**Def 2.2.1:** Two continuous maps are **homotopic** if there exist a continuous map such that:

&

for all . The map is said to be a **homotopy**, and we write g for ‘ is homotopic to ’ or if we wish to specify the homotopy.

**Props 2.2.2** Given a map

**Theorem 2.2.3**  Let be a subspace of , and let be two maps. If, for each , and can be joined by a straight-line segment in , the .

**Corollary 2.2.4** Let be any space, and let be two maps such that for all . Then .

**Def 2.2.5** Given pairs and , two maps of pairs are homotopic if there exists a map of pairs , such that

and for all .

**Def 2.2.6** Two spaces and are **homotopy-equivalent** (or of the same homotopy type) if there exist maps and , such that and , where and are the identity maps of and respectively. In this case if is a homotopy equivalence and is a homotopy inverse of . We write for is homotopy-equivalent to Y' (notice that the symbol has two distinct meanings, depending on the context).

**Def 2.2.11** A subspace of a topological space is a retract of if there exists a map (called a retraction), such that for all . If denotes the inclusion map, then is a deformation refraction (and is a deformation retract of ) if . If also , then is a strong deformation retraction, and is a strong deformation retract of .

**Proposition 2.2.12** If is a deformation retract of , then .

**Definition 2.2.14** A space , homotopy-equivalent to a point, is called contractible.

**Simplicial complexes:**

A simplex is just a generalization to n dimensions of a triangle or tetrahedron, and these are fitted together in such a way that two simplexes meet (if at all) in a common edge or face.

**Def 2.3.1:** A set of points said to be independent if the vectors are linearly independent. Or equivalently

,

Where hence the definition of independence does not depend on the order of the points. Example: are independent if they are not collinear.

**Def 2.3.2.** A geometric n-simplex is the set of points where are independent points in some Euclidean space , and the are real numbers such that for all and . This defines as a subset of ; is given the subspace topology.

**Def vertices.** are called vertices of and are said to span , we write for if we wish to specify the vertices.

**Def interior of .** The subspace of of those points such that is called the interior of (it’s not the same interior as the topological interior) for example a 0-simplex coincides with its interior.

**Def face.** Any subset of vertices , the subspace of of those points linearly dependent on is called a face of .

**Def dimension.** The n in is called the dimension.

**Props 2.3.3.** A geometric n-simplex is a closed convex compact connected subspace of , and is the closure of its interior. A face is a closed subspace of and is itself a simplex. Moreover, a simplex determines its vertices, so that two simplexes coincide if and only if they have the same set of vertices.

**Def Homeomorphism:** Let and topological spaces and then is a homeomorphism if:

* is a bijection
* The inverse function is continuous

**Props 2.3.4.**  and are linearly homeomorphic, that is, there exist a homeomorphism such that

For all points of .

**Def 2.3.5.** A geometrical simplicial complex is a finite set of simplexes, all contained in some Euclidean space . Furthermore

1. If is a simplex of and is a fade of , then is in K.
2. If and are simplexes of , then either is empty, or is a common face of and

The dimension of , , is the maximum of the dimensions of its simplexes.

A subcomplex of is a subset of simplexes of , satisfying property (a) (and hence also (b): see Proposition 2.3.6(c)). In particular, for each the r-skeleton of , , is the subset of simplexes of dimension at most .

A simplicial pair consists of a simplicial complex and a subcomplex Simplicial triples, etc., are similarly defined.

**Def polyhedron of K.** The set of points of that lie in at least one of the simplexes of , topologized as a subspace of , is a topological space, called the polyhedron of , written and if is a subcomplex of then is called a subpolyhedron of .

**Props 2.3.6**

1. If is a simplicial complex, is a closed compact subspace of
2. Every point of is in the interior of exactly one simples of
3. A subcomplex of a simplicial complex is itself a simplicial complex, and is a closed subspace of
4. If and are subcomplexes of , so are and

**Props 2.3.7.**  There exist a subcomplex of , such that . is called the clousure of , written .

**Props 2.3.8.** A subset of is closed if and only if is closed in simplex in .