Physics of Cosmic Structures: week 1 exercises

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To successfully pass the exam, it is important to note that regular exercises labeled with [R] are mandatory. On the other hand, exercises marked with [S] are considered speculative and are not compulsory. You may choose to work on these exercises at your discretion.

It is highly recommended to work on regular exercises independently as it can help build a strong, individual understanding of the topic. However, for speculative exercises, collaborating with others is not only allowed, but also encouraged. Group work can foster creativity and facilitate idea sharing, leading to a more fulfilling learning experience.

Do not hesitate to ask questions.

I. PROBLEM 1 [R]

Take the FRW metric given by:

$$ds^2 = -dt^2 + a(t)\, dl^2 \tag{1}$$

where:

$$dl^2 \equiv \frac{dr^2}{1-Kr^2} + r^2 \left[d\theta^2 + \sin^2\theta d\phi^2\right] \tag{2}$$

Compute: the inverse metric, the Levi-Civita connection, the geodesic equation, the Riemann tensor, the Ricci tensor and scalar and the Einstein tensor. Take a generic perfect fluid stress energy tensor:

$$T^\mu_\nu = \mathrm{diag}(\rho, -P\delta_{ij}) \tag{3}$$

calculate Einstein equations, derive Friedman equations and derive the stress energy continuity equation. Derive the fluid continuity equation from the two Friedman equations.

II. PROBLEM 2 [R]

Take a realistic cosmological model given by:

$$\begin{aligned}
&H_0 = 69[Km/s/Mpc] \\
&\Omega_m = 0.32 \\
&\Omega_b = 0.04 \\
&\rho_{\gamma 0} = 4.4738 \times 10^{-34} g/cm^3
\end{aligned} \tag{4}$$

Consider neutrinos massless and their abundance compatible with the standard model:

$$\rho_{\nu 0} \simeq 0.68 \rho_{\gamma 0} \tag{5}$$

Consider Dark Energy a cosmological constant and curvature to be zero. Solve numerically the continuity equations and Friedmann equations to compute $H(z)$ or $H(a)$, from today to deep into radiation domination. Compute numerically all sort of distance measures by integrating H. Do the same calculation with CAMB. Plot the two results and make sure they agree (to some level of precision). Make log-log plots to appreciate the power law behavior typical of single fluid models at different evolutionary stages.

This exercise is probably most enjoyable when done in python, within a Jupiter notebook.

III. PROBLEM 3 [S]

The FRW background spontaneously breaks time translation invariance. Consider a Lorentz boost with a given velocity, how does that change FRW equations? Look for an updated measurement of the velocity of earth with respect to the CMB frame how much does our velocity biases the measurement of the Hubble constant? Using error propagation calculate how much uncertainty this measurement adds to the determination of the Hubble constant. Is there an effect on the measured temperature of the CMB? Using the last determination of CMB temperature calculate bias end error due to our velocity, if any.