Teoria dei sistemi.

Harmonic Response and Bode plots

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Response to harmonic signals

- We have seen that any LTI system responds to $\cos \omega t$ with

$$|H(jw)| \cos(\omega t + \angle H(jw))$$

- A different story is if the input signal is

$$u(t) = \mathbf{1}(t) \cos(\omega t)$$

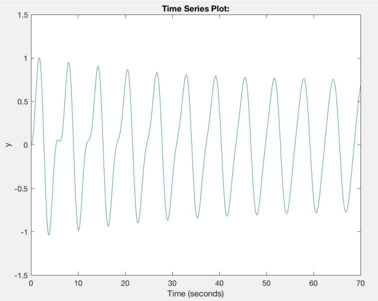
Response to harmonic signals (cont)

- In this case we can see that the response is given by

$$|H(jw)| \cos(\omega t + \angle H(jw)) + f(t)$$

where $f(t)$ is a terms containing the natural modes stemming from the poles of the system
- If the system is BIBO stable the term $f(t)$ vanishes after a transient phase and we end up, at the steady state, with the harmonic response term

$$|H(jw)| \cos(\omega t + \angle H(jw))$$



Frequency Response

Frequency Response

Proposition

The Laplace transform $H(s)$ evaluated at $s = j\omega$ for LTI stable systems accounts for the steady state response to a cosinusoidal signal. Its modulus multiplies the amplitude of the oscillation and the phase corresponds to the phase of the oscillation.

Meaning of poles and zeros

- The physical meaning of poles is quite clear: they give rise to exponential signals.
- As concerns the response to harmonic signals, the poles have an impact on the duration of the transient
- The ploes have a direct impact on the system's open loop stability
- The meaning of the zeros is way less clear.

Meaning of poles and zeros (cont.)

- If a zero is on the imaginary axis (say at $j\omega_0$), then a signal like $\cos\omega_0 t$ does not produce any response at the steady state.
- Generally speaking, for a zero z_i a signal having Laplace transform $U(s) = A/(s - z_i)$ is blocked (blocking property of the zeros).
- The zeros have important implications on the stability of the closed loop system and on the system's performance.

Frequency Response

- The frequency response can be estimated by using different harmonic signals, and measuring for each, amplitude and phase.
- From the experimental data on the frequency response it is possible to reconstruct the transfer function.
- The frequency response is the transfer function evaluated for $s = j\omega$:
$$G(j\omega) = G(s)|_{s=j\omega}$$
- It is now important to see how the shape of $G(j\omega)$ relates to the position of zeros and poles.

Graphical plots of the frequency response

- Since $G(j\omega)$ is a complex function of ω it can be represented in two ways:
  - Magnitude and phase plot (Bode diagrams)
  - Real and imaginary parts (Nyquist diagrams)

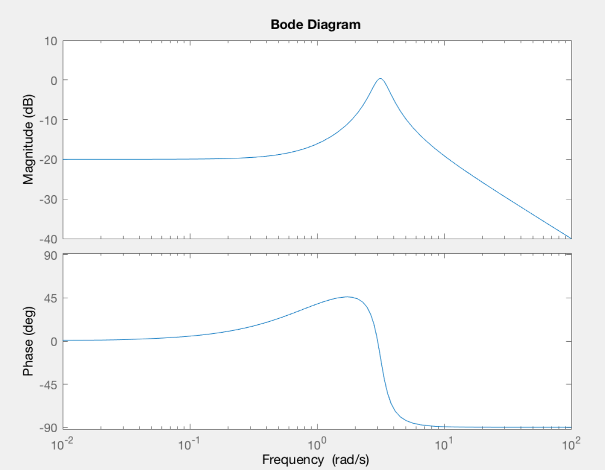
Bode Diagrams

- Bode plots (or diagrams) are graphical representation of the frequency response $G(j\omega)$, which use a logarithmic scale for ω
- They consist of
  - Magnitude plots: i.e., $|G(j\omega)|$ vs $\log\omega$
  - Phase plots: i.e., $\angle G(j\omega)$ vs $\log\omega$
- The use of logarithmic scale has two advantages:
  1. Possibility of spanning with ω across sevral orders of magnitude
  2. Possibility of creating complex siagrams by the simple composition of simpler ones.

Example

Example of bode plots

$$G(s) = \frac{s+1}{s^2+s+10} = |G(s)|\, e^{j\angle G(s)}$$



Magnitude

- The magnitude is defined as

$$|G(j\omega)|_{\mathrm{dB}} = 20\log_{10}|G(j\omega)|$$

- Examples
  - $|G(j\omega)| = 2 \rightarrow |G(j\omega)|_{\mathrm{dB}} = 6$
  - $|G(j\omega)| = 10 \rightarrow |G(j\omega)|_{\mathrm{dB}} = 20$
  - $|G(j\omega)| = 100 \rightarrow |G(j\omega)|_{\mathrm{dB}} = 40$

Bode Form

- In order to draw the Bode diagram, it is convenient to write the transfer function into the following Bode form

$$G(s) = K \frac{\prod_i (1+\tau_{z_i} s) \prod_i (1 + 2\frac{\zeta_i}{\alpha_{ni}} s + \frac{s^2}{\alpha_{ni}^2})}{s^g \prod_i (1+\tau_{p_i} s) \prod_i (1 + 2\frac{\xi_i}{\omega_{ni}} s + \frac{s^2}{\omega_{ni}^2})}$$

- Example

$$\begin{aligned} G(s) &= \frac{s+10}{s^2+4s+2} = \\ &= \frac{10(1+\frac{s}{10})}{2(1+2s+\frac{s^2}{2})} \\ &= 5\frac{(1+\frac{s}{10})}{1+2\frac{\sqrt{2}}{\sqrt{2}}s+\frac{s^2}{\sqrt{2}^2}} \end{aligned}$$

