Pisa, 21/10/2020

Cultural Dynamics: Axelrod Model

Seminar for the Complex Systems class exam

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Report

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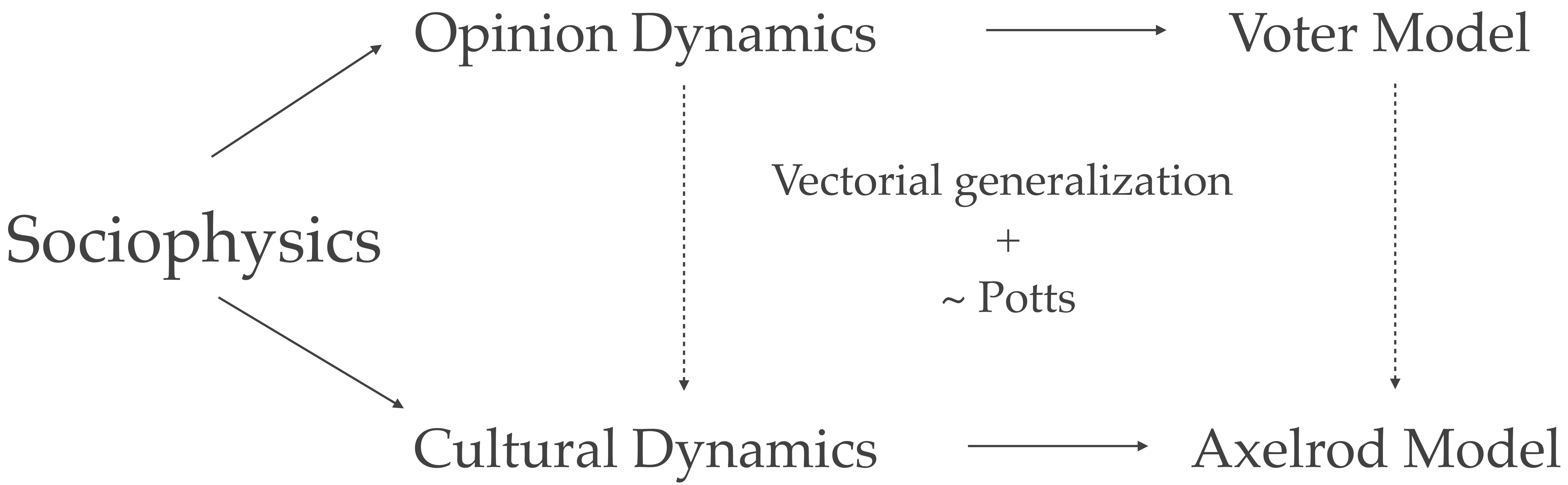
Introduction



Voter Model

Agent $\rightarrow s_i \in \{-1,+1\}$

Nearest neighbor random interaction

$$S_i \leftrightarrow S_j$$

$$\Downarrow$$

$$S_i = S_j$$

A non-equilibrium stochastic processes that can be solved exactly in any dimension *d*

Voter Model

Spin flip probability: $$W_k(S) \equiv W(s_k \to -s_k) = \frac{d}{4}\left(1 - \frac{1}{2d} s_k \sum_j s_j\right)$$

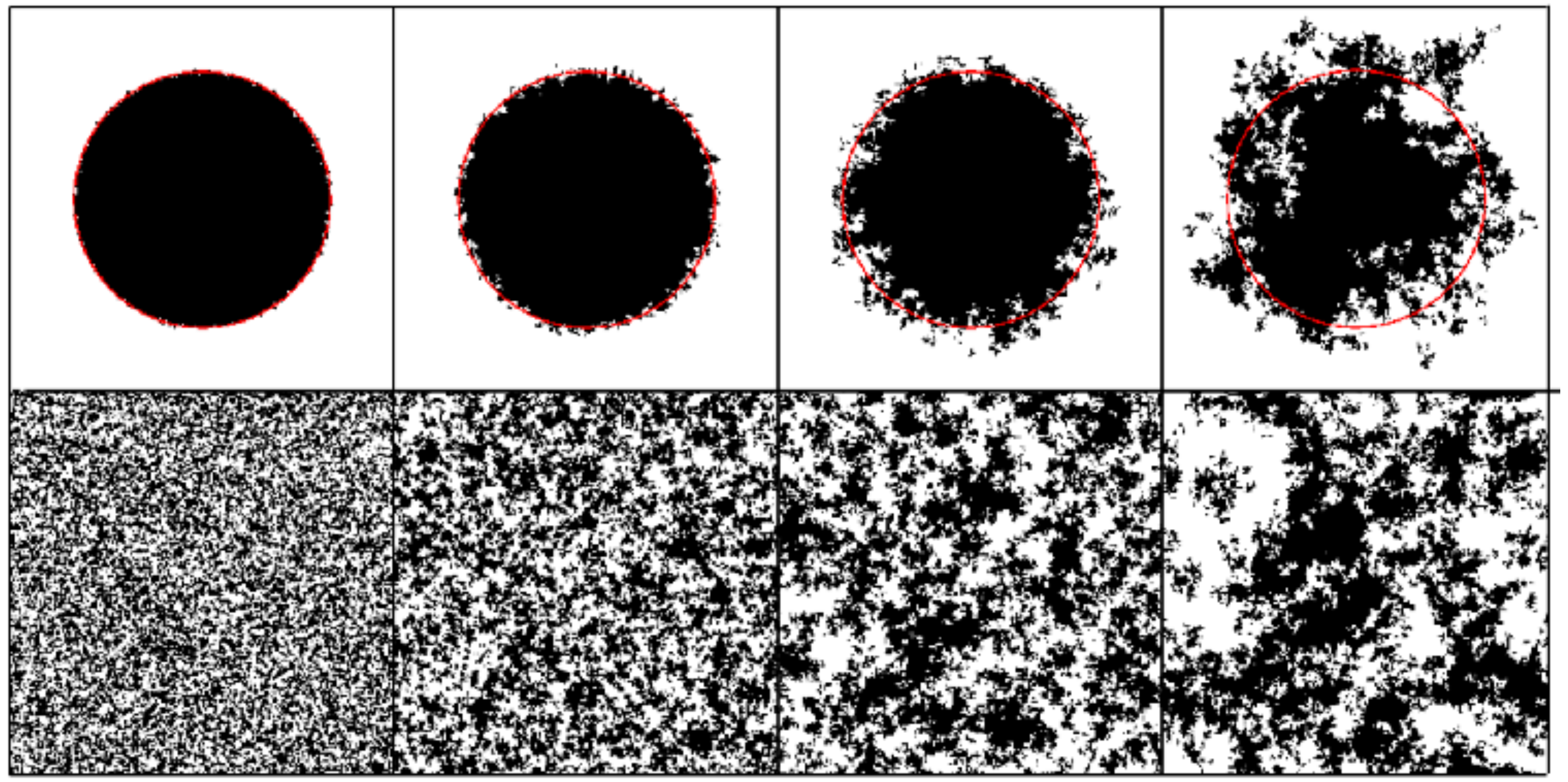
Where j runs over all 2*d* nearest neighbors and the prefactor is chosen to normalize

Master Equation: $$\frac{d}{dt}P(S,t) = \sum_k \left[W_k(S^k)P(S^k,t) - W_k(S)P(S,t)\right]$$

Where S^k equal to $S=\{s_i\}$ except for the flipped spin s_k.

Voter Model

Voter dynamics tends to increase the order of the system $\Rightarrow$ cluster



Interfaces are very rough, fluctuations.

Pressure of the majority only in an average sense.

Axelrod Model

Agent $\rightarrow q_i = \{\, q_{i,0}\,,\, q_{i,1}\,,\, \ldots,\, q_{i,F} \,\}$

Cultural features: "language, art, technical standards and social norms" [1]

$q_{i,f} \in \{\, 0, 1, \ldots, Q \,\}$

Cultural features traits:

No central institutions or mass media, only social interactions.

Axelrod Model

Interaction probability between agent i and j:

$$\omega_{ij} = \frac{1}{F} \sum_{f=1}^{F} \delta_{q_{i,f} q_{j,f}}$$

Also known as overlap

Interaction:

Choose one of the different traits and set the second one equal to the first one:

$$q_{i,f} \neq q_{j,f} \implies q_{j,f} = q_{i,f}$$

Axelrod Model

Main properties:

1. individuals are more likely to interact with others who already share many of their cultural attributes (homophily);
2. interaction increases the number of features that individuals share

Algorithm

Square lattice $L \times L$

$$i \in \{1, \ldots, N = L^2\}$$

$$f \in \{1, \ldots, F\}$$

$$q_{i,f} \in \{1, \ldots, Q\}$$

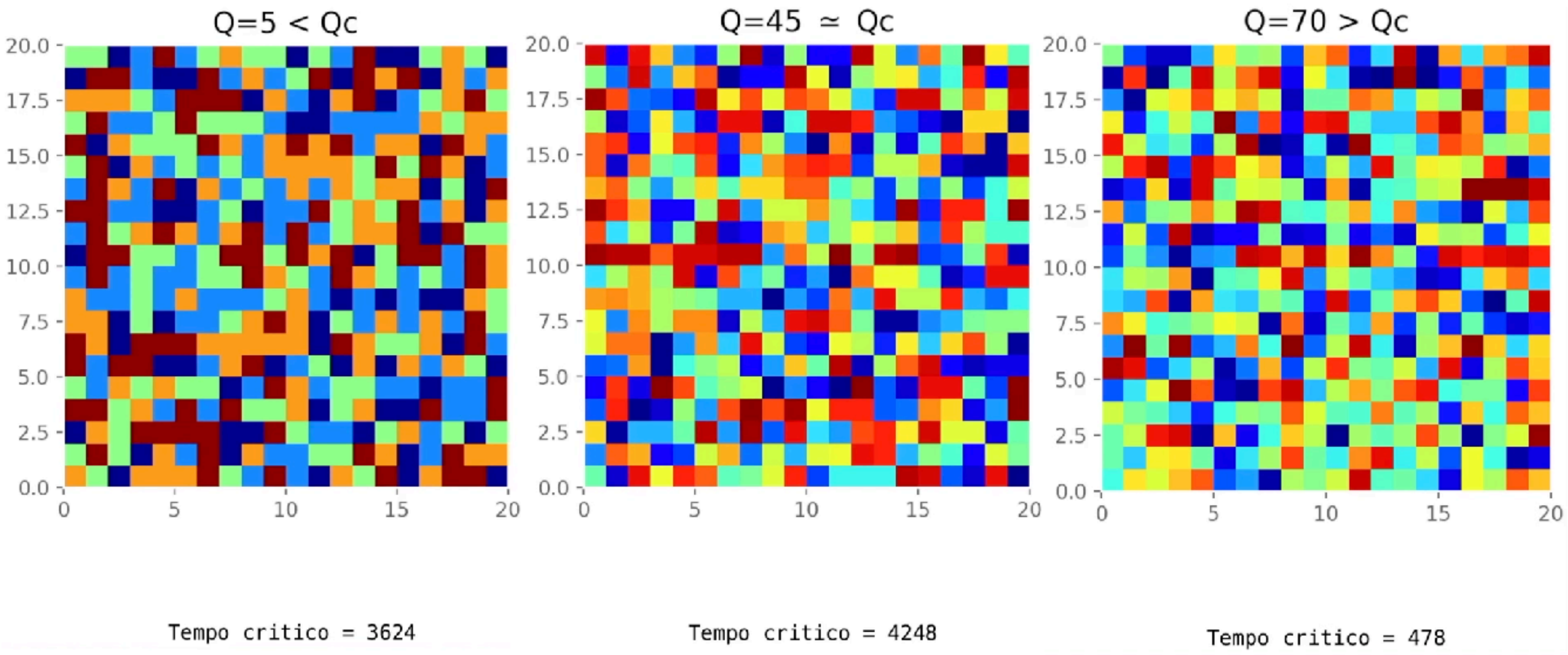
Random initial state

1. i randomly chosen between 0 and N
2. j randomly chosen among the i nearest neighbour
3. f randomly chosen between 0 and F
4. If $q_{i,f} \neq q_{j,f}$:
   A. Next temporal step
5. Otherwise $q_{i,f} = q_{j,f}$:
   A. f' randomly chosen such that $q_{i,f'} \neq q_{j,f'}$
   B. Impose $q_{i,f'} = q_{j,f'}$
   C. Next temporal step

If $\forall i, j \in S \quad q_{i,f} = q_{j,f}$ o $q_{i,f} \neq q_{j,f} \;\; \forall f \quad \Rightarrow \omega_{ij} = 1{,}0 \Rightarrow$ Algorithm stops.

