

Automatic Control

Laboratory practice 4

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Part I - Objectives: analysis of Bode, polar/Nyquist, Nichols diagrams.
Warning: Bode plot in MATLAB may add ± 360 deg in the phase values.

Problem 1

Let us consider the following transfer functions:

1. $H(s) = \frac{-1}{(1+s)(1-s)^2}$

2. $H(s) = \frac{10}{s^2 \left(1 - \frac{s}{2}\right)^2}$

3. $H(s) = \frac{9(s-1)}{s(s^2-9)}$

For each of them, draw the asymptotic Bode plot by hand and draw the Bode plot with MATLAB. Based on the Bode plot, sketch the polar and Nyquist plots by hand. Finally, draw the Nichols diagram with MATLAB.

Problem 2

Given the transfer function

$$H(s) = \frac{1 + s^2}{(s^2 - 4)(s + 4)}$$

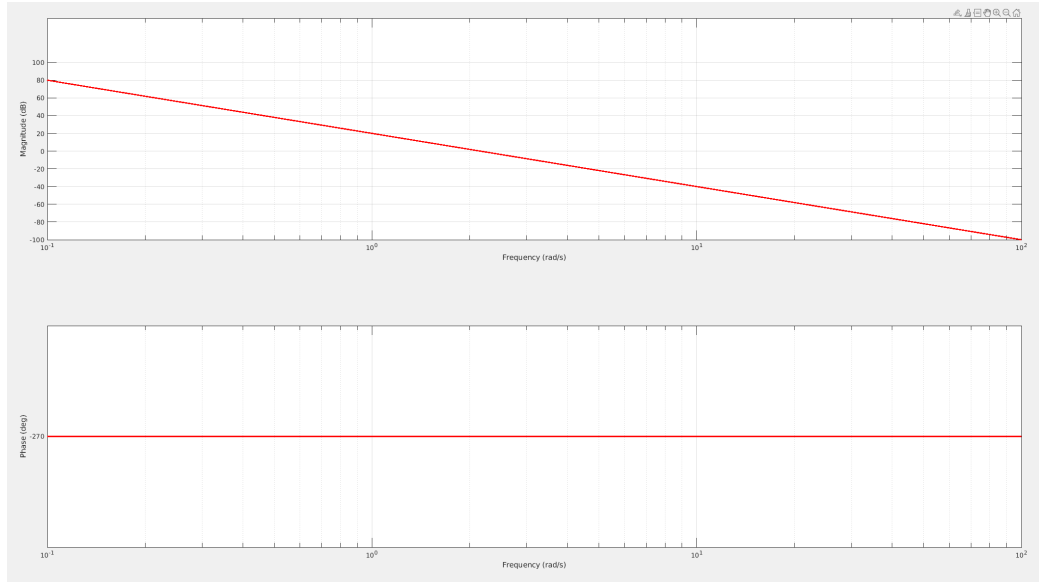
draw its Bode plot with MATLAB. Based on the Bode plot, sketch the polar and Nyquist plots by hand. Then, check their correctness with MATLAB. Finally, draw the Nichols diagram with MATLAB.

(Hints: $H(s)$ has two purely imaginary zeros, $\zeta = 0$.

$H(j\omega) = 0$ for $\omega = 1$, therefore the polar plot crosses the origin.)

Problem 3

Let us consider a minimal LTI system, with real singularities (zero and poles). Its asymptotic Bode plot is the depicted below.



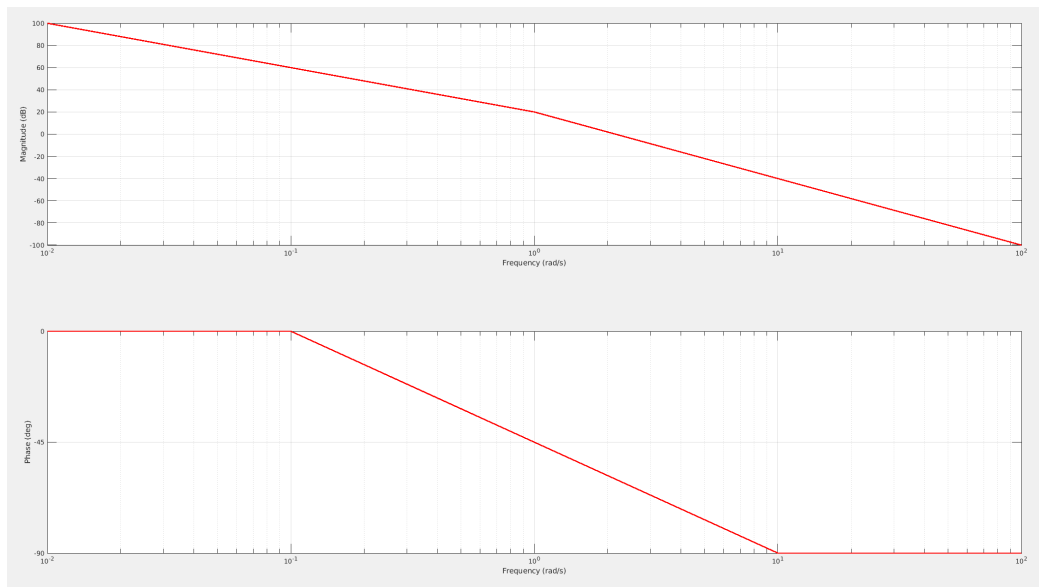
1. Is this system stable?
2. Compute its transfer function $H(s)$
3. Based on the Bode plot, sketch the polar and Nyquist plots by hand. Then, check their correctness with MATLAB. Finally, draw the Nichols diagram with MATLAB.