Basic Building blocks

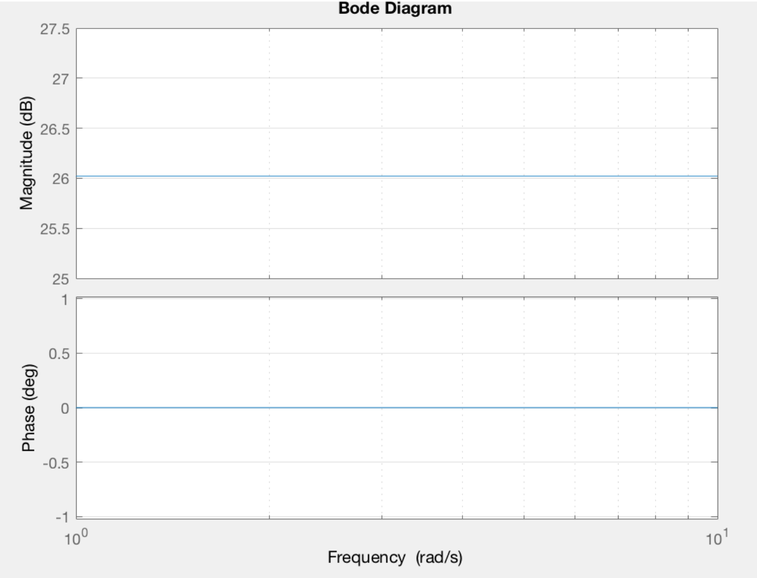
Based on the discussion above, we have the following terms that allow us to construct the bode plot:

- K: the gain, which can be positive or negative
- $s^{\pm 1}$: poles or zeros in the origing
- $(1 + \tau_{z_i} s)$, $(1 + \tau_{p_i} s)$: real zero, real pole
- $(1 + 2\frac{\zeta_i}{\alpha_{ni}} s + \frac{s^2}{\alpha_{ni}^2})$: complex conjugate zero pair
- $(1 + 2\frac{\xi_i}{\omega_{ni}} s + \frac{s^2}{\omega_{ni}^2})$: complex conjugate pole pair

We will now evaluate the contribution to the bode diagram of each of these terms, setting $s = j\omega$. The different contribution will then be composed together.

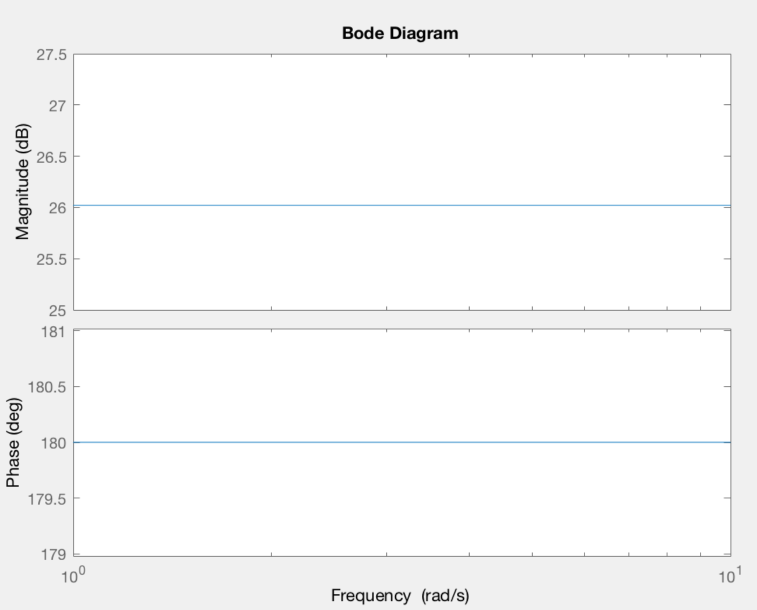
Basic Building blocks

Positive Constant ($K > 0$)



Basic Building blocks

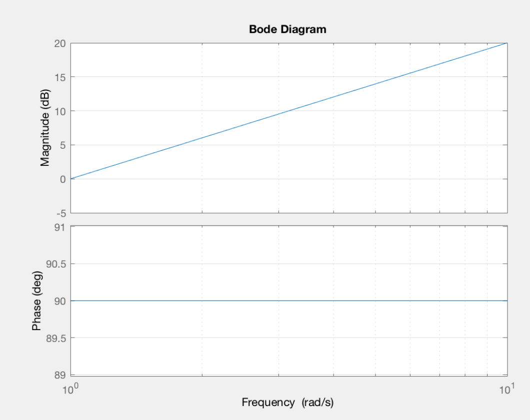
Positive Constant ($K < 0$)



Basic Building blocks

Term $s = j\omega$ (Zero in the origin).

1. The terms tends to zero ($-\infty$ in dB) for $\omega \to 0$
2. Modulus $= 1$ (0 in dB) for $\omega = 0$)
3. Phase $= \pi/2$
4. Slope of magnitude plot is $20dB$ every decade.



Basic Building blocks

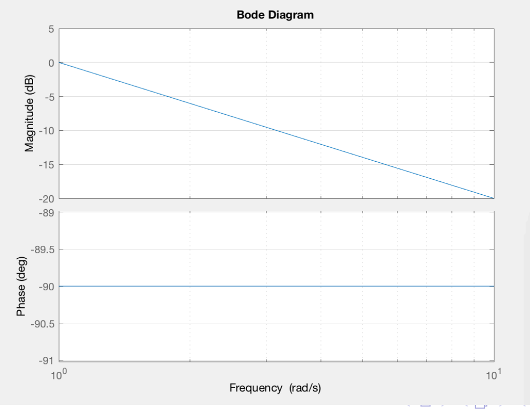
Term $s = j\omega$ (Zero in the origin).

1. The terms tends to zero ($-\infty$ in dB) for $\omega \to 0$
2. Modulus $= 1$ (0 in dB) for $\omega = 0$)
3. Phase $= \pi/2$
4. Slope of magnitude plot is $20dB$ every decade.
5. If the zero has multiplicity h then the slope will be $h \cdot 20dB/decade$

Basic Building blocks

Term $s^{-1} = (j\omega)^{-1}$ (Pole in the origin).

1. The modules tends to ∞ (∞ in dB) for $\omega \to 0$
2. Modulus $= 1$ (0 in dB) for $\omega = 0$)
3. Phase $= -\pi/2$
4. Slope of magnitude plot is $-20dB$ every decade.



