

# Automatic Control

## Laboratory practice 3

M. Canale - S. M. Fosson

March 30, 2025

Objectives: evaluation of the steady state; step response of prototype models.

### Problem 1

---

Consider the LTI system described by the transfer function

$$H(s) = \frac{1}{s^3 + 2s^2 + 5.25s + 4.25}.$$

- (a) Analyse the BIBO stability;  
(b) If possible, compute the steady state response  $y_{ss}(t)$  in the presence of the input

$$u(t) = (3 \sin(0.1t) + 2)\varepsilon(t)$$

- (c) If possible, compute the maximum amplitude  $\bar{u}$  of a sinusoidal input of the form

$$u(t) = \bar{u} \sin(3t)\varepsilon(t)$$

such that the steady state response  $y_{ss}(t)$  satisfies  $|y_{ss}(t)| \leq 1$ .

#### Solution

- (a) The system is BIBO stable. Thus, the steady state response exists.  
(b)  $y_{ss}(t) = (0.7038 \sin(0.1t) - 0.1232) + 0.4706\varepsilon(t)$ .  
(c)  $\bar{u} \leq 17.7658$ .

### Problem 2

---

Consider the 2nd order system described by the transfer function

$$H(s) = \frac{10}{s^2 + 1.6s + 4}.$$

- 
- (a) evaluate the natural frequency  $\omega_n$ , the damping coefficient  $\zeta$  and the time constant  $\tau$  of the poles;
- (b) In MATLAB, plot the step response by using the function `step` and get the following data from the graph:

1. steady state value  $y_\infty$
2. maximum overshoot  $\hat{s}$
3. peak time  $\hat{t}$
4. rise time  $t_r$
5. 5% settling time  $t_{s,5\%}$

**Solution**

(a)  $\omega_n = 2$ ,  $\zeta = 0.4$ ,  $\tau = 1.25$  s

(b)  $y_\infty = 2.5$ ,  $\hat{s} = 25.37\%$ ,  $\hat{t} = 1.7$  s,  $t_r = 1.08$  s,  $t_{s,5\%} = 3.8$  s

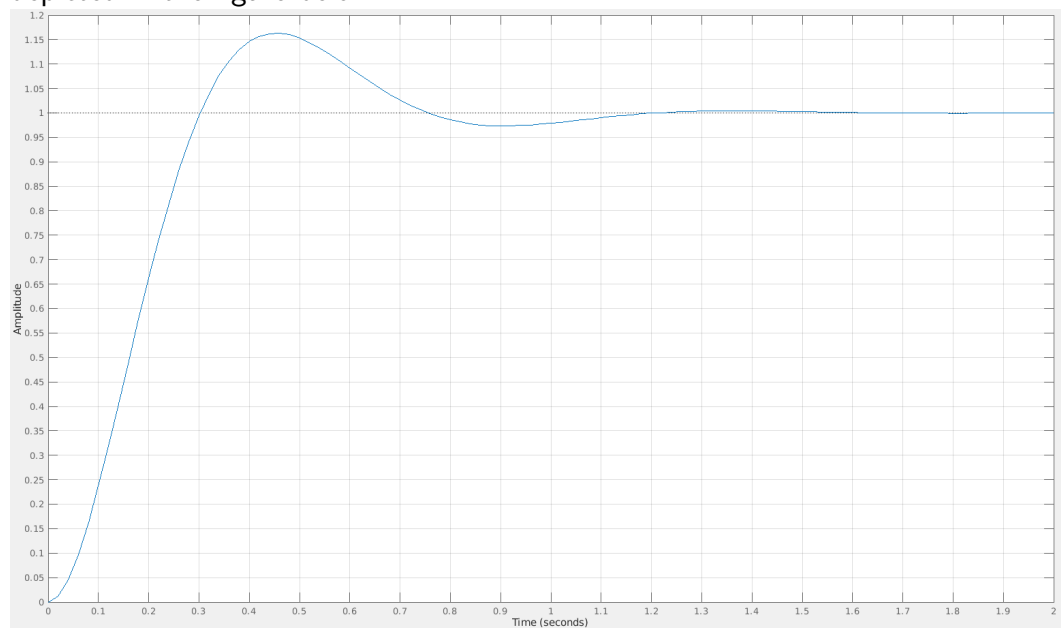
---

## Problem 3

Consider a 2nd order LTI system described by

$$H(s) = K \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}.$$

Its (zero state) output response in the presence of a step input of amplitude 5 is depicted in the figure below.



---

Compute the values of the parameters  $K$ ,  $\omega_n$  and  $\zeta$ .

**Solution**

$$K = 0.2, \omega_n = 8, \zeta = 0.5 \Rightarrow H(s) = \frac{12.8}{s^2 + 8s + 64}$$