Physics of Cosmic Structures: week 8 exercises

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To successfully pass the exam, it is important to note that regular exercises labeled with [R] are mandatory. On the other hand, exercises marked with [S] are considered speculative and are not compulsory. You may choose to work on these exercises at your discretion.

It is highly recommended to work on regular exercises independently as it can help build a strong, individual understanding of the topic. However, for speculative exercises, collaborating with others is not only allowed, but also encouraged. Group work can foster creativity and facilitate idea sharing, leading to a more fulfilling learning experience.

Do not hesitate to ask questions.

I. PROBLEM 1: SUPERHORIZON EVOLUTION OF THE GRAVITATIONAL POTENTIAL [R]

We will derive here the analytic solution for the super-horizon evolution of the gravitational potential that remains valid in the transition from the radiation era to the matter era.

Start from the following Einstein equation:

$$\nabla^2\Phi - 3\mathcal{H}(\Phi' + \mathcal{H}\Phi) = 4\pi G a^2 \delta\rho_m \tag{1}$$

where this expression is written in conformal time and $\delta\rho_m = \sum_a \delta\rho_a$ is total matter density and includes dark matter, baryons and radiation. On super-horizon scales the term $\nabla^2\Phi$ can be dropped and this becomes an equation for the evolution of Φ.

Assuming adiabatic perturbations, show that the superhorizon evolution of the potential in a universe with matter and radiation satisfies:

$$y\frac{d\Phi}{dy} + \Phi = -\frac{4+3y}{6(y+1)}\delta_m \tag{2}$$

where $y \equiv a/a_{\rm eq}$ and $\delta_m = \delta\rho_m/\rho_m$.

Use $\delta_m' = 3\Phi'$ to write this as a closed equation for Φ.

Define

$$u \equiv \frac{y^3\Phi}{\sqrt{1+y}} \tag{3}$$

and show that:

$$\frac{d^2u}{dy^2} + \left[-\frac{2}{y} + \frac{3}{2(y+1)} - \frac{3}{3y+4}\right]\frac{du}{dy} = 0 \tag{4}$$

and by integrating this show that:

$$\Phi(\eta) = \frac{\Phi_i}{10y^3}\left[16\sqrt{1+y} + 9y^3 + 2y^2 - 8y - 16\right] \tag{5}$$

where Φ_i is the primordial value of the gravitational potential.

Confirm that as $y \to \infty$ then $\Phi \to \frac{9}{10}\Phi_i$ to show the change in the gravitational potential during the transition from the radiation era to the matter era.

II. PROBLEM 2: POWER SCALING [R]

Use CAMB to calculate the linear matter power spectrum for the reference cosmological parameters.

Derive and verify the analytic scaling of the different regions of the power spectrum, overplot them.

It might be useful to spline interpolate and differentiate the log power spectrum wrt to $\log k$ to obtain power law indexes...