NEUROVISION®; THE WAY TO MERGE VISUAL REALITY WITH NAVIGATIONAL AND MILITARY SYSTEMS

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Abstract

Historically, the extrapolation of the navigational data into what is perceived as reality by the pilot through the cockpit´s window, and vice versa, has been a major source of human error during flight. Advanced computer generated terrain simulation systems provide real time virtual flights, but the pilot still needs to constantly keep checking visual reality with virtual environment and merge those on his mind. Besides, the field of view is narrow. In order to address this problem the *head up* display superimposes virtual and visual information. Unfortunately, angular inaccuracies related to angular vision coordinates and the changing pilot´s point of view such as head position and distance to the device puts the *"head up"* display on need of improvement. We propose an advanced method to merge visual reality with advanced navigational systems on a wide (180°) or very wide (360°) field of view. The system is able to acquire the tridimensional visual reality as well as infrared view merged with present and future advanced navigational systems and it is also able to deliver it to the pilot in the way the human brain better understands it. The Intruder Planes Projected Trajectory and the 3D shapes of Complex Weather may also be instantaneously understood by the pilot. The exceptional capability of the system to display intruder´s angular position and projected trajectories into the panoramic visual perception of the pilot is shown. Such feature may also be extended to ground or unmanned units. We call this system Neurovision®. This advanced method for stereographic projection, able to generate Riemannian manifolds facilitates and even makes some computer tasks now possible, allowing improvements derived from differential geometry and neuroscience to hopefully provide novel opportunities to avionics and technology in general. Neurovision® enables to merge Panoramic View, Infrared, Navigation Computer Generated Graphics, Complex 3D weather and Intruder´s Planes or Target´s Projected 3D Trajectories for every plane or military units´s 3D *unique* visual perspective.

Introduction

Neurovision® is a method to merge tridimensional visual reality with virtual systems on a 180° or 360° field of view being also able to deliver it to the pilot in the way the human brain better understands it. On Anti- collision and weather systems, it provides unique tridimensional vision of the trajectories of intruder planes and storms. Neurovision® may also be used on unmanned vehicles, ground forces, I.S.R., automotive industry and robotics. With regard to Avionics, NASA has shown that for All-Weather Commercial Aviation Operations, significant improvements in situation awareness, without concomitant increases in workload and display clutter, could be provided by the integration and/or fusion of synthetic and enhanced vision technologies for the pilot-flying and the pilot-not-flying [1-3]. The Neurovision® platform may also upgrade the anti- collision and weather systems such as *GPS, TCAS, Radar, ADS-B,* etc. by providing unique tridimensional vision of the present and predicted trajectories of intruder planes and storms.

Spherical Coordinates System

As shown on Figure 1, in order to define a spherical coordinate system, one must choose two orthogonal directions, the *zenith* and the *azimuth reference*, and an *origin* point in space. These choices determine a reference plane that contains the origin and is perpendicular to the zenith. The spherical coordinates of a point *P* are then defined as follows:

- The *radius* or *radial distance* is the Euclidean distance from the origin *O* to *P*.
- The *inclination* (or *polar angle*) is the angle between the zenith direction and the line segment *OP*.
- The *azimuth* (or *azimuthal angle*) is the signed angle measured from the azimuth reference direction to the orthogonal

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projection of the line segment *OP* on the reference plane.

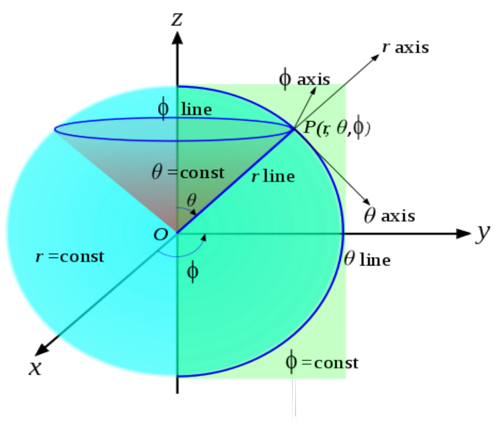

Figure 1. Shows a Pair of Angles where P(r, Θ, Φ)

The sign of the azimuth is determined by choosing what is a *positive* sense of turning about the zenith. This choice is arbitrary, and is part of the coordinate system's definition. The *elevation* angle is 90 degrees (π/2 radians) minus the inclination angle. If the inclination is zero or 180 degrees (π radians), the azimuth is arbitrary. If the radius is zero, both azimuth and inclination are arbitrary. In linear algebra, the vector from the origin *O* to the point *P* is often called the *position vector* of *P* [4-6].

