Formal Languages and Compilers – 2012, session 4

Exercise 1

Say whether

$$\mathcal{L} = \{a^n b^m \mid n + m = 2k \text{ for } n, m, k \geq 0\}$$

is a regular language or not. Justify your answer.

Exercise 2

Let $\mathcal{G}$ be defined as follows:

$$\begin{aligned} S &\rightarrow TUTU \\ T &\rightarrow aT \mid bT \mid \epsilon \\ U &\rightarrow bU \mid cU \mid \epsilon \end{aligned}$$

Say whether $\mathcal{G}$ is LL(1) and in case it is not, define a LL(1) grammar $\mathcal{G}'$ such that $\mathcal{L}(\mathcal{G}') = \mathcal{L}(\mathcal{G})$.

Formal Languages and Compilers session 4

Exercise 1

$$L = \{ a^m b^m \mid m + m = 2k \text{ for } m, m, k \geq 0 \}$$

is a regular language or not. Justify your answer

Come dimostrare che L è un linguaggio regolare

1. riesco a costruire un automa che lo rappresenta (NFA o DFA)
2. rappresentabile con una regular expression
3. generata da una grammatica regolare

* - Il Pumping Lemma invece fornisce una condizione necessaria ma non sufficiente affinché un linguaggio sia regolare o context-free (quindi se la condizione del Pumping Lemma non è soddisfatta il linguaggio con certezza non è regolare)

$S \to aSb \mid B$
$B \to aaB \mid Bbb \mid \varepsilon$

} grammatica non regolare

non è possibile costruire una r.e. o un automa che rappresenti il linguaggio L

Esercizio 1 — Sessione 4

$L = \{ a^n b^m \mid n+m = 2k \text{ for } n, m, k \geq 0 \}$

$2k$ = Pari

$S \to \varepsilon \mid aSb \mid aaS \mid bbS$

Sbb

aa
bb

~~aaS -> aabbS -> aabbaaS~~

$|z| = 4$ — aabb (u v w)

$p = 3$

$|uv| = 3$ — aaaba

$|v| = 2 > 0$

L non è regolare in quanto prodotto da una grammatica context-free ma non regolare.

Infatti l'applicazione del pumping lemma per linguaggi regolari sulla stringa di esempio $w = aabb$ dimostra che il linguaggio non è regolare

Exercise 2

Let G define as follow: $S \to TUTU$

$T \to aT \mid bT \mid \varepsilon$

$U \to bU \mid cU \mid \varepsilon$

Say whether G is LL(1) and in case it is not, define a LL(1) grammar G' such that $\mathcal{L}(G') = \mathcal{L}(G)$

per essere LL(1) una grammatica deve essere:

- non Left-Recursive
- non AMBIGUA
- $FIRST(\alpha)$ e $FIRST(\beta)$ devono esser disgiunti. quindi $FIRST(\alpha) \cap FIRST(\beta) = \{\emptyset\}$
- if $\varepsilon \in FIRST(\beta)$ then $FIRST(\alpha)$ e $FOLLOW(\alpha)$ devono esser disgiunti. e viceversa

$A \to \alpha \mid \beta$

	FIRST	FOLLOW
S	a, b, ε	$
T	a, b, ε	b, c, a, $
U	b, c, ε	a, b, c, $

	a	b	c	$
S	S→TUTU	S→TUTU		S→TUTU
T	T→aT T→ε	T→bT T→ε	T→ε	T→ε
U	U→ε	U→bU U→ε	U→cU U→ε	U→ε

La grammatica non è LL(1) perché ci sono 2 derivazioni per cella.

Ex 2 Session 4

$S \to TUTU$

$T \to aT \mid bT \mid \varepsilon$

$U \to bU \mid cU \mid \varepsilon$

	FIRST	FOLLOW
S	a, b, ε	$
T	a, b, ε	b, c, a, $
U	b, c, ε	a, b, c, $

	a	b	c	$
S	S→TUTU	S→TUTU		S→TUTU
T	T→aT T→ε	T→bT T→ε	T→ε	T→ε
U	U→ε	U→bU U→ε	U→cU U→ε	U→ε

The grammar G is not LL(1) because there are multiply defined entries on the predictive parsing table

continua Ex 2 session 4

$S \to TUTU \} (a|b)^*(b|c)^*(a|b)^*(b|c)^*$

$T \to aT$
$T \to bT$
$T \to \varepsilon$
$\} (a|b)^*$

$U \to bU$
$U \to cU$
$U \to \varepsilon$
$\} (b|c)^*$

$S \to TUTU \to aTETU \to aTEbTcU$
$\to abcU \to abcbU \to abcbcU$
$\to abcbc$

$\varphi(G) = \{$ ae th

$(a|b)^*$

$S \to TUTU \to aT\,aT \to aeT\,aaT = aaaa \in \varphi$
$bbbb \in \varphi$
$cccc \in \varphi$