Implementation

L=20, F=10



Video available on: youtu.be/bV126I_cU1k

Implementation

Order parameters:

$$s_{max} = \max\left(\frac{\#\ \text{of agents with same } q}{\#\ \text{total agents}}\right)$$

nA= # of active bonds

t_c = freezing time

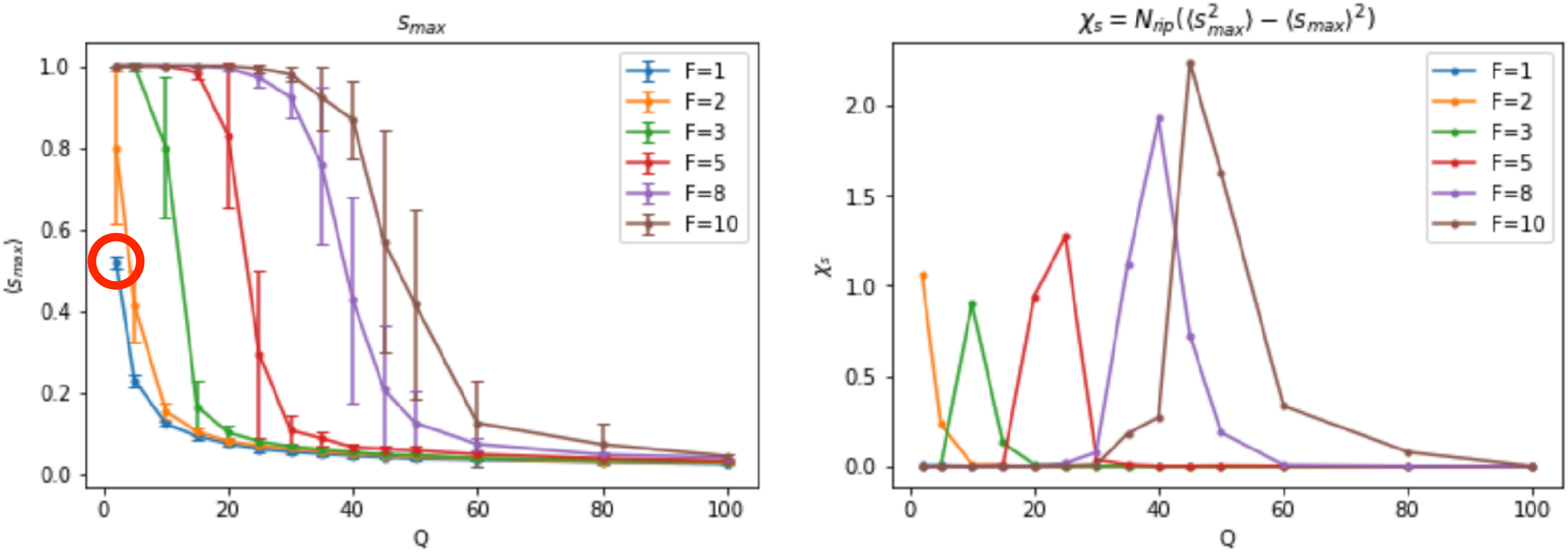
Control parameters:

Q = # of possible values of a feature

L = system size

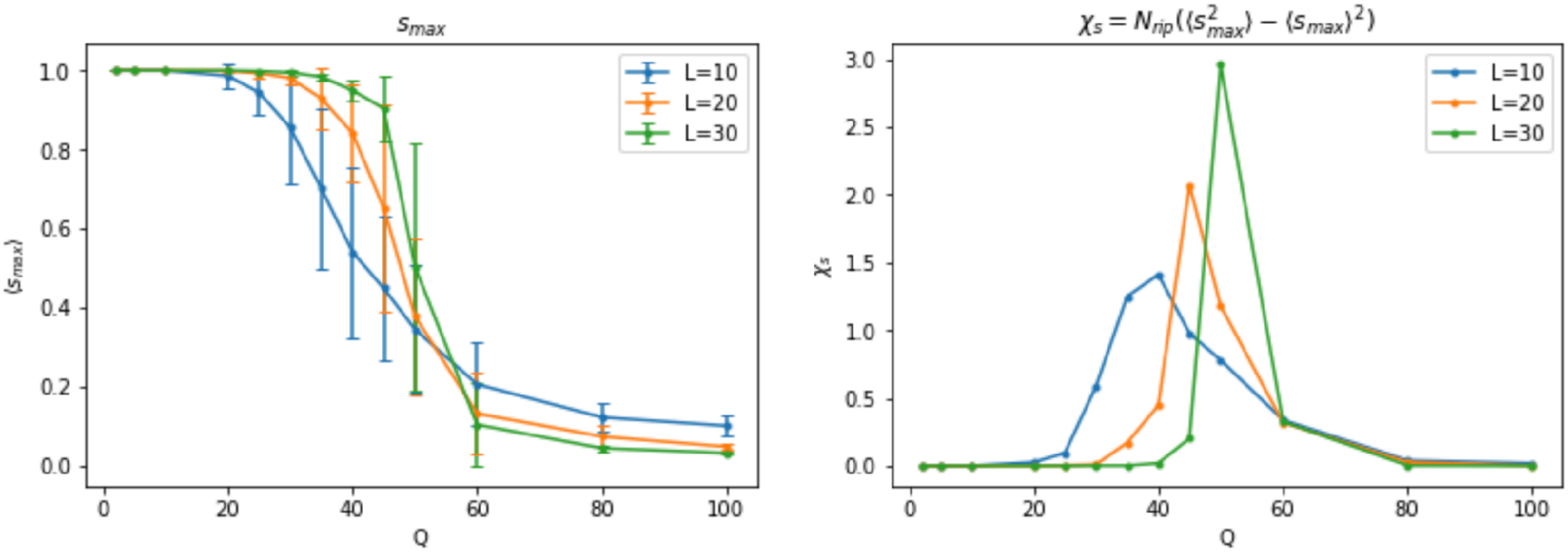
$N = L^2 = \#$ total agents

Absorbing States



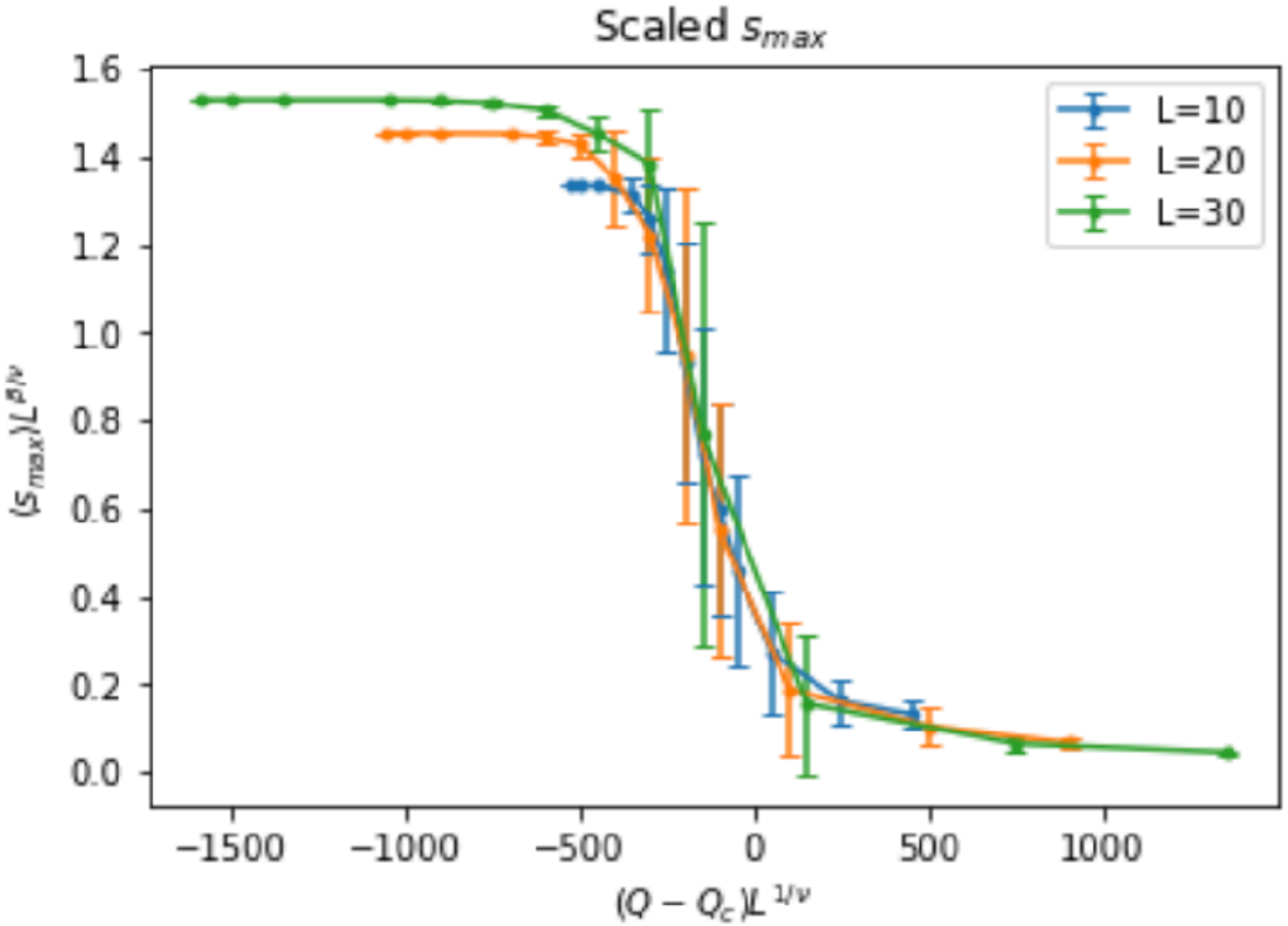
30 realizations, L=20

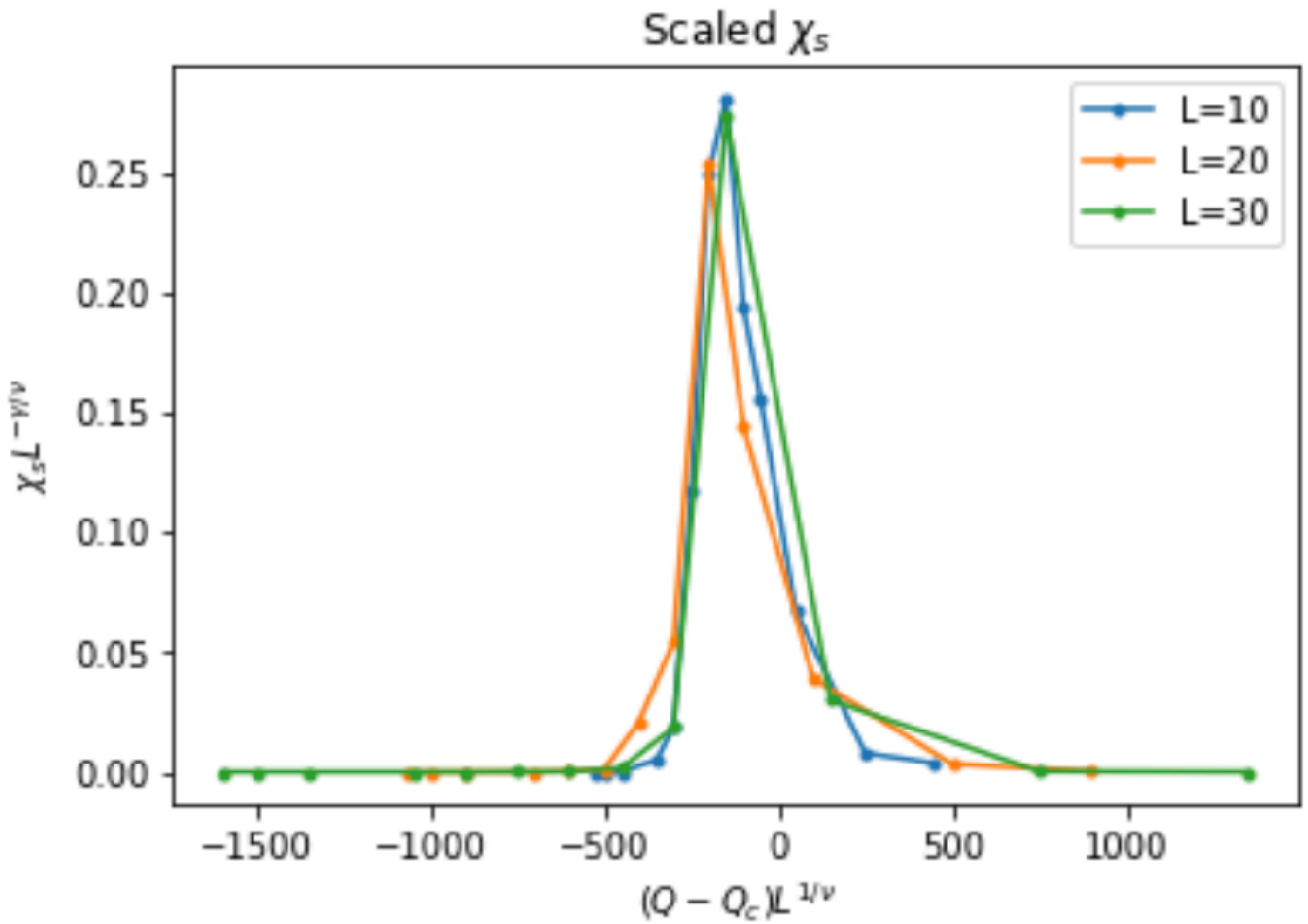
Absorbing States



30 realizations, F=10

Absorbing States

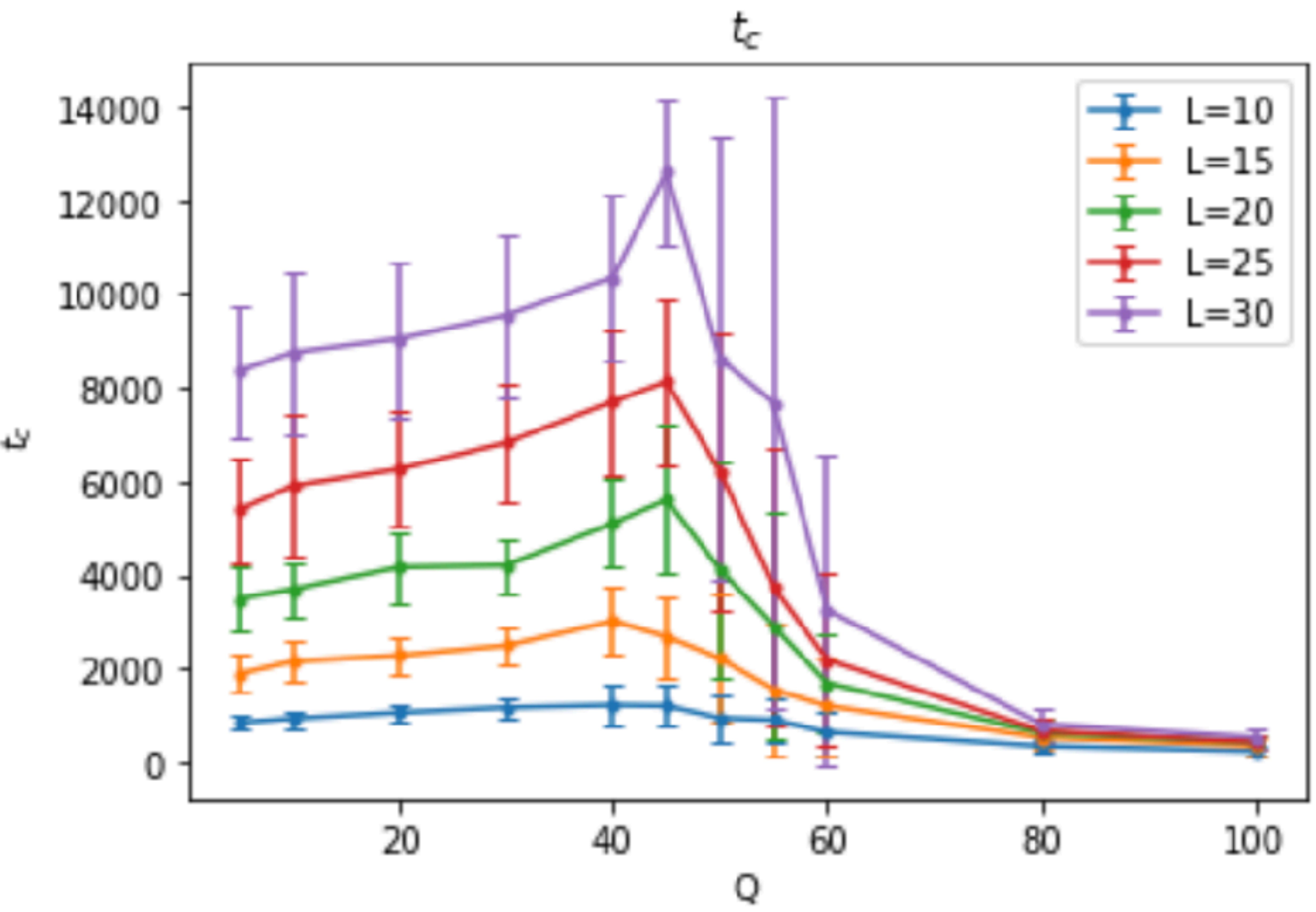


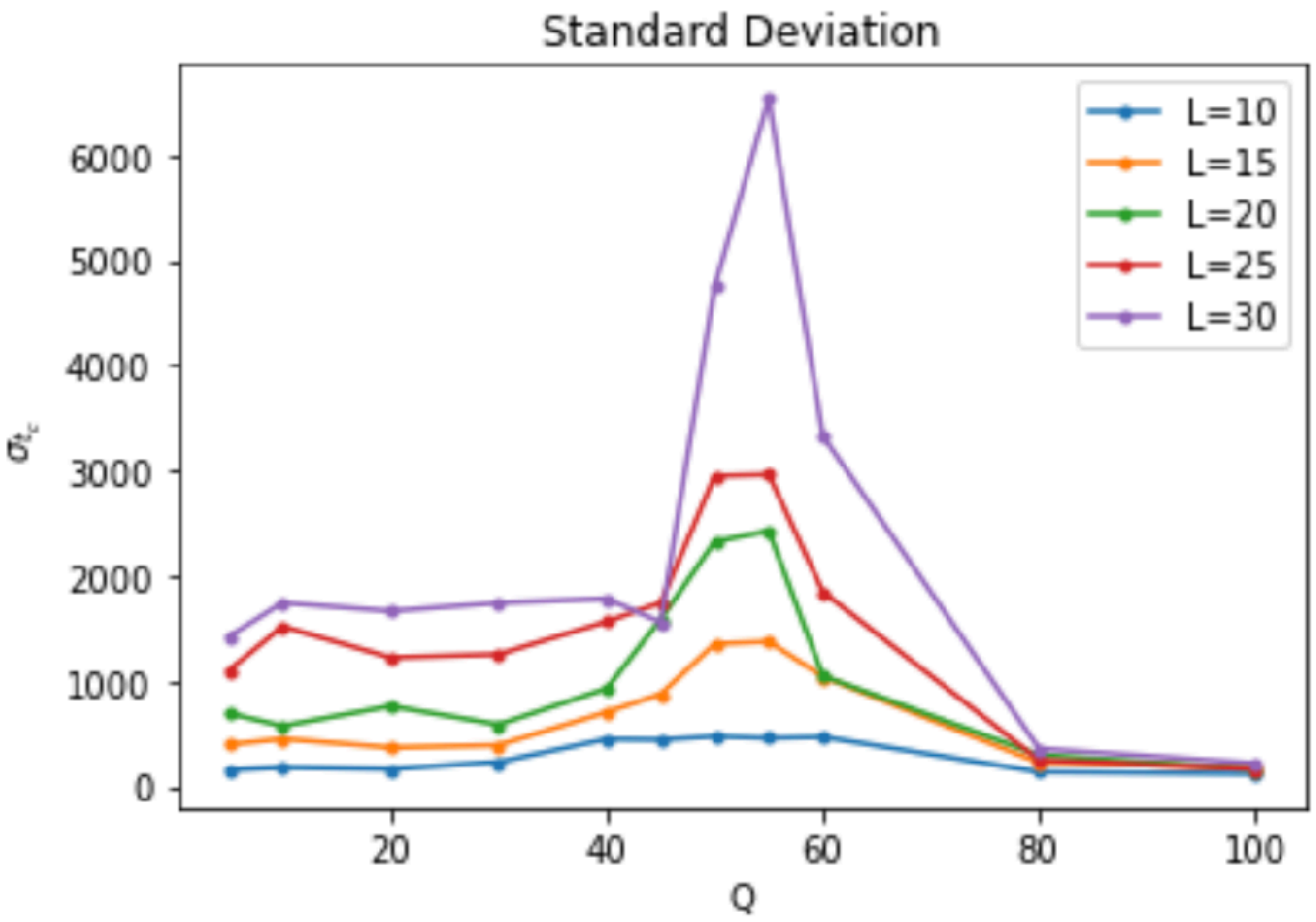


Critical Q: $Q_c \simeq 55$

Critical exponents: $\beta/\nu \simeq 1/8$ e $\gamma/\nu \simeq 0.7$

Freezing time



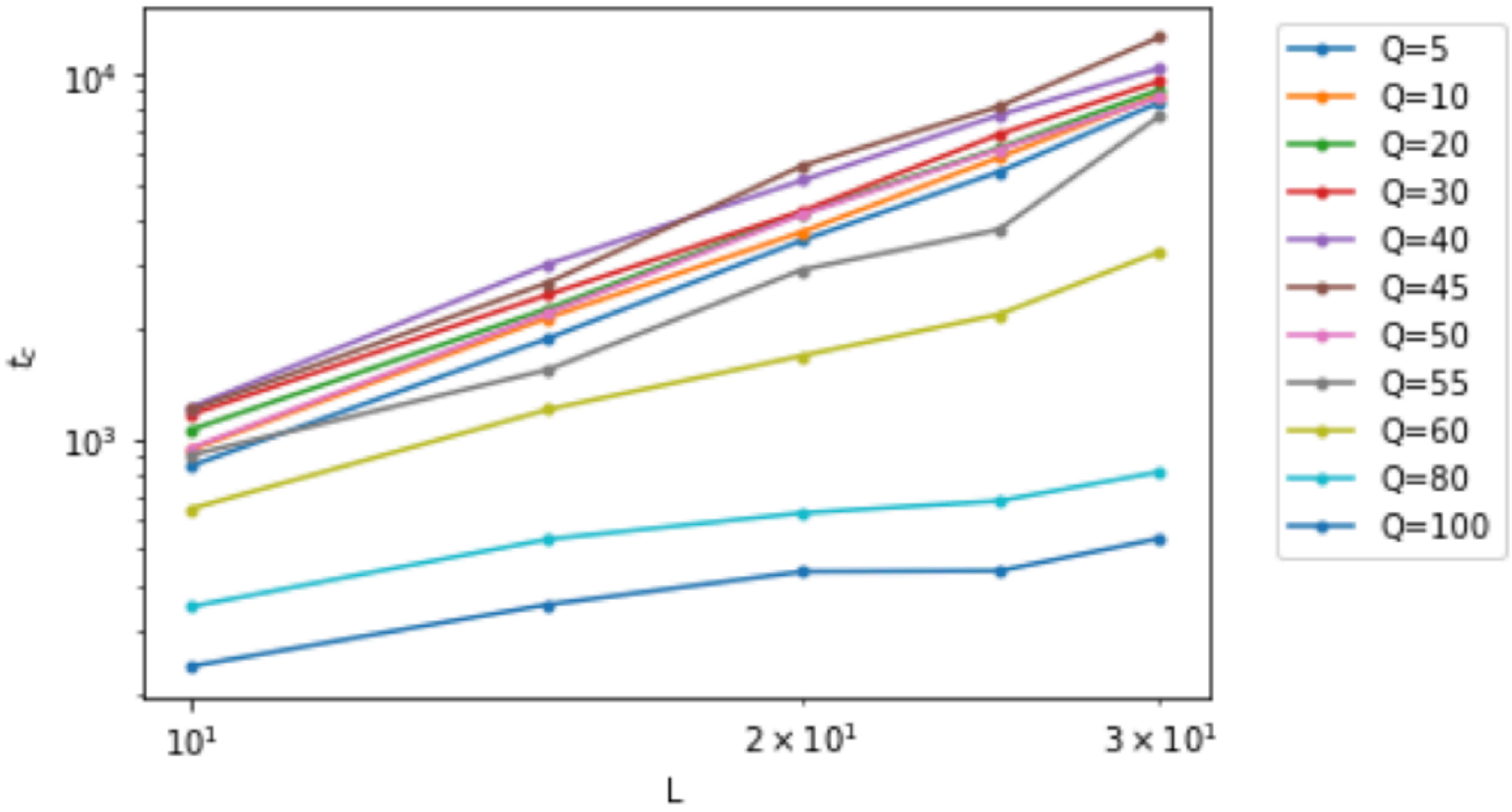


30 realizations, F=10

Freezing time

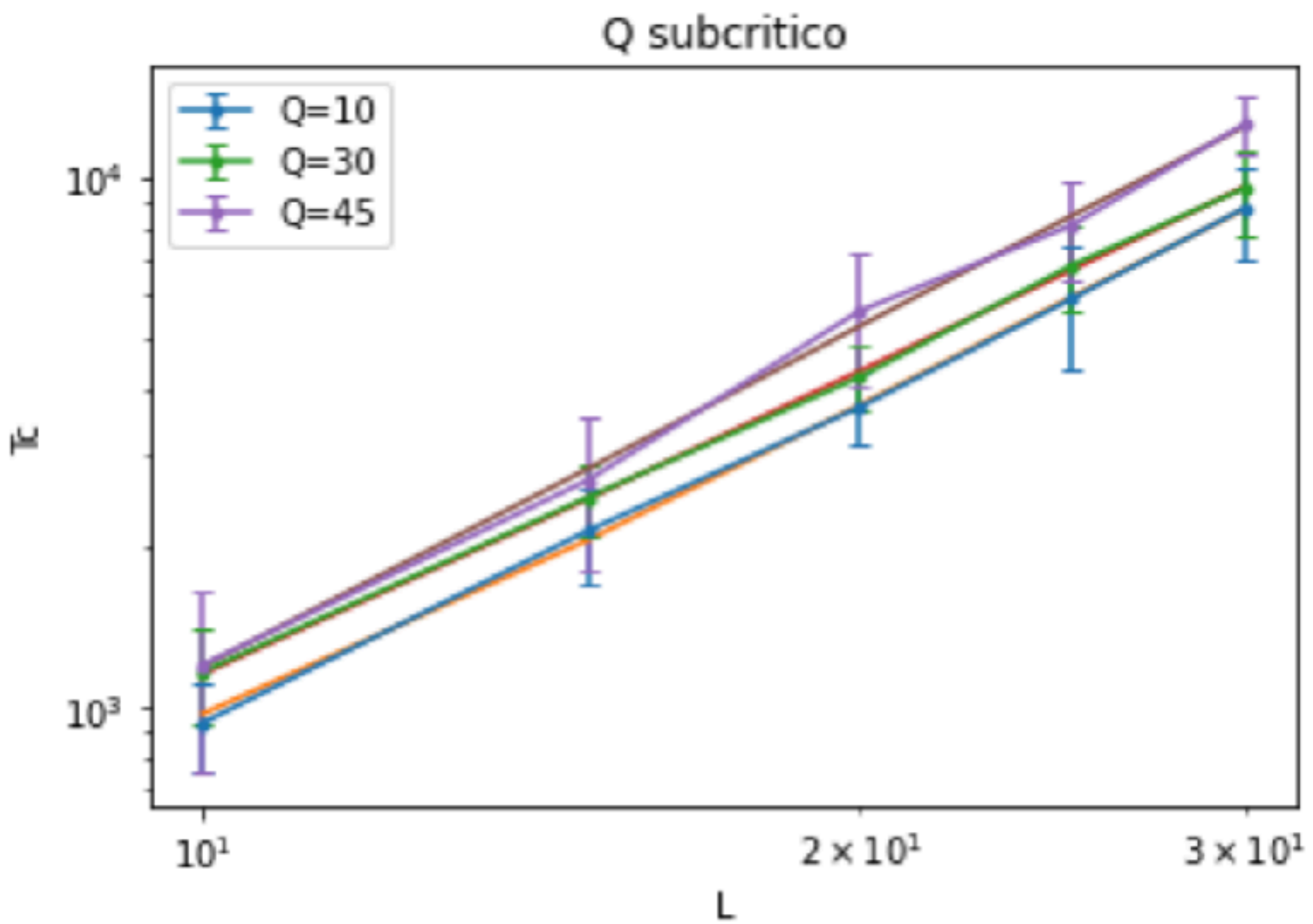
30 realizations, F=10

