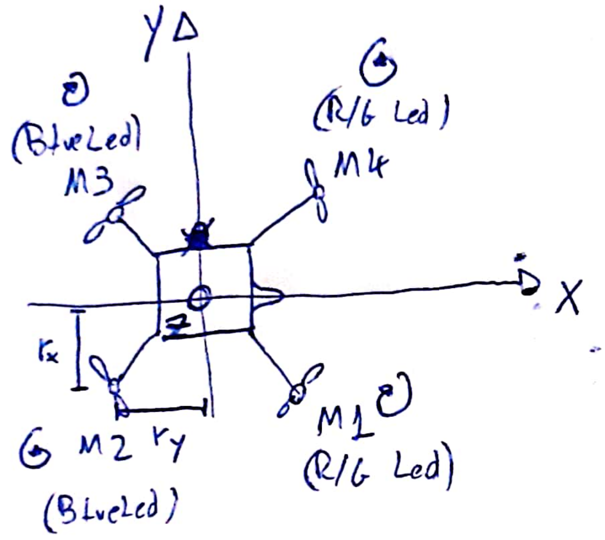
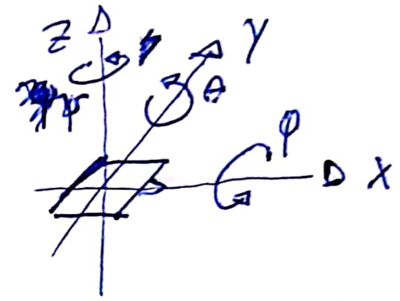
(I)

DINAMICA DRONE (demystifying Crazyflie drone dynamics)

Motori controllati in PWM → velocità angolare, → forza, → momento] System Identification of the Crazyflie 2.0 Nano Quadcopter

BODY FRAME:





φ: ROLL (angle ϕ)

θ: PITCH

ψ: YAW

MOMENTO DI ROLL

$$\tau_x = \sum_{1}^{4} {}_i\, r_i f_i$$

r_i distanza del M_i da asse x

f_i forza espressa da M_i

$$\tau_x = r_x (f_3 + f_4 - f_1 - f_2)$$

MOMENTO DI PITCH

$$\tau_y = \sum_i r_i f_i$$

$$\tau_y = r_y (f_2 + f_4 - f_1 - f_3)$$

MOMENTO DI YAW

$$\tau_z = c (f_2 + f_4 - f_1 - f_3)$$

c → coefficiente di scala del momento

THRUST

$$T = f_1 + f_2 + f_3 + f_4$$

MOTO VERTICALE

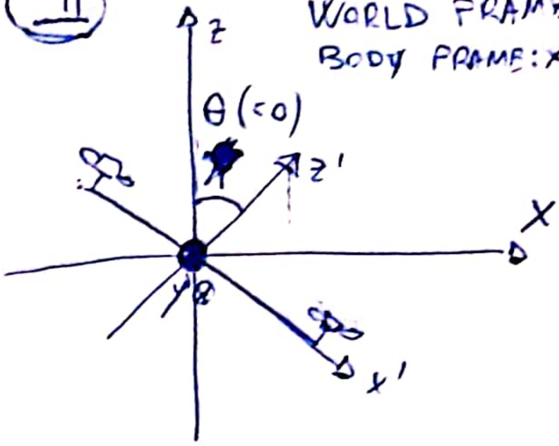
$$m\ddot{z} = T - mg$$

$$\ddot{z} = \frac{T - mg}{m}$$

(in BODY FRAME)

Scansionato con CamScanner

II



WORLD FRAME: xyz
BODY FRAME: x'y'z'

MOTO ORIZZONTALE

$F_x = T \cdot \sin(-\theta) = -T \sin\theta$

$F_z = T \cdot \cos(-\theta) = T \cos\theta$

Similmente per le forze generate quando il drone è inclinato lungo φ invece che θ