~~TUTU~~ ~~aE~~

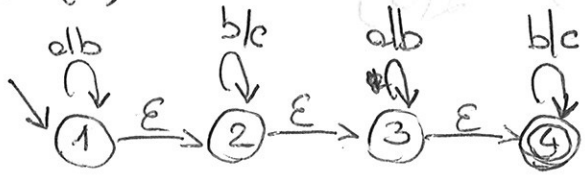
$r = (a|b)^*(b|c)^*(a|b)^*(b|c)^*$

$cc\,aa\,cc \in \varphi$

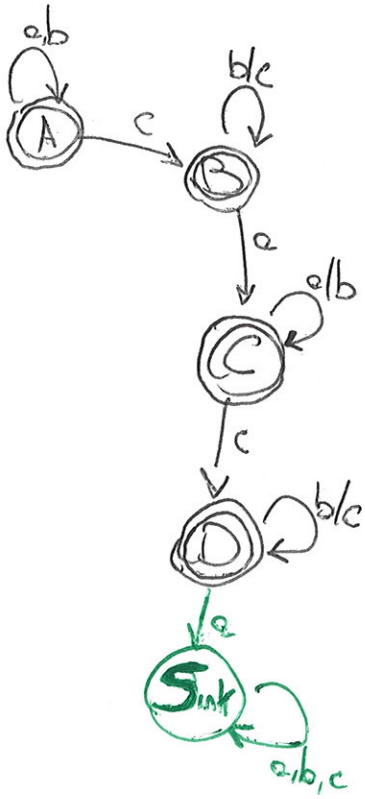
Costruisca una grammatica compatibile con la regular expression e poi verifica che sia LL(1)

$cc\,aa\,cc\,aa \notin \varphi$

$cc\,bb$



	a	b	c
$A = \{1,2,3,4\}$	$\{1,2,3,4\} = A$	$\{1,2,3,4\} = A$	$\{2,3,4\} = B$
$B = \{2,3,4\}$	$\{3,4\} = C$	$\{2,3,4\} = B$	$\{2,3,4\} = B$
$C = \{3,4\}$	$\{3,4\} = C$	$\{3,4\} = C$	$\{4\} = D$
$D = \{4\}$		$\{4\} = D$	$\{4\} = D$



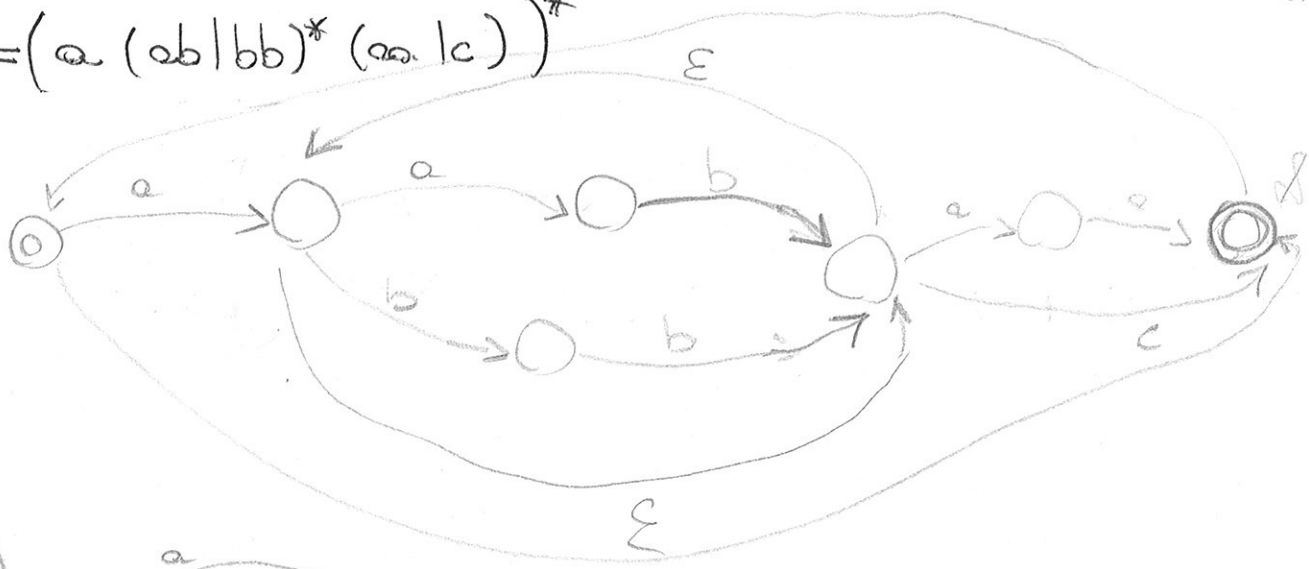
$A \to aA|bA|cB|\varepsilon$
$B \to bB|cB|aC|\varepsilon$
$C \to aC|bC|aD|\varepsilon$
$D \to bD|cD|\varepsilon$