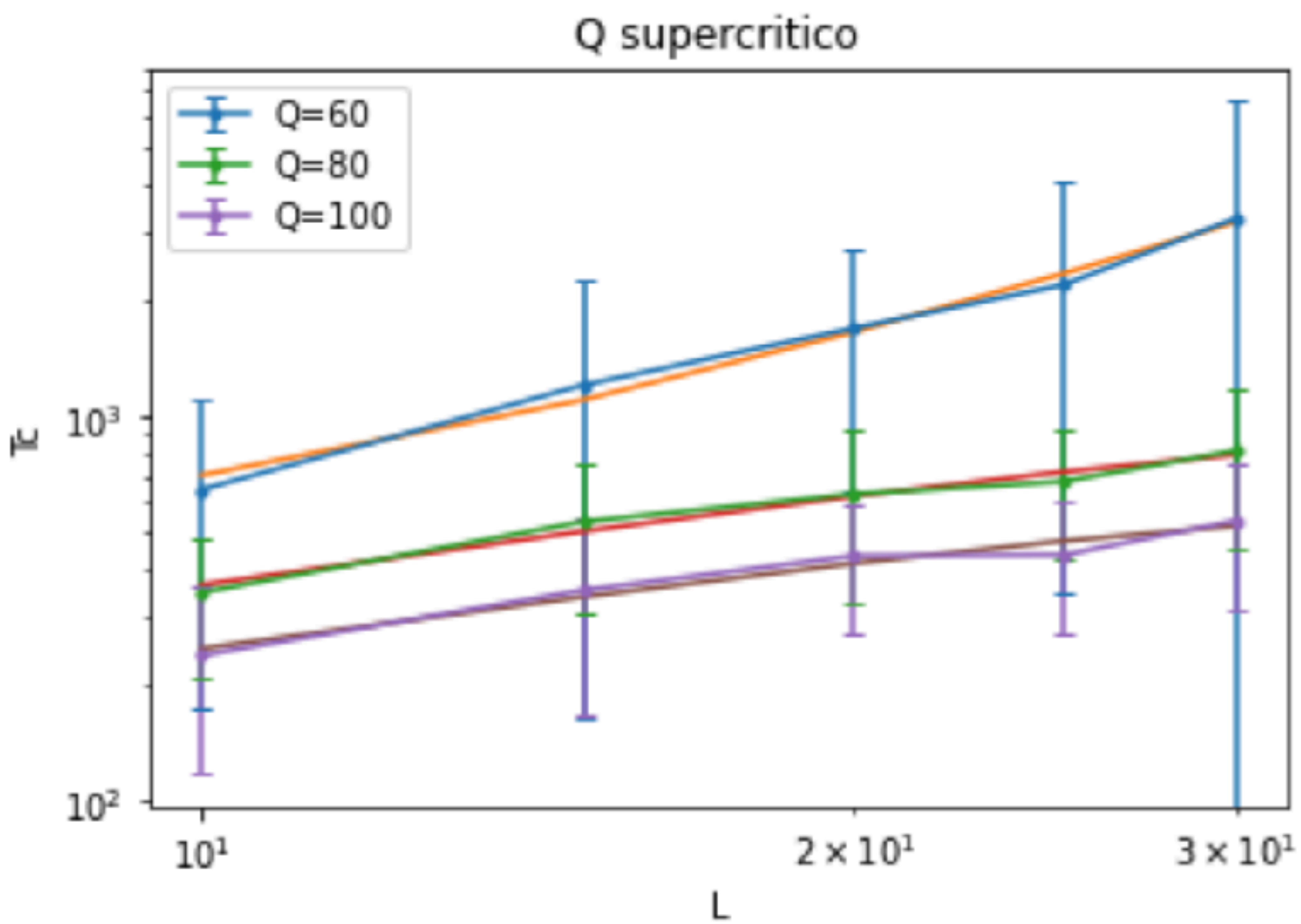
Freezing time



30 realizations, F=10, log-log scale

Freezing time



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Q = 10: Fit Slope = 2.01968
Q = 30: Fit Slope = 1.91801
Q = 45: Fit Slope = 2.1415
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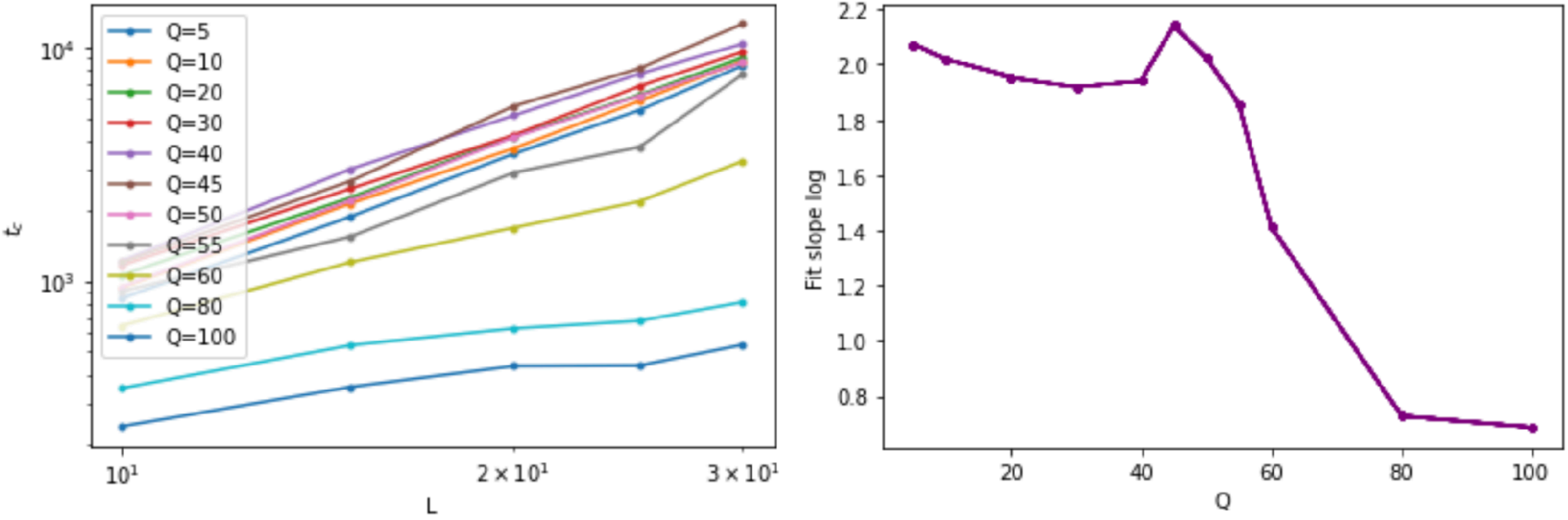
Q = 60: Fit Slope = 1.41161
Q = 80: Fit Slope = 0.72877
Q = 100: Fit Slope = 0.68656
```

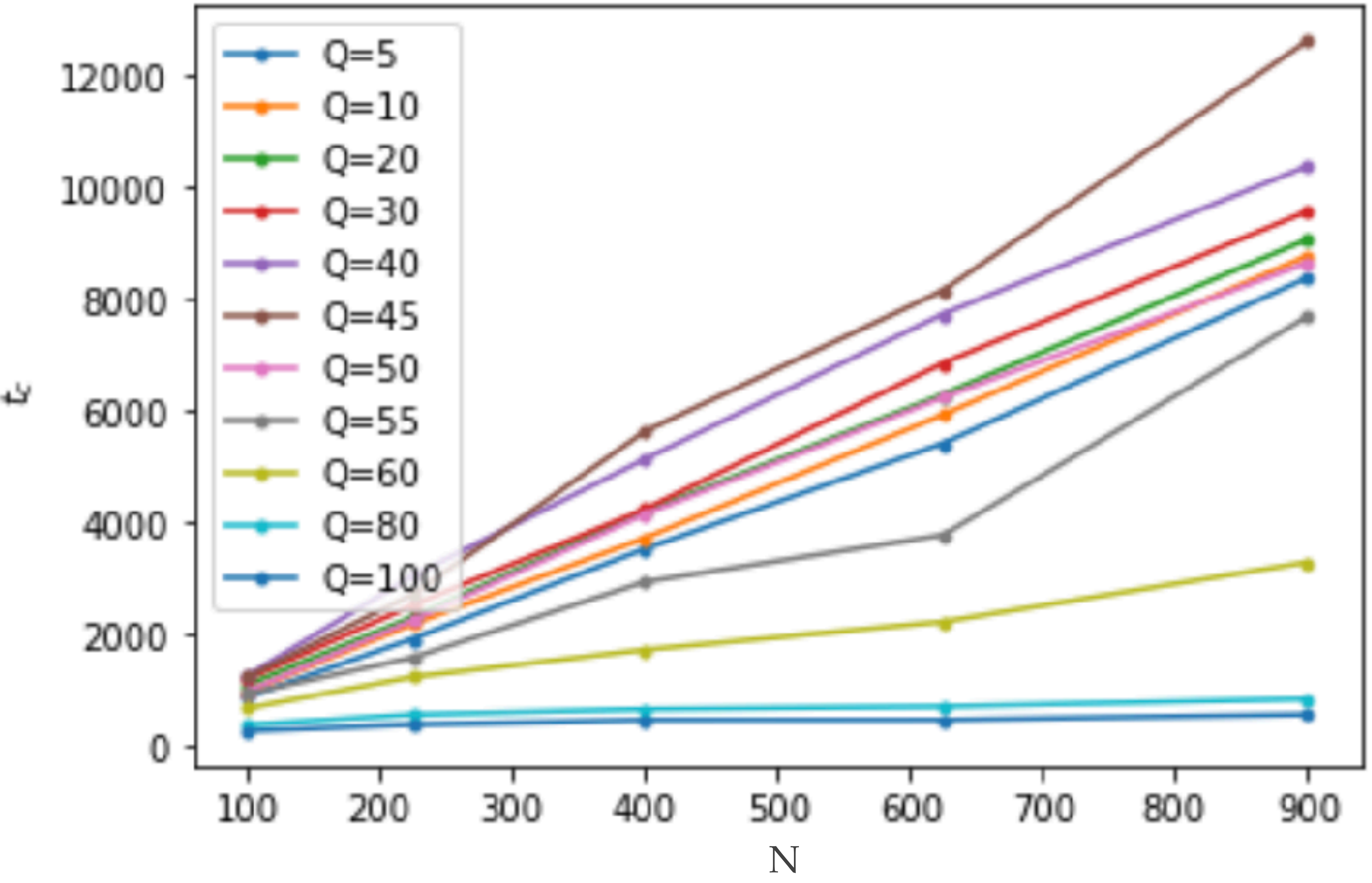
30 realizations, F=10

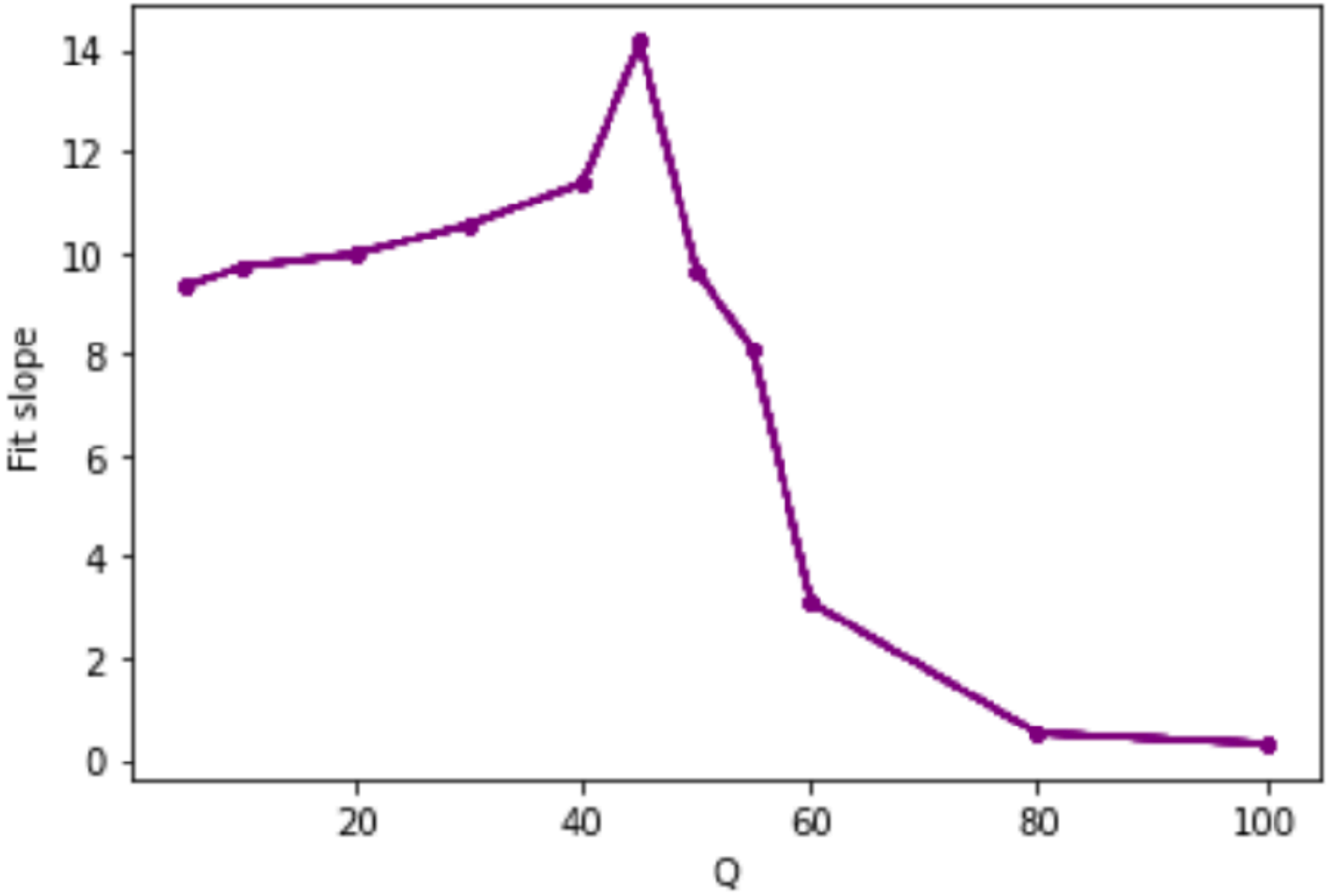
Freezing time



30 realizations, F=10

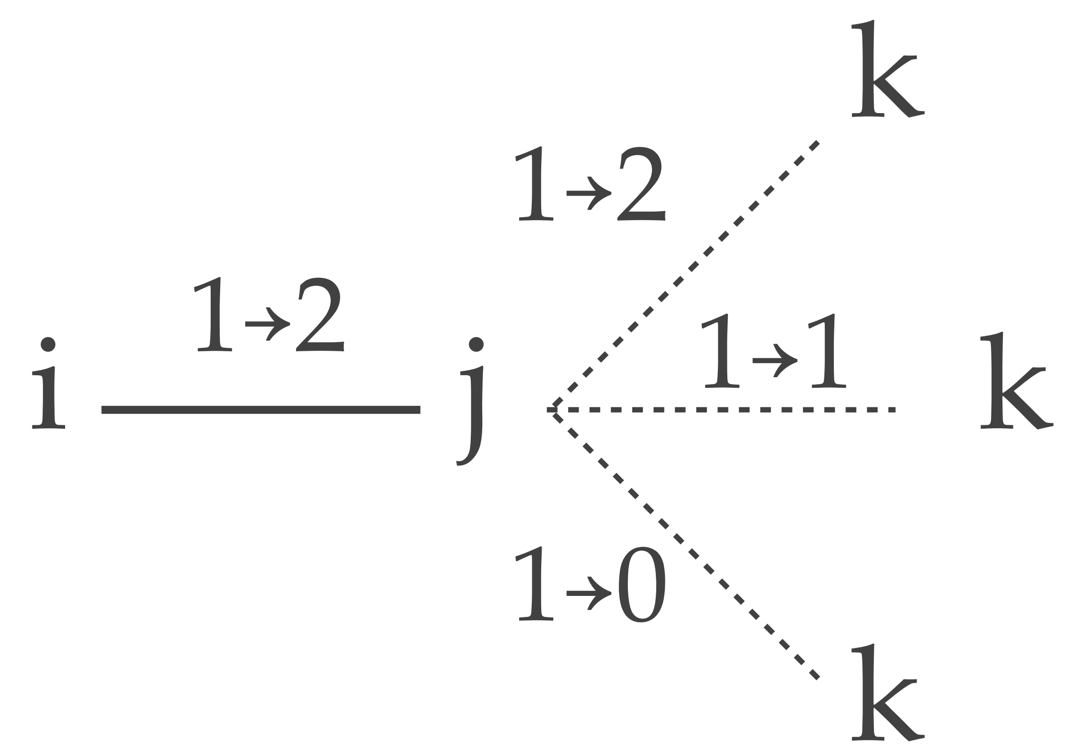
Freezing time





30 realizations, F=10

Axelrod Dynamics