The Trans-Dimensional Coordinates System

Basic Definition.

The objective of the Trans Dimensional TD Coordinates System is to only employ N-1 variables in a N dimension space. The TD Coordinates System also seeks to only employ an N-1 dimension graphic to represent said N dimension space. Of course, an N dimensions coordinates system can be created using N-1 variables by considering one of the variables as a constant. However, in order to approach a tridimensional system as a bi dimensional graph, the TD Coordinates System considers the central point (0,0) in the geometric center of a preferred sphere shaped plane, and as expected, only two variables are employed to locate a point in space [4], by using the matemathical propieties of an hypersphere [5] where, having three dimensions is the 3-sphere: points equidistant to the origin of the Euclidean space $\mathbb{R}^4$ at distance one [7]. If any position is;

$P = (x, y, z, t)$, then

$x^2 + y^2 + z^2 + t^2 = 1$ characterize a point in the 3-sphere [8].

As shown on Figure 2, the perpendicularity of the X-Y plane with the Y-Z plane makes neccesary the use of the hypersphere propieties trough Neurovision®. One of the main advantages of Neurovision comprises the use of a variation in the standard Spherical Coordinates System where both angles **Θ, Φ** define parallel lines on the spherical surface.

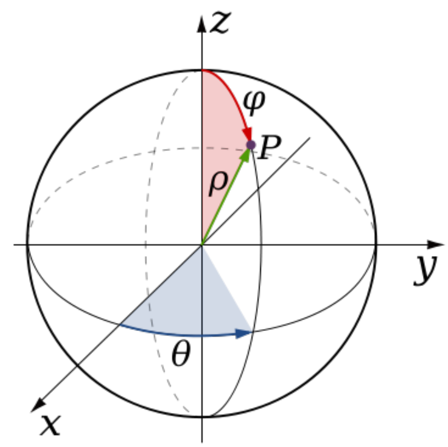

Figure 2. Shows that the Θ Angle exists on the X, Y Bidimensional Plane and the Φ Angle exists on the Y, Z Bidimensional Plane

Stereographic projection

As shown on Figure 3, stereographic projection of the unit sphere from the north pole onto the plane $z = 0$, shown on Figure 3 in cross section. The unit sphere in three-dimensional space $\mathbf{R}^3$ is the set of points (x, y, z) such that $x^2 + y^2 + z^2 = 1$.

Let $N = (0, 0, 1)$ be the "north pole", and let *M* be the rest of the sphere. The plane $z = 0$ runs through the center of the sphere; the "equator" is the intersection of the sphere with this plane. For any point *P* on *M*, there is a unique line through *N* and *P*, and this line intersects the plane $z = 0$ in exactly one point *P*'. Define the stereographic projection of *P* to be this point *P*' in the plane.

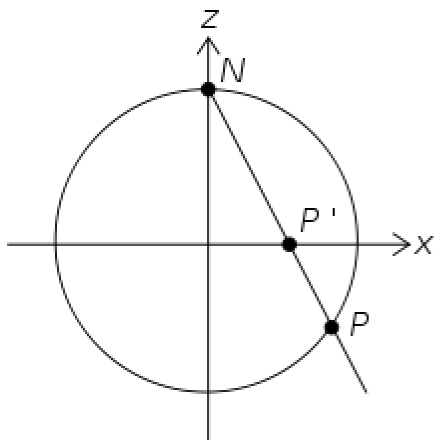

Figure 3. Shows the Stereographic Projection of the Unit Sphere

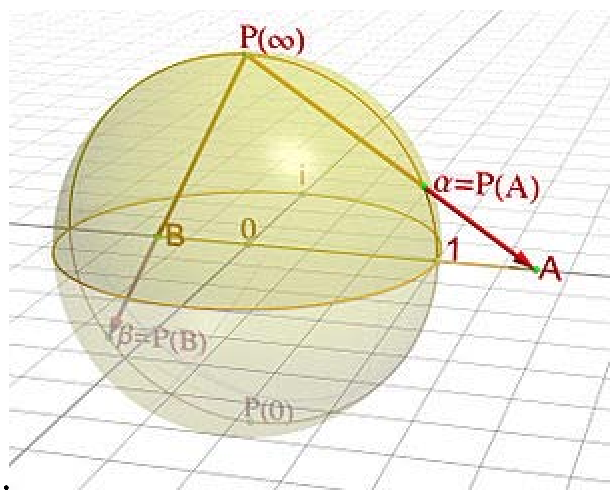

Figure 4. Shows the Tridimensional Unit Sphere Projected

In Cartesian coordinates (x, y, z) on the sphere and (X, Y) on the plane, the projection and its inverse are given by the formulas

