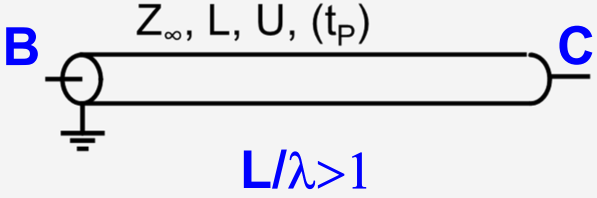
APPLIED ELECTONICS

Part C:

Class exercises 2 with solutions on:

- **Transmission lines**



Parameters

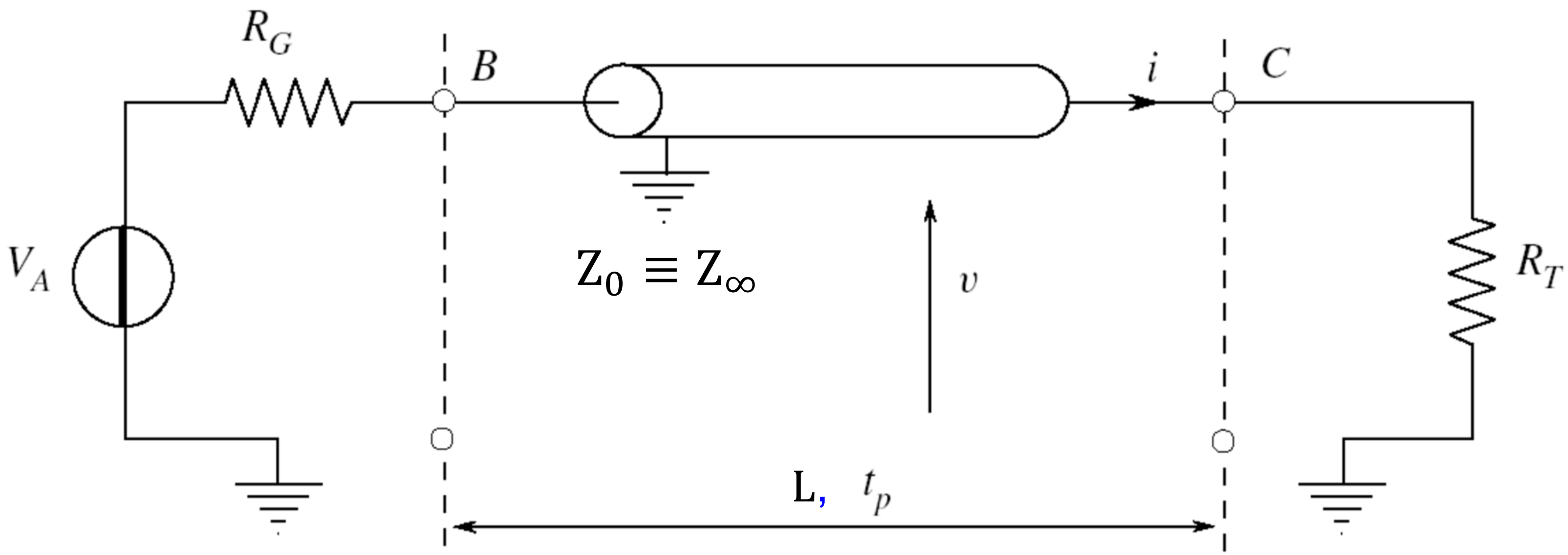
Z_∞: characteristic impedance,
L: length,
U: propagation speed
t_P: propagation time

Problem 1 - Assignment
Lattice diagram

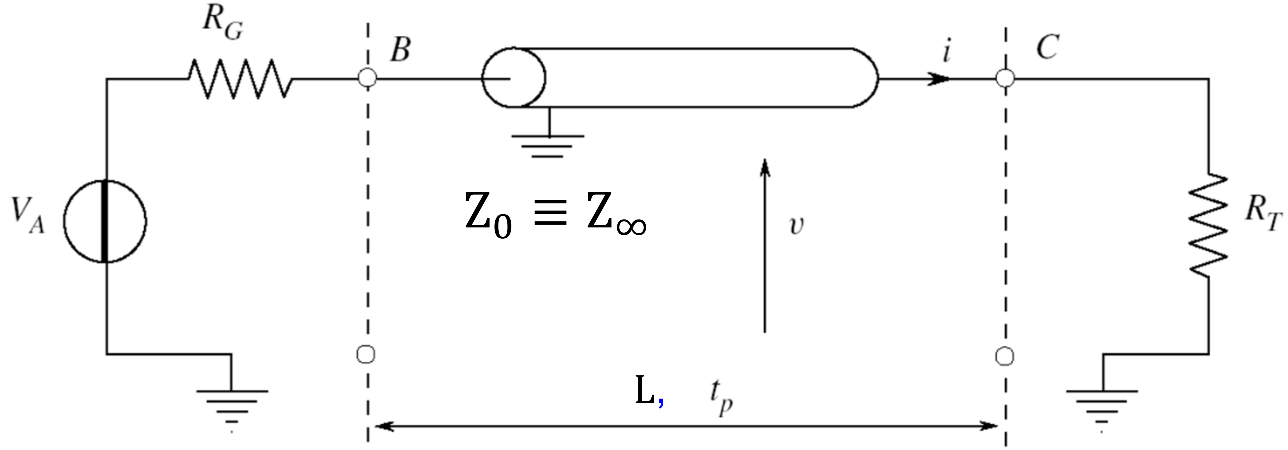
a) Use lattice diagram to plot the voltage at the receiver $V_C(t)$ and at the driver $V_B(t)$ in a $(0 - 4\,t_p)$ time range for L → H transition with V_A from $0\ V$ to $5\ V$ and:

$$R_G = 50\ \Omega,\ \ R_T = \infty,\ \ Z_\infty = 50\ \Omega,\ \ U = 0.8\ c,\ L = 20\ cm$$

b) Repeat the calculation with $R_G = 270\ \Omega$ and $R_G = 15\ \Omega$



Problem 1a,b - Solution



We first calculate the **reflection coefficients** $\Gamma_G \equiv \Gamma_B$ and $\Gamma_T \equiv \Gamma_C$ at the near and far-end respectively:

$$\Gamma_G = \frac{R_G - Z_\infty}{Z_\infty + R_G} = \begin{cases} 0, \text{with } R_G = 50\ \Omega \\ 0.69, \text{with } R_G = 270\ \Omega \\ -0.54, \text{with } R_G = 15\ \Omega \end{cases}; \qquad \Gamma_T = \frac{R_T - Z_\infty}{Z_\infty + R_T} = 1$$

