. Fernández-Canti, "Multiplatform virtual laboratory for engineering education," *International Conference, IEEE*, 2012.
- [4] R. Huzlik, V. Hajek, O. Vitek and P. Bauer, "Virtual laboratory of DC machine and automotive electrical engineering," *Int. Power Electronics and Motion Control Conference*, 2010.
- [5] J. R. Bourne, A. Brodersen, and M. Dawan, *The Influence of Technology on Engineering Education*, CRC Press, Inc., 1995.
- [6] The Math Works Inc., *Virtual Reality Toolbox for use with MATLAB and Simulink: User's Guide (Version 3)*, The Math Works Inc., 2004.
- [7] The Math Works Inc., *MATLAB Creating Graphical User Interface*, The Math Works Inc., 2013.
- [8] Feedback Instruments Ltd., *Digital-Pendulum: Control in a MATLAB Environment (MATLAB 6.5 Version)*, Feedback Instruments Ltd., England, 2004.
- [9] K. Tanaka and H. O. Wang., *Fuzzy control systems design and analysis: a Linear Matrix Inequality approach*, John Wiley and Sons Inc., 2001.
- [10] K. Ogata, *Modern Control Engineering, 3rd ed.*, Practice-Hall, New Jersey, 1997.
- [11] T. Febrianto, "Desain kontrol fuzzy berbasis performansi h $\infty$  dengan batasan *input-output* untuk sistem pendulum-kereta", *Tugas Akhir*, Institut Teknologi Sepuluh Nopember, Surabaya, 2012.
- [12] T. Agustinah, A. Jazidie, and M. Nuh, "Hybrid fuzzy control for swinging up and stabilizing of the pendulum-cart system", *Proc. IEEE Int.Conf. Computer Science and Automation Engineering*, Shanghai-China, June, pp. 109-113, 2011.