Physics of Cosmic Structures: week 5 exercises

Marco Raveri (marco.raveri@unige.it)

To successfully pass the exam, it is important to note that regular exercises labeled with [R] are mandatory. On the other hand, exercises marked with [S] are considered speculative and are not compulsory. You may choose to work on these exercises at your discretion.

It is highly recommended to work on regular exercises independently as it can help build a strong, individual understanding of the topic. However, for speculative exercises, collaborating with others is not only allowed, but also encouraged. Group work can foster creativity and facilitate idea sharing, leading to a more fulfilling learning experience.

Do not hesitate to ask questions.

I. PROBLEM 1: HORIZON TILING OF THE SKY [R]

Calculate the angular size of the horizon at a given redshift, projected on the sky. Do the calculation analytically in matter domination and radiation domination. Do the calculation numerically and plot the results for the reference cosmology used in week 2 exercises (I suggest to use CAMB for distance calculations).

How many horizons do you need to fully tile the sky, as a function of redshift? You can do this the easy way (sky area / horizon area) or the hard way (sphere packing), both are good.

Imagine that you define a test statistic on each of these horizon sized patches, at a given redshift. These are separate universes in practice... What is the scaling of the variance of the test statistic at each redshift?

II. PROBLEM 2.1: SCALAR FIELD EQUATIONS OF MOTION [R]

Start from the scalar field action. Derive the non-perturbative equation of motion and contribution to the stress energy tensor.

You can do this exercise or 2.2.

III. PROBLEM 2.2: SCALAR FIELD EQUATIONS OF MOTION [R]

Use previous results for the perturbed connection. Compute the equation of motion for a scalar field and the scalar field contribution to the stress energy tensor at first order in perturbation theory.

You can do this exercise or 2.1.

IV. PROBLEM 3: SCALAR FIELD DYNAMICS [R]

Take the background scalar field equation. Fix the potential to be $V(\phi) = m^2\phi^2/2$ or something else of your choosing (make it interesting!). Solve numerically the coupled system of field and Hubble equations in vacuum (inflation).

When in need of some cosmological parameters refer to week 2 exercise.

Note: it is better to use a specific time variable: $\ln a$. In this way you do not need to solve FRW equations.

Plot the results for different values of initial conditions.

Is the field slowly rolling? Check the time dependence of slow roll quantities.

Can you find parameters/initial conditions for which slow roll is achieved?