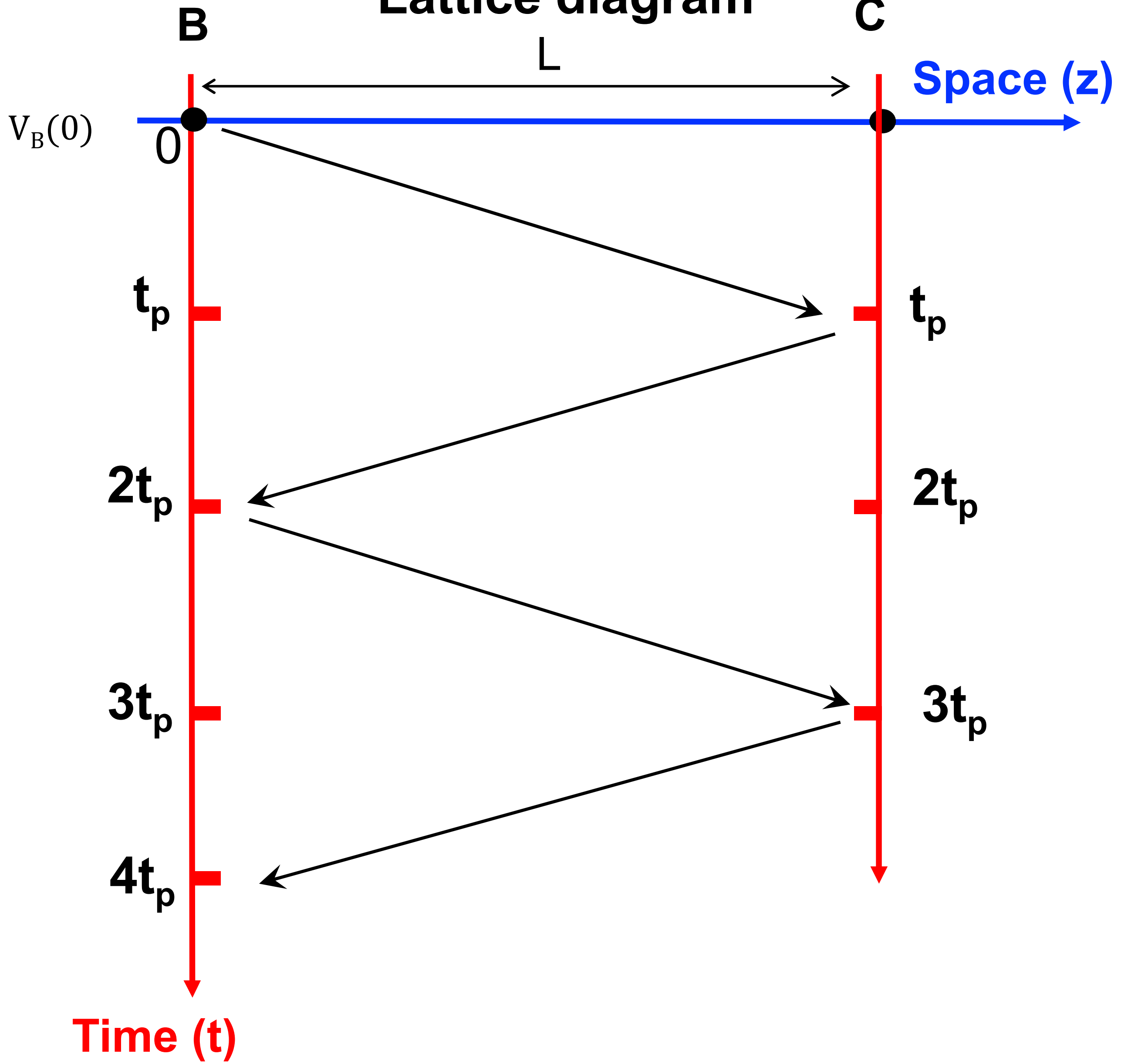
Moreover $t_p = \frac{L}{U} = 0.8$ ns.

At the time instant $t = 0$ the **voltage at the near-end** is:

$$V_B(0) = \frac{Z_\infty}{Z_\infty + R_G} V_A = \begin{cases} 2.5\text{ V}, \text{with } R_G = 50\ \Omega \\ 0.8\text{ V}, \text{with } R_G = 270\ \Omega \\ 3.85\text{ V}, \text{with } R_G = 15\ \Omega \end{cases}$$