$$(X,Y) = \left(\frac{x}{1-z}, \frac{y}{1-z}\right),$$

$$(x,y,z) = \left(\frac{2X}{1+X^2+Y^2}, \frac{2Y}{1+X^2+Y^2}, \frac{-1+X^2+Y^2}{1+X^2+Y^2}\right).$$

In spherical coordinates (φ, θ) on the sphere (with φ the zenith angle, $0 \le \varphi \le \pi$, and θ the azimuth, $0 \le \theta \le 2\pi$) and polar coordinates (R, Θ) on the plane, the projection and its inverse are:

$$(R,\Theta) = \left(\frac{\sin\varphi}{1-\cos\varphi}, \theta\right),$$

$$(\varphi,\theta) = \left(2\arctan\left(\frac{1}{R}\right), \Theta\right).$$

Here, φ is understood to have value π when $R = 0$. Also, there are many ways to rewrite these formulas using trigonometric identities. In cylindrical coordinates (r, θ, z) on the sphere and polar coordinates (R, Θ) on the plane, the projection and its inverse are

$$(R,\Theta) = \left(\frac{r}{1-z}, \theta\right),$$

$$(r,\theta,z) = \left(\frac{2R}{1+R^2}, \Theta, \frac{R^2-1}{R^2+1}\right).$$

The stereographic projection defined in the preceding section sends the "south pole" (0, 0,−1) to (0, 0), the equator to the unit circle, the southern hemisphere to the region inside the circle, and the northern hemisphere to the region outside the circle. The projection is not defined at the projection point N = (0, 0, 1). Small neighborhoods of this point are sent to subsets of the plane far away from (0, 0). The closer P is to (0, 0, 1), the more distant its image is from (0, 0) in the plane. For this reason it is common to speak of (0, 0, 1) as mapping to "infinity" in the plane, and of the sphere as completing the plane by adding a "point at infinity".

Stereographic projection is conformal, meaning that it preserves the angles at which curves cross each other (see Figures 5 and 6). In Figure 5, the grid lines are still perpendicular, but the areas of the grid squares shrink as they approach the north pole. In Figure 6, the grid curves are still perpendicular, but the areas of the grid sectors shrink as they approach the north pole.

On the other hand, stereographic projection does not preserve area; in general, the area of a region of the sphere does not equal the area of its projection onto the plane. The area element is given in (X, Y) coordinates by

$$dA = \frac{4}{(1+X^2+Y^2)^2}\, dX\, dY.$$

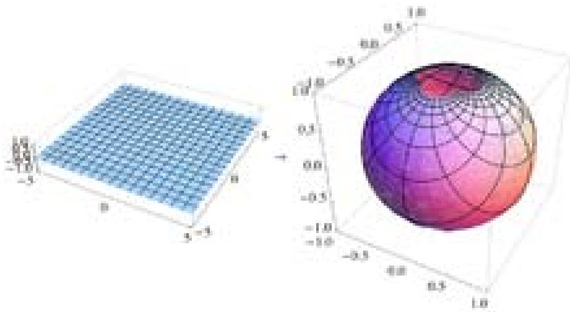

Figure 5. Shows a Cartesian Grid on the Plane That Appears Distorted on the Sphere

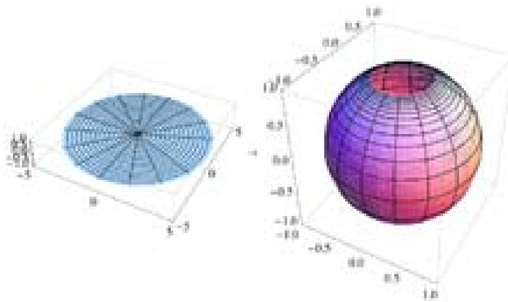

Figure 6. Shows a Polar Grid on the Plane That Appears Distorted on the Sphere

This notion finds utility in projective geometry and complex analysis. On a merely topological level, it illustrates how the sphere is homeomorphic to the one point compactification of the plane. In Cartesian coordinates a point $P(x, y, z)$ on the sphere and its image $P'(X, Y)$ on the plane either both are rational points or none of them:

$$P \in \mathbb{Q}^3 \iff P' \in \mathbb{Q}^2$$

Along the unit circle, where $X^2 + Y^2 = 1$, there is no infinitesimal distortion of area. Near (0, 0) areas are distorted by a factor of 4, and near infinity areas are distorted by arbitrarily small factors. The metric is given in (X, Y) coordinates by

$$\frac{4}{(1+X^2+Y^2)^2}\,(dX^2+dY^2),$$

and is the unique formula found in Bernhard Riemann's *Habilitationsschrift* on the foundations of geometry, delivered at Göttingen in 1854, and entitled *Über die Hypothesen welche der Geometrie zu Grunde liegen*. No map from the sphere to the plane can be both conformal and area-preserving. If it were, then it would be a local isometry and would preserve Gaussian curvature. The sphere and the plane have different Gaussian curvatures, so this is impossible. The conformality of the stereographic projection implies a number of convenient geometric properties. Circles on the sphere that do *not* pass through the point of projection are projected to circles on the plane. Circles on the sphere that *do* pass through the point of projection are projected to straight lines on the plane. These lines are sometimes thought of as circles through the point at infinity, or circles of infinite radius. All lines in the plane, when transformed to circles on the sphere by the inverse of stereographic projection, intersect each other at infinity. Parallel lines, which do not intersect in the plane, are tangent at infinity. Thus all lines in the plane intersect somewhere in the sphere—either transversally at two points, or tangently at infinity. (Similar remarks hold about the real projective plane, but the intersection relationships are different there [9].)

So, as shown on Figures 7, 8 and 9, the ecuatorial stereographic projection of the sphere with standard latitude and longitude lines, as the ones used by the standard spherical coordinates system produces a set of no parallel longitude lines.

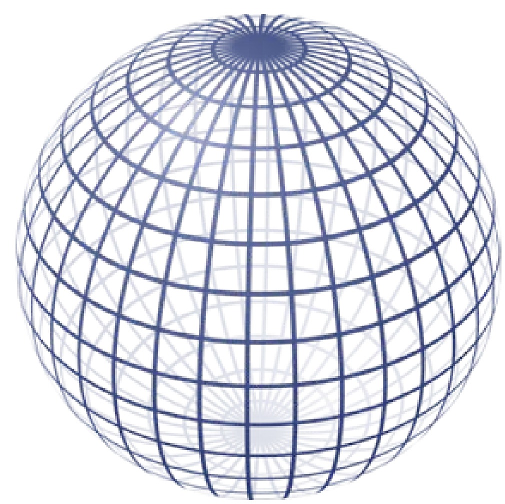

Figure 7. Shows a Sphere with Standard Spherical Coordinates System Longitude and Latitude Lines

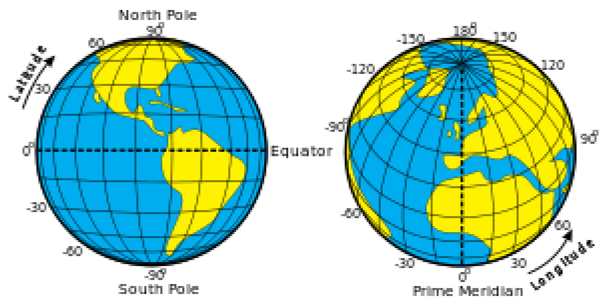

Figure 8. Shows That Latitude Lines Are Not Parallel to Each Other. this Is a Permanent Issue in Mathematics

Figure 9. Shows That the Stereographic Projection Does Not Present Parallel Latitude Lines, (François d'Aiguillon "Opticorum libri philosophis juxta ac mathematicis utiles") 1613, by Rubens.

Thus, breaking one of the postulates for a perfect Euclidian Space although the coordinates are orthogonal, the lines are not parallel [10]. So, the author proposes a simplified method where the stereographic projection of a sphere provides the Euclidean Conditions:

1.- Orthogonal Coordinates.

2.- Parallel latitude and longitude lines.

Such arrange fully complies with the postulates of said Euclidean Space. This arrange is shown on Figures 10 and 11.

