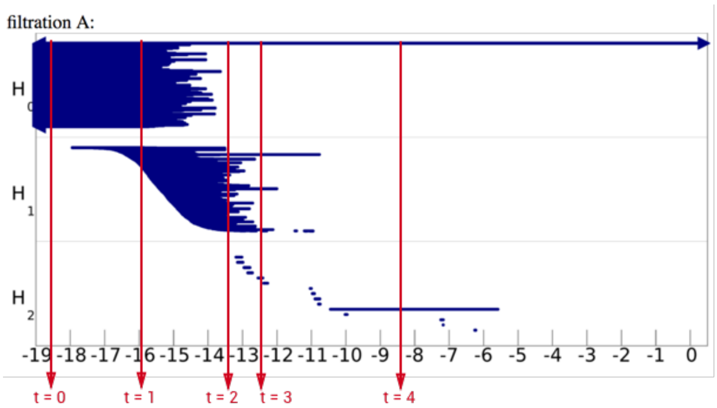
Q8.

Can you infer the topological structure of the spaces underlying the data? To help you in this task, we show pictures of the barcodes obtained on these filtrations below (click on an image to see it at full resolution).

For every filtration, we have identified **different steps** in the filtration, and commented on **what could correpond** to the the given barcodes. We keep in mind that **there can be different interpretation** of these barcodes.

Filtration B, C and D even have some sketches for the filtrations. (see the following pages)

Filtration A



t = 0 Group of points linked without cycles

t = 1 Apparition of the first triangles

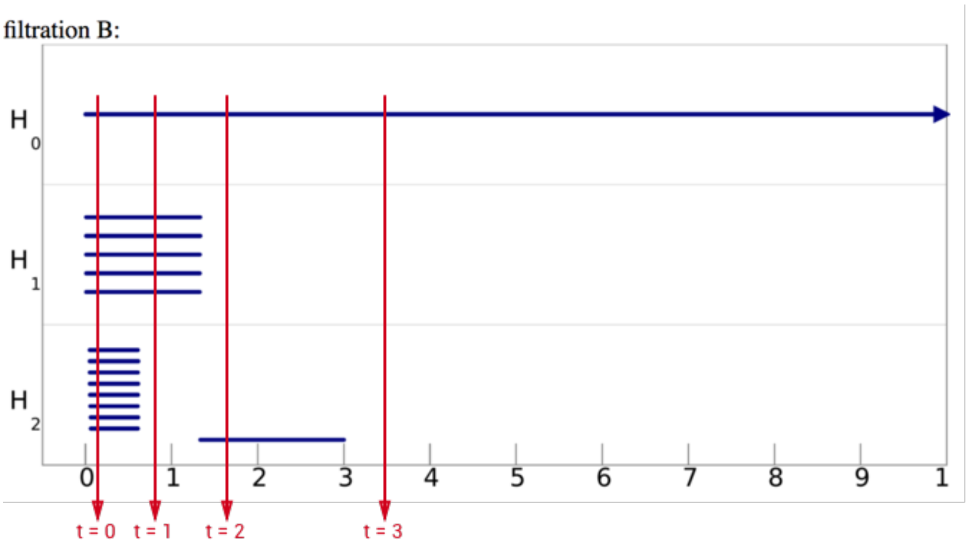
t = 2 All points connected, and some surfaces are added without creating cavities yet !

t = 3 Some cavities are created, while other surfaces are still being added

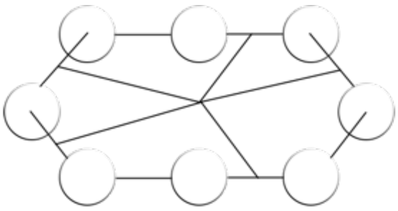
t = 4 Many cavities created before have been filled but one hole remain. We could imagine a ball with a cavity inside.

t = inf The cavities have all been filled and we have like a ball.

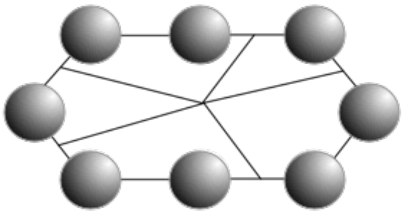
Filtration B



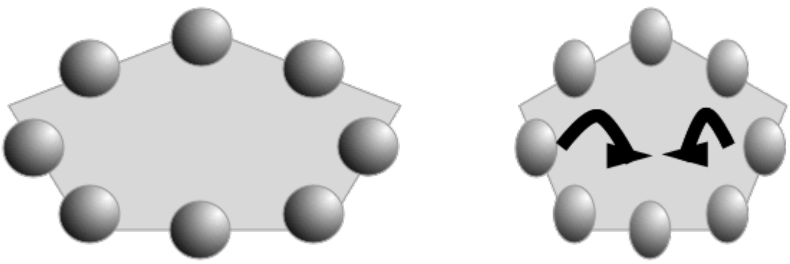
t = 0 8 spheres linked with edges and 5 cycles created by the edges



t = 1 Spheres are filled, the 5 cycles are still here.



