Goniochromism

Sprint 0 - November 2020

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Our PBR

$$L_o(v) = \int_\Omega L_i \; f(l, v) \; (n \cdot v) \; d\omega_o$$

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approximation

$$L_o(v) = \pi \; f(l_c, v) \otimes c_{light}(n \cdot l_c)$$

$c_{light} = intensity * color$ $\qquad color = RGB \in [0, 1]$

Our BRDF

$$f(l, v) = f_{specular}(l, v) + f_{diffuse}(l, v)$$

Our BRDF

$$f(l,v) = f_{specular}(l,v) + f_{diffuse}(l,v)$$

$$f_{lambert}(l,v) = \frac{c_{diff}}{\pi}$$

$$f_{cook-torrance}(l,v) = \frac{F(l,h)g(l,v,h)D(h)}{4(n\cdot l)(n\cdot v)}$$

Our PBR

$$L_{o-specular} = \pi \, \frac{F(l,h)g(l,v,h)D(h)}{4(n \cdot l)(n \cdot v)} \otimes c_{light}(n, l_c)(n \cdot l_c)$$

$$L_o(v) = (L_{o-diffuse})(1 - F(l,h)) + L_{o-specular}$$

Metalness

- metalness = 0 (dielectrics), c diff = base color, c spec = vec3(0.04);
- metalness = 1 (metals), c diff = vec3(0), c spec = base color

Our PBR

$$f_{cook-torrance}(l,v) = \frac{F(l,h)g(l,v,h)D(h)}{4(n \cdot l)(n \cdot v)}$$

Our BRDF

$$D(h) = \frac{\alpha^2}{\pi((n \cdot h)^2 \ (\alpha^2 - 1) + 1)^2}$$

$$g(l, v, h) = G_1(l) \ G_1(v)$$

$$G_1(v) = \frac{n \cdot v}{(n \cdot v)(1 - k) + k}$$

$$f_{schlick}(c_{spec}, l, h) = c_{spec} + (1 - c_{spec}) \cdot (1 - (l \cdot h))^5$$

$$f_{cook-torrance}(l, v) = \frac{F(l, h) g(l, v, h) D(h)}{4(n \cdot l)(n \cdot v)}$$

Paper Approach

The idea

- **Microfacet theory extension** for modeling iridescence
- Focus on iridescence due to **thin films** (of varying thickness)
- **Analytical** (spectral integration) method => real time rendering

