Physics of Cosmic Structures: week 2 exercises

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To successfully pass the exam, it is important to note that regular exercises labeled with [R] are mandatory. On the other hand, exercises marked with [S] are considered speculative and are not compulsory. You may choose to work on these exercises at your discretion.

It is highly recommended to work on regular exercises independently as it can help build a strong, individual understanding of the topic. However, for speculative exercises, collaborating with others is not only allowed, but also encouraged. Group work can foster creativity and facilitate idea sharing, leading to a more fulfilling learning experience.

Do not hesitate to ask questions.

I. PROBLEM 1: DISCOVER COSMIC ACCELERATION [R]

The files attached to this exercise contain the measurements and covariance of Supernovae luminosity distance measurements from arXiv:1710.00845.

These measure the (log) flux of many Supernovae:

$$D_L(z) = \sqrt{\frac{L}{4\pi F}} \tag{1}$$

as a function of redshift. Note that we do not really know L but we do know that it is the same for all objects (i.e. Supernovae are standard candles).

To conform to strange astronomy conventions (magnitudes) we consider:

$$\mu = 5\log_{10} D_L(z) \tag{2}$$

This makes the following sense: we are measuring fluxes to high signal to noise ratio (SNR). The distribution of measured fluxes is going to be Gaussian because of the central limit theorem. The distribution of a ratio of Gaussian variables is not Gaussian. When considering the log, on the other hand, close-to-Gaussianity is guaranteed by high SNR and then everything is a linear combination of Gaussian variables, hence Gaussian.

We start with a Gaussian likelihood for the data x:

$$\ln \mathcal{L} = -\frac{1}{2}(x-\mu)^T\Sigma^{-1}(x-\mu) \tag{3}$$

Note that we neglect all parameter independent normalization factors.

Since we do not really know the true intrinsic luminosity of Supernovae we are not interested in D_0, which depends on the value of the Hubble constant. Then we marginalize analytically over a zero point additive constant:

$$\mathcal{L}_m = \int \mathcal{L}\, d\alpha = \int \exp\left(-\frac{1}{2}(d-\mu+\alpha)\Sigma^{-1}(d-\mu+\alpha)\right) d\alpha \tag{4}$$

where $\alpha \sim \log_{10} D_0$. **Show that**, after some algebra:

$$\ln \mathcal{L}_m = -\frac{1}{2}(d-\mu)\Sigma^{-1}(d-\mu) + \frac{1}{2}\frac{\left[(1)\Sigma^{-1}(d-\mu)\right]^2}{(1)\Sigma^{-1}(1)} \tag{5}$$

where (1) is a vector of ones.

You can now take the calculation of D_L from the previous exercise. Analytic marginalization means that D_L will not depend on H_0. The redshift range that we are considering implies that radiation is negligible. We can neglect curvature. The only free parameter in the standard model is then Ω_m. The abundance of Dark Energy $\Omega_{\rm DE} = 1-\Omega_m$ is fixed by the value of Ω_m.

Write the likelihood in terms of its only input parameter Ω_m. **Grid sample** the likelihood and plot it. Compute the mean and variance of the distribution. You can check your results against arXiv:1710.00845. Have you discovered cosmic acceleration (i.e. $\Omega_m < 1$)?

II. PROBLEM 2: FUN WITH SUPERNOVAE [S]

Can you think of some modification of the luminosity distance formula, perhaps with an extra parameter, and estimate that from data in the same way? What is the physical meaning of your modification? What are the physical implications of your results?

III. PROBLEM 3: MICRO-PHYSICAL INTERPRETATION OF A GENERIC EQUATION OF STATE [S]

The background phase-space distribution function determines the background equation of state. In general, when considering a generic fluid, it is common to consider a generic time dependence for the equation of state $w(t)$, where t can be a generic time coordinate (sometimes we use scale factor, sometimes redshift, ...). What type of phase-space distribution would we need to obtain a generic equation of state? Does this impose some constraints on the values that w can take?