Consider the sphere P(r, Θ, Φ) shown on Figure 10, where Θ, Φ are each one continuous around the one of the x, y, z axes, where Θ and Φ are not around the axe represented in the isocenter (0,0) displayed on the n-1 ecuatorial stereographic projection and are not conformed fully within the boundaries of any of the X-Y, X-Z, Y-Z flat planes. Said sphere provides the localization of 1 point in space where Θ, Φ intersect each other. Please notice that depending on the convention used, up to 8 points in space can be defined with only two variables**.**

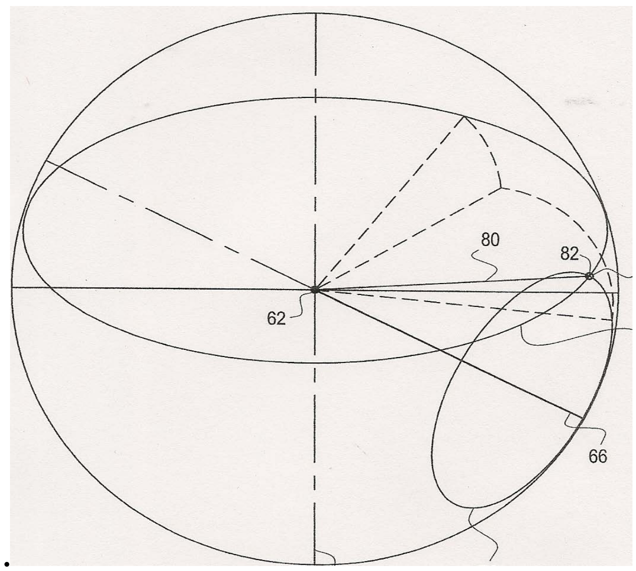

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Figure 10. Shows a Sphere using Trans Dimensional Spherical Coordinates System.

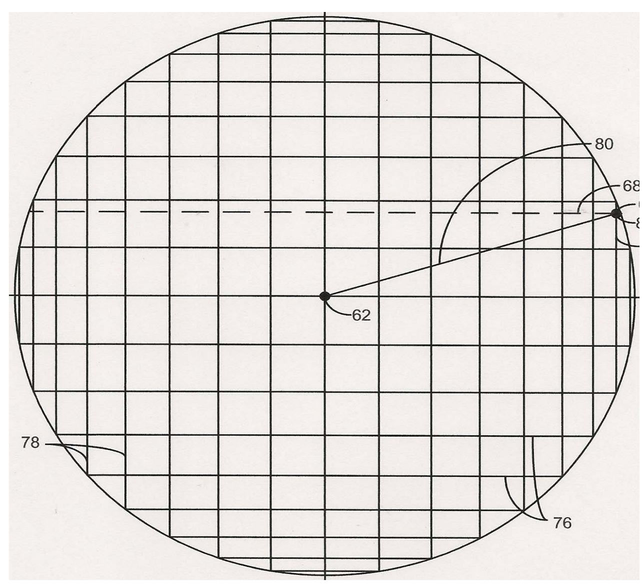

Figure 11. Shows the Stereographic Projection of a Sphere (or any Cylindrical Shaped Body)

As Figure 11 shows, the display of the stereographic projection of the sphere with trans dimensional spherical coordinates provides a unique Euclidean plane. Even with a variable r, changing the shape of the sphere, the stereographic projection remains with parallel and orthogonal lines. The implications of using Trans dimensional spherical coordinates to create a stereographic projection are worth to mention:

$$P \in \mathbb{Q}^3 \iff P' \in \mathbb{Q}^2$$

Thus, allows to create Riemannian manifold *M* (In differential geometry, a (smooth) Riemannian manifold or (smooth) Riemannian space (*M*,*g*) is a real smooth manifold *M* equipped with an inner product g_p on the tangent space T_pM at each point p that varies smoothly from point to point in the sense that if *X* and *Y* are vector fields on *M*, then $p \mapsto g_p(X(p), Y(p))$ is a smooth function. The family g_p of inner products is called a Riemannian metric (tensor). that is geodesically complete if for all $p \in M$, the exponential map $\exp_p$ is defined for all $v \in T_pM$, i.e. if any geodesic $\gamma(t)$ starting from *p* is defined for all values of the parameter $t \in$ R. (The Hopf-Rinow theorem asserts that *M* is geodesically complete if and only if it is complete as a metric space) [9].