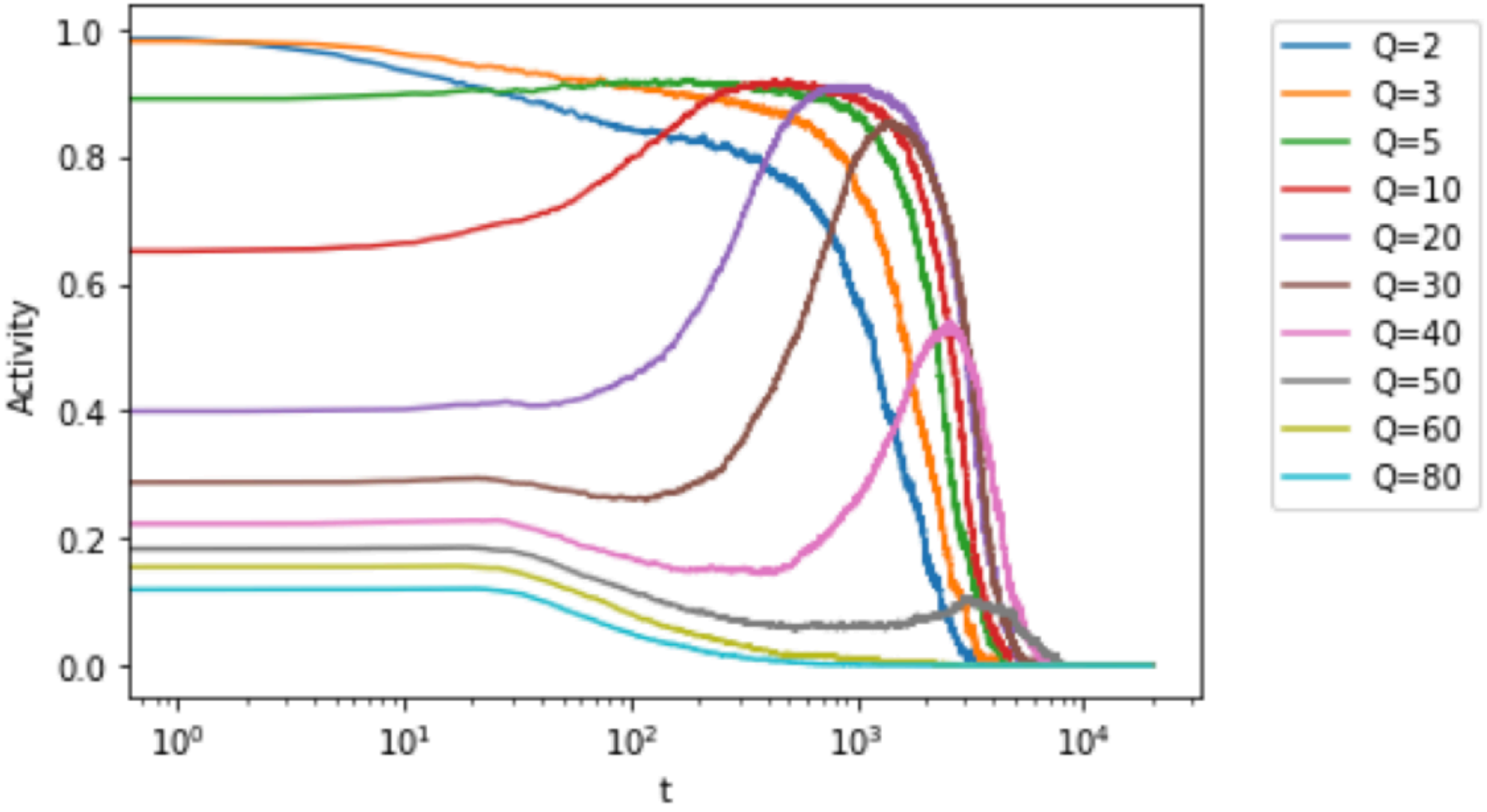
- Direct interaction

$$\Rightarrow \omega_{ij} \to \omega_{ij} + \frac{1}{F}$$

- Indirect interaction

$$\Rightarrow \omega_{jk} \to \omega_{jk}\,,\ \omega_{jk} \pm \frac{1}{F}$$

k nearest neighbour of j



30 realisations, L=20, F=10

Master Equation

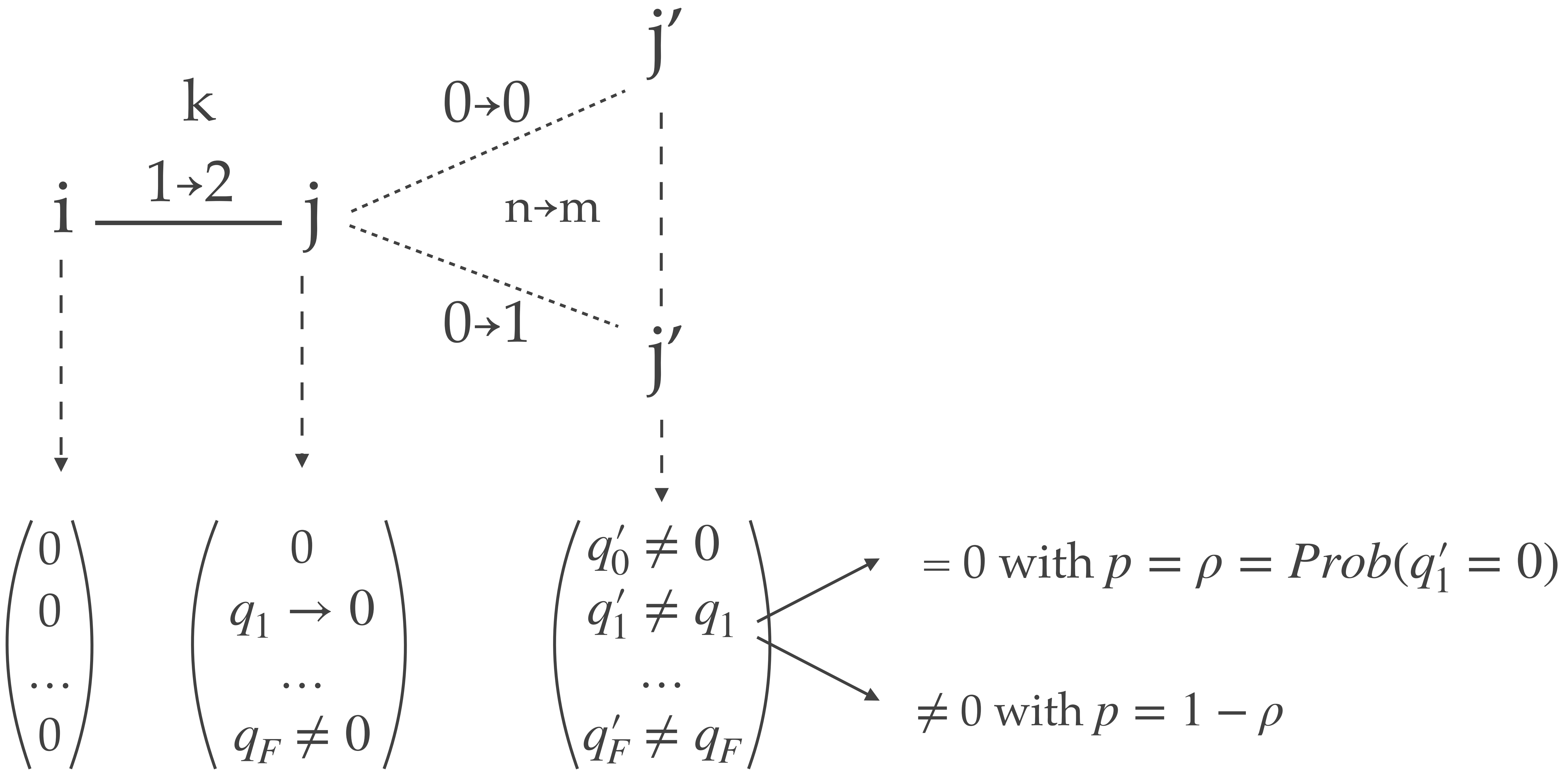
$$\frac{dP_m}{dt} = \sum_{k=1}^{F-1} \frac{k}{F} P_k [\delta_{m,k+1} - \delta_{m,k} + (g-1) \sum_{n=0}^{F} (P_n W_{n,m}^{(k)} - P_m W_{m,n}^{(k)})]$$

- $\frac{k}{F} P_k$ probabilità di interazione
- Se $k = m - 1$ si crea un collegamento m
- Se $k = m$ viene distrutto
- Interazioni indirette

$$P_m(0) = \binom{F}{m} \rho_0^m (1-\rho_0)^{F-m}$$