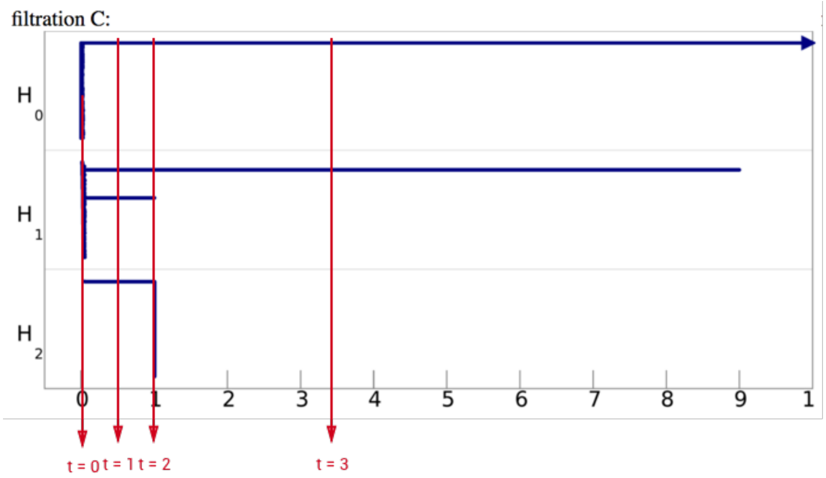
t = 2 The surface between the edges are filled, (to kill 1-cycle)
And the figure is folded to create a hole (a new 2-cycle)



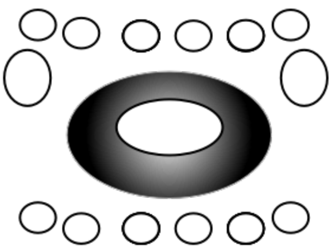
It creates a sphere will balls on it

t = 3 The sphere is filled ! It's a ball with balls on it !

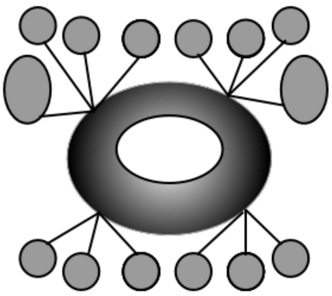
Filtration C



t = 0   A torus with distant circles around it.

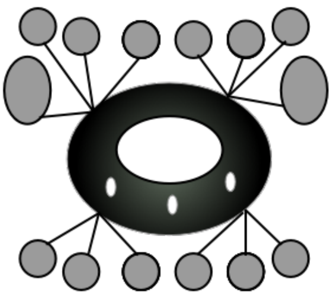


t = 1   The circle are filled, and linked to the torus, so that the homology is still as a torus



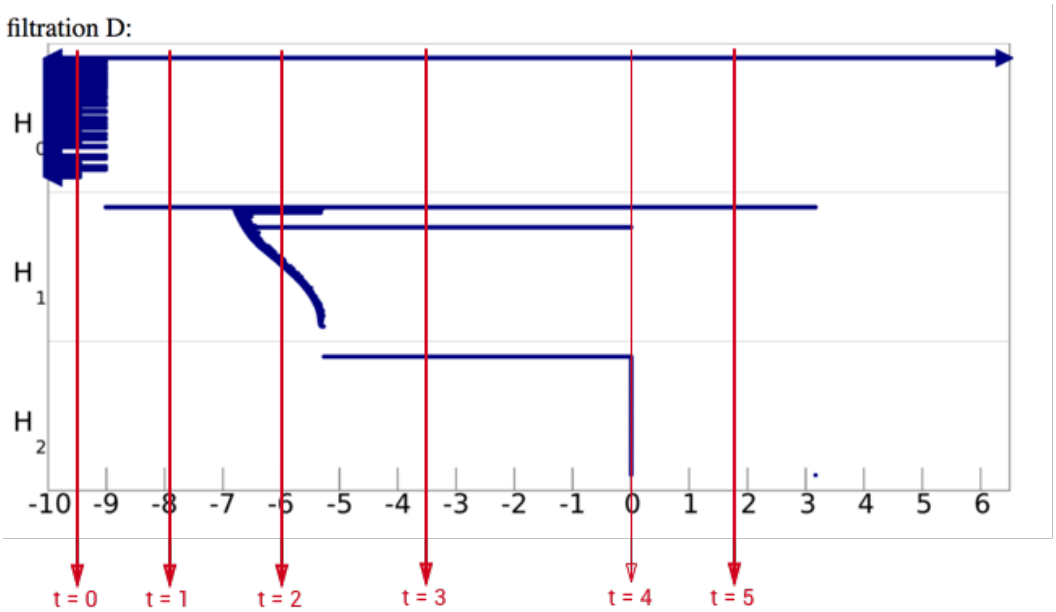
$$H^* = \mathbb{K} \oplus \mathbb{K}^2 \oplus \mathbb{K}$$

t = 2   The interior of the torus starts to be filled but many holes appear inside at t = 2



t = 3   The interior of the torus is completely filled at t = 3 !

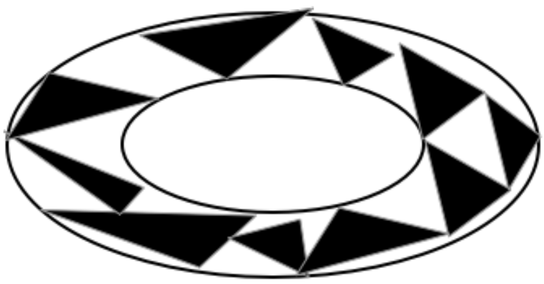
Filtration D



t = 0 Group of points linked without cycles

t = 1 One cycle that links all points (for example a circle)

t = 2 Edges are added between points to create triangles, but these 1-cycles are quickly killed because faces appear also. It takes the shape of a torus (to prepare for t = 3)



t = 3 All faces are filled and we have our torus !

t = 4 Like in filtration C. The interior of the torus starts to be filled but many holes appear.

t = inf The interior of the torus is completely filled at t = 5 !