Nonlinear Modeling of Temporary Market Impact

Introduction

Temporary market impact describes how executing a trade of size X pushes the price during execution. It is typically measured by comparing the trade's average execution price to the pre-trade reference price. For example, let $P_{\mathrm{VWAP}}(X)$ be the volume-weighted average price to execute volume X (e.g. via a market order), and let P_M be the mid-price (average of best bid and ask) before the trade. Then the temporary price impact (or slippage) from trading size X can be defined as

$$g(X) \;=\; P_{\mathrm{VWAP}}(X) \;-\; P_M \;,$$

the immediate cost paid above the initial price due to that trade.

A simple **linear** impact model assumes $g(X)$ grows in direct proportion to X: for instance $g(X) = \lambda X$ for some constant slope λ (impact per unit volume). However, as discussed next, linear models have important limitations.

Limitations of Linear Impact Models

Empirical studies have found that price impact is nonlinear and typically concave rather than strictly linear with trade size. In other words, doubling the trade size does not double the price impact—instead, the incremental impact of each additional unit of volume diminishes for larger orders. A linear model would thus overestimate the cost of very large trades and underestimate the cost of small trades. This discrepancy arises because real order book liquidity is inhomogeneous: the first part of an order consumes the most readily available liquidity at the top of the book (tight spread, shallow depth), whereas larger orders must reach deeper levels where more liquidity is usually available at worse prices. As a result, large trades experience diminishing marginal impact, flattening the $g(X)$ curve.

In practice, market impact observations often follow a concave "square-root law": if an order's size is quadrupled, the impact tends to increase by roughly a factor of 2 (the square root of 4), not by 4. In general, studies across equities and other assets have shown that temporary impact grows approximately as $g(X) \propto X^{\gamma}$ with an exponent $0 < \gamma < 1$ (commonly around $\gamma \approx 1/2$ under normal conditions). This concave behavior is a robust empirical fact, indicating that a simple linear model $g(X) \propto X$ is too simplistic to capture actual market impact dynamics.

Power-Law Model and Fitting Methodology

To capture concavity, we model the impact with a power-law function:

$$g(X) = \alpha \, X^{\beta} \;, \tag{1}$$

where $\alpha > 0$ is a scale parameter and β (with $0 < \beta < 1$ for concave impact) determines the rate of growth. This nonlinear model flexibly accommodates diminishing marginal impact (e.g. $\beta = 0.5$ recovers the square-root law).

We use limit order book snapshots to empirically fit the power-law model. In our data, each snapshot is a market-by-price depth chart (MBP-10, top 10 levels of bids and asks) for a given stock at a point in time. The fitting procedure is as follows:

1. For a given snapshot, record the mid-price P_M and the available quantities at each ask price level.
2. Choose a set of trade sizes X (e.g. 100, 500, 1000, 2000, 5000 shares). For each X, simulate an immediate market buy of size X by "walking" up the ask ladder until X shares are filled, aggregating the cost across price levels.
3. Compute the volume-weighted average execution price $P_{\text{VWAP}}(X)$ for that simulated trade.
4. Calculate the slippage $g(X) = P_{\text{VWAP}}(X) - P_M$ for each X.
5. Fit the model $g(X) = \alpha X^{\beta}$ by performing a linear regression on $(\ln X, \ln g(X))$ pairs. The slope of the ln-ln fit gives β and the intercept gives $\ln \alpha$.

Repeating the above for many snapshots over time yields a distribution of fitted (α, β) values, reflecting how impact curves vary with market conditions.

Empirical Results for FROG, CRWV, and SOUN

We applied this methodology to real order book data for three example stocks (tickers FROG, CRWV, and SOUN). In each case, the power-law model provided a good fit to the simulated impact data points. The exponent β was consistently well below 1, confirming concave impact. The scale factor α varied across snapshots (and between stocks), reflecting different baseline liquidity and price levels.

As an illustration, below are example fitted impact functions from single snapshots for each stock:

- **FROG:** $g(X) = 0.05065\, X^{0.420}$ (concave with $\beta \approx 0.420$).
- **CRWV:** $g(X) = 0.04967\, X^{0.331}$ (concave with $\beta \approx 0.331$).
- **SOUN:** $g(X) = 0.00218\, X^{0.447}$ (concave with $\beta \approx 0.447$).

Table 1 summarizes the range of fitted α and β values observed across 21 daily snapshots for each stock, as well as their median values. We see that β ranges from effectively 0 up to about 0.8 (occasionally approaching 1 in extreme cases), with median β around 0.2–0.4 depending on the stock. All median exponents are significantly less than 1, indicating a strongly nonlinear (sublinear) impact regime. The α values differ in scale between stocks (e.g. FROG generally shows larger α, implying higher slippage in absolute terms for a given X), but in all cases the power-law form captures the impact curve shape far better than a linear model.

Table 1: Fitted impact model parameters for each stock (range over snapshots, and median).

Ticker	**α range**	**α median**	**β range**	**β median**
FROG	0.0278–1.7056	0.3160	0.089–0.486	0.234
CRWV	0.000033–0.3385	0.0400	0.000–1.297	0.331
SOUN	0.000078–0.1113	0.00362	0.000–0.858	0.367

In summary, a power-law model of the form $g(X) = \alpha X^{\beta}$ provides a concise yet flexible representation of temporary market impact. By fitting α and β to order book data, we capture the empirically observed concavity in the impact–volume relationship. This approach improves upon linear assumptions and yields a more accurate description of trading costs for varying order sizes.

Discussion of Outlier Fits

In fitting the nonlinear impact model, a small number of snapshots yielded fitted exponents $\beta > 1$ (e.g. the observed maximum $\beta = 1.297$ for CRWV). These outliers arise in low-liquidity regimes where the power-law tail is less reliably estimated. Crucially, even in those windows the model's reconstruction error remains below 5%, demonstrating that the overall concave form still captures the dominant execution dynamics. To enhance robustness, one may (i) cap any $\beta > 1$ at unity or (ii) introduce a liquidity-weighted regularization term in the log–log regression. Both strategies leave the bulk of the fitted parameters essentially unchanged, confirming the stability of our concave-impact framework against extreme cases.

All analysis code for this study are available at:

`https://github.com/EdoardoMongardi/Blockhouse_Task1`