

**Prove that Language  $L = \{0^n: n \text{ is a perfect square}\}$  is irregular.**

**Solution:**

L is infinite. Suppose L is also regular. Then according to pumping lemma there exists an integer  $n$  such that for every string  $w$  in  $L$  where  $|w| \geq n$ , we can break  $w$  into three strings  $w = xyz$  such that:

$|y| > 0$  ,  $|xy| \leq n$  and for all  $k \geq 0$  , the string  $xy^kz$  is also in  $L$ .

Choose  $w$  to be  $w = 0^s$  where  $s = n^2$  that is it is a perfect square.

Let  $w = 0000000000000000 \dots 00000 = xyz$ . (The length of  $w = s = n^2$  in this case.)  
 Let  $|xy| \leq n$  and  $|y| = k$ .

That is  $w = 0000 \underline{0^k} 000\dots$

X
y
z

$$\text{So, } |xyz| = |xz| + |y| = (n^2 - k) + (k)$$

From pumping lemma, I can pump  $y$  any number of times and the new string should also belong to the language. Suppose I pump  $y$  twice then, the new string should belong to the language that is it should have length that is a perfect square but,

$$|xy^2z| = |xz| + 2|y| = (n^2 - k) + 2k = n^2 + k$$

where  $n^2 + k < n^2 + n$  ( $k \leq n$  as  $|xy| \leq n$  and  $|y| = k$ )

$$\begin{aligned}
 &< n^2 + 2n + 1 \\
 &= (n+1)(n+1)
 \end{aligned}$$

and  $n^2 + k > n^2$  (As  $k > 0$ )

$$\Rightarrow n^2 < n^2 + k < (n+1)^2$$

- ⇒  $n^2 + k$  is not a perfect square
- ⇒  $xy^2z$  is not in  $L$
- ⇒ This is a contradiction to the pumping lemma
- ⇒ So, our initial assumption must have been wrong that is  $L$  is not regular.



## Steps to solve Pumping Lemma problems:

1. If the language is finite, it is regular (quiz3-section1), otherwise it might be non-regular.
2. Consider the given language to be regular
3. State pumping lemma
4. Choose a string  $w$  from language, choose smartly :')
5. Partition it according to constraints of pumping lemma in a generic way
6. Choose a pumping factor  $k$  such that the new 'pumped' string is not part of the language given.
7. State the contradiction and end the proof.

## How to remember what pumping Lemma says:

(As ahsan mentioned)

**Pumping Lemma alternates between “for all” and “there is at least one” or “for every” or “there exists”. Notice:**

**For every** regular language  $L$

**There exists** a constant  $n$

**For every** string  $w$  in  $L$  such that  $|w| \geq n$ ,

**There exists** a way to break up  $w$  into three strings  $w = xyz$  such that  $|y| > 0$ ,  $|xy| \leq n$  and

**For every**  $k \geq 0$ , the string  $xy^kz$  is also in  $L$ .

## What does this imply?

- ⇒ **I can choose the language I want to prove** (this is trivial...obviously in exams you have no choice but you can pick up a language of choice during practice)
- ⇒ **I cannot choose  $n$  that is I have to keep it arbitrary; I have to use a generic value.** (So I cannot choose 4 or 16 as the length of my string It has to be defined as a function of  $n$ )
- ⇒ **I can choose  $w$ , with any combination of alphabets. The only constraint is that the chosen string should belong to the language under consideration.** (So I can have 000000... or 111111 ... or the combination of the two even if the alphabet comprises of any number of letters in the first or second solution as long as length chosen arbitrarily is perfect square...but I cannot have all aaaaa... in  $a^n b^n$  language since it does not belong to the language.. I can choose only those strings that conform to the language's general construction)
- ⇒ **I cannot choose the division of my string. It has to be arbitrary and general and should conform to the pumping lemma constraints.** (SO, I cannot choose the length of  $y$  to be 1 or 2 It has to be a generic, choose a variable  $k$  or something)
- ⇒ **I can choose my pumping factor  $k$ .** (This means that I can choose to pump any number of times I want to. I can choose to pump 0 times 1 time or even put a constraint on it like I have not pumped more than  $n$  times like sir did in class in the factorial proof). **If you want to complete a proof your pumping factor should lead to a contradiction to pumping lemma but the point is you can pump any times you want like in prime language proof in the book you could have pumped  $p-m$  times or  $2p-2m$  times or  $3p-3m$  times ...no constraint there.**

I hope this helps in clarifying confusions!!