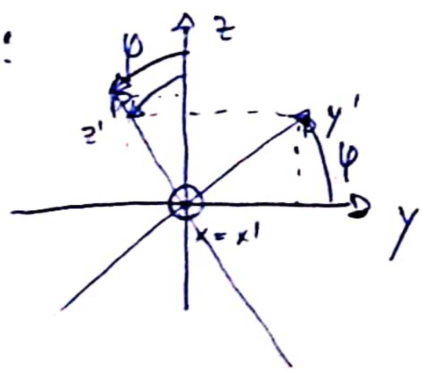
L'equazione completa della dinamica sarà:

$$\begin{pmatrix} m\ddot{x} \\ m\ddot{y} \\ m\ddot{z} \end{pmatrix} = m\begin{pmatrix} 0 \\ 0 \\ -g \end{pmatrix} + R^{3\times3}\begin{pmatrix} 0 \\ 0 \\ T \end{pmatrix} \Rightarrow \begin{pmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -g \end{pmatrix} + \frac{R^{3\times3}}{m}\begin{pmatrix} 0 \\ 0 \\ T \end{pmatrix}$$

IDENTIFICARE R (matrice di rotazione)

$$R = R_x(\varphi) \cdot R_y(\theta) \cdot R_z(\psi) \qquad P' = RP$$

R_x:



WORLD FRAME xyz
BODY FRAME x'y'z'

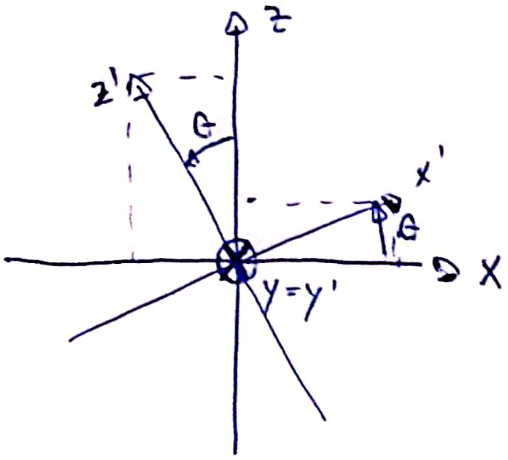
$x' = x$

$y' = \cos\varphi\, y + \sin\varphi\, z$

$z' = -\sin\varphi\, y + \cos\varphi\, z$

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c\varphi & s\varphi \\ 0 & -s\varphi & c\varphi \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

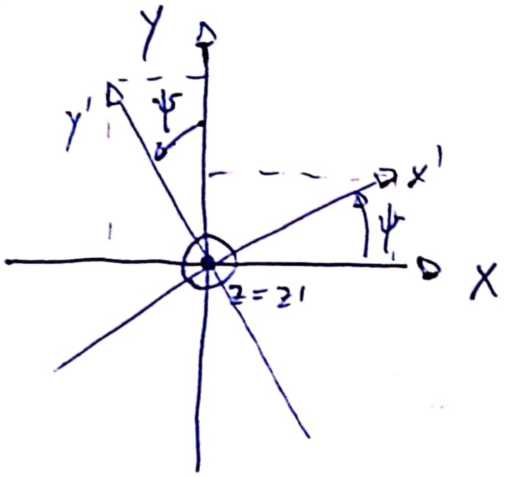
R_y:



$$x' = \cos\theta\, x + \sin\theta\, z$$
$$y' = y$$
$$z' = -\sin\theta\, x + \cos\theta\, z$$

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} c\theta & 0 & s\theta \\ 0 & 1 & 0 \\ -s\theta & 0 & c\theta \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

R_z



$$x' = \cos\psi\, x + \sin\psi\, y$$
$$y' = -\sin\psi\, x + \cos\psi\, y$$
$$z' = z$$