A Riemannian metric (tensor) makes it possible to define various geometric notions on a Riemannian manifold, such as angles, lengths of curves, areas (or volumes), curvature, gradients of functions and divergence of vector fields. In many instances, in order to pass from a linear-algebraic concept to a differential-geometric one, the smoothness requirement is very important. Below are some examples of how differential geometry is applied to other fields of science and mathematics. In physics, four uses will be mentioned: Differential geometry is the language in which Einstein's general theory of relativity is expressed. According to the theory, the universe is a smooth manifold equipped with a pseudo-Riemannian metric, which describes the curvature of space-time. Understanding this curvature is essential for the positioning of satellites into orbit around the earth. Differential geometry is also indispensable in the study of gravitational lensing and black holes. Differential forms are used in the study of electromagnetism. Differential geometry has applications to both Lagrangian mechanics and Hamiltonian mechanics. Symplectic manifolds in particular can be used to study Hamiltonian systems. Riemannian geometry and contact geometry have been used to construct the formalism of geometrothermodynamics which has found applications in classical equilibrium thermodynamics. In economics, differential geometry has applications to the field of econometrics [11]. Geometric modeling (including computer graphics) and computer-aided geometric design draw on ideas from differential geometry. In engineering, differential geometry can be applied to solve problems in digital signal processing [12]. In probability, statistics, and information theory, one can interpret various structures as Riemannian manifolds, which yields the field of information geometry, particularly via the Fisher information metric. In structural geology, differential geometry is used to analyze and describe geologic structures. In computer vision, differential geometry is used to analyze shapes [13]. In image processing, differential geometry is used to process and analyse data on non-flat surfaces [14]. Grigori Perelman's proof of the Poincaré conjecture using the techniques of Ricci flows demonstrated the power of the differential-geometric approach to questions in topology and it highlighted the important role played by its analytic methods. In wireless communications, Grassmanian manifold is used for beamforming techniques in multiple antenna systems [15].

Neurophysiologic Considerations

It is obvious that technologies need to display information in order to be intuitively understood by the user. Having this in mind, it is important to consider that the visual data acquisition for the pilot on each eye resembles a hemispheric field that is only limited by facial protuberant bones as the ones on the nose. So, the Central Nervous System works and intuitively understands 180° x 180° vision. It is an evolution´s adaptation result that provides higher survival rates. In nature it is not random to use hemispheric vision (frontally located eyes on predators for distance and speed calculations) and spherical vision (laterally located eyes for improved situational awareness). Besides, the eye instantaneously interacts with the brain to provide the angular localization (**Θ, Φ)** of any object of interest that is observed. Please notice that the sensorial layer of the eye, named retina, in order to provide simultaneously said hemispheric vision and a higher resolution on the view of the object of interest, also has developed a dedicated zone named Macula that has the highest density of receptors to light on the retina. Interestingly, in humans, retinal cells are also specialized on "where" the target of regard is in space. They are concerned with depth perception, high contrast sensitivity and fast temporal resolution (M class ganglion cell). Of course, the most accurate r is given inside the brain by trigonometric calculations provided by the intermingling of data from both eyes [16].

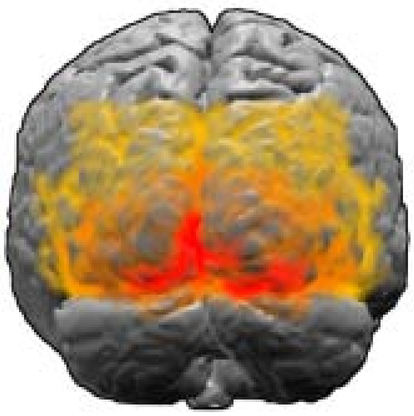

Figure 12, Shows the Rectangular Shape of the Visual Human Cortex

The final receiver of the light sensed by the retina and macula is the visual cerebral cortex. It should be noticed that the golden ratio is found in the bidimensional view of said visual cerebral cortex as shown on Figure 12 and has been used also in the design of TV sets and all kind of visual displays [17]. Finally but most important, the human 180 x 180 visual field magnifies what is in the center of the sight without 180 x 180 loss of situational awareness in the same way Neurovision® works.

Neurovision®

Neurovision® is based on Pat. Pending Methods to generate advanced differential geometry applications on a common platform, so, enabling the integration and/or fusion of present and future technologies, making computer tasks faster and some of them even now possible. Human interaction is also visually more neurophysiology compatible. With regard to Avionics, Neurovision® presents the opportunity to intermingle present (Synthetic Vision, Enhanced Vision) / future (Intruder Trajectories, 3D Weather) vision technologies with panoramic visual reality on a single display. Neurovision ® might also be leaving the head-up display inaccuracies in the past.

