TABLE T.1
FOURIER TRANSFORMS

Definitions

Transform
$$V(f) = \mathscr{F}[v(t)] = \int_{-\infty}^{\infty} v(t)e^{-j2\pi ft}\,dt$$

Inverse transform
$$v(t) = \mathscr{F}^{-1}[V(f)] = \int_{-\infty}^{\infty} V(f)e^{j2\pi ft}\,df$$

Integral theorem

$$\int_{-\infty}^{\infty} v(t)w^*(t)\,dt = \int_{-\infty}^{\infty} V(f)W^*(f)\,df$$

Theorems

Operation	Function	Transform
Superposition	$a_1v_1(t) + a_2v_2(t)$	$a_1V_1(f) + a_2V_2(f)$
Time delay	$v(t - t_d)$	$V(f)e^{-j\omega t_d}$
Scale change	$v(\alpha t)$	$\frac{1}{\lvert\alpha\rvert}V\left(\frac{f}{\alpha}\right)$
Conjugation	$v^*(t)$	$V^*(-f)$
Duality	$V(t)$	$v(-f)$
Frequency translation	$v(t)e^{j\omega_c t}$	$V(f-f_c)$
Modulation	$v(t)\cos(\omega_c t + \phi)$	$\frac{1}{2}[V(f-f_c)e^{j\phi} + V(f+f_c)e^{-j\phi}]$
Differentiation	$\frac{d^n v(t)}{dt^n}$	$(j2\pi f)^n V(f)$
Integration	$\int_{-\infty}^{t} v(\lambda)\,d\lambda$	$\frac{1}{j2\pi f}V(f) + \frac{1}{2}V(0)\,\delta(f)$
Convolution	$v * w(t)$	$V(f)W(f)$
Multiplication	$v(t)w(t)$	$V * W(f)$
Multiplication by t^n	$t^n v(t)$	$(-j2\pi)^{-n}\frac{d^n V(f)}{df^n}$

Transforms

Function	$v(t)$	$V(f)$
Rectangular	$\Pi\left(\frac{t}{\tau}\right)$	$\tau \operatorname{sinc} f\tau$
Triangular	$\Lambda\left(\frac{t}{\tau}\right)$	$\tau \operatorname{sinc}^2 f\tau$
Gaussian	$e^{-\pi(bt)^2}$	$(1/b)e^{-\pi(f/b)^2}$
Causal exponential	$e^{-bt}u(t)$	$\dfrac{1}{b + j2\pi f}$
Symmetric exponential	$e^{-b\|t\|}$	$\dfrac{2b}{b^2 + (2\pi f)^2}$
Sinc	$\operatorname{sinc} 2Wt$	$\dfrac{1}{2W}\Pi\left(\dfrac{f}{2W}\right)$
Sinc squared	$\operatorname{sinc}^2 2Wt$	$\dfrac{1}{2W}\Lambda\left(\dfrac{f}{2W}\right)$
Constant	1	$\delta(f)$
Phasor	$e^{j(\omega_c t + \phi)}$	$e^{j\phi}\,\delta(f - f_c)$
Sinusoid	$\cos(\omega_c t + \phi)$	$\frac{1}{2}[e^{j\phi}\,\delta(f - f_c) + e^{-j\phi}\,\delta(f + f_c)]$
Impulse	$\delta(t - t_d)$	$e^{-j\omega t_d}$
Sampling	$\sum_{k=-\infty}^{\infty} \delta(t - kT_s)$	$f_s \sum_{n=-\infty}^{\infty} \delta(f - nf_s)$
Signum	$\operatorname{sgn} t$	$1/j\pi f$
Step	$u(t)$	$\dfrac{1}{j2\pi f} + \frac{1}{2}\,\delta(f)$