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} c\psi & s\psi & 0 \\ -s\psi & c\psi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$R_y \cdot R_x = \begin{pmatrix} c\theta & 0 & s\theta \\ 0 & 1 & 0 \\ -s\theta & 0 & c\theta \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & c\varphi & s\varphi \\ 0 & -s\varphi & c\varphi \end{pmatrix} = \begin{pmatrix} c\theta & -s\theta s\varphi & s\theta c\varphi \\ 0 & c\varphi & s\varphi \\ -s\theta & -c\theta s\varphi & c\theta c\varphi \end{pmatrix}$$

$$R_z \cdot R_y \cdot R_x = \begin{pmatrix} c\psi & s\psi & 0 \\ -s\psi & c\psi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c\theta & -s\theta s\varphi & s\theta c\varphi \\ 0 & c\varphi & s\varphi \\ -s\theta & -c\theta s\varphi & c\theta c\varphi \end{pmatrix} =$$

$$= \begin{pmatrix} c\psi c\theta & s\psi c\varphi - c\psi s\theta s\varphi & c\psi s\theta c\varphi + s\psi s\varphi \\ -s\psi c\theta & s\psi s\theta s\varphi + c\psi c\varphi & c\psi s\varphi - s\psi s\theta c\varphi \\ -s\theta & -c\theta s\varphi & c\theta c\varphi \end{pmatrix}$$

$$R = R_x(\varphi)\cdot R_y(\theta)\cdot R_z(\psi) =$$

IV

$$= R_x(\varphi)\cdot\begin{pmatrix} c\theta & 0 & s\theta \\ 0 & 1 & 0 \\ -s\theta & 0 & c\theta \end{pmatrix}\cdot\begin{pmatrix} c\psi & s\psi & 0 \\ -s\psi & c\psi & 0 \\ 0 & 0 & 1 \end{pmatrix} = R_x(\varphi)\cdot\begin{pmatrix} c\theta c\psi & c\theta s\psi & s\theta \\ -s\psi & c\psi & 0 \\ -s\theta c\psi & -s\theta s\psi & c\theta \end{pmatrix} =$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c\varphi & s\varphi \\ 0 & -s\varphi & c\varphi \end{pmatrix}\begin{pmatrix} c\theta c\psi & c\theta s\psi & s\theta \\ -s\psi & c\psi & 0 \\ -s\theta c\psi & -s\theta s\psi & c\theta \end{pmatrix} = \cancel{\begin{matrix} c\theta c\psi & c\theta s\psi & s\theta \\ -c\varphi s\psi - s\varphi s\theta c\psi & c\varphi c\psi - s\varphi s\theta s\psi & \end{matrix}}$$

$$= \begin{pmatrix} c\theta c\psi & c\theta s\psi & s\theta \\ -c\varphi s\psi - s\varphi s\theta c\psi & c\varphi c\psi - s\varphi s\theta s\psi & s\varphi c\theta \\ s\varphi s\psi - c\varphi s\theta c\psi & -s\varphi c\psi - c\varphi s\theta s\psi & c\varphi c\theta \end{pmatrix}$$

Torna uguale alla $R_{1,2,3}(\phi,\theta,\psi) = R_1(\phi)\cdot R_2(\theta)\cdot R_3(\psi)$, ma poiché il PITCH ($\theta$) è invertito, i termini che presentano sin e sono ribaltati in segno (mentre $\cos(\theta) = \cos(-\theta)$)

$$R_L = \begin{pmatrix} 1 & \psi & \theta \\ -\psi & 1 & \varphi \\ -\theta & -\varphi & 1 \end{pmatrix} \Rightarrow$$

Linearizzazione

si suppone $c\alpha \to 1$ e $s\alpha \to \alpha$

$s\alpha s\beta \to \alpha\beta$, che è $<< \alpha$, quindi

$s\alpha s\beta \to 0$ per semplicità

- $s\alpha \to \alpha$ ha 1% errore per $\alpha = 14°$
- $c\alpha \to 1$ ha 1% errore per $\alpha = 8,2°$

EQUAZIONE DELLA DINAMICA CARTESIANA AGGIORNATA: V

$$\begin{pmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -g \end{pmatrix} + \frac{1}{m} \begin{pmatrix} // & // & s\theta \\ // & // & s\varphi c\theta \\ // & // & c\varphi c\theta \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ T \end{pmatrix} =$$

$$= \begin{pmatrix} 0 \\ 0 \\ -g \end{pmatrix} + \frac{T}{m} \cdot \begin{pmatrix} s\theta \\ s\varphi c\theta \\ c\varphi c\theta \end{pmatrix}$$

STUDIO ACCELERAZIONE ANGOLARE

~~Supponiamo~~ Svolgo l'analisi rispetto BODY FRAME per semplicità.