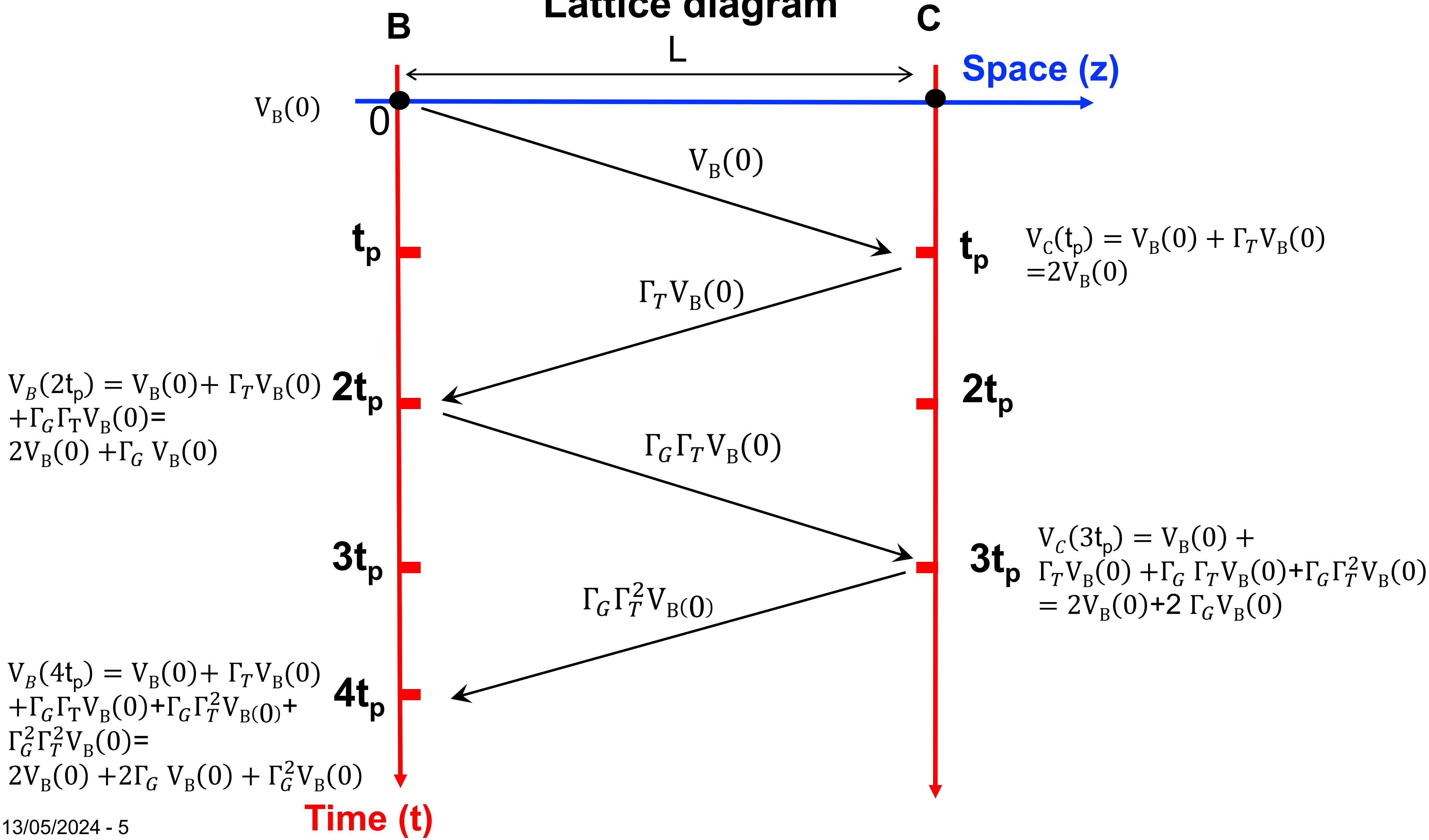
Problem 1 - Solution

Lattice diagram

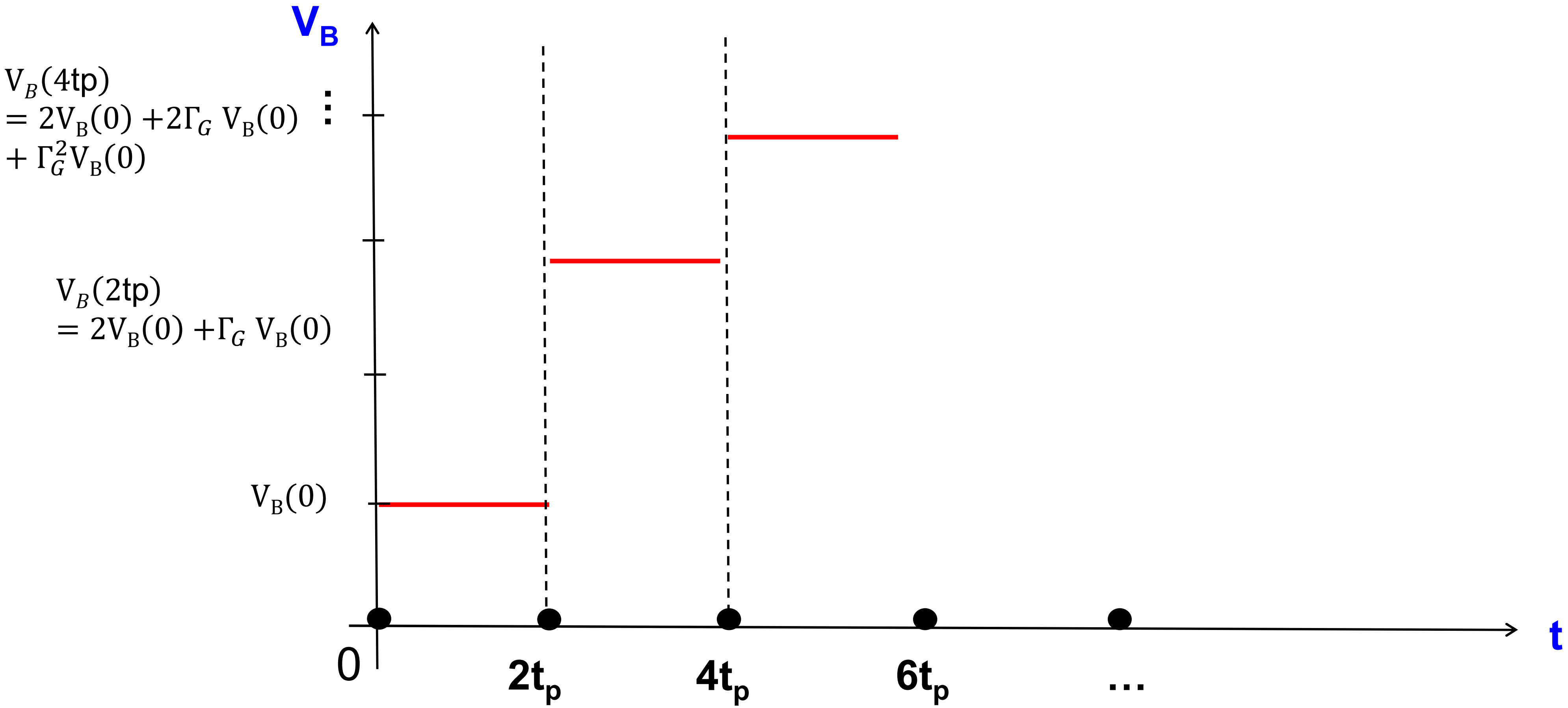


Problem 1 - Solution

Lattice diagram



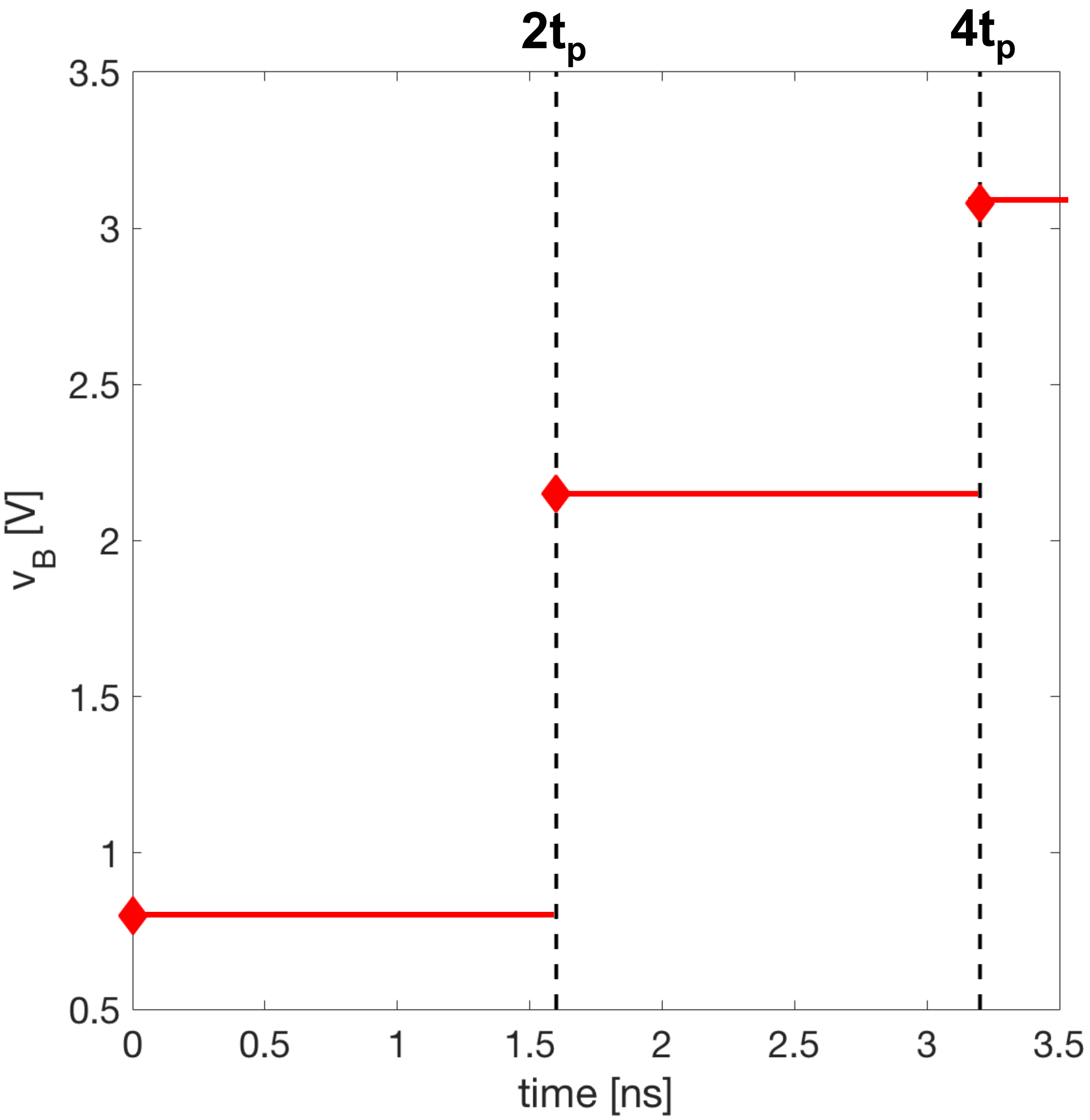
Problem 1a,b - Solution



Problem 1a,b - Solution

Staircase behaviour

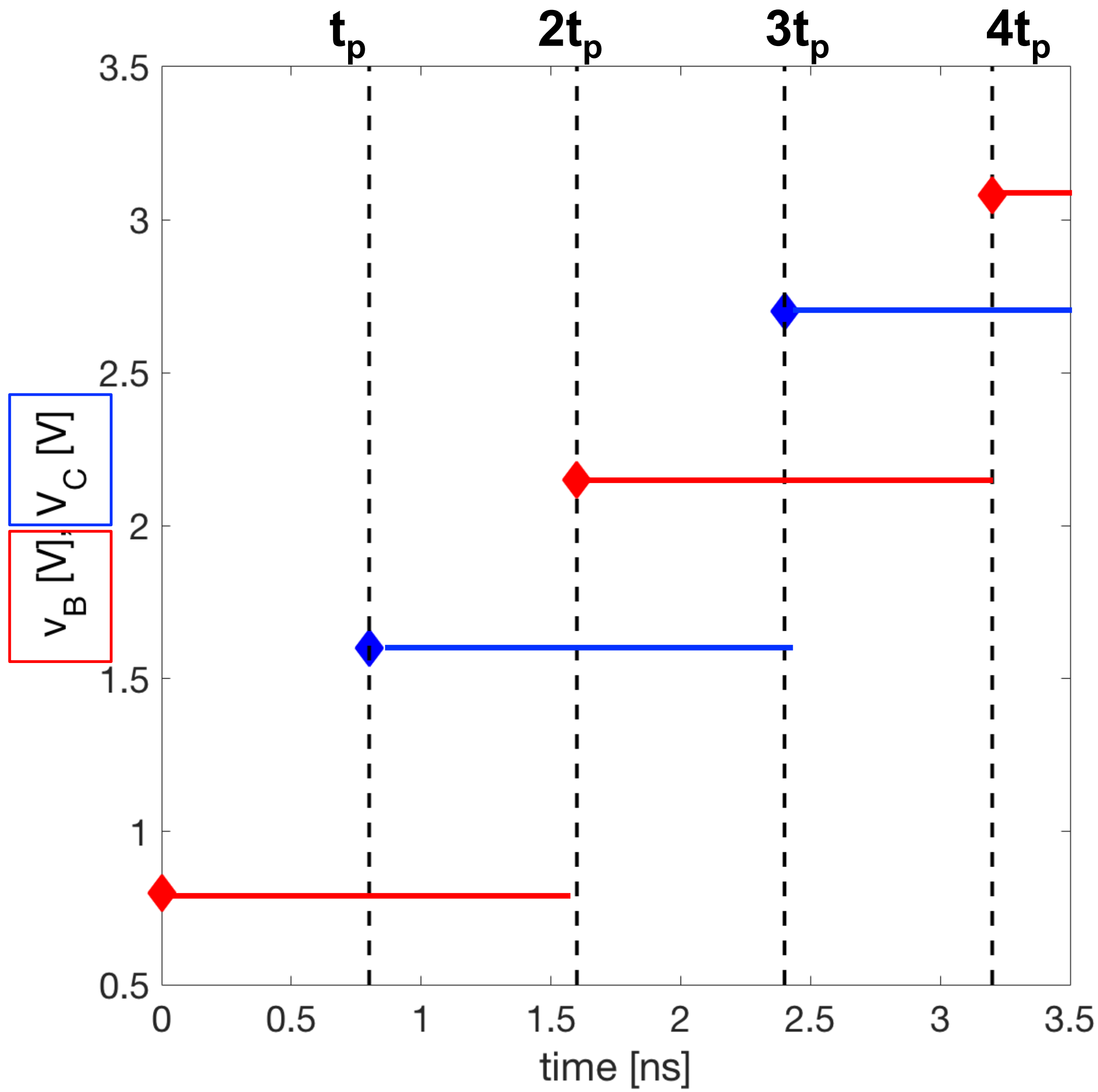
$R_G = 270\ \Omega$



Problem 1a,b - Solution

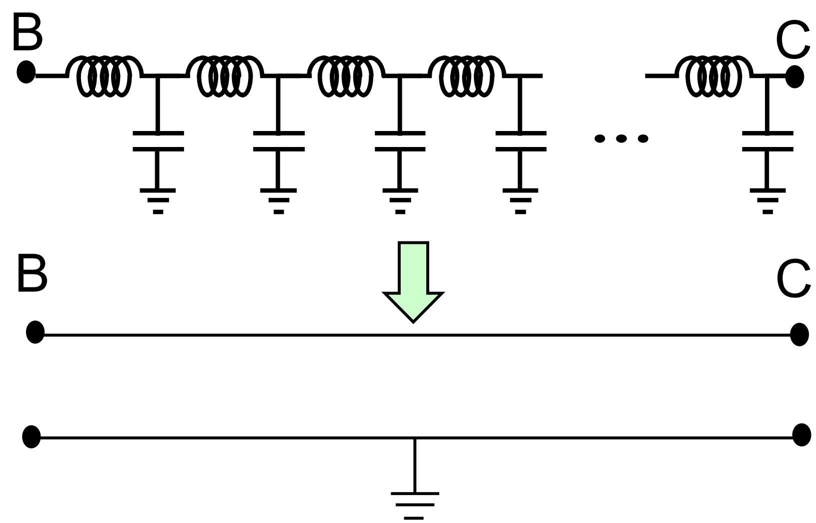
Staircase behaviour

$R_G = 270\ \Omega$



Note that at steady state the transmission line behaves as an equipotential wire. Thus for $\Gamma_T = 1$:

$$V_B(\infty) = V_C(\infty) = V_A$$



Problem 2 – Assignment
Incident Wave Switching

a) Calculate the driver output resistance (R_O) for IWS driving for a connection with:

L → H transition with $V_A = 0\ V \rightarrow 4\ V$
Receiver threshold $V_T = 2.5\ V$
Z_∞= 70 Ω and open circuit termination

b) Explain why this configuration can give multiple transition for a receiver placed at the far-end. Indicate how multiple transitions can be eliminated

c) Draw qualitatively the voltage $V_C(t)$ in the case a capacitor C_L is connected at the far end.

Problem 2a,b - Solution

Since **IWS** requires $V_B(0) > V_T$ for L→H transition and:

$$V_B(0) = \frac{Z_\infty}{Z_\infty + R_O} V_A$$

using the problem data, we get the following constraint on R_O:

$$\boxed{R_O < Z_\infty \left(\frac{V_A}{V_T} - 1 \right) = 70\ \Omega \left(\frac{4\ \text{V}}{2.5\ \text{V}} - 1 \right) = 42\ \Omega}$$

