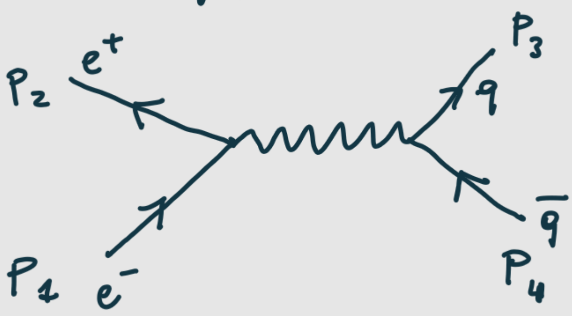
$e^+e^- \longrightarrow q\bar{q}$



The matrix element is given by the formula

$$M_f = \bar{v}(p_2)\, ie\gamma^\mu u(p_1)\ \frac{-ig_{\mu\nu}}{(p_1+p_2)^2}\ \bar{u}(p_3)\, ieQ_f\gamma^\nu v(p_4)$$

$$\Rightarrow |M_f|^2 = \frac{1}{4}\ \frac{e^4 Q_f^2}{q^4}\ g_{\mu\nu}\, g_{\alpha\beta}\, T^{\mu\nu\alpha\beta} \quad (1)$$

where $T^{\mu\nu\alpha\beta} = \sum_{\substack{\text{all}\\\text{spins}}} \bar{v}(p_2)\gamma^\mu \underbrace{u(p_1)\bar{u}(p_1)}_{\not{p}_1}\gamma^\alpha v(p_2)\bar{u}(p_3)\gamma^\nu \underbrace{v(p_4)\bar{v}(p_4)}_{\not{p}_4}\gamma^\beta u(p_3) =$

$$\Rightarrow T^{\mu\nu\alpha\beta} = \sum_{s_2}\bar{v}(p_2)\gamma^\mu\not{p}_1\gamma^\alpha v(p_2)\sum_{s_3}\bar{u}(p_3)\gamma^\nu\not{p}_4\gamma^\beta u(p_3)$$

$$= Tr\left[\gamma^\mu\not{p}_1\gamma^\alpha\not{p}_2\right] Tr\left[\gamma^\nu\not{p}_4\gamma^\beta\not{p}_3\right]$$

So

$$Tr\left[\gamma^\mu\not{p}_1\gamma^\alpha\not{p}_2\right] = p_{1\eta}\,p_{2\varepsilon}\, Tr\left[\gamma^\mu\gamma^\eta\gamma^\alpha\gamma^\varepsilon\right] =$$

$$= p_{1\eta}\,p_{2\varepsilon}\, 4\left(g^{\mu\eta}g^{\alpha\varepsilon} - g^{\mu\alpha}g^{\eta\varepsilon} + g^{\mu\varepsilon}g^{\eta\varepsilon}\right) =$$

$$= 4\left(p_1^\mu p_2^\alpha - g^{\mu\alpha}(p_1p_2) + p_1^\alpha p_2^\mu\right)$$

Similarly

$$Tr\left[\gamma^\nu\not{p}_4\gamma^\beta\not{p}_3\right] = 4\left(p_4^\nu p_3^\beta - g^{\nu\beta}(p_4p_3) + p_4^\beta p_3^\nu\right)$$

So now we can evaluate the term in (1)

$$g_{\mu\nu}g_{\alpha\beta}T^{\mu\nu\alpha\beta} = g_{\mu\nu}g_{\alpha\beta}\left(p_1^\mu p_2^\alpha - g^{\mu\alpha}p_1\cdot p_2 + p_1^\alpha p_2^\mu\right)\cdot\left(p_4^\nu p_3^\beta - g^{\nu\beta}p_4p_3 + p_4^\beta p_3^\nu\right)\cdot 16 =$$

$$= 16\Big[(p_1p_4)(p_2p_3) - \cancel{(p_1p_2)(p_3p_4)} + (p_1p_3)(p_2p_4) - \cancel{(p_1p_2)(p_3p_4)} +$$

$$+ \cancel{4(p_1p_2)(p_3p_4)} - \cancel{(p_1p_2)(p_3p_4)} + (p_1p_3)(p_2p_4) - \cancel{(p_1p_2)(p_3p_4)} +$$

$$+ (p_1p_4)(p_2p_3)\Big] = 32\left[(p_1p_4)(p_2p_3) + (p_1p_3)(p_2p_4)\right]$$

Using Mandelstam variables

$$s = (p_1+p_2)^2 = (p_3+p_4)^2 = 2p_1p_2 = 2p_3p_4$$

$$t = (p_1-p_3)^2 = (p_2-p_4)^2 = -2p_1p_3 = -2p_2p_4 = -\frac{s}{2}(1-\cos\theta)$$

$$u = (p_1-p_4)^2 = (p_2-p_3)^2 = -2p_1p_4 = -2p_2p_3 = \frac{s}{2}(1+\cos\theta)$$

$$g_{\mu\nu}\, g_{\alpha\beta}\, T^{\mu\nu\alpha\beta} = 32\left(\frac{u^2}{4}+\frac{t^2}{4}\right)$$