Solution

Unstable, $H(s) = \frac{10}{s^3}$.

Problem 4

Let us consider a minimal LTI system, with real singularities (zero and poles). Its asymptotic Bode plot is the depicted below (remark: in the phase plot, $0 = -360$ deg)



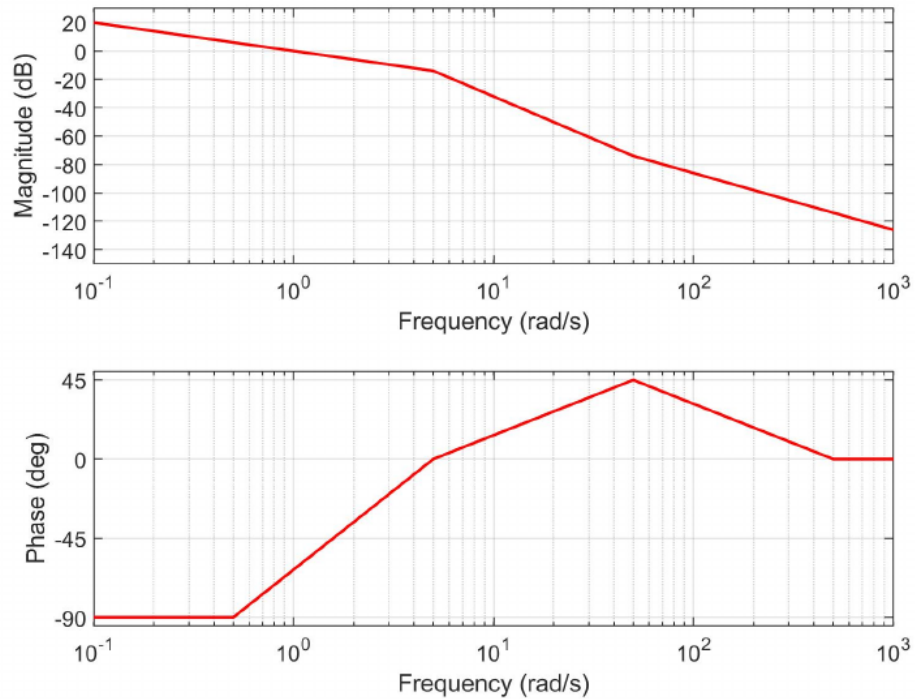
1. Is this system stable?
2. Compute its transfer function $H(s)$
3. Based on the Bode plot, sketch the polar and Nyquist plots by hand. Then, check their correctness with MATLAB. Finally, draw the Nichols diagram with MATLAB.

Solution

Unstable, $H(s) = \frac{-10}{s^2(1+s)}$.

Problem 5

Let us consider a minimal LTI system, with real singularities (zero and poles). Its asymptotic Bode plot is the depicted below.



1. Is this system stable?
2. Given the input $u(t) = 2\epsilon(t)$ compute the analytical expression of the output response $y(t)$.
3. If possible, evaluate the steady state $y_{ss}(t)$ in the presence of the input $u(t) = 2\epsilon(t)$.

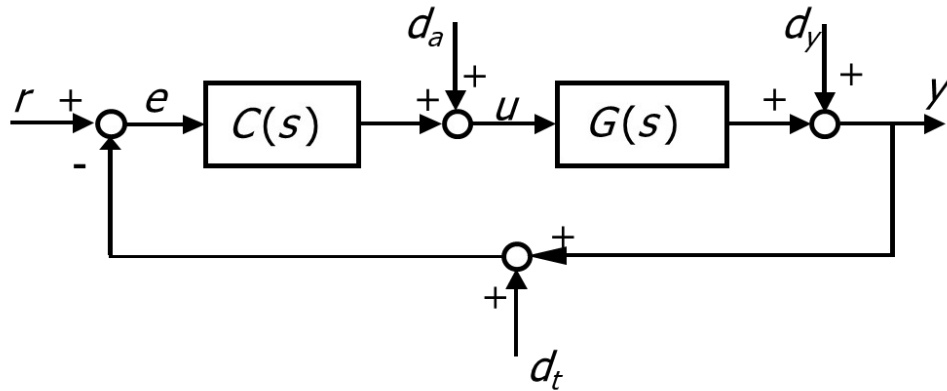
Solution

Unstable, no steady state.

$$y(t) = (0.76 + 2t - 0.76e^{5t} + 1.8te^{5t})\epsilon(t) \quad \left[H(s) = \frac{1 - \frac{s}{50}}{s(1 - \frac{s}{5})^2} \right]$$

Problem 5

Let us consider the feedback control system:



where $G(s) = \frac{1}{(1+s)^2}$ and $C(s) = \frac{(1+s)^2}{s(1+\frac{s}{4})}$. We assume that $G(s)$ and $C(s)$ are minimal.