Basic Building blocks

Term $s^{-1} = (j\omega)^{-1}$ (Pole in the origin).

1. The modulus tends to ∞ (∞ in dB) for $\omega \to 0$
2. Modulus $= 1$ (0 in dB) for $\omega = 0$)
3. Phase $= -\pi/2$
4. Slope of magnitude plot is $-20dB$ every decade.
5. If the multiplicity of the pole in the origin is h, the slope is $-h \cdot 20dB/decade$

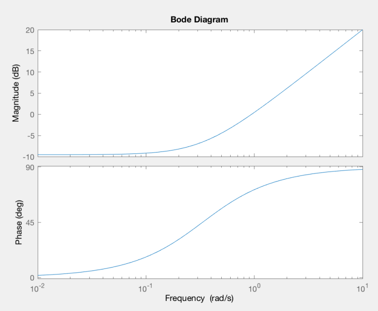
Basic Building blocks

Term $(1+s\tau) = (1+j\omega\tau)$: real zero.

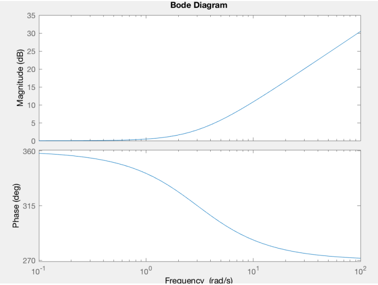
1. For $\omega \to 0$, the modules is 1 (0 dB), and the phase is 0
2. For $\omega\tau \gg 1$
   - the magnitude tends to infinity with a slope of $20 db/decade$
   - for $\omega = 1/\tau$ the modulus ($\sqrt{1+\omega^2\tau^2}$ is $\sqrt{2}$ $(3dB)$.
   - the phase tends to $-\pi/2$ if τ is negative and to $\pi/2$ if τ is positive.
   - the phase is $\pm\pi/4$ for $\omega = 1/\tau$

Basic Building blocks

Term $(1 + s\tau) = (1 + j\omega\tau)$: real zero.



$\tau > 0$



$\tau < 0$

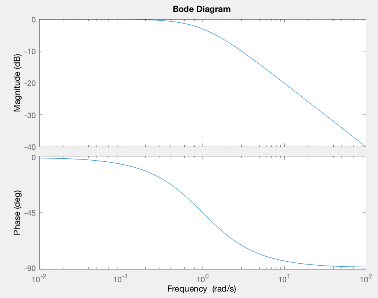
Basic Building blocks

Term $\frac{1}{(1+s\tau)} = \frac{1}{(1+j\omega\tau)}$: real pole

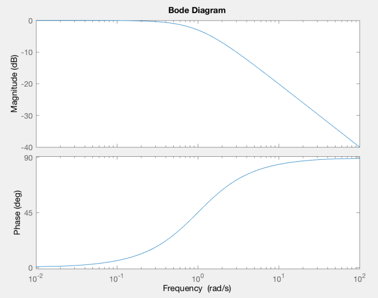
1. For $\omega \to 0$, the modules is 1 (0 dB), and the phase is 0
2. For $\omega\tau \gg 1$
   - the magnitude tends to infinity with a slope of $-20db/decade$
   - for $\omega = 1/\tau$ the modulus ($\frac{1}{\sqrt{1+\omega^2\tau^2}}$ is $\frac{1}{\sqrt{2}}$ $(-3dB)$.
   - the phase tends to $-\pi/2$ if τ is positive and to $\pi/2$ if τ is negative.
   - the phase is $\pm\pi/4$ for $\omega = 1/\tau$

Basic Building blocks

Term $\frac{1}{(1+s\tau)} = \frac{1}{(1+j\omega\tau)}$: real pole



$\tau > 0$



$\tau < 0$

Asymptotic Diagrams

- In order to simplify the drawing of bode plots, it is possible to plot an approximate diagram using the asympotic behaviour
- The asymptotic diagram is constructed by drawing line segments corresponding to small values of ω and to high values of ω
- The break point in the magnitude diagram corresponds to to point where the asymptotic phase plot becomes tangent to the actual phase plot.
- Let us go back to the example of a real pole

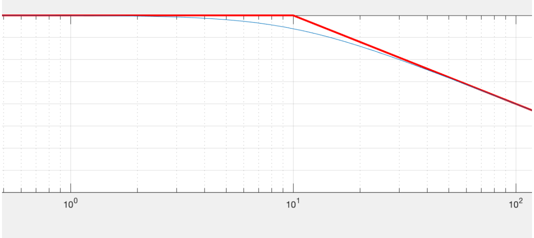
Basic Building blocks

Term $\frac{1}{(1+s\tau)} = \frac{1}{(1+j\omega\tau)}$: real pole

- $|G(j\omega)| = \frac{1}{\sqrt{1+(\omega\tau)^2}}$
- $\omega\tau \ll 1 \rightarrow |G(j\omega)||_{dB} \approx -20\log_{10} 1 = 0$
- $\omega\tau \gg 1 \rightarrow |G(j\omega)||_{dB} \approx 20\log_{10}\frac{1}{|\omega\tau|} =$ $20\log_{10}\frac{1}{|\tau|} - 20\log_{10}(\omega)$, which is 0 for $\omega = \frac{1}{\tau}$

Basic Building blocks

Term $\frac{1}{(1+s\tau)} = \frac{1}{(1+j\omega\tau)}$: real pole

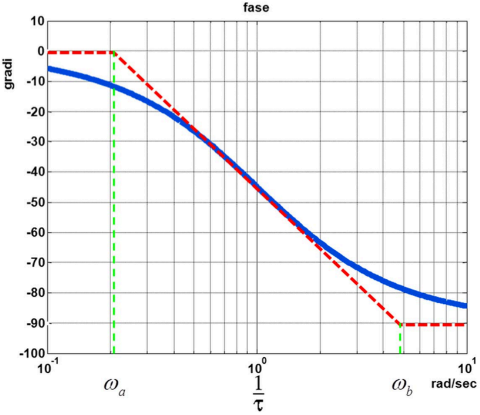


- In the breakpoint the value of the actual bode plot is $-3dB$.