Putting everything together

$$|\overline{M}|^2 = \frac{e^4 Q_f^2}{s^2}(u^2+t^2)$$

so:

$$\left(\frac{d\sigma}{d\Omega}\right)_{C.M.} = \frac{1}{64\pi^2 s}|\overline{M}|^2 = \frac{e^4}{16\pi^2}\frac{Q_f^2}{4s^2}(u^2+t^2) = \frac{Q_f^2\alpha^2}{8s}(1+\cos^2\theta)$$

and

$$\frac{d\sigma}{d\cos\theta} = Q_f^2\frac{\alpha^2\pi}{4s}(1+\cos^2\theta)$$

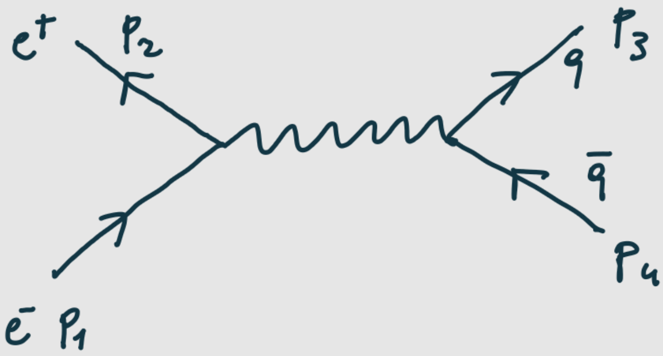
If we consider other flavours we should make the following change

$$Q_f \to \sum_f Q_f^2$$

where the sum can be performed over quarks with mass m, such that

$$2m < s.$$

Now we include the exchange of Z boson



$$M_Z = \bar{v}(p_2)\,\frac{-i g_w}{2\sqrt{2}}\,\gamma^\mu (V_e - \gamma^5 A_e)\, u(p_1)\,\frac{-i g_{\mu\nu}}{q^2 - m_Z^2}\,\bar{u}(p_3)\,\cdot$$

$$\cdot \frac{i g_w}{2\sqrt{2}}\,\gamma^\nu (V_f - \gamma^5 A_f)\, v(p_4) =$$

$$= \frac{i g_w^2}{8(p^2 - m_Z^2)}\,\bar{v}(p_2)\gamma^\mu (V_e - \gamma^5 A_e)\, u(p_1)\,\frac{g^{\mu\nu}}{p^2 - m_Z^2}\,\bar{u}(p_3)\gamma^\nu (V_f - \gamma^5 A_f)\, v(p_4)$$

Putting the photon and Z element together

$$|M_\gamma + M_Z|^2 = |M_\gamma|^2 + |M_Z|^2 + 2\,\mathrm{Re}\,(M_\gamma^* M_Z)$$

$$|M_Z|^2 = \frac{1}{4}\,\frac{g_w^4}{64\,(p^2 - m_Z^2)^2}\, g_{\mu\nu}\, g_{\alpha\beta}\, T^{\mu\nu\alpha\beta}$$

where

$$T^{\mu\nu\alpha\beta} = \sum_{\text{spins}} \bar{v}_{p_2}\gamma^\mu (V_e - \gamma^5 A_e)\, u(p_1)\,\bar{u}(p_1)\,\gamma^\alpha (V_f - \gamma^5 A_f)\, v_{p_2}\,\cdot$$

$$\cdot\, \bar{u}_{p_3}\gamma^\nu (V_f - \gamma^5 A_f)\, v_{p_4}\,\bar{v}_{p_4}\,\gamma^\beta (V_f - \gamma^5 A_f)\, u_{p_3} =$$

$$= \mathrm{Tr}\left[\gamma^\mu (V_e - \gamma^5 A_e)\,\not{p}_1\,\gamma^\alpha (V_f - \gamma^5 A_f)\,\not{p}_2\right] \cdot \quad \rightarrow 1$$

$$\cdot\, \mathrm{Tr}\left[\gamma^\nu (V_f - \gamma^5 A_f)\,\not{p}_4\,\gamma^\beta (V_f - \gamma^5 A_f)\,\not{p}_3\right] \quad \rightarrow 2$$

1

$$= P_{1\eta} P_{2\varepsilon} \{ V_e^2 Tr[\gamma^\mu \gamma^\eta \gamma^\alpha \gamma^\varepsilon] + A_e^2 Tr[\gamma^\mu \gamma^5 \gamma^\eta \gamma^\alpha \gamma^5 \gamma^\varepsilon] +$$

$$- V_e A_e [Tr[\gamma^\mu \gamma^\eta \gamma^\alpha \gamma^5 \gamma^\varepsilon] + Tr[\gamma^\mu \gamma^5 \gamma^\eta \gamma^\alpha \gamma^\varepsilon]]\} =$$