$$\rho_0 = \text{Prob}[q_{i,f} = q_{j,f}] = 1/Q$$

Master Equation



F=3: $\quad W^{(0)}_{n,m} = W^{(0)}_{m,n} = 0 \quad W^{(3)}_{n,m} = W^{(3)}_{m,n} = 0 \quad W^{(1)}_{n,m} = W^{(2)}_{n,m} \quad \forall m, n$

Master Equation

$$W_{0,0} = 1 - \rho \qquad W_{0,1} = \rho$$

$$W_{1,0} = \frac{1}{3} \qquad W_{1,1} = \frac{2}{3}(1-\rho) \qquad W_{1,2} = \frac{2}{3}\rho$$

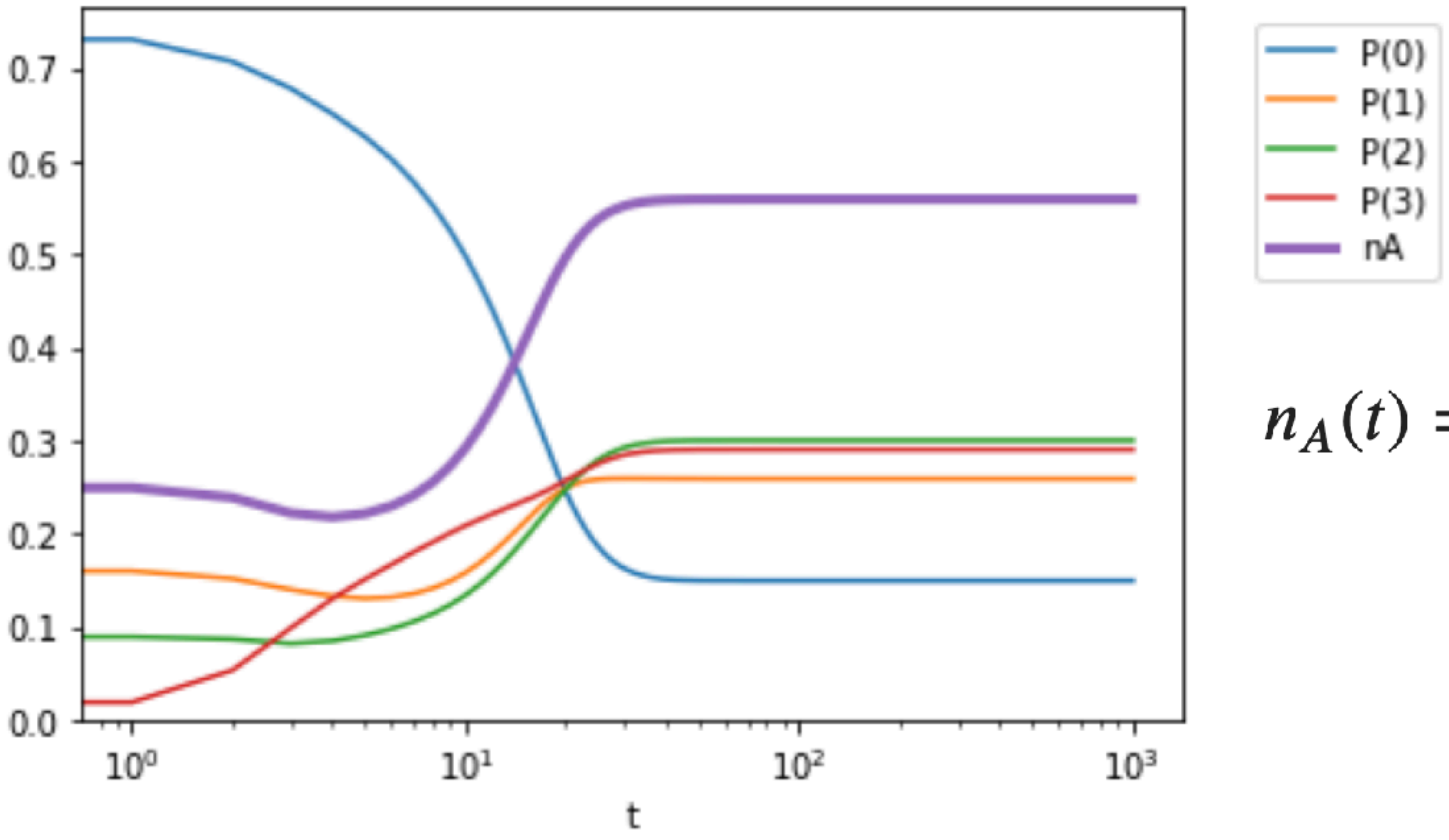
$$W_{2,1} = \frac{2}{3} \qquad W_{2,2} = \frac{1}{3}(1-\rho) \qquad W_{2,3} = \frac{1}{3}\rho$$

$$W_{3,2} = 0 \qquad W_{3,3} = 1$$

$$\rho = \sum_k kP_k/F$$