1. Compute analytically the time response of the control input $u(t)$, when $r(t)$ is a unit step function and the other inputs $d_a(t)$ and $d_y(t)$ are set to zero.
2. Is this feedback control system well-posed?
3. Check the stability of the feedback control system by using the definition.
4. Check the stability of the feedback control system by using the Nyquist criterion.
5. If possible, compute analytically the steady state response $e_{ss}(t)$ of the tracking error $e(t)$ when $d_y(t) = 0.5 \sin(t)\epsilon(t)$, while the other inputs $r(t)$ and $d_a(t)$ are set to zero.
6. If possible, compute analytically the steady state response $y_{ss}(t)$ of the controlled output $y(t)$ when $r(t) = 3\epsilon(t)$, $d_y(t) = 2\epsilon(t)$ and $d_a(t) = 0$.
7. Perform a numerical simulation of the system with Simulink. Verify the analytical expressions obtained at 1., 5., 6. through the numerical results.

Solution

$u(t) = (3e^{-2t} - 2te^{-2t} + 1)\epsilon(t)$. The system is well-posed and stable.
 $e_{ss}(t) = 0.4123 \sin(t - 2.2531)$, $y_{ss}(t) = 3\epsilon(t)$.

Problem 6

Packet information flow in a router working under TCP/IP can be modelled, in a suitable working condition, through the following transfer function (see C. V. Hollot et al., "Analysis and Design of Controllers for AQM Routers Supporting TCP Flows", IEEE TRANSACTIONS ON AUTOMATIC CONTROL, VOL. 47, NO. 6, pp. 945-959, 2002)

$$G(s) = \frac{q(s)}{p(s)}, \quad q(s) = \frac{c^2}{2N}e^{-sR}, \quad p(s) = \left(s + \frac{2N}{R^2c}\right) \left(s + \frac{1}{R}\right)$$

where

- q = queue length (packets)
- p = probability of packet mark/drop
- c = link capacity (packets/sec)
- N = load factor (number of TCP sessions) item R = round trip time (sec)

The objective of an active queue management (AQM) algorithm is to choose automatically the packet mark/drop probability p , so that the sender can tune the window size to keep the queue length at a constant level. This system can be represented by the standard feedback structure reported in the previous figure.

Several AQM algorithms are available, but the one that has received special attention is the random early detection (RED) algorithm. The RED algorithm dynamics can be approximated through the linear controller

$$C(s) = \frac{\ell}{1 + \frac{s}{k}}.$$

1. Given the following controller and plant parameter values

$$\ell = 1.86 \cdot 10^{-4}, \quad k = 0.005, \quad c = 3750, \quad N = 60, \quad R = 0.246$$

build a suitable Simulink scheme to, simulate the given AQM. In particular, evaluate the transient performance in terms of maximum overshoot, 10 –

90% rise time and settling time 1%, the steady state tracking error and the maximum amplitude of the input magnitude $\max |u(t)|$ when the reference signal is a step function with amplitude 1 and the other inputs are set to 0.

2. Repeat the previous points with

$$C(s) = K_c \frac{1 + \frac{s}{z}}{s}, \quad K_c = 9.64 \cdot 10^{-6}, \quad z = 0.53$$

Solution

$$\begin{aligned} \hat{s} = 0, \quad t'_r &\approx 34.59 \text{ sec}, \quad t_{s,1\%} \approx 74.12, \quad |e_r^\infty| = 0.09, \quad \max |u(t)| = 1.67 \cdot 10^{-5} \\ \hat{s} = 0, \quad t'_r &\approx 2.99 \text{ sec}, \quad t_{s,1\%} \approx 6.49, \quad |e_r^\infty| = 0, \quad \max |u(t)| = 2.24 \cdot 10^{-5} \end{aligned}$$

Problem 7

Let us consider a feedback control system with loop transfer function

$$L(s) = \frac{1}{s \left(1 + \frac{s}{2}\right)^3}$$

1. Verify that the feedback control system is stable by using the Nyquist criterion
2. Verify that the feedback control system is stable by using the Nichols diagram of $L(s)$
3. Compute the gain margin and the phase margin
(hints: $\angle L(j\omega_0) = -180^\circ$ for $\omega_0 = 1.15 \text{ rad/s}$; $L(j\omega_c) = -1$ for $\omega_c = 0.8$)
4. Verify that the feedback control system with loop transfer function

$$L_\Delta(s) = 3L(s)$$

is unstable

Solution

Gain margin: 5 dB

Phase margin: 24.6 deg