Exercise 1

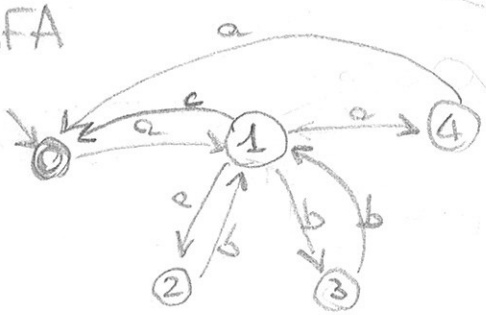
Appello

3 6 7 8
1 2 4

$r = (a\,(ab \mid bb)^*\,(aa \mid c))^*$

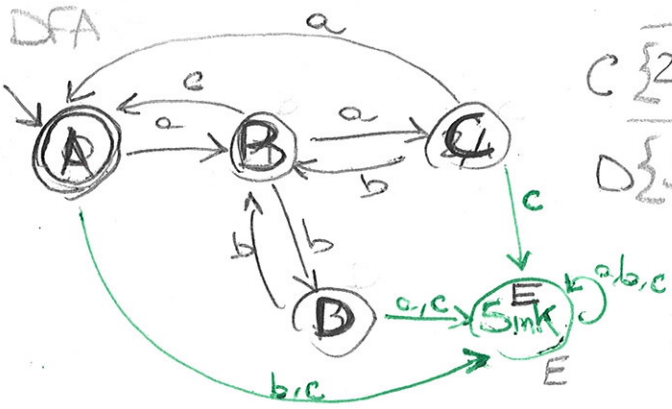


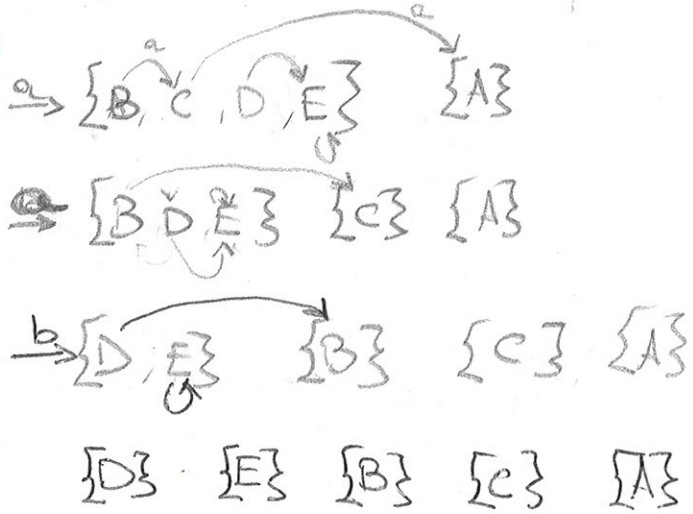
NFA



NFA	DFA		
	a	b	c
A {0}	{1}	\	\
B {1}	{2,4}	{3}	{0}
C {2,4}	{0}	{1}	\
D {3}	\	{1}	\