Main factors

- Multiple scattering
- Polarization
- Phase changes

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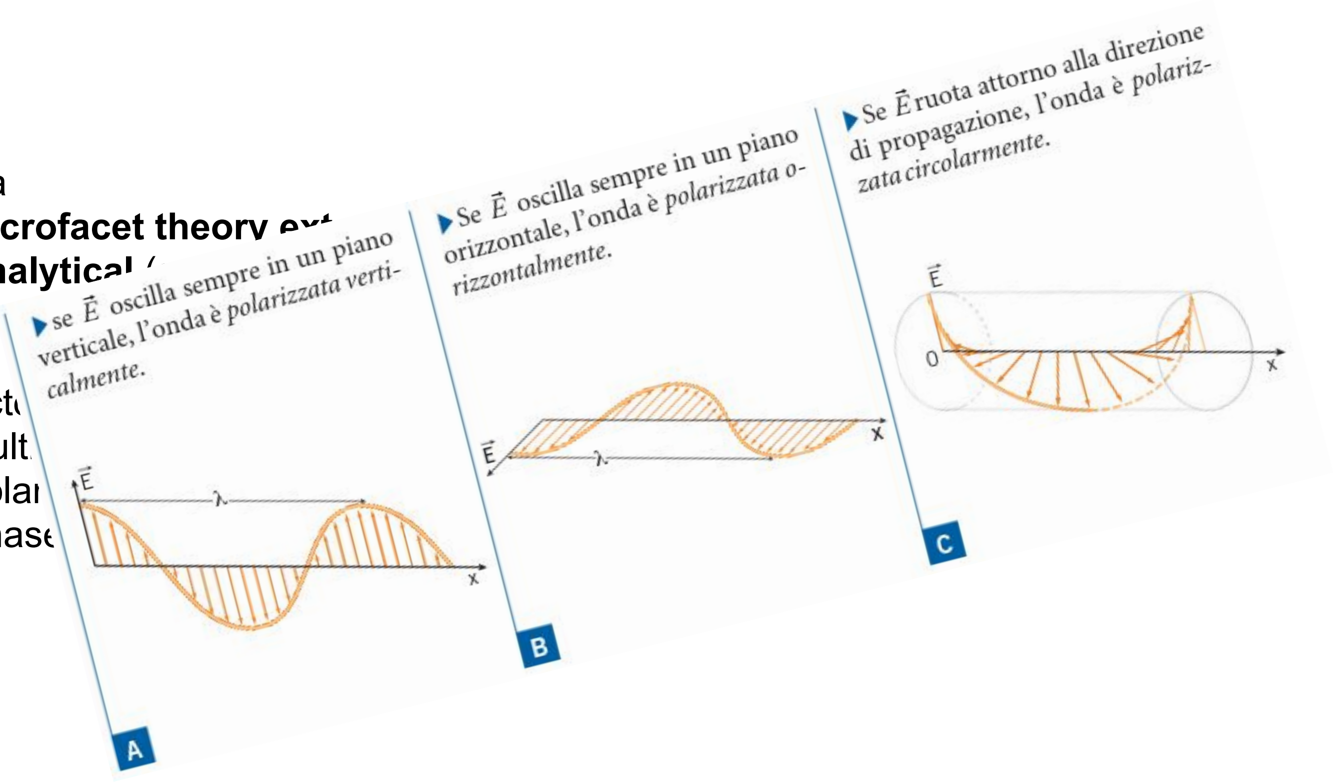
Main fact

- Mult
- Polar
- Phase



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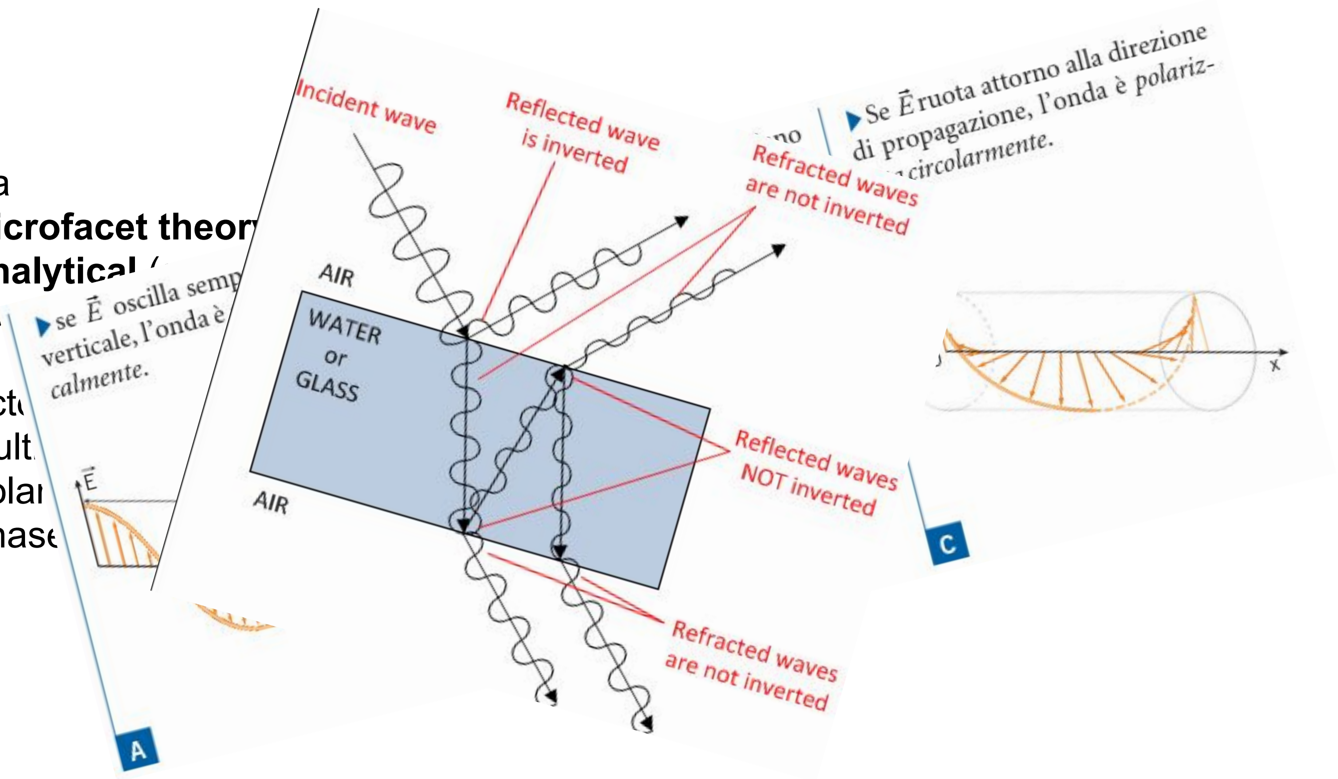