Since the reflection coefficient at the near-end (driver side) is given by:

$$\Gamma_B = \frac{R_O - Z_\infty}{Z_\infty + R_O} < 0$$

we get negative terms that sums up at the far-end (receiver side) that may cause the voltage at the far-end $V_C(t)$ to became smaller than V_T and in turn **multiple transitions**. Multiple transitions can be eliminated by matched far-end termination: $R_T = Z_\infty$.

Problem 2c – Hint for solution

For a first approximation analysis, the far-end capacitor with a capacitance C_L can be considered a short circuit when the step arrives at the termination ($\Gamma_T = -1$), and an open circuit ($\Gamma_T = 1$) after the transient. Therefore at $t = t_P$ (for the far-end) and $t = 2t_P$ (for the near-end) the waveform corresponds to **short circuit** at the far-end. For $t >>$ transient time associated to the capacitor charge the waveforms will correspond to an **open line**. During the transient at termination we expect an exponentially increasing voltage V_C.

<u>NOTE: We will see an example of this behaviour during LAB2</u>

Problem 3 – Assignment

Transmission times and total skew time

Consider a backplane with $L_U = 8\ \mathrm{nH/cm}$, $Z_\infty = 85\ \Omega$ (without capacitive load), length $L = 48\ \mathrm{cm}$, open circuit termination, and 24 connectors. Each board that can be inserted in the connectors has an input capacitance of $35\ \mathrm{pF}$. The system can have from 2 to 24 connected boards.