$$I \dot{\omega} + \omega \times (I \omega) = M$$

- ω = velocità angolare rilevata dal drone $(p, q, r)^T$
- I = matrice di inerzia (rilevata in System Identification)
- M = momenti di forze esterni $= (\tau_x, \tau_y, \tau_z)^T$

L'accelerazione angolare si ottiene risolvendo il sistema di tipo $Ax = b$: $\rightarrow \dot{\omega} = (\dot{p}, \dot{q}, \dot{r})^T$

$$I \dot{\omega} = M - \omega \times I \omega$$

NOTA:

$V \times W = C(v) \cdot W$ dove $v, w = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

$$C(v) = \begin{bmatrix} 0 & -V_3 & V_2 \\ V_3 & 0 & -V_1 \\ -V_2 & V_1 & 0 \end{bmatrix} \rightarrow \text{cross-product Matrix}$$

VARIAZIONE ANGOLI DI EULERO DA VELOCITÀ ANGOLARI (VI)

$$\begin{pmatrix} \dot{\varphi} \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix} = \dot{u} \qquad \begin{pmatrix} p \\ q \\ r \end{pmatrix} = \omega' \rightarrow \text{Body Frame}$$

(aggiungo ' per specificare)

$$\omega' = E'_{123}(u)\,\dot{u}$$ → dove E'_{123} è la matrice di variazione degli angoli di Eulero coniugata (Body Frame) nella sequenza (1,2,3)

NB: ricordare che si differenzia dall'effettiva E'_{123} perché θ è invertito

$$\underline{\dot{u} = E'^{-1}_{123}\,\omega'}$$

$$E'_{123}(\phi,\theta,\psi) = \begin{bmatrix} 1 & 0 & +s\theta \\ 0 & c\phi & c\theta s\phi \\ 0 & -s\phi & c\theta c\phi \end{bmatrix}$$

NOTA
$\boxed{\varphi \equiv \phi}$

$$E'^{-1}_{123}(\phi,\theta,\psi) = \frac{1}{c\theta}\begin{bmatrix} c\theta & -s\phi s\theta & -c\phi s\theta \\ 0 & c\phi c\theta & -s\phi c\theta \\ 0 & s\phi & c\phi \end{bmatrix}$$

$$\begin{pmatrix} \dot{\varphi} \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix} = \frac{1}{c\theta}\begin{bmatrix} c\theta & -s\phi s\theta & -c\phi s\theta \\ 0 & c\varphi c\theta & -s\varphi c\theta \\ 0 & s\varphi & c\varphi \end{bmatrix} \cdot \begin{pmatrix} p \\ q \\ r \end{pmatrix}$$

MODELLO SISTEMA DINAMICO

- STATI

$$x^T = [x \quad y \quad z \quad \dot{x} \quad \dot{y} \quad \dot{z} \quad \varphi \quad \theta \quad \psi \quad \dot{\varphi} \quad \dot{\theta} \quad \dot{\psi}]$$

Sono tutti ottenibili dalle variabili di logging, inclusi $[\dot{\varphi} \; \dot{\theta} \; \dot{\psi}]^T$ che si ricavano dai noti $[p \; q \; r]^T$

- INGRESSI

$$u^T = [U_1 \quad U_2 \quad U_3 \quad U_4]$$

$U_1 \rightarrow$ forza totale rivolta verso l'asse z' positivo ($\underline{U_1 = T - mg}$)

$U_2 \rightarrow$ momento di ~~pitch~~ roll (τ_x)

$U_3 \rightarrow$ momento di pitch (τ_y)

$U_4 \rightarrow$ momento di yaw (τ_z)

- USCITE

$$y^T = [x \quad y \quad z \quad \varphi \quad \theta \quad \psi] \rightarrow$$ da cui si calcola l'errore dal riferimento da inseguire

- SISTEMA

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

A $\rightarrow$ matrice degli stati (di sistema)

B $\rightarrow$ " degli ingressi

C $\rightarrow$ " delle uscite

D $\rightarrow$ " di feed-forward (percorso)

