

Exam Practice

A comprehensive set of practice questions on all topics covered on the final exam

Version 1

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Disclaimer

The problems within this document are heavily inspired off of the math textbook as well as those that were given in class.

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1 Remainder Theorem

1.1. $2x^n + ax^2 - 6$ leaves a remainder of -7 when it is divided by $x - 1$. It also leaves a remainder of 129 when divided by $x - 3$. What is a and n given that $n \in \mathbb{N}$

Required Knowledge

1. Factor Theorem
2. Basic algebra ($2 \cdot 1^n = 2$)

1.1. Solutions

Step I: Solving for a

$f(1) = -7$	By the remainder theorem
$2(1)^n + a - 6 = -7$	Plug in the 1.
$2 + a - 6 = -7$	$2 \cdot 1^n = 2$
$a = -3$	Solve for a .

Step II: Solving for n

$f(-3) = 129$	By the remainder theorem
$2(-3)^n + (-3)(-3)^2 - 6 = 129$	Plug in -3
$(-3)^n = 81$	Simplify
$n = 4$	No need for logs for this particular exponent problem.

Final Answer

$$n = 4$$

$$a = -3$$

2 Polynomial Rules

2.1. A polynomial has

1. A degree of 5
2. A leading coefficient of 2
3. x^4 has a coefficient of 3.
4. The y -intercept is 5.
5. Its only two real zeroes are $\frac{1}{2}$ and $1 \pm \sqrt{2}$.
6. Its other roots are $m \pm ni$, $n > 0$.

What is m and n ?

Required Knowledge

1. Sum of roots
2. Product of roots

2.1. Solutions

Step I: Sum of Roots

$$\Sigma = \frac{-a_{n-1}}{a_n}$$

$$\frac{-3}{2}$$

$$0.5 + 1 + \sqrt{2} + 1 - \sqrt{2} + m + ni + m - ni = \frac{-3}{2}$$

$$2.5 + 2m = \frac{-3}{2}$$

$$2m = -4$$

$$m = -2$$

Using sum of roots

Equate to the actual sum of roots to solve.

Solve for m

Step II: Product of Roots

$$\Pi = \frac{(-1)^n a_0}{a_n}$$

$$\Pi = \frac{-5}{2}$$

$$\left(\frac{1}{2}\right)(1 + \sqrt{2})(1 - \sqrt{2})(m - ni)(m + ni) = \frac{-5}{2}$$

$$\left(\frac{1}{2}\right)(1 - 2)(m^2 - (ni)^2) = \frac{-5}{2}$$

$$\left(\frac{-1}{2}\right)(m^2 + n^2) = \frac{-5}{2}$$

$$-(4 + n^2) = \frac{-5}{2}$$

$$-4 - n^2 = -5$$

$$-n^2 = -1$$

$$n = \pm 1$$

$$n = 1$$

Using the product of roots formula

Equate the product of roots to the formula

Simplify

Plug in the value of m determined earlier.

$n < 0$ because of the question

Final Answer

$$m = -2$$

$$n = 1$$

2.2. Find all values of the real parameter m , $m \neq 0$, for which the equation $(mx)^2 + 3mx + 1 - m = 0$ has no solutions.

Required Knowledge

1. Calculating the discriminant
2. Quadratic inequalities ($\Delta < 0$)

2.2. Solutions

Step I: Stating what is known

$$\begin{aligned}(mx)^2 + 3mx + 1 - m &= 0 \\ m^2x^2 + 3mx + 1 - m &= 0 \\ ax^2 + bx + c \\ a &= m^2 \\ b &= 3m \\ c &= 1 - m\end{aligned}$$

Step II: Find when $\Delta = 0$

$$\begin{aligned}\Delta &= 0 \\ b^2 - 4ac &= 0 && \text{Expand } \Delta \\ (3m)^2 - 4m^2(1 - m) &= 0 && \text{Plug in } a, b, \text{ and } c. \\ 3^2m^2 - 4m^2 + 4m^3 &= 0 \\ 4m^3 + 5m^2 &= 0 \\ m^2(4m + 5) &= 0 && \text{Simplify} \\ m \in \{0, -\frac{5}{4}\} &&& \text{Solve for roots by factoring}\end{aligned}$$

Step III: Find when $\Delta < 0$

This is done using graphical analysis, as shown below using Geogebra. A sketch of the function on a number line is also possible. The idea is to see when it is below the x-axis.