Driver/receiver CMOS: $V_{DD} = 3.3\ \mathrm{V}$; $R_O = 95\ \Omega$; $V_{IH} = 2\ \mathrm{V}, V_{IL} = 1\ \mathrm{V}$.

a) Calculate t_p in the cases of 2 and 24 connected boards.

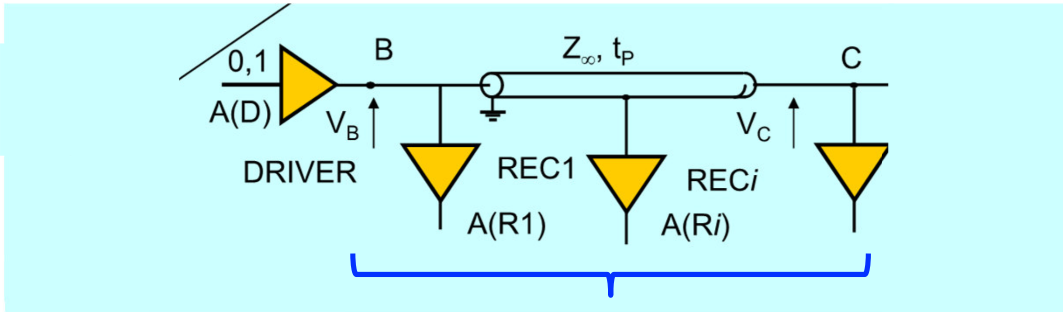
b) Calculate t_{TXmin} and t_{TXmax} for 2 connected boards in the two extremes of the line

c) Calculate t_{TXmin} and t_{TXmax} with 24 connected boards

d) Calculate the maximum R_{OH} to drive the line in IWS mode in the case of 24 inserted boards

Problem 3a - Solution

a) Consider the following **Backplane scheme**:



Max 24 boards equally spaced

If consider **2 boards** connected to the transmission line and we assume that this configuration **does not load the line with a capacitance load**:

$$t_p = \frac{L}{U} \text{ where } U = \frac{1}{\sqrt{L_U C_U}} \text{ and } C_U = \frac{L_U}{Z_\infty^2}$$

Using the problem data:

$$C_U = \frac{8\ \text{nH/cm}}{(85\ \Omega)^2} = 1.1\ \text{pF/cm}; U = \frac{1\ \text{cm}}{\sqrt{8\ \text{nH} \times 1.1\ \text{pF}}} = 0.11 \times 10^{11}\ \frac{\text{cm}}{\text{s}}$$

then:

$$\boxed{t_p = \frac{48\ \text{cm}}{1.1 \times 10^{10}\ \frac{\text{cm}}{\text{s}}} = 4.4\ \text{ns}}$$

Problem 3a - Solution

In case of a number of boards $n > 2$ the total capacitance per unit length is:

$$C'_U = C_U + \frac{C_i n}{L}$$

from which:

$$U' = \frac{1}{\sqrt{L_U C'_U}}; \; t'_p = \frac{L}{U'}$$

Using the problem data:

$$C'_U = \frac{8\ \mathrm{nH/cm}}{(85\ \Omega)^2} + \frac{35\ \mathrm{pF} \times 24}{48\ \mathrm{cm}} = 18.6\ \frac{\mathrm{pF}}{\mathrm{cm}}; U' = \frac{1\ \mathrm{cm}}{\sqrt{8\ \mathrm{nH} \times 18.6\ \mathrm{pF}}}$$

$$= 0.026 \times 10^{11}\,\frac{\mathrm{cm}}{\mathrm{s}}$$

then:

$$t'_p = \frac{48\ \mathrm{cm}}{0.026 \times 10^{11}\,\frac{\mathrm{cm}}{\mathrm{s}}} = 18.46\ \mathrm{ns}$$

Problem 3b - Solution

b) In presence of **2 connected boards at the extremes** of the line in order to determine t_{TXmax} and t_{TXmin} for L → H transition we consider that the voltage at the near-end and far-end at multiple of t_p. At near-end we have:

$$V_B(0) = \frac{Z_\infty}{Z_\infty + R_O} V_{DD} = \frac{85\ \Omega}{85\ \Omega + 95\ \Omega} 3.3\ V = 1.4\ V > V_{IL} \Longrightarrow \mathbf{t_{TXmin,B} = 0}$$

$$V_B(2t_p) = V_B(0) + \Gamma_C V_B(0) + \Gamma_B \Gamma_C V_B(0) = 3.2\ V > V_{IH} \Longrightarrow \mathbf{t_{TXmax,B} = 2t_p}$$

where we used:

$$\Gamma_B = \frac{R_O - Z_\infty}{Z_\infty + R_O} = \frac{95\ \Omega - 85\ \Omega}{85\ \Omega + 95\ \Omega} = 0.06; \Gamma_C = \frac{+\infty - Z_\infty}{Z_\infty + \infty} = 1\ (\text{open circuit})$$

The voltage at the far-end is:

$$V_C(t_p) = 2V_B(0) = 2.8\ V > V_{IH} \Longrightarrow \mathbf{t_{TXmax,C} = t_p}$$

Finally the total **skew time** is: $\boxed{t_k = t_{Txmax} - t_{Txmin} = 2t_p = 2 \times 4.4\ \text{ns} = 8.8\ \text{ns}}$

Problem 3c - Solution

c) In presence of **24 boards** we have:

$$Z'_\infty = \sqrt{\frac{L_U}{C'_U}} = \sqrt{\frac{L_U}{C_U}}\sqrt{\frac{1}{\left(1+\frac{nC_i}{C_U L}\right)}} = \frac{Z_\infty}{4.1} = 20.7\ \Omega;\ \ \Gamma'_B = \frac{R_O - Z'_\infty}{Z'_\infty + R_O} = \frac{95\ \Omega - 20.7\ \Omega}{20.7\ \Omega + 95\ \Omega}$$

$$= 0.64$$

In order to determine t_{TXmax} and t_{TXmin} we calculate the voltage at the near-end and far-end at multiple of t'_p = 18.46 ns that we have calculated at point a):

Problem 3c - Solution

c) In presence of **24 boards** we have:

$$Z'_\infty = \sqrt{\frac{L_U}{C'_U}} = \sqrt{\frac{L_U}{C_U}}\sqrt{\frac{1}{\left(1+\frac{nC_i}{C_U L}\right)}} = \frac{Z_\infty}{4.1} = 20.7\ \Omega;\ \ \Gamma'_B = \frac{R_O - Z'_\infty}{Z'_\infty + R_O} = \frac{95\ \Omega - 20.7\ \Omega}{20.7\ \Omega + 95\ \Omega}$$
$$= 0.64$$

In order to determine t_{TXmax} and t_{TXmin} we calculate the voltage at the near-end and far-end at multiple of t'_p = 18.46 ns that we have calculated at point a):

$$V_B(0) = \frac{Z'_\infty}{Z'_\infty + R_O} V_{DD} = \frac{20.7\ \Omega}{20.7\ \Omega + 95\ \Omega} 3.3\ V = 0.6\ V < V_{IL} \Longrightarrow \text{no switch}$$

$$V_C(t'_p) = V_B(0) + \Gamma_C V_B(0) = 1.2\ V > V_{IL} \Longrightarrow \mathbf{t_{TXmin,C} = t'_p = 18.46\ ns}$$

$$V_B(2t'_p) = V_B(0) + \Gamma_C V_B(0) + \Gamma'_B \Gamma_C V_B(0) = 1.58\ V < V_{IH},\ 1.58\ V > V_{IL}$$
$$\Longrightarrow \mathbf{t_{TXmin,\mathit{B}} = 2t'_p}$$

$$V_C(3t'_p) = V_B(0) + \Gamma_C V_B(0) + \Gamma'_B \Gamma_C V_B(0) + \Gamma_C \Gamma_C \Gamma'_B V_B(0) = 1.97\ V < V_{IH}$$