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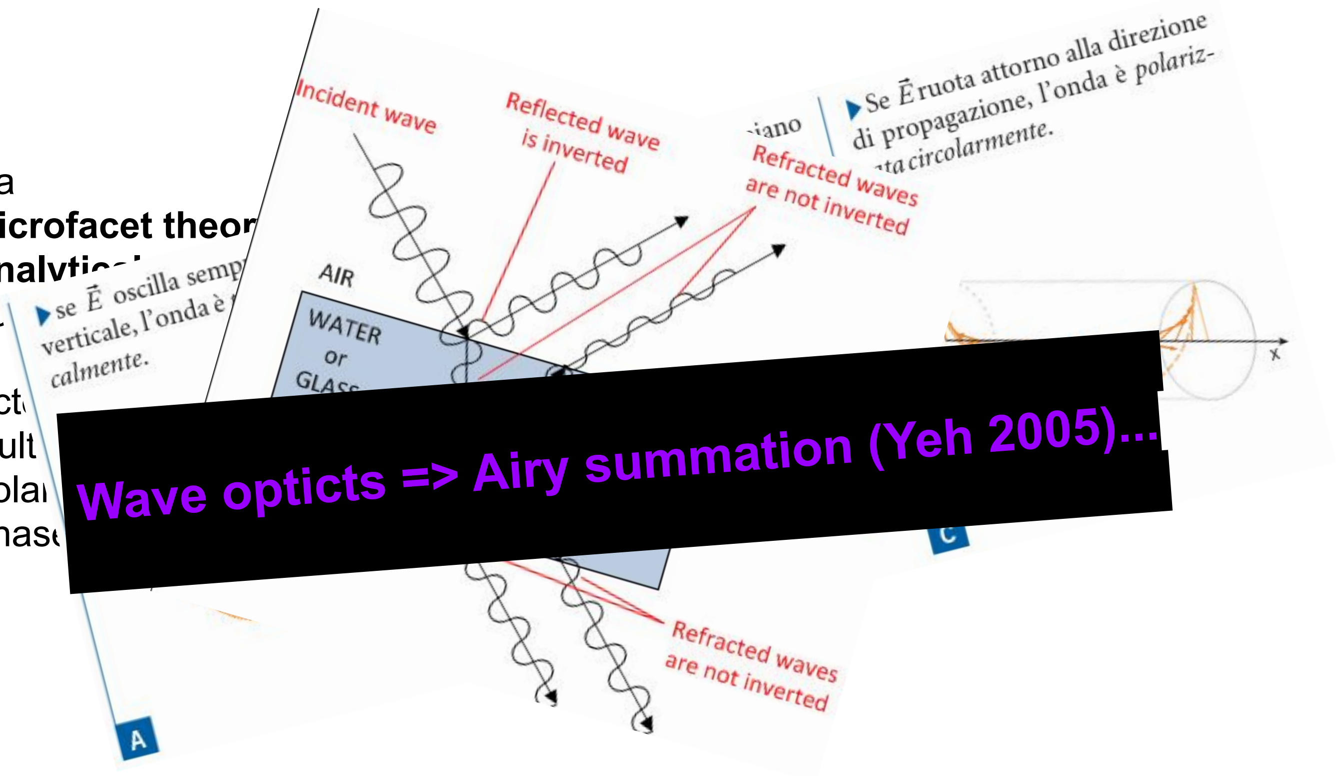