Optics

There are different optical ways to generate a panoramic view. Optical distortion therefore is expected. Neurovisión provides ways to obtain optically and/or computer generated panoramic views without undesired distortion. As example, please notice Figures 13 to 16 that display the same picture. The differences accordingly to the visual treatment are shown. The optical technologies for real time, panoramic 180° vision are mainly wide angle lens.

Figure 13. Labeled #1, Typical Distorted Panoramic View Obtained with Standard Wide Angled Lens

Figure 14. Labeled #2, Even Focusing the Horizon, the Distortion Is Still Present

One of the major flaws of this kind of devices is the distortion produced when the horizon is away from the focal point, as shown on the Figure 13.The other issue is that even with the horizon line centered, as shown on Figure 14, the optical distortion is also noticeable making the superimposition of virtual data non reliable. Please notice that the 180° field of view picture shown on Figure 15 is now distortion free using some of the features of Neurovision®.

Figure 15. Labeled #3, It Is Possible to Achieve a Distortion Free Image on a 180° Panoramic View

Figure 16. Labeled #4, A Sample of How Neurovision ® Provides a 180° Panoramic Platform

Besides, by optionally using additional features of the Neurovision ® technologies, that employs visual references in the same way the human brain percives reality, you will see on picture labeled as number 4 that the center of the picture also provides a distortion free magnification. Said magnification has been centered to show the pilot a higher range of visibility focused where the trajectory of the plane is headed.

Avionic Applications

Trans dimensional algorithms are over simplified for explanatory purposes. Many other applications away from this Avionics document are also part of Neurovision®

1.- Panoramic Vision without distortion in a glance.

2.- Computer generated navigation images and panoramic vision and/or night vision may be superimposed without distortion in the same display.

3.- Flight trajectories of intruder planes and weather are easily understood by the pilot.

Figure 17 shows an example of distortion free panoramic vision. Drones, planes and helicopters may get benefit of this situational awareness enhancement by using standard or Neurovision® related distortion fix algorithms. Figure 18 shows the same 180° wide panoramic view but additionally the pilot is now provided with magnified region of interest without losing the 180° wide view. He or she may now *¨see the tree and the forest¨* at the same time. In other words, a wider situational awareness and a safer focus are now concurrent in the same way the brain works. Any aircraft moving on that wide view remains visually detected. Additionally, since there is an increasing need of merging the reality comprised through the windows, as shown on Figure 19 with the navigational data displayed by the cockpit instruments as shown on Figure 20. Previous attempts have given rise to "head up" devices. Unfortunately the intrinsic inaccuracies related to the changing pilot´s point of view (head position and distance from the transparent display), using both eyes, etc., leaves this need not fully satisfied.

Figure 17. Panoramic Optical View Using Neurovision® in Order to Acquire a Distortion Free Picture

Figure 18. A Panoramic Optical View using Customized Neurovision® Features

Figure 19. A Cockpit´s Window View

Figure 20. A Navigation Monitor that includes Synthetic Vision

Please notice that the panoramic vision that the plane windows provide when the pilot looks ahead is sensed by the low resolution peripheral retinal receptors.

The wide angle of vision that the cockpit´s windows also provide has neither been yet matched by the displays on the cockpit´s console.

Neurovision® now proposes to discharge that workload from the pilot and show him/her the real land scape through merging instruments as shown on the Figure 21.

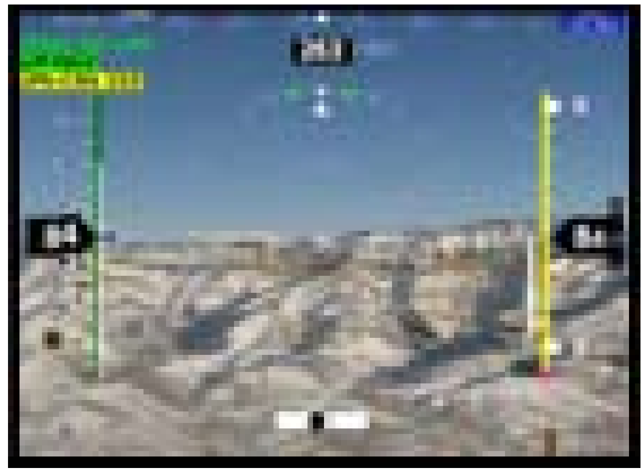

Figure 21. A Narrow Optic View with Superimposed Navigation Intruments

However, it would be even nicer and safer to have a panoramic 180 degrees of real (or infrared vision) as the one shown on Figure 22 and everything else merged together in the same monitor. Please take a look at Figure 23 where Neurovision® also proposes to accurately overlay real 180º vision with the navigation instruments (HEDI). Optionally, Synthetic Vision and any other navigation technologies may also be included. The monitor also improves the GPS, TCAS, Radar, ADS-B, etc., separated displays by showing the present and projected intruder planes trajectories (green, yellow or red straight lines).