Problem 3d - Solution

Analogously:

$$V_B(4t'_p) = V_B(0) + \Gamma_C V_B(0) + \Gamma'_B \Gamma_C V_B(0) + \Gamma_C^2 \Gamma'_B V_B(0) + \Gamma_C^2 \Gamma_B'^2 V_B(0) = 2.21\ \mathrm{V} > V_{IH}$$

$$\Rightarrow \mathbf{t_{TXmax,B} = 4t'_p}$$

$$V_C(5t'_p) = V_B(0) + \Gamma_C V_B(0) + \Gamma'_B \Gamma_C V_B(0) + \Gamma_C^2 \Gamma'_B V_B(0) + \Gamma_C^2 \Gamma'_B \Gamma'_B V_B(0) + \Gamma_C^3 \Gamma'_B \Gamma'_B V_B(0)$$

$$= 2.45\ \mathrm{V} > V_{IH} \Rightarrow \mathbf{t_{TXmax,C} = 5t'_p = 5 \times 18.46\ ns = 92.3\ ns}$$

Finally, the total **skew time** is: $t_k = t_{TXmax} - t_{TXmin} = 4t'_p = 4 \times 18.46\ \mathrm{ns} = 73.84\ \mathrm{ns}$

Note that all the intermediate receivers have $t_{TXmin,i} > t_{TXmin,C}$ (since $t_{TXmin,B} > t_{TXmin,C}$) and $t_{TXmax,i} < t_{TXmax,C}$ (since $t_{TXmax,B} < t_{TXmax,C}$).

d) In the case of **24 inserted boards** we have:

$$V_B(0) = \frac{Z'_\infty}{Z_\infty + R_{OH}} V_{DD} = \frac{20.7\ \Omega}{20.7\ \Omega + R_{OH}} 3.3\ \mathrm{V}$$

Hence the values of R_{OH} that **drive a IWS** are given by the following relation:

$$\frac{20.7\ \Omega}{20.7\ \Omega + R_{OH}} 3.3\ V > V_{IH} = 2\ \mathrm{V}$$

Then their maximum value is:

$$\frac{3.3\ \mathrm{V} \times 20.7\ \Omega}{\ldots}$$

Problem 4 - Assignment

A track on a PCB backplane has characteristic impedance $Z_0 = 95\ \Omega$ (with no load), wave propagation speed $U = 0.65\ c$. The track length is $L = 30\ \text{cm}$, without terminations, and 15 equally spaced devices are connected to the track. The total capacitive load of these devices increases the distributed track capacitance (towards GND) by a factor 20 (loaded unity capacitance = unloaded capacitance $\text{x}\ 20$).
The interface uses CMOS circuits, with power supply $5\ \text{V}$, and the following parameters: $V_{OH} = 4\ \text{V}, I_{OH} = -16\ \text{mA}, V_{OL} = 0.8\ \text{V}, I_{OL} = 16\ \text{mA}, V_{IH} = 2.7\ \text{V}, V_{IL} = 1.3\ \text{V}$.
Further assumptions: the PCB tracks can be considered lossless transmission lines, and linear equivalent circuits can be used for drivers and receivers.

Problem 4 - Assignment

a) Find the characteristic impedance Z_0' and the propagation speed U' of the loaded track, and evaluate the propagation time t_P over the full length of the connection, for a fully loaded track.
(propagation speed $U = 1/\sqrt{L_U C_U}$)

b) Find the equivalent output resistance of drivers, for H and L states (respectively R_{OH} and R_{OL}), and find the minimum and maximum transmission times t_{TXmin} and t_{TXmax} from a driver placed at one end and a receiver placed at any position along the connection, for fully loaded track without termination in case of L→H transition.

c) Evaluate the equivalent driver output resistance R_{OH}' required to operate a receiver connected at any intermediate point of the track in IWS (Incident Wave Switching) for the L → H transition, with $NM = 100\ mV$, and line driven from one end.
Using drivers with equivalent output resistance R_{OH}', what should be connected at the opposite end to guarantee correct operation?

Problem 4 - Assignment

d) Draw the Information and Control signals (INF, STB) at Driver and at Receiver, and the destination register clock CK for a **synchronous** write cycle and evaluate cycle duration with the following parameters.
- Interconnection: $t_K = 25\ \mathrm{ns}, t_{TXmin} = 20\ \mathrm{ns}$
- Receiver register: $t_{SU} = 10\ \mathrm{ns}, t_H = 5\ \mathrm{ns}$

e) Draw the Information and Control signals (INF, STB, ACK) at Driver and at Receiver, and the destination register clock CK for an **asynchronous** transfer cycle and evaluate cycle duration with the following parameters.
- Interconnection: $t_K = 25\ \mathrm{ns}, t_{TXmin} = 20\ \mathrm{ns}$
- Receiver register: $t_{SU} = 10\ \mathrm{ns}, t_H = 5\ \mathrm{ns}$

Problem 4a - Solution

a) We first obseve that:

$$U = 0.65\ c = 0.65 \times 3 \times 10^8 \frac{m}{s} = 0.65 \times 3 \times 10^8 \times 10^2 \times 10^{-9} \frac{cm}{ns} = 19.5 \frac{cm}{ns}$$

From the definition of the **characteristic impedance**, denoting with $C'_U = 20C_U$ the loaded unity capacitance, in case of loaded track:

$$\boxed{Z'_0 \equiv Z'_\infty = \sqrt{\frac{L_U}{C'_U}} = \sqrt{\frac{1}{20}}\sqrt{\frac{L_U}{C_U}} = \sqrt{\frac{1}{20}}\, Z_O = 21.24\ \Omega}$$

Analogously from the definition of the **propagation speed,** in case of loaded track:

$$U' = \sqrt{\frac{1}{L_U C'_U}} = \sqrt{\frac{1}{20}}\sqrt{\frac{1}{L_U C_U}} = \sqrt{\frac{1}{20}}\, U = 4.36\ \frac{cm}{ns}$$

Thus in case of fully loaded track the **propagation time** t_P is:

$$\boxed{t_P = \frac{L}{U'} = \frac{30\ cm}{4.36\ cm/ns} = 6.88\ ns}$$

Problem 4b - Solution

b) The equivalent **output resistances of drivers**, for H and L states are given by:

High State $V_O = V_{DD} + I_{OH}R_{OH} \Longrightarrow$ $\boxed{R_{OH} = \frac{V_{DD} - V_{OH}}{-I_{OH}} = \frac{5\text{ V} - 4\text{ V}}{16\text{ mA}} = 62.5\ \Omega}$

Low State $V_O = R_{OL}I_O \Longrightarrow$ $\boxed{R_{OL} = \frac{V_{OL}}{I_{OL}} = \frac{0.8\text{ V}}{16\text{ mA}} = 50\ \Omega}$

We use the previous results to calculate the voltage V_B at the driver-end for L→ H transition at the initial time $t = 0$:

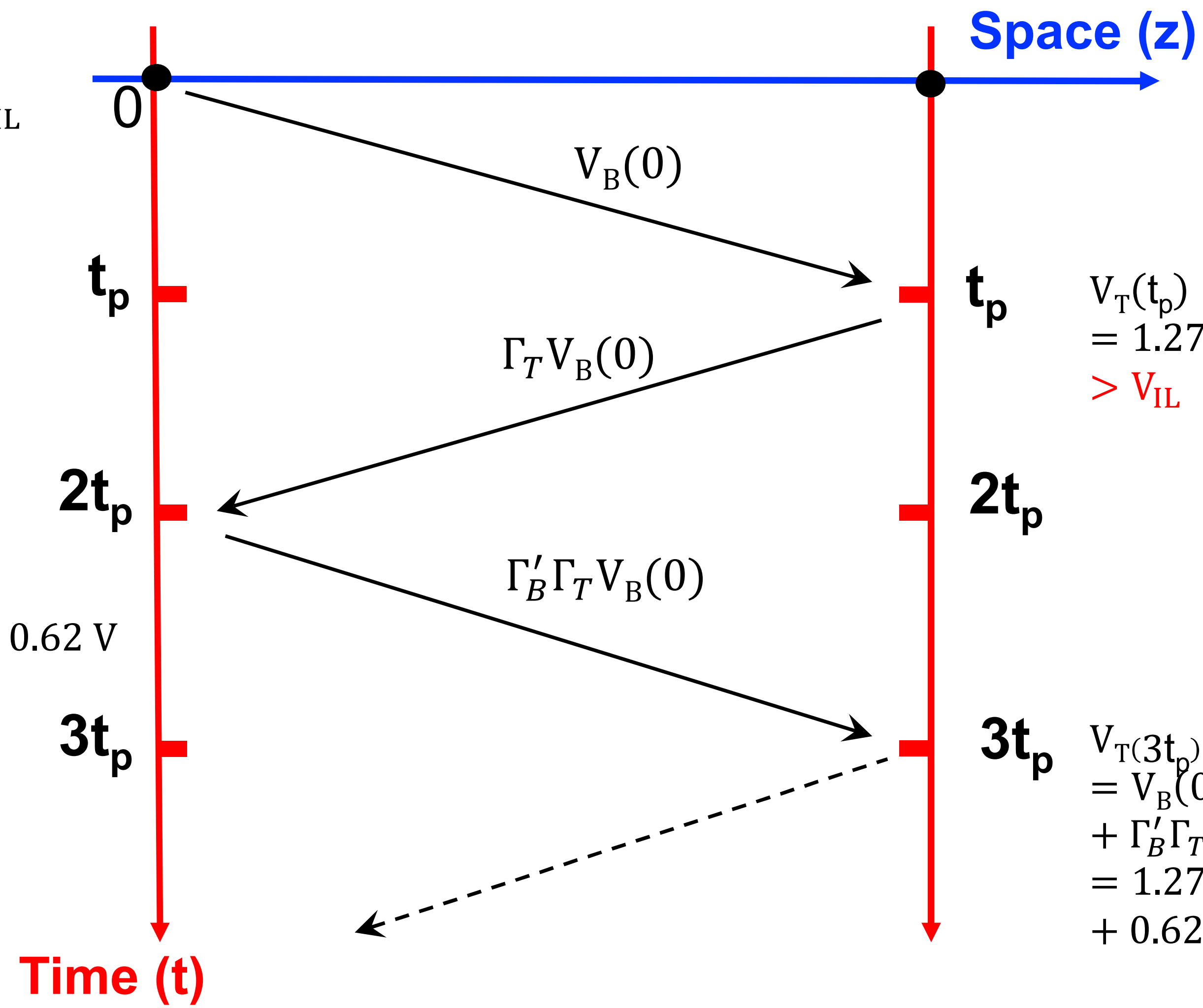
$$V_B(0) = \frac{Z_0'}{Z_0' + R_{OH}} V_{DD} = \frac{21.24\ \Omega}{21.24\ \Omega + 62.5\ \Omega} \times 5\text{ V} = 1.27\text{ V}$$

The reflection coefficients at the driver or near-end and at the line termination are respectively:

$$\boxed{\Gamma_B' = \frac{R_{OH} - Z_0'}{Z_0' + R_{OH}} = \frac{62.5\ \Omega - 21.24\ \Omega}{62.5\ \Omega + 21.24\ \Omega} = 0.49; \Gamma_T = 1}$$

Problem 4b - Solution

Using linear models and with the help of the **lattice diagram** we can evaluate the voltages at **driver**, at **termination**, and **along the line**.



$V_B(0) = 1.27\ \text{V} < V_{IL}$

$V_T(t_p) = V_B(0) + \Gamma_T V_B(0) = 1.27\ \text{V} + 1.27\ \text{V} = 2.54\ \text{V} > V_{IL}$

$V_B(2t_p) = V_B(0) + \Gamma_T V_B(0) + \Gamma_B' \Gamma_T V_B(0) = 1.27\ \text{V} + 1.27\ \text{V} + 0.62\ \text{V} = 3.16\ \text{V} > V_{IH}$

$V_T(3t_p) = V_B(0) + \Gamma_T V_B(0) + \Gamma_B' \Gamma_T V_B(0) + \Gamma_T \Gamma_B' \Gamma_T V_B(0) = 1.27\ \text{V} + 1.27\ \text{V} + 0.62\ \text{V} + 0.62\ \text{V} = 3.78\ \text{V} > V_{IH}$

Problem 4b - Solution

The following table describes what happens on the interconnection over time:

time	Voltage at driver (V)	Travelling wave (V)	Directi on	Voltage at termination (V)	Remarks
0	1,27				First step does not cross Vil
		1,27	D → T		First step travelling towards Termination
t_p				1,27+1,27 = 2,54	First step reflected from Term. First step + refl crosses Vil at Term. : Ttxmin = Tp
		1,27	D ← T		First reflection travels to Driver
$2t_p$	1,27+1,27+ 0,62 = 3,16				Reflected wave at driver; Vih crossed for RX at Driver
		0,62	D →T		Second refllect. towards Term
$3t_p$				2,54+0,62+0,62 = 3,78	Vih crossed for RX at Term Ttxmax = 3 Tp All RX switched, not worth to continue further the analysis

Problem 4b - Solution

Hence the **minimum** and **maximum propagation times for a receiver at the termination** are given by:

$$\boxed{t_{TX\,min} = 6.88\ \text{ns} \ \text{ and } \ t_{TXmax} = 3 \times 6.88\ \text{ns} = 20.64\ \text{ns}}$$

and thus we have the **skew time**:

$$t_k = t_{TX\,max} - t_{TXmin} = 13.76\ \text{ns}$$

This also corresponds to the maximum skew time of any intermediate receiver.

Note: For H → L transition, the value of R_{OL} is different, therefore also the first step amplitude and the reflection coefficient at the driver-end are different. All wave amplitudes must be evaluated with the new data.

Problem 4c - Solution

c) In order to have IWS at near-end point of the track the first step must be higher than:

$$V_{IH} + NM = 2.7\ V + 0.1\ V = 2.8\ V$$

Hence R'_{OH} must satisfy the relation:

$$2.8\ V = \frac{Z'_0}{Z'_0 + R'_{OH}} V_{DD} = \frac{21.24\ \Omega}{21.24\ \Omega + R'_{OH}} 5\ V$$