Wave optics => Airy summation (Yeh 2005)...

Paper BRDF

$$\rho(\omega_o, \omega_i; \lambda) = \frac{F(h \cdot \omega_i; \lambda) G(\omega_o, \omega_i) D(h)}{4(\omega_o \cdot n)(\omega_i \cdot n)}$$

Paper BRDF

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outgoing

wavelength

incoming

Paper BRDF

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outgoing

incoming

wavelength

$$F = \frac{1}{2}(F^{\perp} + F^{\|})$$

Fresnel reflectance w.r.t. light waves polarized perpendicularly and parallaly to the plane

Paper BRDF

$$\rho(\omega_o, \omega_i; \lambda) = \frac{\boldsymbol{R}(h \cdot \omega_i; \lambda) G(\omega_o, \omega_i) D(h)}{4(\omega_o \cdot n)(\omega_i \cdot n)}$$

outgoing

wavelength

incoming

$$F = \frac{1}{2}(F^{\perp} + F^{\|})$$

Fresnel reflectance w.r.t. light waves polarized perpendicularly and parallaly to the plane

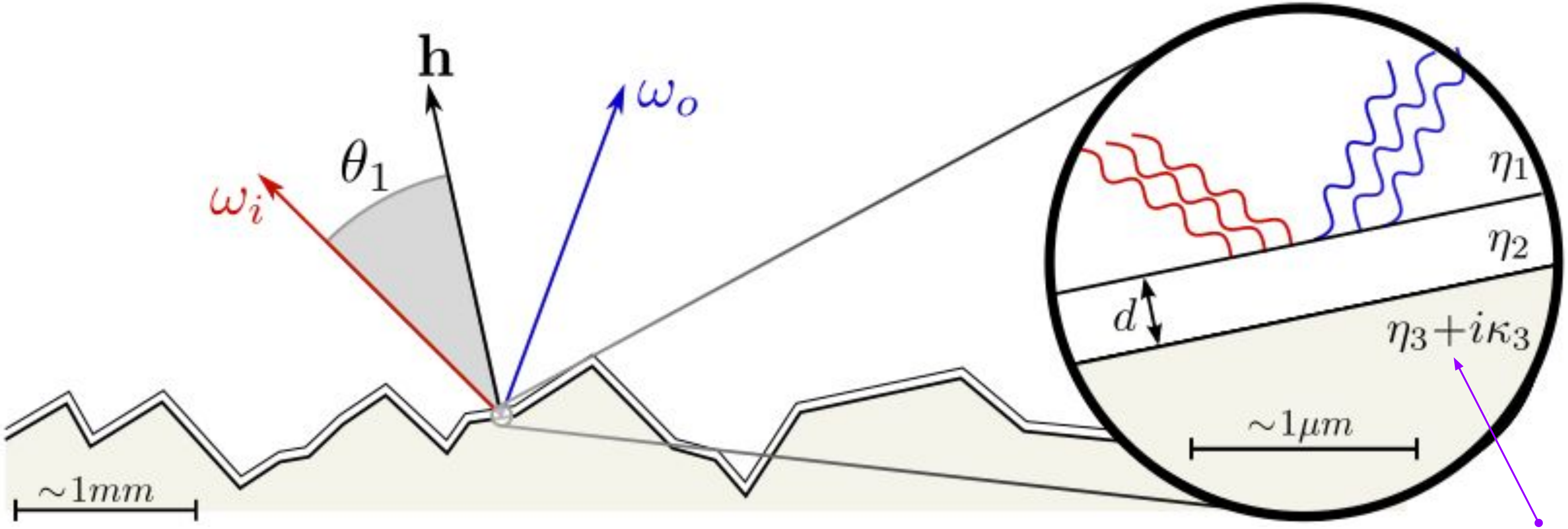
Paper BRDF

$\rho(\omega_o, \omega_i; \lambda) =$ $R(h \cdot \omega_i; \lambda)G($ [redacted] (h) / $4($ [redacted] $)$

The same but replacing Fresnel with Airy Reflectance

f_{cook} [redacted] $=$ [redacted] $(l, h)g(l, v, h)D(h)$ / $4(n \cdot l)(n \cdot v)$

Thin Film



complex refraction index

Airy Notation

Reflectance (for a given wavelength/polarization)

$$R = \frac{A_o}{A_i}$$

Power of **i**ncoming/**o**utgoing waves

Power is proportional to the wave **amplitude**

$$A_{o,i} \propto |a_{o,i}|^2$$

$$R = \frac{|a_o|^2}{|a_i|^2} = |r|^2$$

r is a complex reflection coefficient

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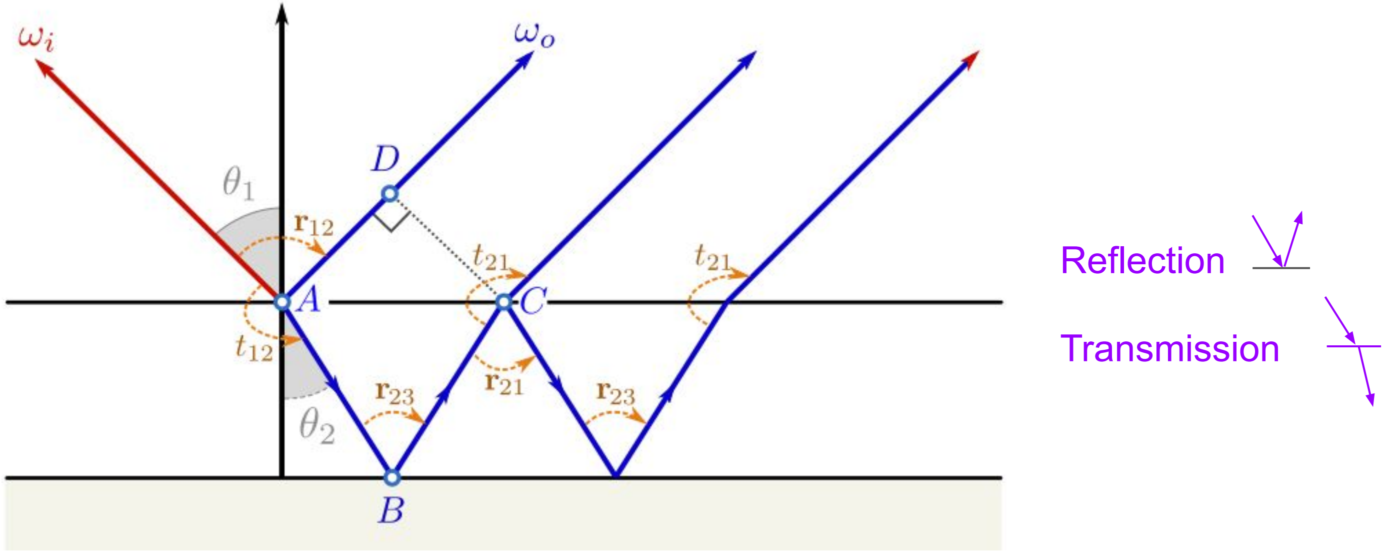
$$A_{o,i} \propto |a_{o,i}|^2$$

$$\boxed{R = \frac{1}{2}(|r^{\perp}|^2 + |r^{\|}|^2) \quad R = \frac{|a_o|^2}{|a_i|^2} = |r|^2}$$