Basic Building blocks

Term $\frac{1}{(1+s\tau)} = \frac{1}{(1+j\omega\tau)}$: real pole

For the phase plot remember that the value is $-\pi/4$ for $\omega = 1/\tau$.



Basic Building blocks

Term $\frac{1}{(1+s\tau)} = \frac{1}{(1+j\omega\tau)}$: real pole

- How to find ω_a and ω_b?
- In the point $1/\tau$ the tangent is:

$$
\begin{aligned}
\left.\frac{d\angle G(j\omega)}{d\log_{10}\omega}\right|_{\omega=1/\tau} &= \left.\frac{d-\arctan\omega\tau}{d\omega}\frac{d\omega}{d\log_{10}\omega}\right|_{\omega=1/\tau} = \\
&= \left.\frac{-\tau}{1+(\omega\tau)^2}\frac{de^{\log_e\omega}}{d\log_{10}\omega}\right|_{\omega=1/\tau} = \\
&= \left.\frac{-\tau}{1+(\omega\tau)^2}\frac{de^{\frac{\log_{10}\omega}{\log_{10}e}}}{d\log_{10}\omega}\right|_{\omega=1/\tau} = \\
&= \left.\frac{-\tau}{1+(\omega\tau)^2}\frac{1}{\log_{10}e}e^{\frac{\log_{10}\omega}{\log_{10}e}}\right|_{\omega=1/\tau} = \\
&= \left.\frac{-\omega\tau}{1+(\omega\tau)^2}\frac{1}{\log_{10}e}\right|_{\omega=1/\tau} = \\
&= -\frac{1}{2}\frac{1}{\log_{10}e}
\end{aligned}
$$

Basic Building blocks

Term $\frac{1}{(1+s\tau)} = \frac{1}{(1+j\omega\tau)}$: real pole

- How to find ω_a and ω_b?
- We need to find the intesection between the phase asymptotes and the tangent in $\omega = \frac{1}{\tau}$

$$\frac{\pi/4}{\log_{10}\frac{1}{\tau} - \log_{10}\omega_a} = \frac{1}{2 * \log_{10} e} \quad \rightarrow \frac{1}{\tau\omega_a} = \log_{10}\left(\pi/2\log_{10} e\right)$$

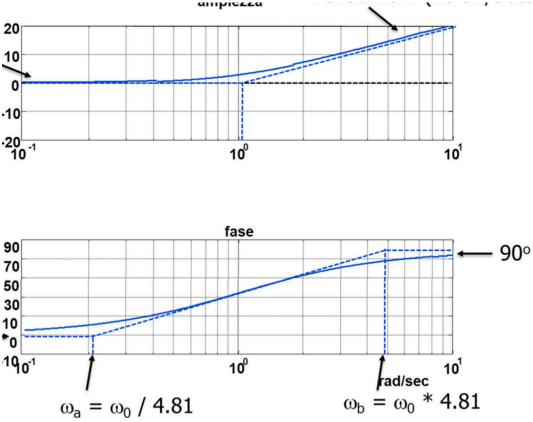
$$\frac{\pi/4}{\log_{10}\omega_b - \log_{10}\frac{1}{\tau}} = \frac{1}{2 * \log_{10} e} \quad \rightarrow \tau\omega_b = \log_{10}\left(\pi/2\log_{10} e\right)$$

Break Points

$$\omega_a = \frac{1/\tau}{4.81}, \;\; \omega_b = 4.81 \cdot \frac{1}{\tau}$$

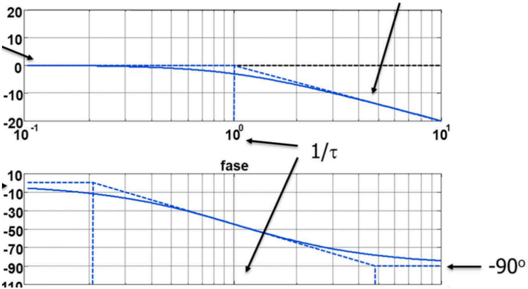
Asymptotic Diagram – Summary

- The summary of what we just said is that it is possible to simply draw a simplified version of the diagram by the asymptotes.
- Example: $G(j\omega) = (1 + j\omega\tau)$



Asymptotic Diagram – Summary

- Example: $G(j\omega) = \frac{1}{1+j\omega\tau}$



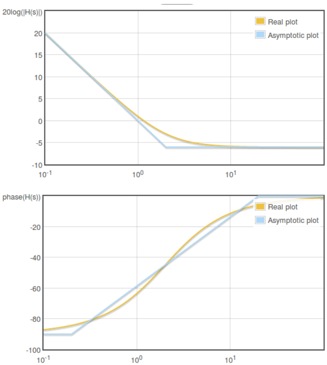
Asymptotic Diagram – Example

- Example: $G(s) = 0.5\frac{s+2}{(s+0.1)(s+10)}$
- Bode form: $G(s) = \frac{1+0.5s}{(1+10s)(1+0.1s)}$

$$
\begin{aligned}
G(s) &= G_1(s)G_2(s)G_3(s)\\
G_1(s) &= 1 + 0.5s\\
G_2(s) &= \frac{1}{1+10s}\\
G_3(s) &= \frac{1}{1+0.1s}
\end{aligned}
$$