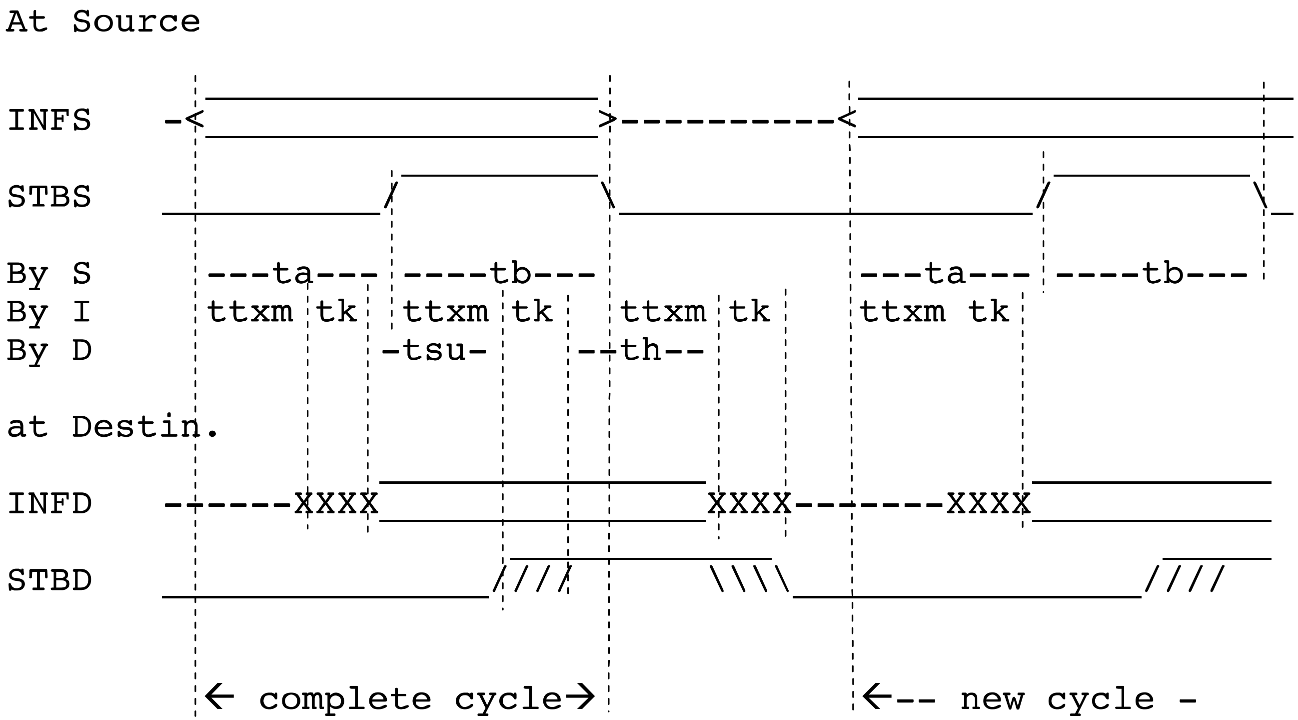
From which we obtain: $\boxed{R'_{OH} = 16.7\ \Omega}$

This value of R'_{OH} is lower than Z'_O, and causes negative reflection coefficient at driver, with oscillating waveforms on the line, and possible multiple crossing of thresholds. Reflections at far end must be avoided by placing a matched termination resistor, or at least reduced with a "almost matched" termination. Note that a driver at intermediate position is loaded by two Z'_O lines in parallel (left and right branch); the output resistance R''_{OH} must be lower than R'_{OH} in order to have IWS. Hence, in this case we get:

$$2.8\ V = \frac{Z'_O/2}{Z'_O/2 + R''_{OH}} V_{DD} = \frac{10.62\ \Omega}{10.62\ \Omega + R''_{OH}} 5\ V \Longrightarrow \boxed{R''_{OH} = 8.3\ \Omega}$$

Problem 4d - Solution

d) Timing diagram



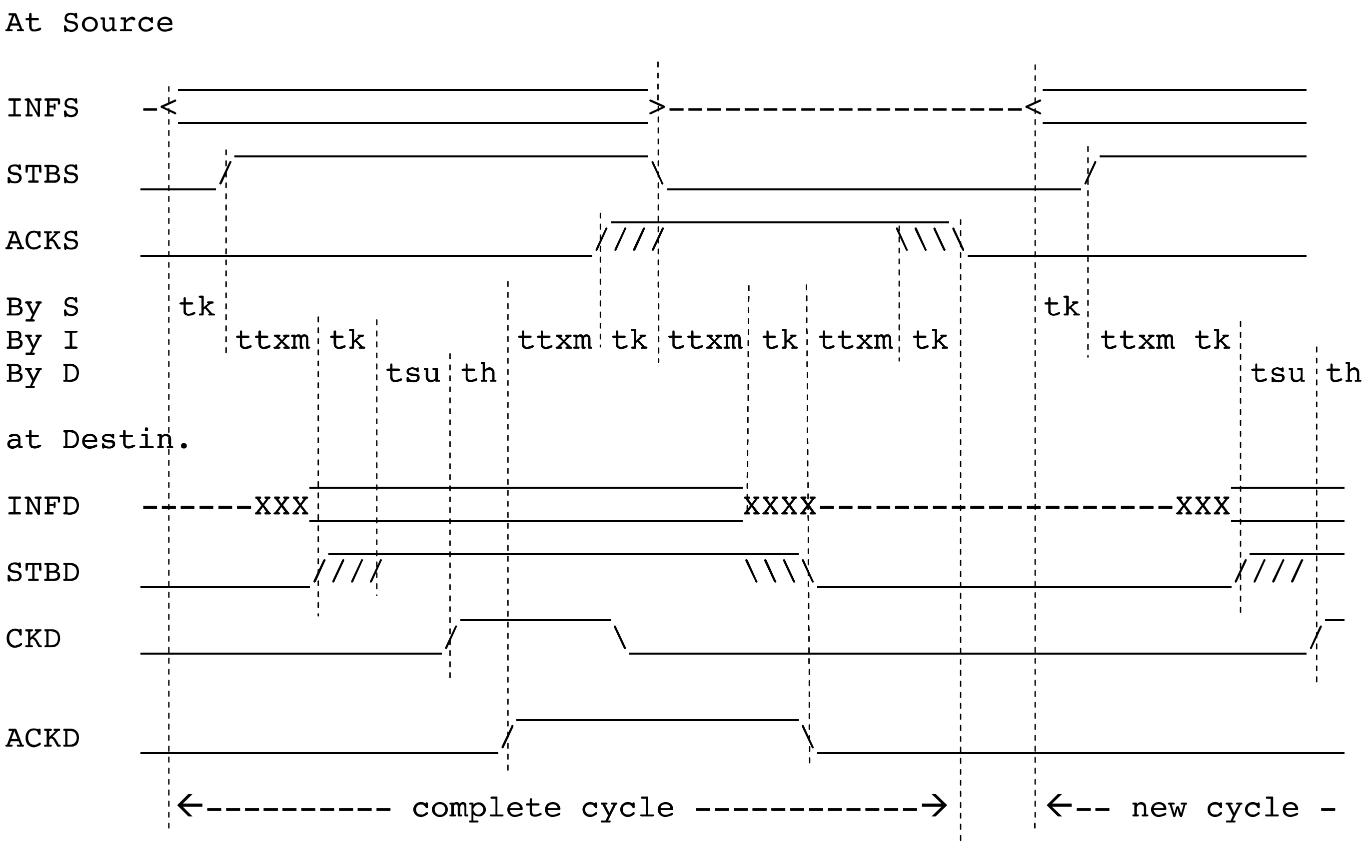
The **total cycle time** is:

$$T_{WR} = t_A + t_B = 2 \times t_K + t_H + t_{SU} = 65 \text{ ns}$$

We observe that the clock CK for the destination register is the Strobe signal STB (at destination). Total cycle time is independendent from t_{TXmin}, and related only with t_K, t_{SU}, and t_H.

Problem 4e - Solution

e) Timing diagram



The **total cycle time** is in the worst case ($t_{TX} = t_{TXmin} + t_k$):

$$\boxed{T_{WR} = 4 \times (t_K + t_{TXmin}) + t_K + t_H + t_{SU} = 220 \text{ ns}}$$

We observe that the CK for the destination register is CKD, generated by the **destination interface circuit**.