DFA





a b c

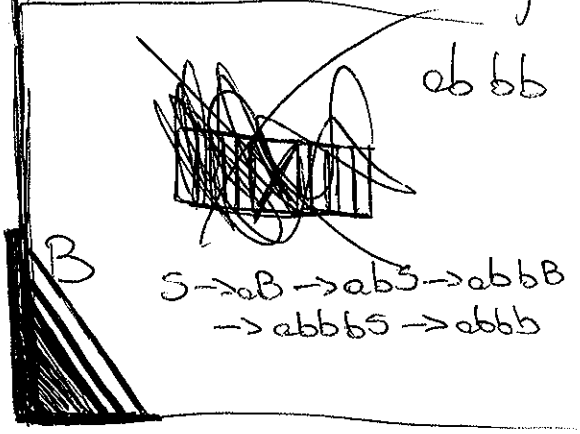
ε
a aa*
a c *

a ab* aa
a ab* c
a bb* aa
a b b* c

$r_2 = (ab|bb)^*$

$S \to aB | bB | \varepsilon$

$B \to bS$



$S \to aB | \varepsilon$

$B \to aC | bC$

$C \to bD$

$D \to aE | c$

$E \to a$

$S \to aB \to aaC \to aa.$

$r_3 = aa. | c$

$S \to aB | c$

$B \to a$

$S \to aB | \varepsilon | aX$

$B \to aC | bC$

$C \to bB | bX$

a abab c

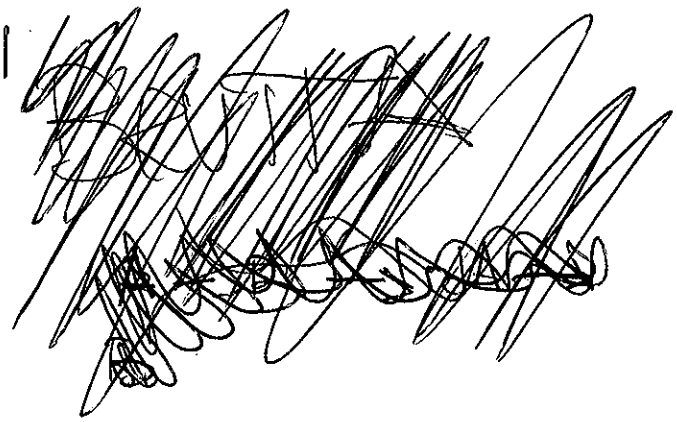
$S \to aB \to a\ aC \to a\ abB$

$\to a\ ab\ aC \to aab\ abX$

$\to a\ ab\ ab\ cS \to aababc$

$X \to aW | cS$

$W \to aS$



Appello 1

Esercizio 2

G' $S \to A \mid \varepsilon$ ok

$A \to AB \mid AB\,e \mid B \mid bB$

G' $B \to dA \mid d \mid Be$

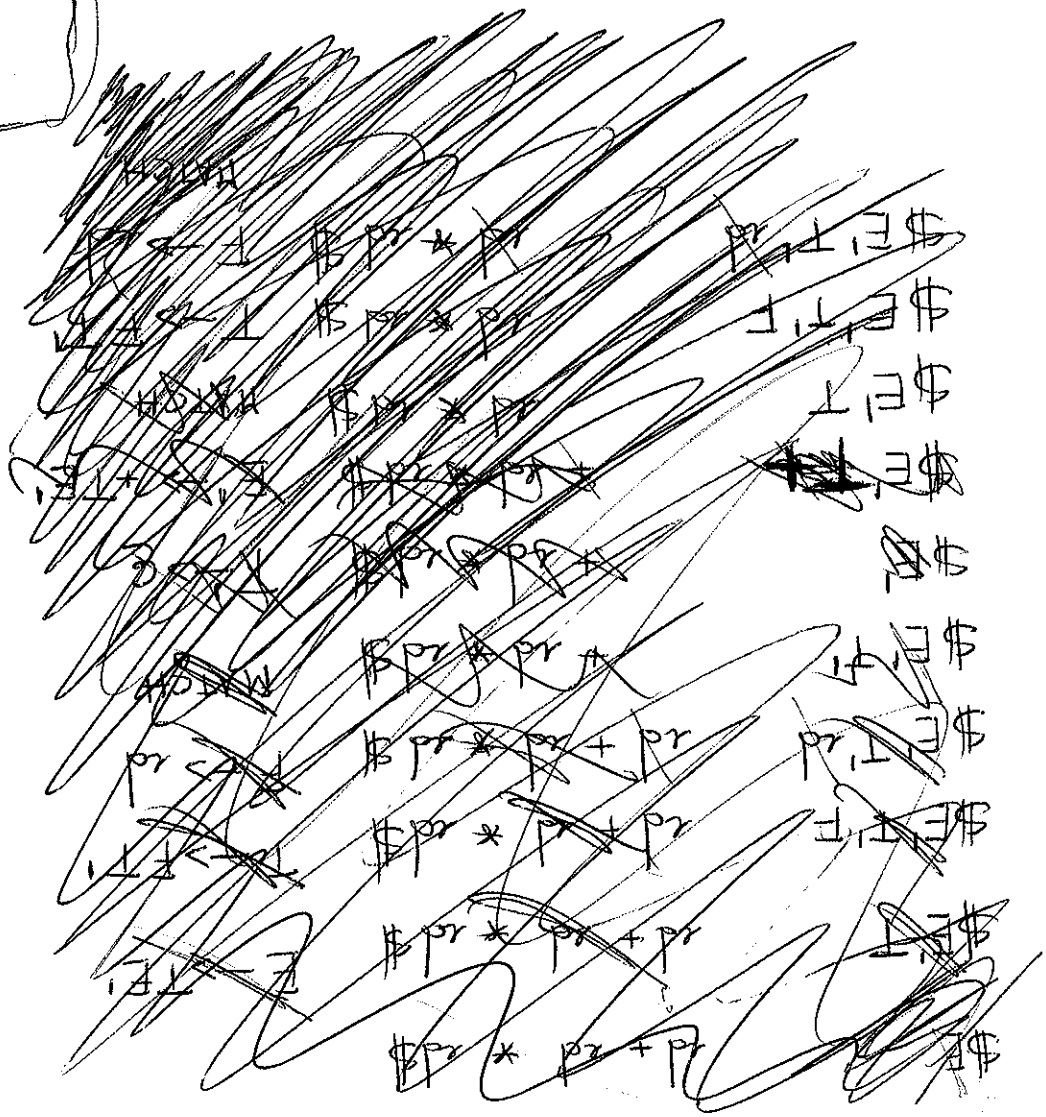
$S \to A \mid \varepsilon$

$A \to BA' \mid bBA'$

$A' \to BA' \mid BeA' \mid \varepsilon$

$B \to dAB' \mid dB'$

$B' \to eB' \mid \varepsilon$



	FIRST	FOLLOW
S	ε b d	$
A	b d	$ e d
A'	ε d	$ e d
B	d	d e $
B'	e ε	$ e d

$S \to A \mid \varepsilon$

$A \to BA' \mid bBA'$

$A' \to BA' \mid BeA' \mid \varepsilon$

$B \to dAB' \mid dB'$

$B' \to eB' \mid \varepsilon$

LL(1) TABLE

	b	d	e	$
S	S→A	S→A		
A	A→bBA'	A→BA'		
A'		A'→BA' A'→BeA'		
B		B→dAB B→dB'		
B'			B'→eB'	

G' IS NOT SLR