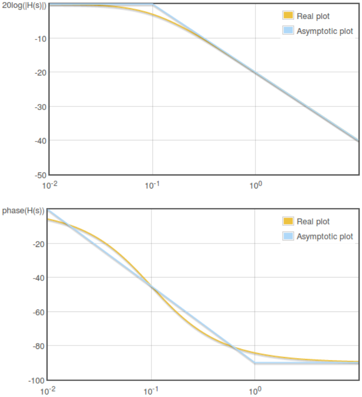
Asymptotic Diagram – Example

$$G_1(s) = 1 + 0.5s$$



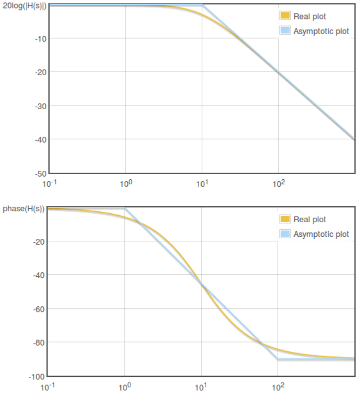
Asymptotic Diagram – Example

$$G_2(s) = \frac{1}{1 + 10s}$$



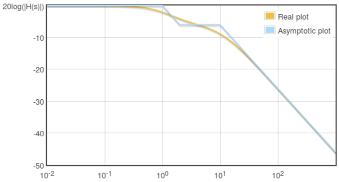
Asymptotic Diagram – Example

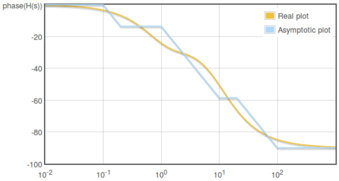
$$G_3(s) = \frac{1}{1 + 0.1s}$$



Asymptotic Diagram – Example

$$G(s) = G_1(s)G_2(s)G_3(s)$$





Basic Building Blocks

- Let us now consider a couple of complex conjugate poles:

$$G(s) = \frac{1}{\left(1 + \frac{s^2}{\omega_n^2} + 2\delta\frac{s}{\omega_n}\right)^m}$$

$$G(j\omega) = \frac{1}{\left(1 - \frac{\omega^2}{\omega_n^2} + 2\delta\frac{j\omega}{\omega_n}\right)^m}$$

- Let us consider the case $m = 1$ (the case $m > 1$ is treated similarly using the additive property of magnitude and phase)

Basic Building Blocks - Complex pair

- Magnitude

$$20\log_{10}|G(j\omega)| = 20\log_{10}\frac{1}{\sqrt{\left(1-\frac{\omega^2}{\omega_n^2}\right)^2 + 4\delta^2\frac{\omega^2}{\omega_n^2}}}$$

- Phase

$$\angle G(j\omega) = -\arctan\frac{2\delta\frac{\omega}{\omega_n}}{1-\frac{\omega^2}{\omega_n^2}}$$

Complex pole pair - Magnitude

- For $\omega/\omega_n \ll 1$

$$20\log_{10}|G(j\omega)| = 20\log_{10}\frac{1}{\sqrt{\left(1-\frac{\omega^2}{\omega_n^2}\right)^2 + 4\delta^2\frac{\omega^2}{\omega_n^2}}} \approx 0$$

- For $\omega/\omega_n \gg 1$

$$20\log_{10}|G(j\omega)| = 20\log_{10}\frac{1}{\sqrt{\left(1-\frac{\omega^2}{\omega_n^2}\right)^2 + 4\delta^2\frac{\omega^2}{\omega_n^2}}}$$
$$\approx 40\log_{10}\omega_n - 40\log_{10}\omega$$

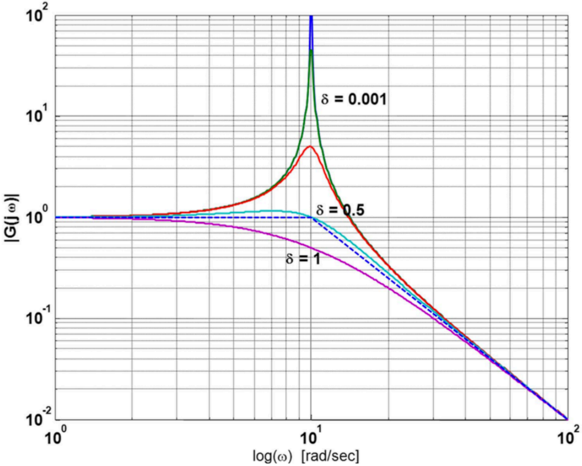
  The amplitude decreases with a rate of -40 dB/decade
- We can still build and asymptotic diagram, but it can be very far from the actual one (the deviation could be infinite)

Complex pole Pair - Magnitude

- Cases:

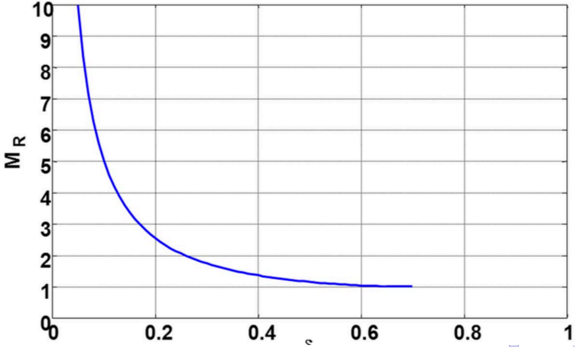
$$
\begin{aligned}
0 \leq \delta \leq 1/\sqrt{2} \rightarrow &\quad \text{The amplitude plot has a maximum} \\
0 \leq \delta \leq 1/2 \rightarrow &\quad \text{The curve lies above the asymptotic plot} \\
&\quad \text{(intesection with x axis greater than } \omega_n\text{)} \\
1/2 \leq \delta \leq 1/\sqrt{2} \rightarrow &\quad \text{Intersection with x axis smaller than } \omega_n \\
1/\sqrt{2} \leq \delta \leq 1 \rightarrow &\quad \text{The curve is below the aymptotic plot} \\
&\quad \text{(no intersection with x axis)}
\end{aligned}
$$

Complex Poles Pair - Example



Complex Poles Pair - Resonance Peak

- The maximum value (when exists) is called resonance peak
- The ω for which it is attained is called resonance pulsation
- If we set $u = \omega/\omega_n$, the resonance pulsation can be found by minimising $(1-u^2)^2 + 4\delta^2 u^2$
- By computing the derivative and setting to zero, one finds the resonance pulsation ω_R as $\omega_R = \omega_n\sqrt{1-2\delta^2}$ and he correspondign peak: $\frac{1}{2\delta\sqrt{1-\delta^2}}$



