• AITOR

$\left(\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x}\right)\left(\frac{\partial u}{\partial t} - c\frac{\partial u}{\partial x}\right)$

• **ADVECTION (WAVE)** : $\frac{\partial u}{\partial t} - c\frac{\partial u}{\partial x} = 0$

→ Explicit FTCS : 1st order in time | 2nd order in space (subtract 2 taylor $x \pm \Delta x$)
stability : $\frac{2 D \Delta t}{\Delta x^2} \leq 1$

→ LAX : replace $u_{m,j}$ by $\frac{1}{2}(u_{m+1,j} + u_{m-1,j})$
stability : $\frac{D\Delta t}{\Delta x} \leq 1$ but if $\frac{D\Delta t}{\Delta x}$ is diffusive

→ STAGGERED LEAPFROG : 2nd order in space and time
stability : $c\frac{\Delta t}{\Delta x} \leq 1$ NO diffusion

• **DIFFUSION (HEAT)** : $\frac{\partial u}{\partial t} = D\frac{\partial^2 u}{\partial x^2}$

→ Explicit FTCS : 1st order in time 2nd order in space (add 2 taylor $x \pm \Delta x$)
stability : $\frac{2 D \Delta t}{\Delta x^2} \leq 1$

→ FULLY IMPLICIT : like FTCS but the spacial derivatives are at $j+1$
stability : always stable

→ CRANK-NICHOLSON : 2nd order in space ( ) and time (subtract 2 taylor $t \pm \Delta t$)
→ we center at time step $j + \frac{1}{2}$ in both space and time

→ 1st order expansion on $\left.\frac{\partial^2 u}{\partial x^2}\right|_{m,j+1}$ → $\left.\frac{\partial^2 u}{\partial x^2}\right|_{m,j+\frac{1}{2}} = \frac{1}{2}\left[\left.\frac{\partial^2 u}{\partial x^2}\right|_{m,j+1} + \left.\frac{\partial^2 u}{\partial x^2}\right|_{m,j}\right]$

$$= \frac{1}{2}\left[\frac{u_{m+1,j+1} - 2u_{m,j+1} + u_{m-1,j+1}}{\Delta x^2} + \frac{u_{m+1,j} - 2u_{m,j} + u_{m-1,j}}{\Delta x^2}\right]$$

BLACK - SCHOLES

- EXPLICIT FTCS : 1st order in time 2nd order on space $\rightarrow \frac{\partial V}{\partial S}$, $\frac{\partial^2 V}{\partial S^2}$
  stability : $N_t \geq T \cdot N_S^2 \cdot \sigma^2$

- FULLY IMPLICIT : Same as explicit but evaluated at $j+1$ in space.
  stability : Always stable

- CRANK-NICHOLSON : average of implicit and explicit methods

$$\frac{V_{m,j+1} - V_{m,j}}{\Delta t} = \frac{1}{2}\left[\underset{\text{IMPLICIT}}{\ldots \text{ evaluated at } j+1}\right] + \frac{1}{2}\left[\underset{\text{EXPLICIT}}{\text{evaluated at } j}\right] + O(\Delta t, \Delta S^2)$$

- STABILITY : $u_{m,j} = \xi^j(k)\, e^{ikm\Delta x}$ $\rightarrow$ Substitute and check $|\xi(k)| \leq 1$ for stability

  - $e^{ia} = \cos(a) + i\sin(a)$ | $e^{-ia} = \cos(a) - i\sin(a)$
  - $z = a + ib$ $\quad |z|^2 = z \cdot z^*$ dove $z^* = a - ib$