$$= P_{1\eta} P_{2\varepsilon} \{ (V_e^2 + A_e^2) Tr[\gamma^\mu \gamma^\eta \gamma^\alpha \gamma^\varepsilon] + 2 V_e A_e Tr[\gamma^\mu \gamma^\eta \gamma^\varepsilon \gamma^5]\} =$$

$$= P_{1\eta} P_{2\varepsilon} \{ (V_e^2 + A_e^2) (g^{\mu\eta} g^{\alpha\varepsilon} - g^{\mu\alpha} g^{\eta\varepsilon} + g^{\mu\varepsilon} g^{\alpha\eta}) 4 +$$

$$+ 2 V_e A_e (-4i) \varepsilon^{\mu\eta\alpha\varepsilon} \}$$

2

Similar to 1

$$= P_{4\eta} P_{3\varepsilon} \{ (V_f^2 + \gamma^5 A_f^2) (g^{\nu\eta} g^{\beta\varepsilon} - g^{\nu\beta} g^{\eta\varepsilon} + g^{\nu\varepsilon} g^{\beta\eta}) \cdot 4 +$$

$$- 8i V_f A_f \varepsilon^{\nu\eta\beta\varepsilon} \}$$

$$g_{\mu\nu} g_{\alpha\beta} T^{\mu\nu\alpha\beta} = (V_e^2 + A_e^2)(V_f + A_f)^2 \cdot (\text{contribute similar to } g) +$$

$$+ A_f V_f V_e V_f \cdot A + B$$

$\hookrightarrow = 0$ for tensor symmetry

$$A \propto P_{1\eta} P_{2\varepsilon} \varepsilon^{\mu\eta\alpha\varepsilon} P_{4\rho} P_{3\theta} \varepsilon^{\nu\rho\beta\theta} g_{\mu\nu} g_{\alpha\beta} =$$

$$= P_{1\eta} P_{2\varepsilon} P_{3\theta} P_{4\rho} \varepsilon^{\mu\eta\alpha\varepsilon} \varepsilon^{\mu\rho\alpha\theta} =$$

$$= (P_1 P_4)(P_2 P_3) - (P_1 P_3)(P_2 P_4) =$$

$$= \frac{s^2}{16} \left(1 + \cos\theta - (1 - \cos\theta) \right) = \frac{s^2}{8} \cos\theta$$

so

$$|M_Z|^2 = \frac{1}{4}\frac{g_w^2}{64(p^2-m_Z^2)}\Big\{(V_e^2+A_e^2)(V_f^2+A_f^2)\,4\,s^2(1+\cos^2\theta)+$$
$$-8s^2 V_e A_e V_f A_f \cos\theta\Big\}$$

Interference terms.

$$M_\gamma^* M_Z = -ie^2\frac{Q_f}{q^2}\frac{ig^2 w}{8(p^2-m^2)}\left(\bar v_{p_4}\gamma^\nu u_{p_3}\, g_{\mu\nu}\, \bar u_{p_1} v_{p_2}\right)\cdot$$
$$\cdot\, \bar v(p_2)\gamma^\alpha(V_e-\gamma^5 A_e)u(p_1)\frac{g_{\alpha\beta}}{p^2-m_Z^2}\,\bar u(p_3)\gamma^\beta(V_f-\gamma^5 A_f)v(p_4)$$

$$\sum M_\gamma^* M_Z = \frac{e^2 Q_f g_w^2}{8q^2(p^2-m^2)^2}\, g_{\mu\nu}\, g_{\alpha\beta}\, T^{\mu\nu\alpha\beta}$$

$$T^{\mu\nu\alpha\beta} = \mathrm{Tr}\left[\gamma^\nu \not{p}_3\gamma^\beta(V_f-\gamma^5 A_f)\not{p}_4\right]\cdot$$
$$\mathrm{Tr}\left[\gamma^\mu\not{p}_2\gamma^\alpha(V_e-\gamma^5 A_e)\not{p}_1\right]$$

$$\Rightarrow g_{\mu\nu}g_{\alpha\beta}T^{\mu\nu\alpha\beta} = g_{\mu\nu}g_{\alpha\beta}\left(V_f\,\mathrm{Tr}[\gamma^\nu\not{p}_3\gamma^\beta\not{p}_4]-A_f\,\mathrm{Tr}[\gamma^\alpha\not{p}_3\gamma^\beta\gamma^5\not{p}_4]\right)\cdot$$
$$\cdot\left(V_e\,\mathrm{Tr}[\gamma^\mu\not{p}_2\gamma^\alpha\not{p}_1]-A_e\,\mathrm{Tr}[\gamma^\mu\not{p}_2\gamma^\alpha\gamma^5\not{p}_1]\right)=$$
$$\propto V_f V_e\cos^2\theta + \underbrace{(V_f A_e + A_e V_f)(\dots)}_{\to 0 \text{ symmetry}} +$$
$$+ A_e A_f \cos\theta$$

so we will have

$$\frac{d\sigma}{d\cos\theta}\propto A\cos^2\theta + B\cos\theta$$

If we compare this formula to the one with only γ exchange, there a new part proportional to $\cos\theta$ which contains the term proportional to the axial current.