Complex Poles Pair - Phase Plot

The asymptotic diagram is quite easy:

1. The phase changes between 0 and $-\pi$
2. The phase is $-\pi/2$ on the breakpoint
3. The connection on the asymptote can be found computing the tanget of $\angle G(j\omega) = \phi(u) = -\arctan \frac{2\delta u}{1-u^2}$ with $u = \omega/\omega_n$:
4. The tangent is as follows:

$$\left.\frac{d\phi}{d\log_{10}\omega}\right|_{\omega=\omega_n} = \left.\frac{d\phi}{du}\frac{du}{d\log_{10}\omega}\right|_{u=1}$$
$$= -\frac{1}{\delta \log_{10} e}$$

Complex Poles Pair - Phase Plot

The intesection between the asymptotes and the tangent is as follows:

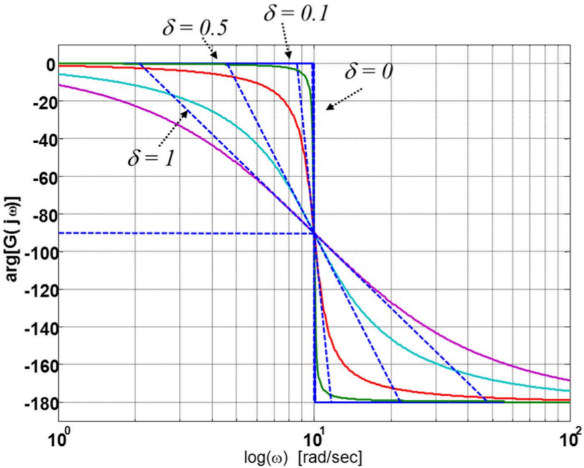
$$
\begin{aligned}
\frac{1}{\delta \log_{10} e} &= \frac{\pi/2}{\log_{10}\omega_n - \log_{10}\omega_a} &&= \frac{\pi/2}{\log_{10}\omega_b - \log_{10}\omega_n} \\
\log_{10}\frac{\omega_n}{\omega_a} &= \log_{10}\frac{\omega_b}{\omega_n} &&= \frac{\pi\delta\log_{10} e}{2} \\
\frac{\omega_n}{\omega_a} &= \frac{\omega_b}{\omega_n} &&= 4.81^{\delta}
\end{aligned}
$$

Which leads us to:

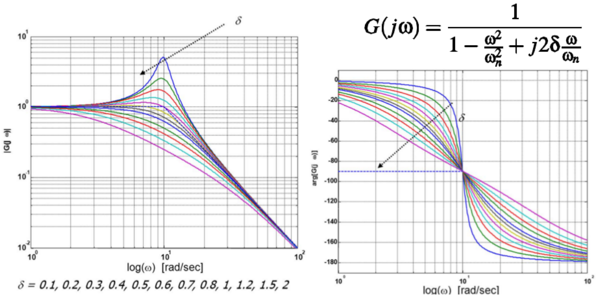
Break Points

$$\omega_a = \frac{1}{4.81^{\delta}}\omega_n, \;\; \omega_b = \left(4.81^{\delta}\right)\cdot\omega_n$$

Complex Poles Pair - Phase Plot

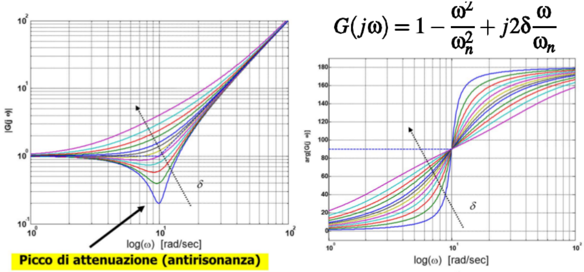


Complex Pole Pair - Summary



Complex Zero Pair - Summary

Reasoning in the same way, we can work with a complex conjugate pair of zeros:



Complex Zero Pair - Summary

Important Observation

For poles (and zeros) the sign of the pole (of the zero) has an impact only on the phase plot, but it does not change the magnitude plot. In particular, if $\delta < 0$ the phase will be positive for poles and negative of zeros (the opposite for $\delta > 0$).

Summary

For poles (zeros) with negative real part:

- Every pole(zero) determines a break point from which the slope of the magnitude decreases (increases) by 20dB/decade.
- The phase decreases (increases) asymptotically of $\pi/2$
- In the break point the phase decreases (increases) of $\pi/4$
- For a complex conjugate pair of poles we can have a resonance peak.

For poles (zeros) in the origin with multiplicity n

- The initial slope is -$n \cdot 20$ dB/decade ($+n \cdot 20$ db/decade).
- The initial phase is -$n \cdots \pi/2$ ($+n \cdot \pi/2$)
- The contibution of the poles (zeros) must take vale K for $\omega = 1$

Bode diagram – Example 1

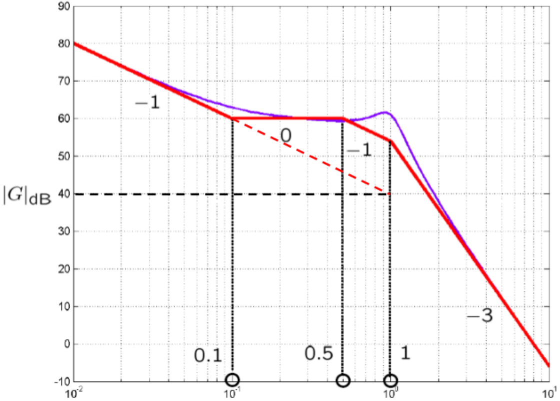
Consider

$$G(s) = \frac{100(1+10s)}{s(1+2s)(1+0.4s+s^2)}$$

Basic data:

$$\begin{aligned} K &= 100(40dB) \\ z_1 &= -0.1 \\ p_1 &= 0 \\ p_2 &= -0.5 \\ p_{3,4} &= -0.2 \pm j\sqrt{0.96}(\omega_n = 1, \delta = 0.2) \end{aligned}$$

