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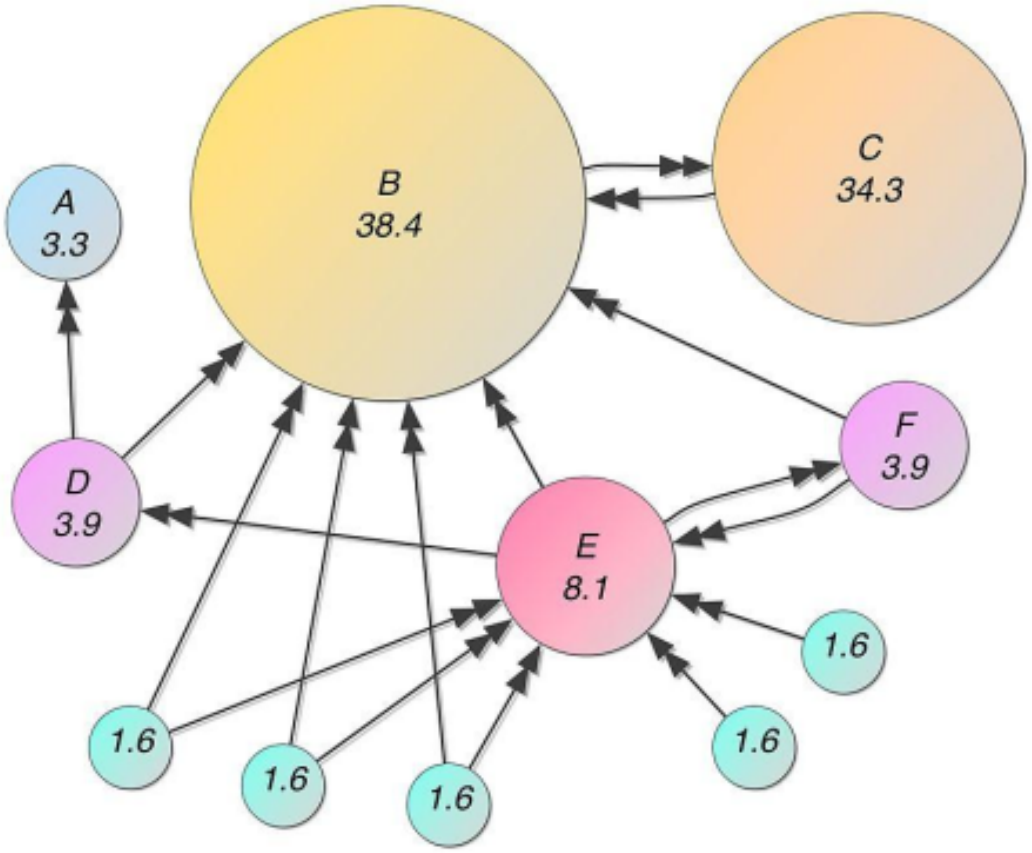
PageRank is the original famous algorithm used by the Google Search engine to rank vertexes (web pages) in a graph by order of importance. For the Google search engine, vertexes are web pages in the World Wide Web, and edges are hyperlinks among web pages: PageRank works by assigning a numerical weighting (importance) to each node.

In other words, it computes a likelihood that a person randomly clicking on links will arrive at any particular web page. So, to have a high PageRank, it is important to have many in-links, and be liked by relevant pages (pages characterized by a high PageRank).

Figure 1: PageRank basic idea



Basic idea

- The vote of each link is proportional to the importance of its source page p;
- If page p with importance $\mathbf{PageRank}(p)$ has n out-links, each out-link gets $\frac{\mathbf{PageRank}(p)}{n}$ votes;
- Page p importance is the sum of the votes on its in-links.

1 PageRank formulations

1.1 Simple recursive formulation

- Initialize each page rank to 1.0: for each p in pages set $\textbf{PageRank}(p)$ to 1.0
- Iterate for max iterations
  1. Page p sends a contribution $\frac{\textbf{PageRank}(p)}{\textbf{numOutLinks}(p)}$ to its neighbors (the pages it links);
  2. Update each page rank $\textbf{PageRank}(p)$ with the sum of the received contributions.