because some prodoction are LEFT FACTORING

Esercizio 3 — Appello 1 — 2012

Say wether or not the language

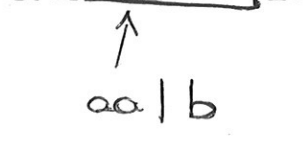
$L = \{ a^i a^i b^j \mid i,j \geq 0 \text{ and if } i,j > 0 \text{ then } i \neq j \}$

is a context-free language. Justify your answer.

$S \to aa \mid b \mid aaBb \mid$

$B \to aa \mid aaA$

aa
b
$a^1 a^1 aab^1 \leftarrow i=2 \quad j=1$



aa | b

$a^{2i} b^j \quad i,j \geq 0 \quad$ if $i,j > 0$ then $i \neq j$

$S \to aa \mid bb \mid aaSb$

ϵ ~~aab~~ b aa

~~aab~~ aabbb

aabb aaaabbb aaaab ~~aa~~ b

$L(G) = \{ a^i a^i b^j$

~~abb~~

aaa b

$S \to \varepsilon \mid aa \mid b \mid aaSb$

$S \to$ ~~aaSb | aaA~~ aa | b | ~~ε~~ | A

$A \to$ ~~aab | aaAbb~~ ~~aaAb~~ ~~aaAbbb~~ | ϵ

aaaaAb | aaAbb | ϵ | aaA

aaaaBb

aaBbb

aaaaaab

se vgl aab $\longrightarrow$ aabb 2b

$L = \{ a^i b^j \mid i \neq j \}$

$S \to aS \mid Ba$

$B \to bB$

S

$S \to aSbb \mid aSb$

aaabb

deve aggiungere sempre almeno 2 a

$i=2$ $j=3$

aaaabbb

U V W

$|z| = 7$

$p = 5$

ababa|ababa

$S \to aSa \mid bSb \mid \varepsilon$

ababa ababa $|z| = 10$

U V W

$S \to aSa \to abSba \to$
$abaSaba \to ababSbaba$
$\to ababaSbaba$
$\to ababaabbaba$

aba | baabaa | baba

v^2

aaabb $|z| = 5$

U V W

$p = 4$