Airy Summation



$$r = r_{12} + t_{12} r_{23} t_{21} e^{i\Delta\phi} + t_{12} r_{23} r_{21} r_{23} t_{21} e^{2i\Delta\phi} + \ldots$$

Airy Summation

r_{12}

(Complex) Reflection coefficient of light coming from medium 1 reflected by the surface of medium 2

$$r = r_{12} + t_{12} r_{23} t_{21} e^{i\Delta\phi} + t_{12} r_{23} r_{21} r_{23} t_{21} e^{2i\Delta\phi} + \ldots$$

Airy Summation

r_{12} (Complex) Reflection coefficient of light coming from medium 1 reflected by the surface of medium 2

t_{12} Transmission ...

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Airy Summation

r_{12} (Complex) Reflection coefficient of light coming from medium 1 reflected by the surface of medium 2

$$OPD = d_1\eta_1 - d_2\eta_2$$

t_{12} Transmission ...

$\Delta\phi$ Phase shift due to **OPD** (Optical Path Difference)

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k number of inter-reflections

Airy Summation

Optical path difference [edit]

The OPD corresponds to the phase shift undergone by the light emitted from two previously coherent sources when passed through mediums of different refractive indices. For example, a wave passed through glass will appear to travel a greater distance than an identical wave in air. This is because the source in the glass will have experienced a greater number of wavelengths due to the higher refractive index of the glass.

The OPD can be calculated from the following equation:

$$\mathrm{OPD} = d_1 n_1 - d_2 n_2$$

where d_1 and d_2 are the distances of the ray passing through medium 1 or 2, n_1 is the greater refractive index (e.g., glass) and n_2 is the smaller refractive index (e.g., air).

$$r = \ldots t_{12} r_{23} r_{21} r_{23} t_{21} e^{2i\Delta\phi} + \ldots$$

Airy Summation

$$r = r_{12} + t_{12}r_{23}t_{21}e^{i\Delta\phi} + t_{12}r_{23}r_{21}r_{23}t_{21}e^{2i\Delta\phi} + \ldots$$

$$r = r_{12} + \sum_{k=1}^{+\infty} t_{21}r_{23}[r_{21}r_{23}]^{k-1}t_{21}e^{ik\Delta\phi}$$ Geometric series

$$\boxed{= r_{12} + \frac{t_{12}r_{23}t_{21}e^{i\Delta\phi}}{1 - r_{21}r_{23}e^{i\Delta\phi}}}$$

Phase Shift

$$\Delta\phi = 2\pi\upsilon\mathcal{D} = 2\pi\frac{1}{\lambda}\mathcal{D}$$

Phase Shift

$$\Delta\phi = 2\pi\upsilon\mathcal{D} = 2\pi\frac{1}{\lambda}\mathcal{D}$$

k-th order OPD

$$\mathcal{D}(k) = k\mathcal{D}$$

1st order OPD

$$\mathcal{D} = 2\ \eta_2\ d\cos\theta_2$$

Phase Shift

$$\Delta\phi = 2\pi\upsilon\mathcal{D} = 2\pi\frac{1}{\lambda}\mathcal{D}$$

k-th order OPD

$$\mathcal{D}(k) = k\mathcal{D}$$

1st order OPD

Film thickness

$$\mathcal{D} = 2\ \eta_2\ d\cos\theta_2$$

Snell's law

$$cos\ \theta_2 = \sqrt{1 - \frac{\eta_1^2}{\eta_2^2}(1 - cos^2\theta_1)}$$

Now on I’m Confused!