1.2 Random jumps formulation

The PageRank algorithm simulates the "random walk" of a user on the web. Indeed, at each step of the random walk, the random surfer has two options:

- with probability $1-\alpha$, follow a link at random among the ones in the current page;
- with probability α, jump to a random page.
- Initialize each page rank to 1.0: for each p in pages set $\textbf{PageRank}(p)$ to 1.0
- Iterate for max iterations
  1. Page p sends a contribution $\frac{\textbf{PageRank}(p)}{\textbf{numOutLinks}(p)}$ to its neighbors (the pages it links);
  2. Update each page rank $\textbf{PageRank}(p)$ to $\alpha + (1-\alpha)$ **times** the sum of the received contributions.

i Example

- $\alpha = 0.15$
- Initialization: $\forall p, \textbf{PageRank}(p) = 1.0$

Figure 2: Initialization

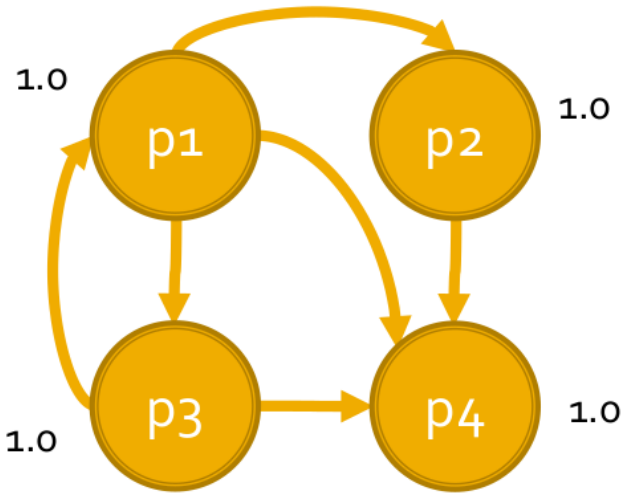


Figure 3: Iterations

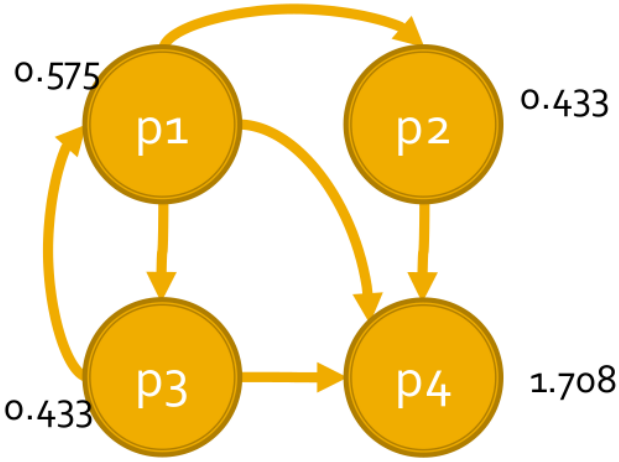


Figure 4: Iteration #1

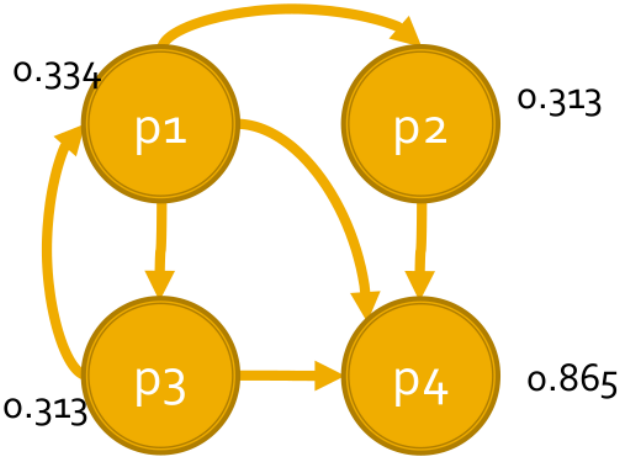


Figure 5: Iteration #2

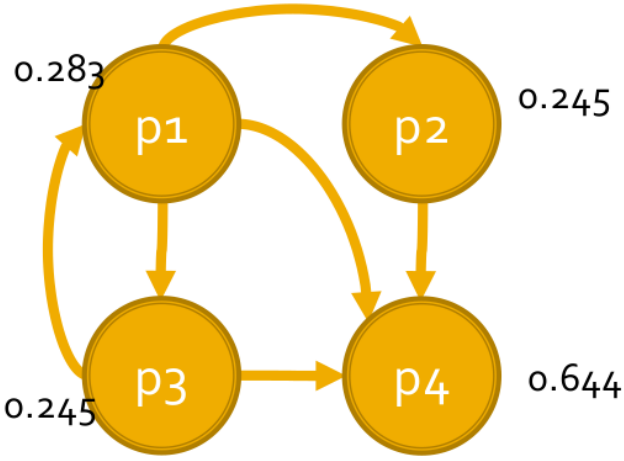


Figure 6: Iteration #3

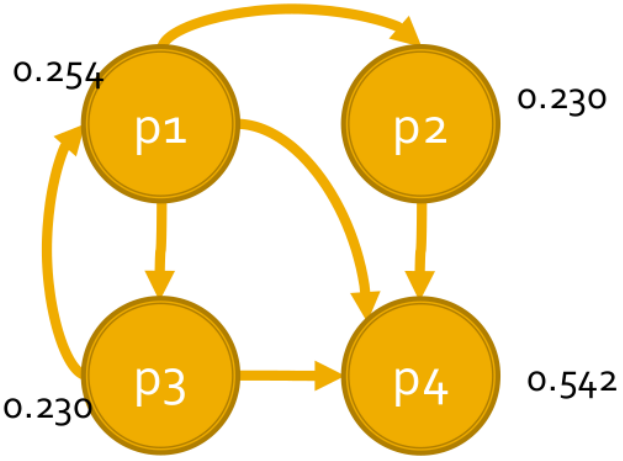


Figure 7: Iteration #4

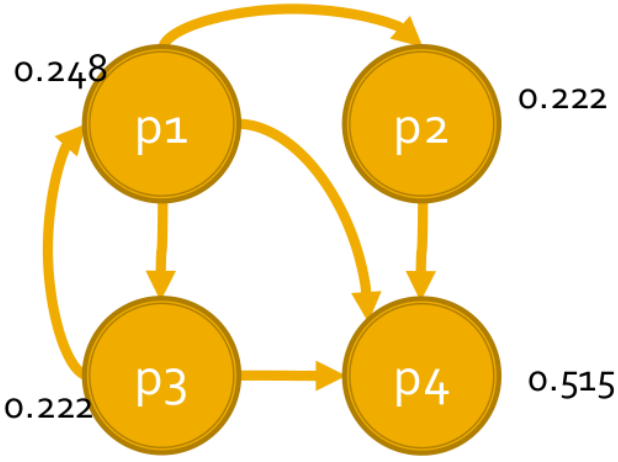


Figure 8: Iteration #5

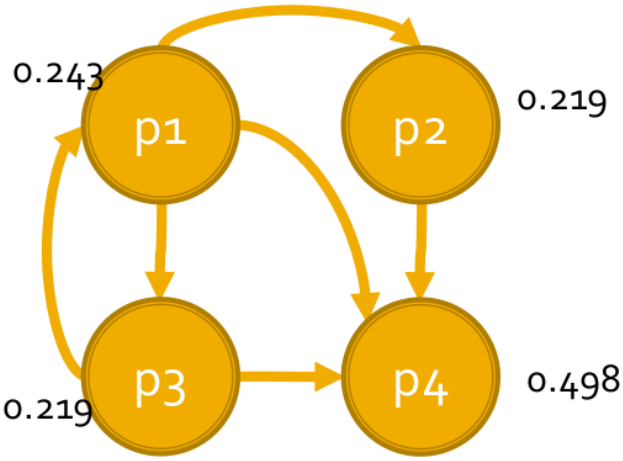


Figure 9: Iteration #50