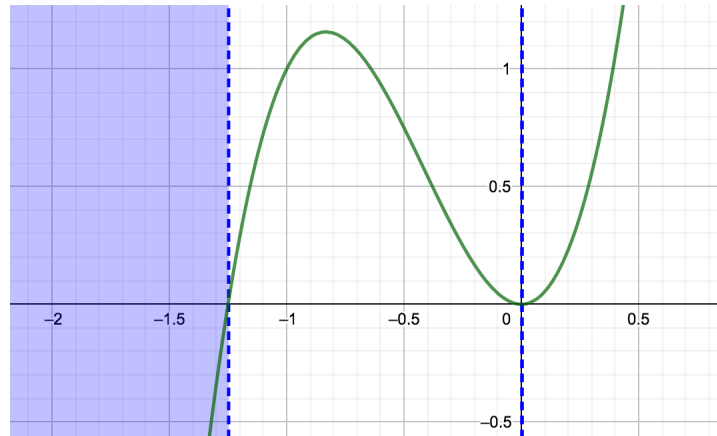


Figure 1: Graphical analysis

Final Answer

$m < \frac{-5}{4}$. This means that whenever m is less than and **not** equal to $\frac{-5}{4}$, the function has no real roots. Note that the quadratic function mapping Δ and the function itself are completely separate.

2.3. Given the polynomial $-3x^5 - x^4 + 5x^3 + 9x^2 - 11x + 20 = 0$, determine the following:

Required Knowledge

1. Complex conjugate root theorem
2. Descartes's rule of signs
3. Sum of roots
4. Product of roots

Possible Real	Possible Imaginary	Total
1	4	5
3	2	5
5	0	5

Table 1: Possible Real, Imaginary, and Total Roots for the Polynomial

(a) *The possible real roots*

2.3. Solutions

The complex conjugate root theorem suggests that there must be at least two imaginary. If that is the case, there must be three real. If there are four imaginary, there must be 1 real, and if there are zero imaginary roots, there are five real. See Table 1.

Final Answer

5 or 3 or 1

(b) *The possible imaginary roots*

2.3. Solutions

A very similar approach. See Table 1.

Final Answer

4 or 2 or 0.

(c) *The possible positive real roots*

2.3. Solutions

Here, we count the number of sign changes. There are many ways to do this. If we color the positive terms **red** and the negative terms **blue**, we can clearly see that there are three sign changes.

$$-3x^5 - x^4 + 5x^3 + 9x^2 - 11x + 20 = 0$$

Final Answer

By Descartes' rule of signs, therefore, there are 3 or 1 possible positive real roots for this polynomial.

(d) *The possible negative roots*

2.3. Solutions

Here, Descartes' rule is also used, but we look at the sign changes for $p(-x)$, as opposed to $p(x)$.

$$p(-x) = 3x^5 - x^4 - 5x^3 + 9x^2 + 11x + 20$$

There are only two sign changes.

Final Answer

According to Descartes' rule of signs, there could be 2 or 0 roots.

(e) *The possible rational roots*

2.3. Solutions

By the rational roots theorem, the roots of this polynomial are as follows:

$$\text{p.r.z} = \frac{\pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20}{\pm 1, \pm 3}$$

This is not an answer. The list of possible rational zeroes can be expressed as such:

Final Answer

$$\text{p.r.z.} = \left\{ \pm 1, \pm \frac{1}{3}, \pm 2, \pm \frac{2}{3}, \pm 4, \pm \frac{4}{3}, \pm 5, \pm \frac{5}{3}, \pm 10, \pm \frac{10}{3}, \pm 20, \pm \frac{20}{3} \right\}$$

(f) *The sum of roots*

2.3. Solutions

$\Sigma = \frac{1a_{n-1}}{a_n}$	The formula for the sum of roots
$\Sigma = \frac{-1}{3}$	Plug in and simplify

Final Answer

$$\Sigma = \frac{-1}{3}$$

(g) *The product of roots*

2.3. Solutions

$\Pi = \frac{(-1)^n a_0}{a_n}$	The formula for the product of roots
$\Pi = \frac{20}{3}$	Plug everything into it

Final Answer

$$\Pi = \frac{20}{3}$$

2.4. Polynomial $g(x)$ is defined as:

$$g(x) = x^3 + 5x^2 + px + q$$

It's roots are ω , 2ω , and $\omega + 3$. Find the values of p and q .

