

$x(t) \rightarrow$ Population at time t

↓ If Both parents are Hipsters the offspring will be Hipster

• ELSE $\rightarrow \pi\%$ of being Hipster

$b \rightarrow$ birth rate

$d \rightarrow$ death rate

b, d are constants

$$\frac{dx(t)}{dt} = (b-d)x(t)$$

$$\frac{dx_i(t)}{dt} = (b-d)x_i(t) + \pi b(x(t) - x_i(t))$$

$x_i(t) \rightarrow$ Number of Hipsters at time t

$$P(t) = \frac{x_i(t)}{x(t)} \rightarrow \text{percentage of Hipsters at time } t$$

Chain rule :

$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$$

$$\begin{aligned} \frac{dp(t)}{dt} &= \frac{dp}{dx} \cdot \frac{dx}{dt} + \frac{dp}{dx_i} \cdot \frac{dx_i}{dt} = \\ &= (x_i \cdot (-1)x^{-2}) \cdot (b-d)x + \frac{1}{x} \cdot [(b-d)x_i + \pi b(x - x_i)] \end{aligned}$$

$$= -\frac{x_i}{x} \cdot (b-d) + \frac{x_i}{x} \cdot (b-d) + \frac{1}{x} \pi b(x - x_i)$$

$$= \pi b(1 - p)$$