VARIAZIONE DEGLI STATI

VIII

$$\dot{X}\begin{cases}\dot{x} = \dot{x}\\ \dot{y} = \dot{y}\\ \dot{z} = \dot{z}\\ \ddot{x} = \frac{T}{m} s\theta \xrightarrow{L} \frac{T}{m}\theta \cancel{\rightarrow \frac{T-mg}{m}}\\ \ddot{y} = \frac{T}{m} s\varphi\, c\theta \xrightarrow{L} \frac{T}{m}\varphi\\ \ddot{z} = \frac{T}{m} c\varphi\, c\theta - g \xrightarrow{L} \frac{T}{m} - g \rightarrow \frac{T-mg}{m} = \frac{U_1}{m}\\ \dot{\varphi} = \dot{\varphi}\\ \dot{\theta} = \dot{\theta}\\ \dot{\psi} = \dot{\psi}\\ \ddot{\varphi} = \frac{\tau_x}{I_x} = \frac{U_2}{I_x}\\ \ddot{\theta} = \frac{\tau_y}{I_y} = \frac{U_3}{I_y}\\ \ddot{\psi} = \frac{\tau_z}{I_z} = \frac{U_4}{I_z}\end{cases}$$

0 per linearizzazione

$$\ddot{x} \approx \frac{T}{m}\theta = \cancel{\frac{T-mg}{m}}\theta + \frac{mg}{m}\theta \approx g\theta$$

$$\ddot{y} \simeq \frac{T}{m}\varphi \xrightarrow{\text{IDEM}} \approx g\varphi$$

ESPRESSIONE MATRICIALE

$$\underbrace{\begin{bmatrix}\dot{P}\\ \ddot{P}\\ \dot{u}\\ \ddot{u}\end{bmatrix}}_{12\times1} = \underbrace{\begin{bmatrix} I^{3\times3} & \varnothing^{3\times3} & \varnothing^{3\times3} & \varnothing^{3\times3}\\ \varnothing^{3\times3} & \varnothing^{3\times3} & \begin{smallmatrix}\varnothing & g & \varnothing\\ g & \varnothing & \varnothing\\ \varnothing & \varnothing & \varnothing\end{smallmatrix} & \varnothing^{3\times3}\\ \varnothing^{3\times3} & \varnothing^{3\times3} & \varnothing^{3\times3} & I^{3\times3}\\ \varnothing^{3\times3} & \varnothing^{3\times3} & \varnothing^{3\times3} & \varnothing^{3\times3}\end{bmatrix}}_{12\times12} \cdot \underbrace{\begin{bmatrix}P\\ \dot{P}\\ u\\ \dot{u}\end{bmatrix}}_{12\times1} + \underbrace{\begin{bmatrix} & \varnothing^{3\times4} & & \\ \begin{smallmatrix}0\\0\\1/m\end{smallmatrix} & & \varnothing^{3\times3} & \\ & \varnothing^{3\times4} & & \\ 0 & 1/I_x & 0 & 0\\ 0 & 0 & 1/I_y & 0\\ 0 & 0 & 0 & 1/I_z\end{bmatrix}}_{12\times4} \cdot \underbrace{\begin{bmatrix}U_1\\ U_2\\ U_3\\ U_4\end{bmatrix}}_{4\times1}$$

~~VARIAN~~ RISOLUZIONE SISTEMA INGRB551

IX

$$U_1 = T - mg = f_1 + f_2 + f_3 + f_4 - mg$$

$$U_2 = \tau_x = -r_x f_1 - r_x f_2 + r_x f_3 + r_x f_4$$

$$U_3 = \tau_y = r_y f_2 - r_y f_2 - r_y f_3 + r_y f_4$$

$$U_4 = \tau_z = -c f_1 + c f_2 - c f_3 + c f_4$$

NB: $r_x = r_y$ distanza dal motore all'asse cartesiano, ovvero distanza tra 2 motori adiacenti/2

$$\bar{U} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -d & -d & d & d \\ d & -d & -d & d \\ -c & c & -c & c \end{bmatrix} \cdot \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{bmatrix} = \begin{bmatrix} mg \\ 0 \\ 0 \\ 0 \end{bmatrix}$$