$$\frac{dP_m}{dt} = \sum_{k=1}^{F-1} \frac{k}{F} P_k [\delta_{m,k+1} - \delta_{m,k} + (g-1) \sum_{n=0}^{F} (P_n W_{n,m}^{(k)} - P_m W_{m,n}^{(k)})]$$

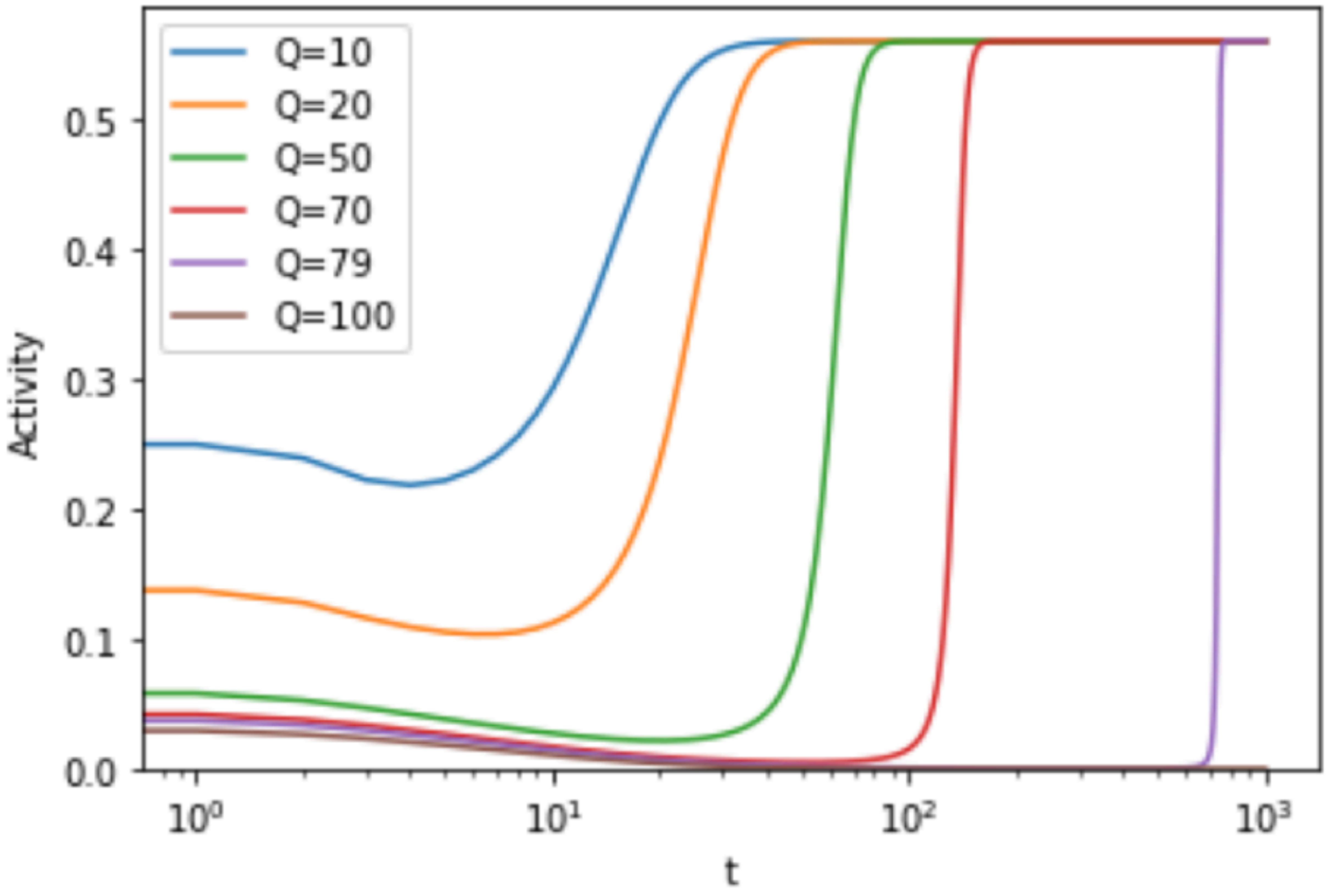
Master Equation



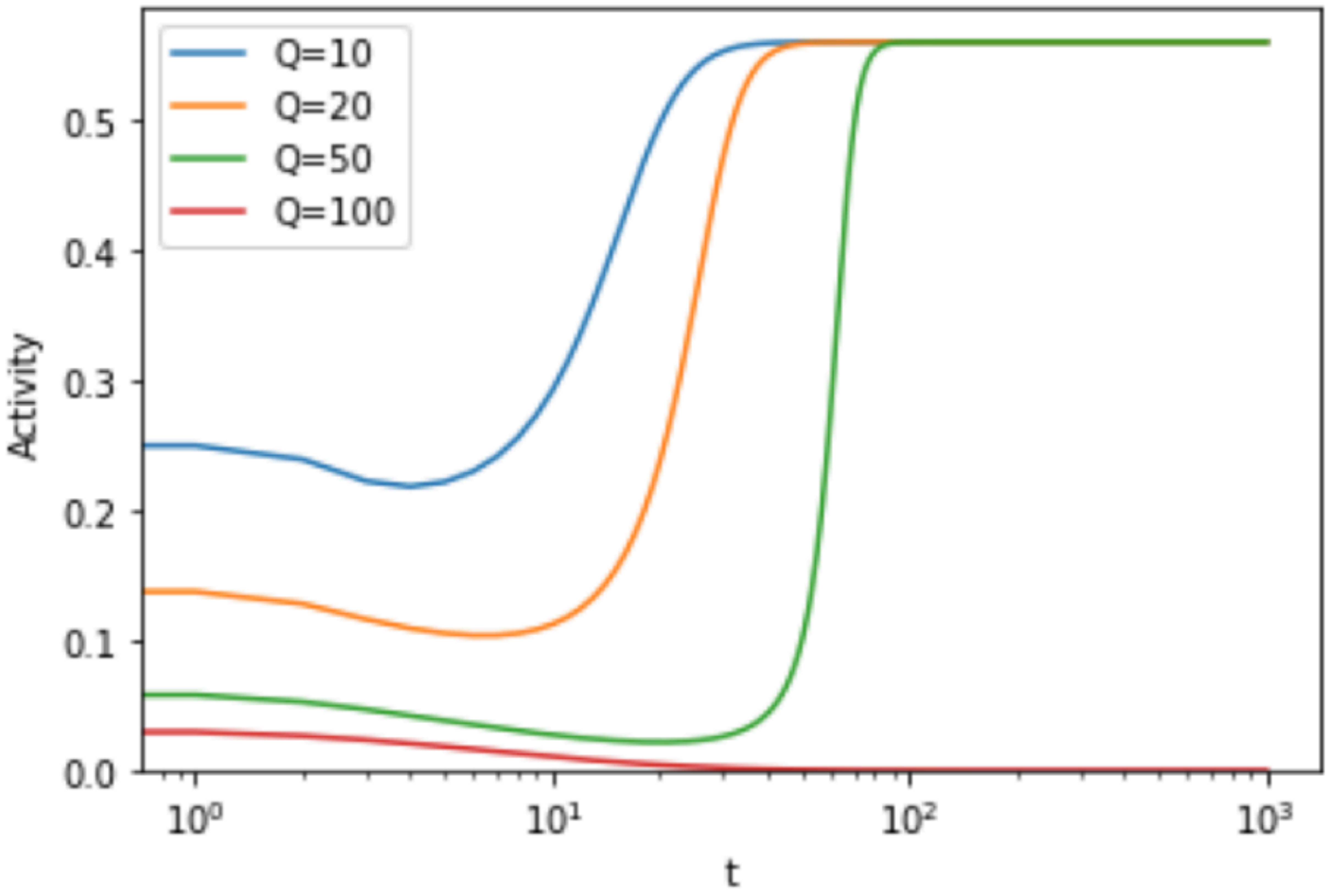
$$n_A(t) = \sum_{k=1}^{F-1} P_k(t)$$

F=3, Q=10

Master Equation



F=3

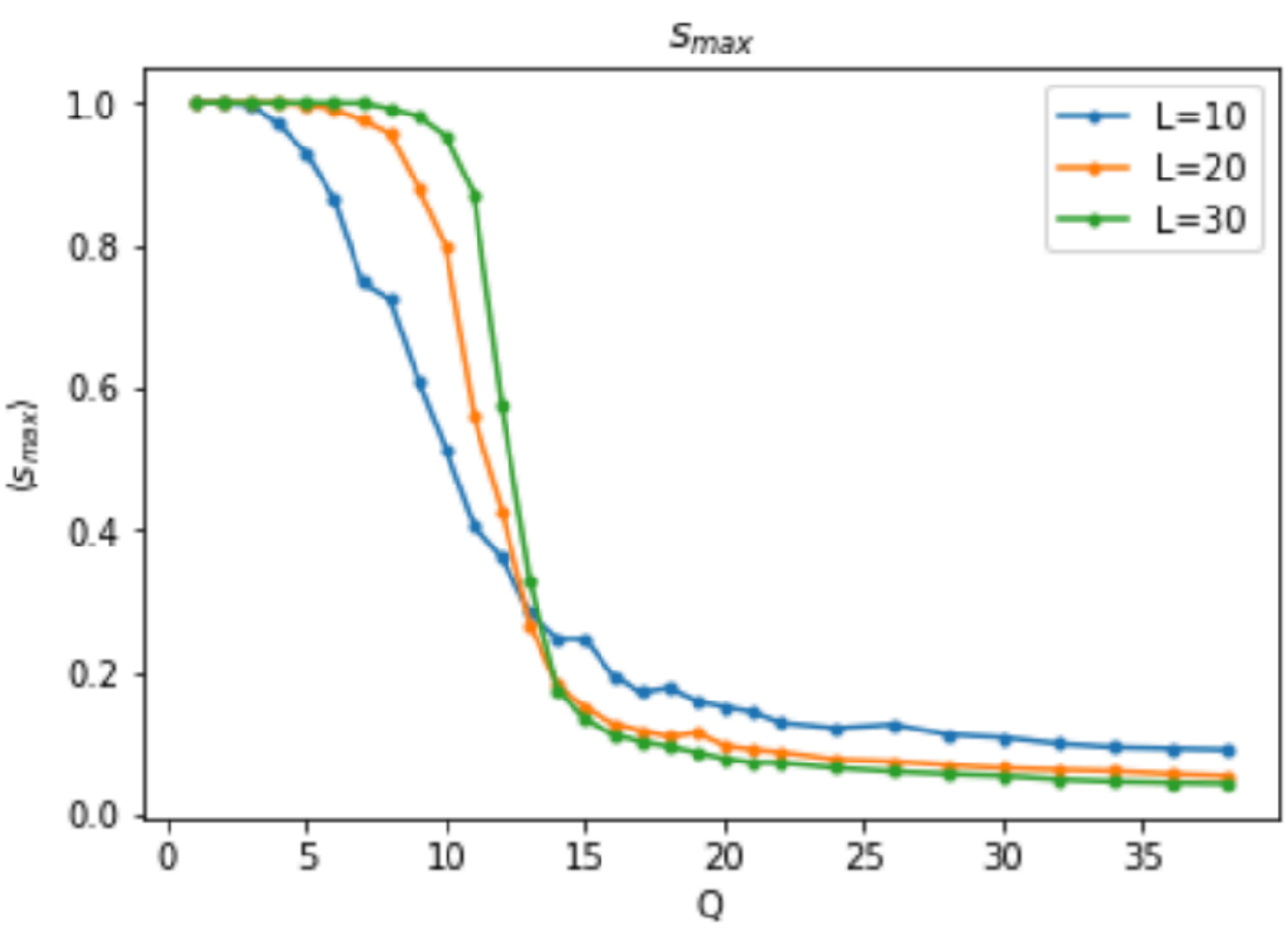


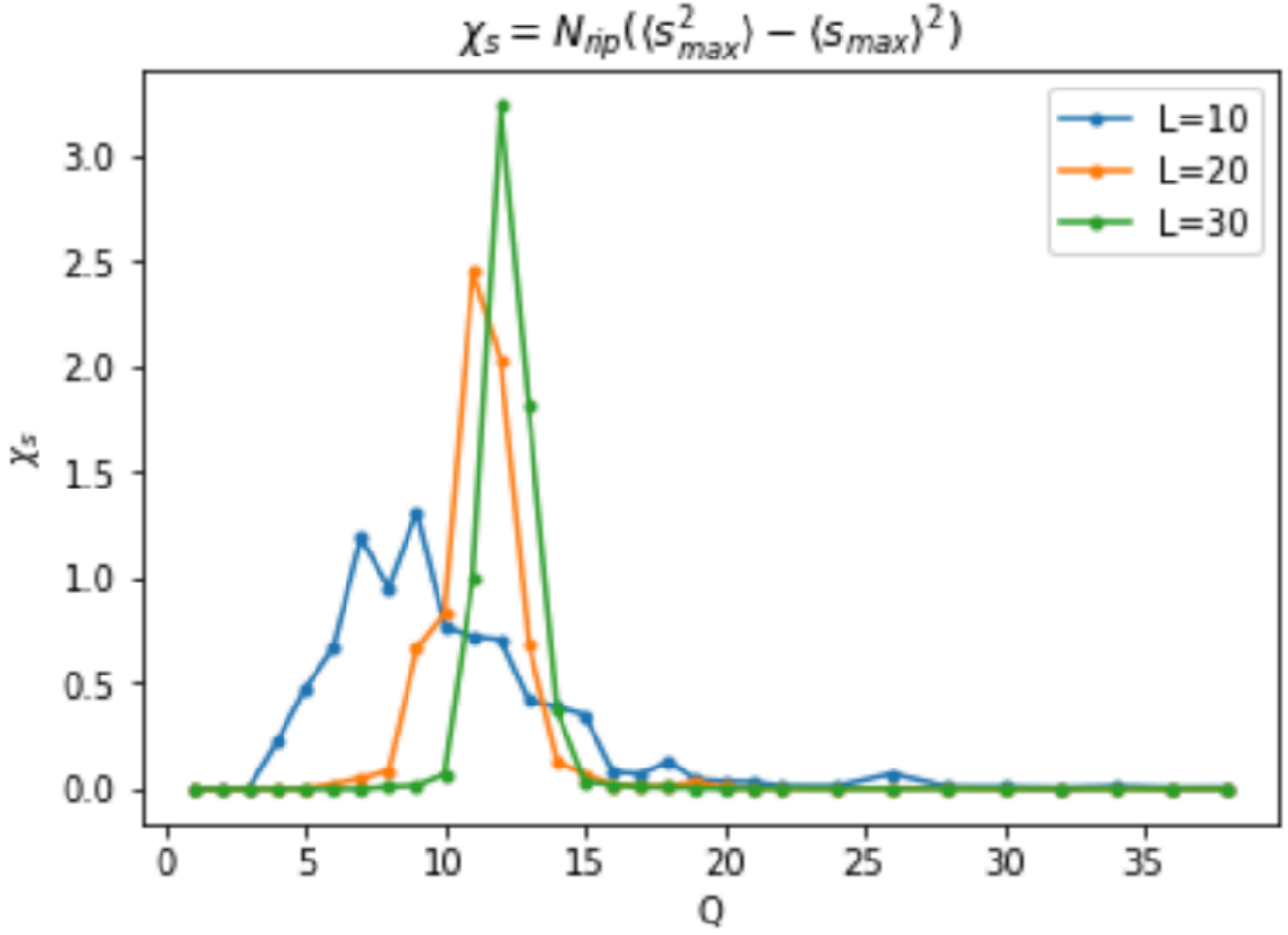
F=3

Master Equation

F=3 F=10

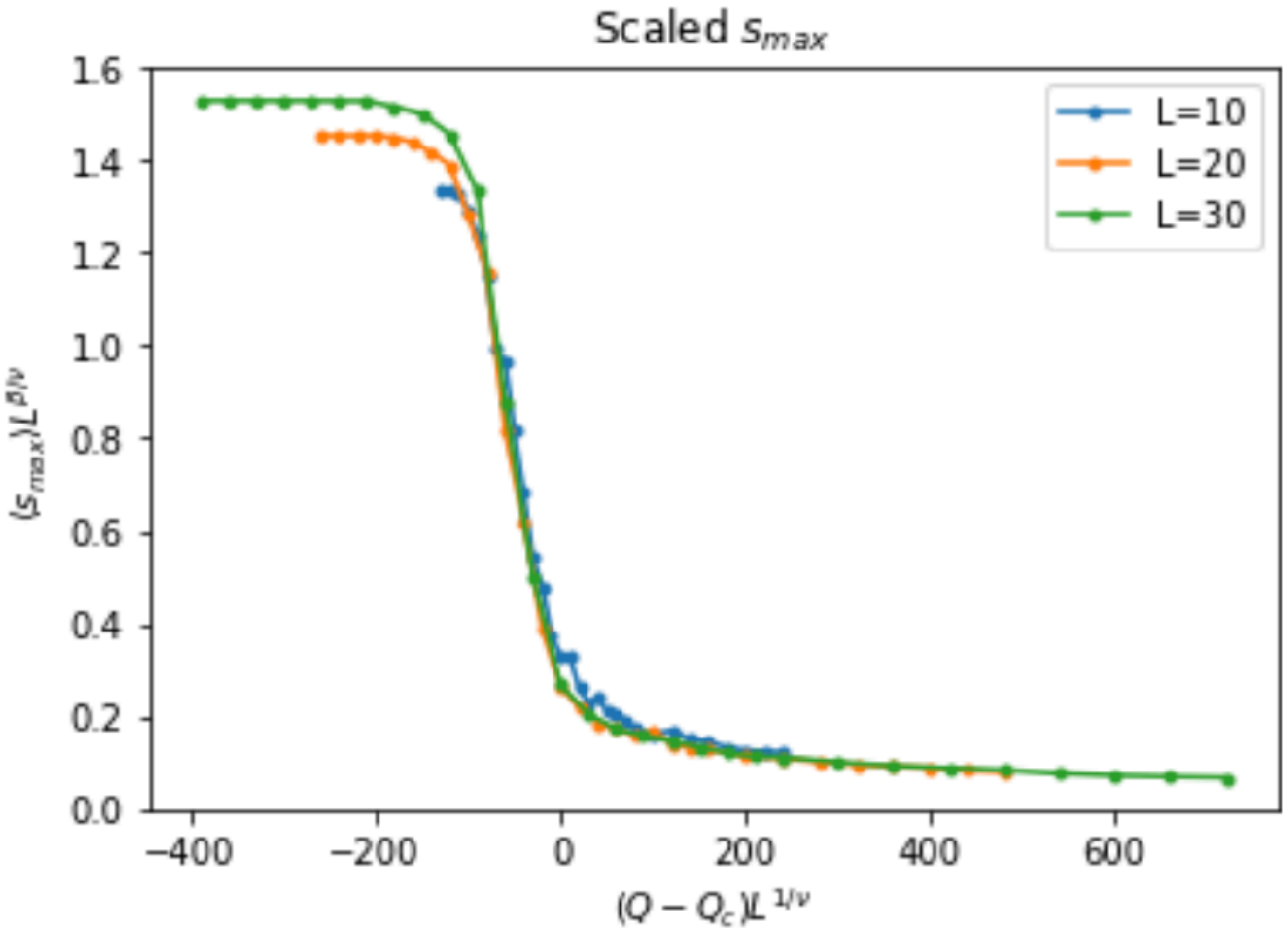
Master Equation

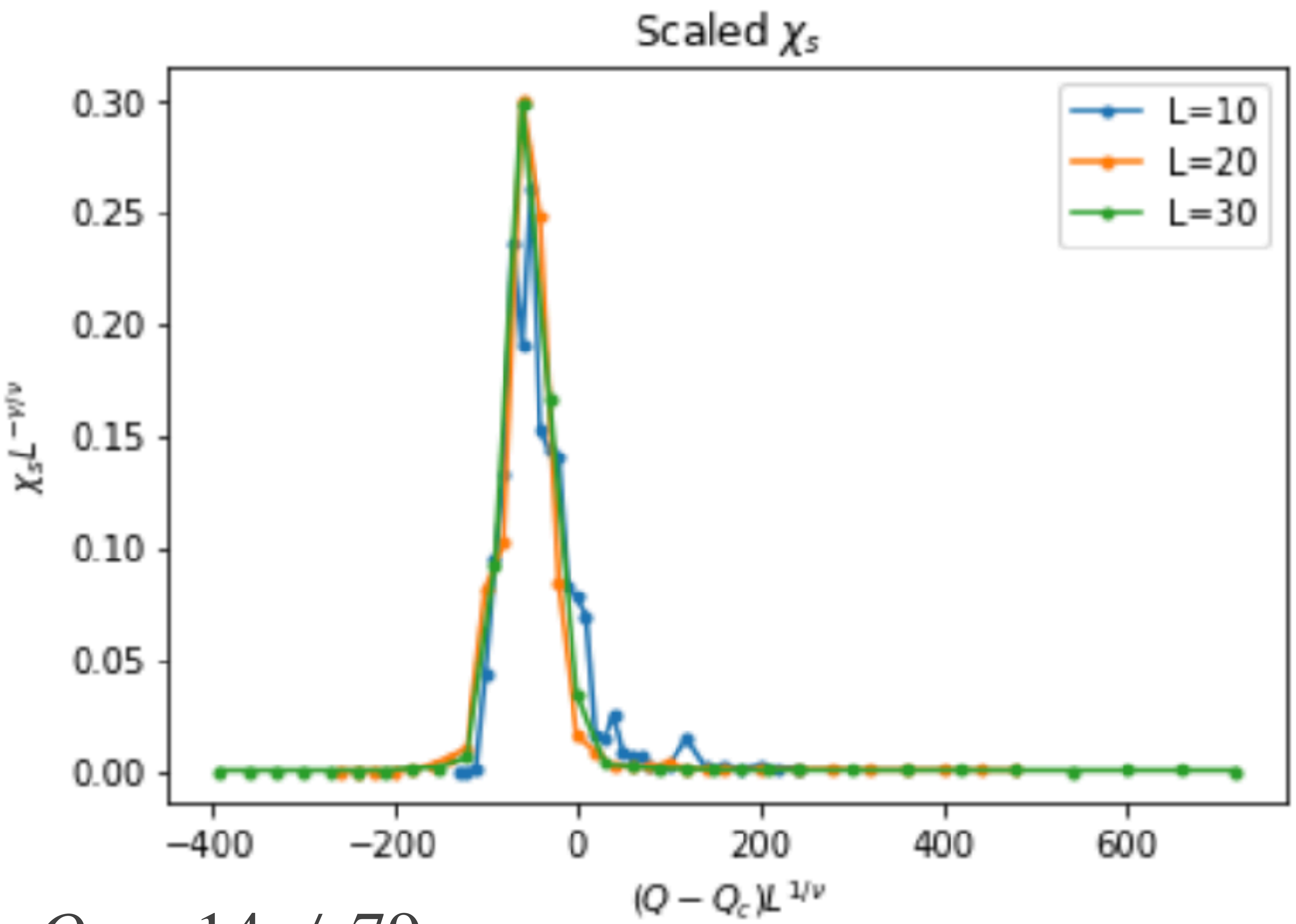




F=3

Differences



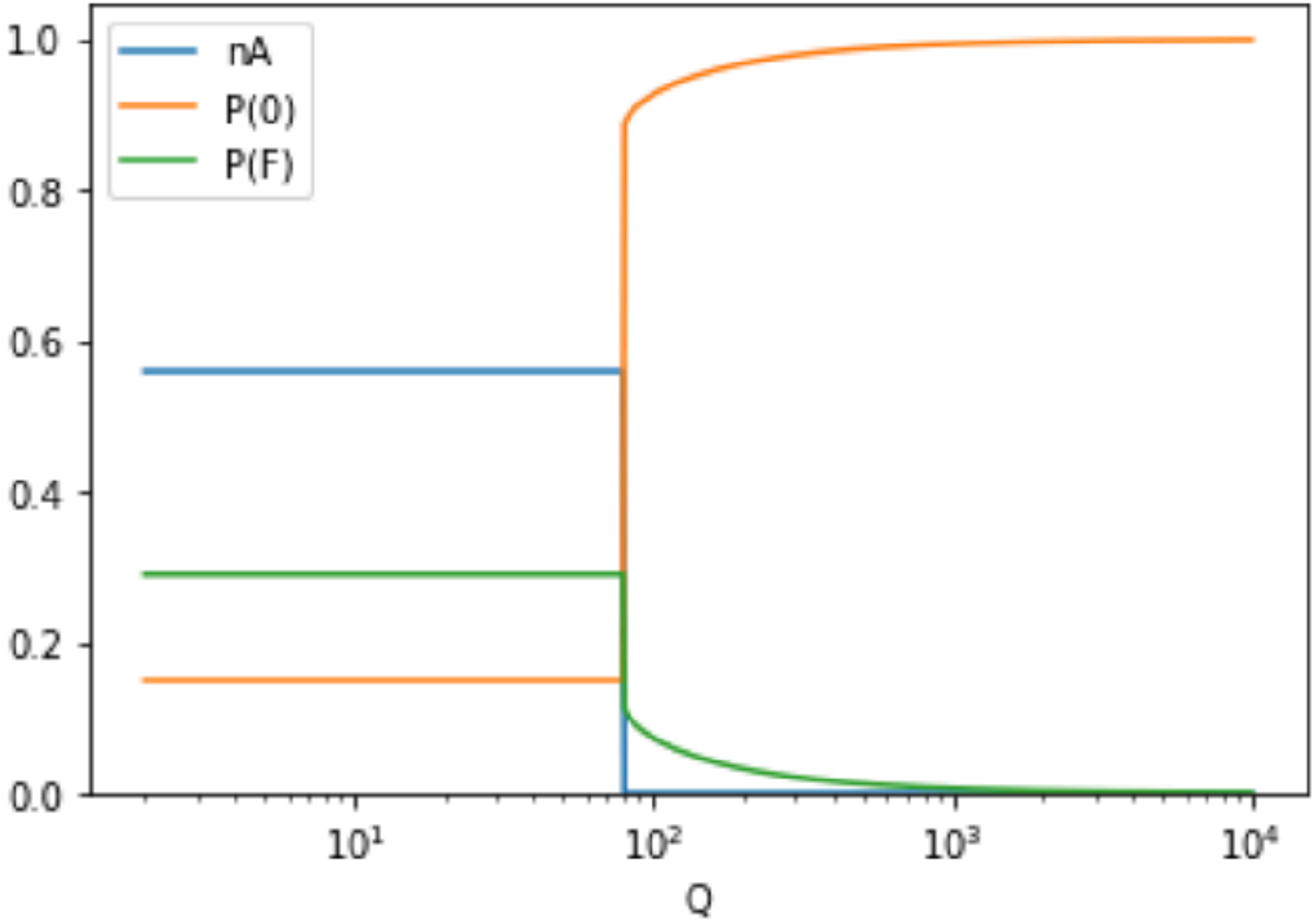


Critical Q: $Q_c \simeq 14 \neq 79$

Critical exponents: $\beta/\nu \simeq 1/8$ e $\gamma/\nu \simeq 0.7$

Master Equation

Discontinuous phase transition



Conclusion

Comparison simulation - Master Equation qualitatively correct.

Variants:

- Implementation on complex network
- Property 2 stringent
- Non trivial probabilities for $q_{i,f}$
- Correlations between features

Bibliography

[1] Axelrod, R., 1997, J. Conflict Resolut. 41(2), 203.

[2] C. Castellano, M. Marsili, and A. Vespignani, 2000, Phys. Rev. Lett. 85(16), 3536.

[3] C. Castellano, S. Fortunato and V. Loreto, “Statistical Physics of Social Dynamics”, 2009, Rev. Mod. Phys. 81, 591.

Report of this presentation, with explanation and codes, on the web page:
https://edoarder.github.io/Sistemi_Complessi/

Thanks for your attention