Figure 22. The Same Picture, but Now on Panoramic View

Figure 23. A Panoramic Optical View (POV) with Superimposed Navigation Instruments, IPPT, and Optionally Added Synthetic Vision

Please notice that on Figure 23;

a) Synthetic Vision may now be apparent by color coded lines only, in order to visually assist the pilot with the terrain/graphics merging.

b) The plane´s path view may now be magnified without sacrifice of the width of the view, as a safety addition. Please notice that the wide panoramic view is still present, providing the pilot with a superior situation awareness in a glance. (The pilot does not have to see sideways to understand the view; a golden ratio is additionally now being used).

c) Intruder´s Planes Projected Trajectory (IPPT) for the following *n* seconds may also be displayed and it is easily understood by the pilot in a 3D context.

Figure 24 shows said panoramic view with 3D complex weather conditions view, night vision, navigation instruments (HEDI, etc.), Synthetic Vision and IPPT. Neurovisión**®** is also a platform for future applications. Military aircrafts may also have a 360° x 360° vision.

Target Users:

A.- Drones

B.- Helicopters

C.- General Aviation.

D.-Military Aviation.

Figure 24. A Panoramic Optical Vision View with Customized Advanced Applications

Finally, Neurovision® is also able to rapidly share the angular position of objects of interest such as hostiles, suspects, targets, etc. to *each individual point of view of every member of the team*. As shown on Figure 25, a helicopter, (or drone, or plane) has detected at least two heat signatures from unknown people hiding on a corn field. Instead of using a laser beam to point out the findings to the soldiers standing up outside said field, Neurovision® skips the obvious inconveniences of doing that and is able to send the Angular Position of said heat signatures directly to the individual aiming displays of every soldier showing him/her where to go or aim/shoot, as shown on Figure 26. Neurovision® is also able to pin point every hostile position when first detected by ground personnel by simply sharing said information amongst the individual Neurovision®´s aiming displays. Please notice that on Figure 26 the two yellow spots on the left upper quadrant on the display, are *the angular position for that particular soldier* of the two heat signatures shared by the helicopter. The red spot on the lower right quadrant indicates an unknown person on the back and right of that particular soldier.

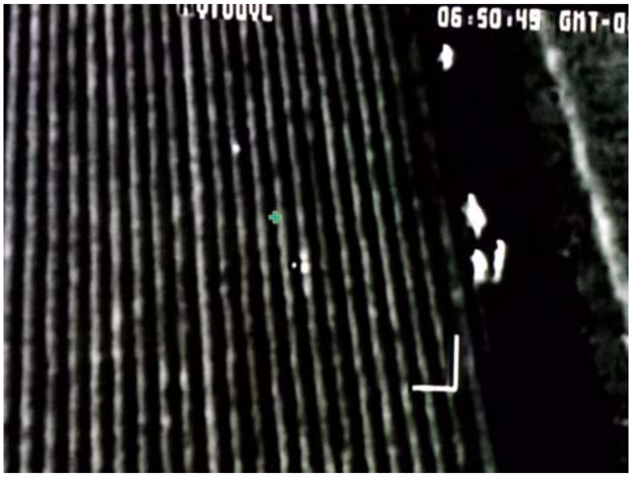

Figure 25. Aa Night Aerial View of Two Hostiles Hidden on a Corn Field. Four Ground Personnel Units Stands Out of the Corn Field Looking for Them

Figure 26. A Neurovision® Display Attached in Parallel to the Aim Scope of One of the Soldiers Searching the Corn Field

Conclusions

The novel method for stereographic projection, able to generate Riemann´s manifolds facilitates and even makes some computer tasks now possible. The improvements derived from differential geometry and neuroscience hopefully may provide novel opportunities to avionics and technology in general. Neurovision® enables to merge wide visual, infrared, computer generated graphics, complex 3D weather and Intruder´s Planes Projected Trajectory on the pilot´s and team members 3D *individual* visual perspective.

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