Example – Magnitude



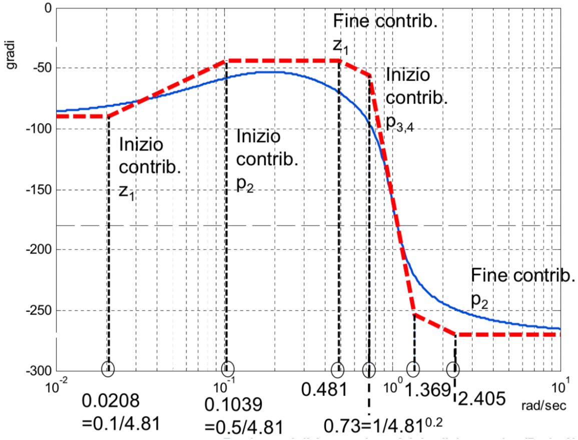
Example – Phase plot

For the Phase plot we have to compute the break points:

- z_1: $\omega_{1a} = 0.1/4.81 = 0.0208$, $\omega_{1b} = 0.1 \cdot 4.81 = 0.481$
- p_2: $\omega_{2a} = 0.5/4.81$, $\omega_{2b} = 0.5 \cdot 4.81 = 2.405$
- $p_{3,4}$: $\omega_{3a} = 1/(4.81^{0.2}) = 0.73$, $\omega_{3b} = 1 \cdot 4.81^{0.2} = 1.369$

Attention: The pole in the origin introduces a phase shift of $\pi/2$

Example – Phase



Bode diagram – Example 2

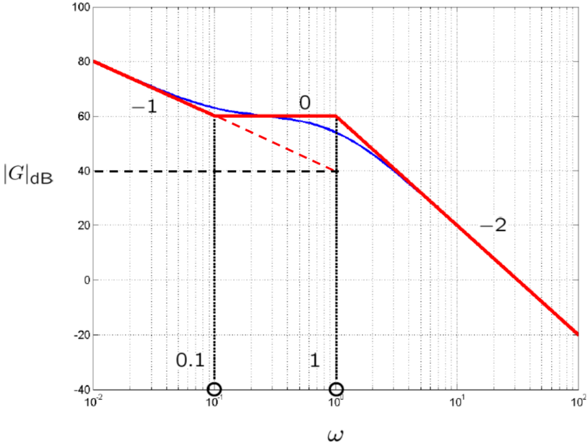
Consider

$$G(s) = \frac{100(1+10s)}{s(1+s)^2}$$

Basic data:

$$\begin{aligned} K &= 100(40dB) \\ z_1 &= -0.1 \\ p_1 &= 0 \\ p_2 = p_3 &= -1 \end{aligned}$$

Example 2 – Magnitude



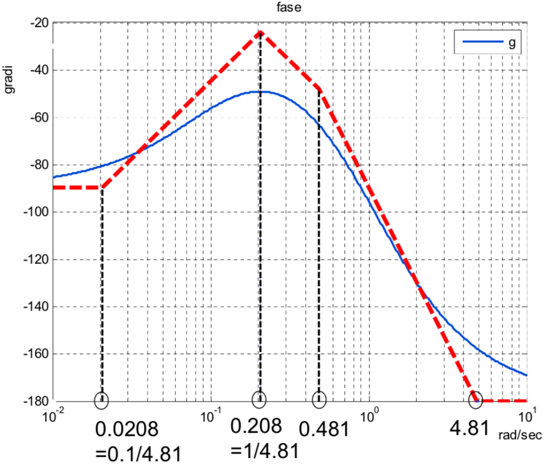
Example 2 – Phase plot

For the Phase plot we have to compute the break points:

- z_1: $\omega_{1a} = 0.1/4.81 = 0.0208$, $\omega_{1b} = 0.1 \cdot 4.81 = 0.481$
- $p_{2,3}$: $\omega_{2a} = 1/4.81$, $\omega_{2b} = 1 \cdot 4.81 = 4.81$

Attention: The pole in the origin introduces a phase shift of $\pi/2$.
The pole with multiplicity 2 determines a phase change equal to π

Example 2 – Phase



Bode diagram – Example 3

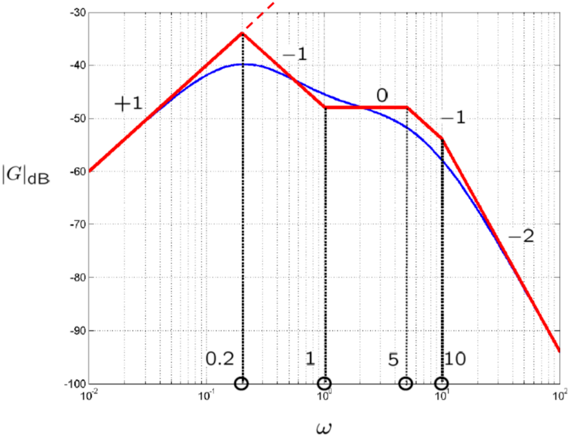
Consider

$$G(s) = \frac{0.1s(1+s)}{(1+5s)^2(1+0.2s)(1-0.1s)}$$

Basic data:

$$\begin{aligned} K &= 0.1(-20dB) \\ z_1 &= 0 \\ z_2 &= -1 \\ p_1 = p_2 &= -0.2 \\ p_3 &= -5 \\ p_4 &= 10 \end{aligned}$$

Example 3 – Magnitude

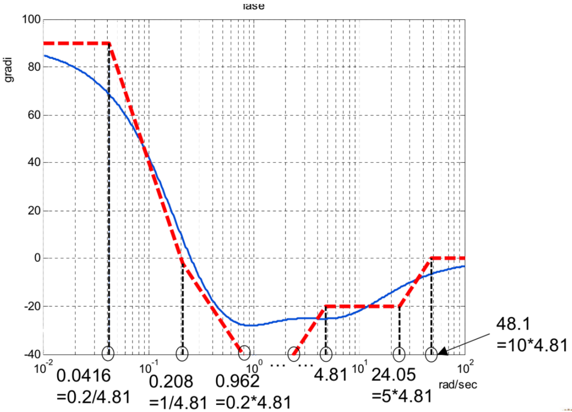


Example 3 – Phase plot

For the Phase plot we have to compute the break points:

- z_2: $\omega_{2a} = 1/4.81 = 0.208$, $\omega_{2b} = 1 \cdot 4.81 = 0.481$
- $p_{1,2}$: $\omega_{1a} = 0.2/4.81 = 0.0416$, $\omega_{2b} = 0.2 \cdot 4.81 = 0.962$
- p_3: $\omega_{3a} = 5/4.81 = 1.0395$, $\omega_{3b} = 5 \cdot 4.81 = 24.05$
- p_4: $\omega_{4a} = 10/4.81$, $\omega_{4b} = 10 \cdot 4.81 = 48.1$

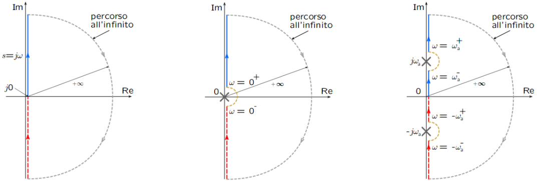
Example 3 – Phase



Nyquist Plots

A different and useful way to represent the tansfer function is by a polar diaggram.

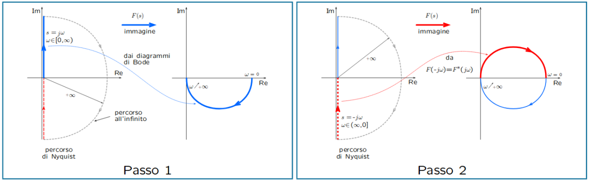
- GIven our transfer function $G(s)$, we follow a path along the imaginary axis leaving to the left (by circlets of infinitesimal radius) the poles on the imaginary axis



Nyquist Plots

A different and useful way to represent the tansfer function is by a polar diaggram.

- By noting that $G(-j\omega) = G^{\star}(j\omega)$,
- We can therefore construct the plot for positive frequencies and complete it with the symmetric plot correpsonign to the negative path.
- The two branches join on the origin for strictly proper functions



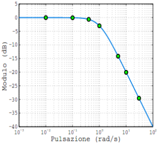
Nyquist Plots

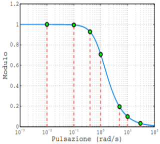
The Easiest way to construct the Nyquist diagram is to draw teh bode diagram first (moving to a linear scale for the magnitude) and then use the plot for the Nyquist.

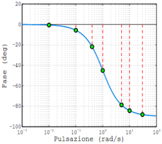
Example:

$$G(s) = \frac{1}{s+1}.$$

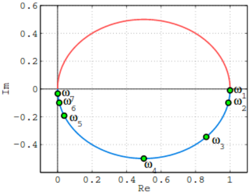
Bode Plots.







Nyquist Plot

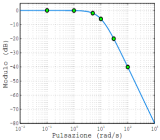


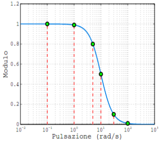
Nyquist Plots

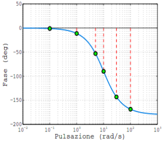
Example 1:

$$G(s) = \frac{100}{(s+10)^2} = \frac{1}{(1+s/10)^2}.$$

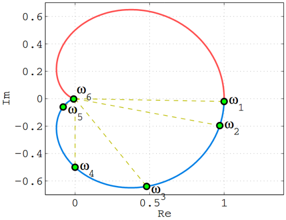
Bode Plots.







Nyquist Plot

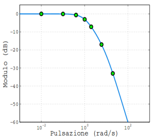


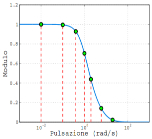
Nyquist Plots

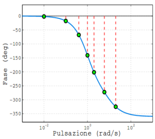
Example 2:

$$G(s) = \frac{10(1-s)}{(s+1)^2(s+10)} = \frac{1-s}{(s+1)^2(1+s/10)}.$$

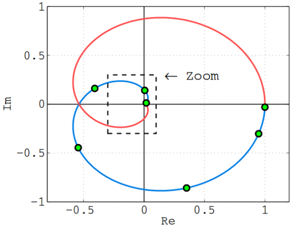
Bode Plots.







Nyquist Plot

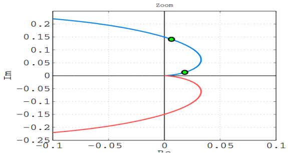


Nyquist Plots

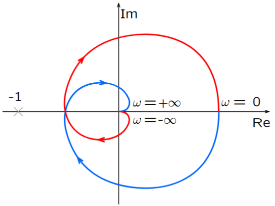
Example 2:

$$G(s) = \frac{10(1-s)}{(s+1)^2(s+10)} = \frac{1-s}{(s+1)^2(1+s/10)}.$$

Nyquist Plot (close up):



Nyquist Plot (qualitative):

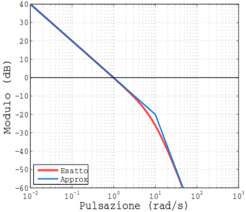


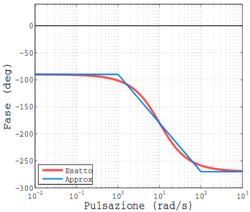
Nyquist Plots

Example 3:

$$G(s) = \frac{100}{s(s+10)^2} = \frac{1}{s(1+s/10)^2}.$$

In this case we have a pole in the origin with multiplicity 1
Bode Plots.





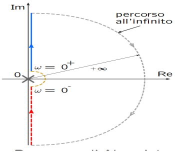
The Bode plots reveal that the Nyquist plots makes half a rotation at infinity clockwise when moving from $\omega = 0^-$ to $\omega = 0^+$

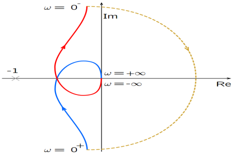
Nyquist Plots

Example 3:

$$G(s) = \frac{100}{s(s+10)^2} = \frac{1}{s(1+s/10)^2}.$$

Nyquist Plot (qualitative)





Nyquist Plot (Exact)