Spectral Integration

Online rendering engines use small discrete set of spectral bands (three for RGB)

j **band**

s_j associated **sensitivity function** $s_R \; s_G \; s_B$ for RGB

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Online rendering engines use small discrete set of spectral bands (three for RGB)

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s_j associated **sensitivity function** $s_R \; s_G \; s_B$ for RGB

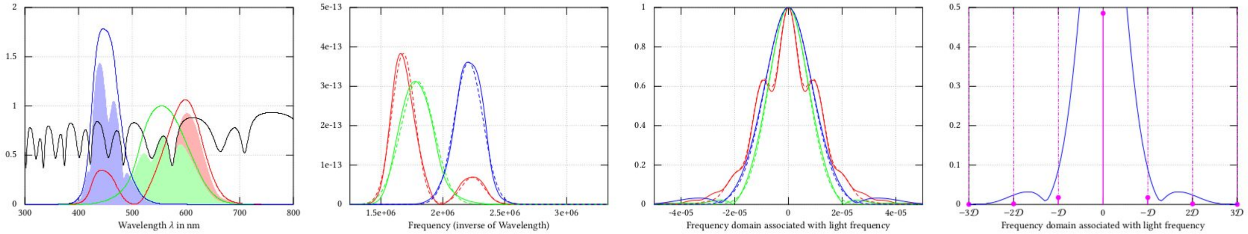
$$L_j^{\uparrow}(\omega_0) = \int \frac{s_j(\lambda)}{||s_j||} \int_{\Omega} \rho(\omega_0, \omega_i; \lambda) \; L^{\downarrow}(\omega_i; \lambda)(\omega_i \cdot n) d\omega_i d\lambda$$

$$L_j^{\uparrow}(\omega_0) \approx \int_{\Omega} \boxed{\rho_j(\omega_o, \omega_i)} \; L_j^{\downarrow}(\omega_i) \; (\omega_i \cdot n) \; d\omega_i$$

Assumes that material and lighting are not spectrally correlated

Spectral Integration

$$R_j(h \cdot \omega_i) = \int R(h \cdot \omega_i; \lambda) \frac{s_j(\lambda)}{||s_j||} d\lambda$$



(a) Spectral integration problem (b) Scaled sensitivity functions (c) moduli in Fourier (d) Integration in Fourier

Fig. 5. The spectral integration problem is illustrated in (a) for the CIE XYZ color space: the reflectances $\{R_X, R_Y, R_Z\}$ (light colored areas) are obtained by integrating the product of each sensitivity curve (s_X in red, s_Y in green, s_Z in blue) with the spectral Airy reflectance R (in black). In our approach, the sensitivity functions are first re-expressed in terms of $\nu = 1/\lambda$ in (b), yielding $\{S_X, S_Y, S_Z\}$. They are then transformed in Fourier space: the moduli $\{|\hat{S}_X|, |\hat{S}_Y|, |\hat{S}_Z|\}$ are shown in (c). Since the Fourier transform $\hat{R}$ of the Airy reflectance term is composed of diracs, the integral in Fourier becomes analytical as shown in (d) for the evaluation of R_Z. The dashed curves in (b) and (c) show Gaussian fits, which provide reasonable approximations in practice.

Derive Airy Reflectance

$$r = \sum_{k=0}^{+\infty} c_k e^{i\phi_k}$$

$$\phi_k = k(\Delta\phi + \phi_{23} + \phi_{21}) - \phi_{21}$$
$$c_k = t_{12} r_{23} [r_{21} r_{23}]^{k-1} t_{21}$$
$$c_0 = -r_{21} \qquad \phi_0 = \phi_{21}$$

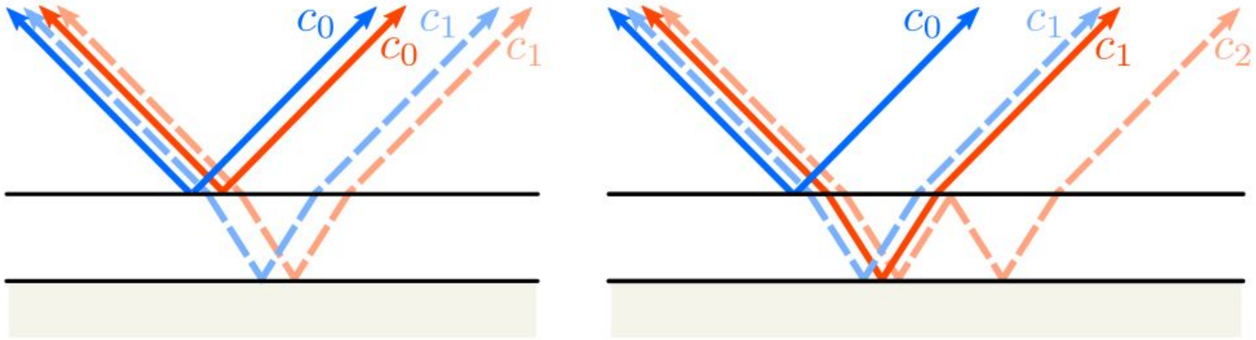
$$R = C_0 + 2 \sum_{m=1}^{+\infty} C_m \, cos(m\Phi)$$

$$C_m = \sum_{k=0}^{+\infty} c_k c_{k+m}$$

$$C_0 = \sum_{k=0}^{+\infty} c_k^2$$

m is the offset in orders between pairs of light paths

Derive Airy Reflectance



$$C_0 = c_0 c_0 + c_1 c_1 + \dots \qquad C_1 = c_0 c_1 + c_1 c_2 + \dots$$

$$R = C_0 + 2 \sum_{m=1}^{+\infty} C_m \, cos(m\Phi)$$

$$C_m = \sum_{k=0}^{+\infty} c_k c_{k+m}$$

$$C_0 = \sum_{k=0}^{+\infty} c_k^2$$

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Derive Airy Reflectance

$$R_j = \int R(v) \frac{s_j \, \frac{1}{v}}{||s_j|| \, v^2} \, dv$$

Derive Airy Reflectance

$$R_j = \int R(v) \underbrace{\frac{s_j \ \frac{1}{v}}{||s_j|| \ v^2}}_{} \, dv$$

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Derive Airy Reflectance

$$R_j = \int R(v) \underbrace{\frac{s_j \; \frac{1}{v}}{||s_j|| \; v^2}} dv$$

$$S_j(v) = \frac{s_j \; \frac{1}{v}}{||s_j|| \; v^2}$$

Fourier Transform

Complex Conjugate

$$R_j = \int \hat{R}(\mu)\hat{S}_j(\mu)^{\star} d\mu$$

Derive Airy Reflectance

$$R_j = \int R(v) \underbrace{\frac{s_j \ \frac{1}{v}}{||s_j|| \ v^2}} dv$$

$$S_j(v) = \frac{s_j \ \frac{1}{v}}{||s_j|| \ v^2}$$

$$R_j = \int \hat{R}(\mu)\hat{S}_j(\mu)^{\star} d\mu$$

$$\boxed{R_j = C_0 + 2\sum_{m=1}^{+\infty} C_m \begin{bmatrix} cos(m\phi_2) \\ sin(m\phi_2) \end{bmatrix}^T \begin{bmatrix} \Re_j(m\mathcal{D}) \\ \Im_j(m\mathcal{D}) \end{bmatrix}}$$

PAPER - MAIN RESULT

Derive Airy Reflectance

$$R_j = \int R(v) \frac{s_j \frac{1}{v}}{||s_j|| \; v^2} \, dv$$

$$S_j(v) = \frac{s_j \frac{1}{v}}{||s_j|| \; v^2}$$

$$R_j = \int \hat{R}(\mu) \hat{S}_j(\mu)^{\star} d\mu$$

$$\begin{bmatrix} cos(m\phi_2) \\ sin(m\phi_2) \end{bmatrix}^T \begin{bmatrix} \mathfrak{R}_j(m\mathcal{D}) \\ \mathfrak{I}_j(m\mathcal{D}) \end{bmatrix}$$

ER - MAIN RESULT

Credits

- A Practical Extension to Microfacet Theory for the Modeling of Varying Iridescence
- Bug icon

Thanks