Required Knowledge

1. Sum of roots
2. Product of roots

2.4. Solutions

Step I: Define what we have

$$a_n = 1$$

$$a_{n-1} = 5$$

$$a_0 = q$$

Step II: Sum of Roots

$$\Sigma = \frac{-a_{n-1}}{a_n} \quad \left| \begin{array}{l} \text{The formula for sum of roots} \\ \Sigma = -5 \end{array} \right.$$

Step III: Equate the Sum of Roots to the Given Roots

$$\omega + 2\omega + \omega + 3 = -5 \quad \left| \begin{array}{l} \text{The sum of all roots is -5.} \\ \omega = -2 \end{array} \right. \quad \left| \begin{array}{l} \text{Simplify} \end{array} \right.$$

Step IV: Product of Roots

$$\Pi = \frac{(-1)^n a_0}{a_n} \quad \left| \begin{array}{l} \text{The formula for product of} \\ \text{roots} \\ = -q \end{array} \right. \quad \left| \begin{array}{l} \text{Plug in and simplify} \end{array} \right.$$

Step V: Equate the product of roots to $-q$

$$\omega \cdot 2\omega \cdot (\omega + 3) = -q \quad \left| \begin{array}{l} \text{The product of roots is equal} \\ \text{to } -q. \end{array} \right.$$

$$2\omega^3 + 6\omega^2 = -q \quad \left| \begin{array}{l} \text{Simplify} \end{array} \right.$$

$$2(-2)^3 + 6(-2)^2 = -q \quad \left| \begin{array}{l} \text{Plug in } \omega = -2 \end{array} \right.$$

$$q = -8 \quad \left| \begin{array}{l} \text{Simplify} \end{array} \right.$$

Step VI: Solve for p

By factor theorem, we know that $g(\omega)$ will yield 0. This, we solve for p when $g(\omega) = 0$.

$$g(\omega) = 0$$

$$(-2)^3 + 5(-2)^2 - 2p - 8 = 0 \quad \left| \begin{array}{l} \text{Plug in for } x = \omega \\ -8 + 20 - 2p - 8 = 0 \\ 4 = 2p \\ p = 2 \end{array} \right.$$

Final Answer

$$\omega = -2$$

$$q = -8$$

$$p = 2$$

Checking Your Answer

Solving for p

This is the most straightforward method to find the sum and product of roots. That said, other alternate methods also do exist. At least, when it comes to solving for p , for instance, one could use the following formula:

$$p = x_2x_3 + x_2x_1 + x_3x_1$$

$$x_1 = \omega = -2$$

$$x_2 = 2\omega = -4$$

$$x_3 = 1$$

$$p = (-4)(1) + (-4)(-2) + (1)(-2)$$

$$p = -4 + 8 - 2$$

$$p = 2$$

Formula for finding p , where x_i are each one of the three roots of the parabola.

Plug in the roots using the values obtained from omega earlier.

Simplify

Solve for p

Construct the function and solve for its roots

$$x^3 + 5x^2 + px + q$$

The polynomial as it is from the question

$$x^3 + 5x^2 + 2x - 8$$

Subbing p and q .

In theory, ω , or -2 , should be a factor of this polynomial.

$$\begin{array}{r|rrrr}
 & 1 & 5 & 2 & -8 \\
 -2 & & -2 & -6 & 8 \\
 \hline
 & 1 & 3 & -4 & 0
 \end{array}$$

Figure 2: -2 is a factor of the polynomial

We are left with $x^2 + 3x - 4$, which factors to $(x + 4)(x - 1)$. The factors of this polynomial therefore match with their expected results.

✓ The solution has been verified and is correct.

3 Sequences and series

3.1. The sides of a square are 16cm in length. A new square is formed by joining the midpoints of the adjacent sides and shading two, as in Figure 3.

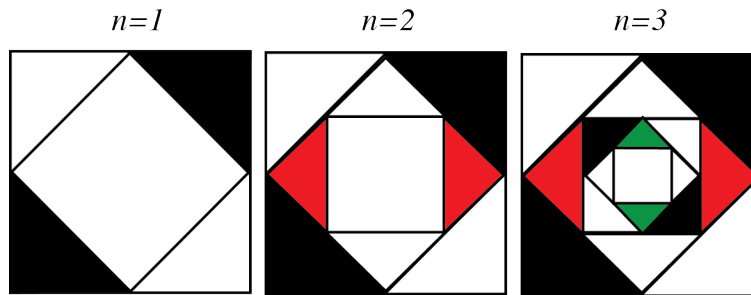


Figure 3: Different Iterations of the fractal

Required Knowledge

1. Geometric series with a fixed amount of terms.
2. Geometric series with an infinite amount of terms.

(a) What is the total area of the shaded region if the process is continued for a total of 10 iterations?

3.1. Solutions

Step I: Setting the problem up

What can be helpful in this situation is to create a table for each of the iterations of the fractal, and to do some calculations on the side:

→ calculations for $n = 1$

Divide the square into 8 triangular sections as shown in Figure 4.

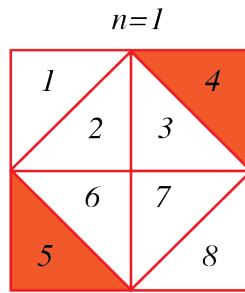


Figure 4: Diving the square into several sections

Notice $\frac{2}{8}$, or $\frac{1}{4}$ of these triangular sections are shaded. This means that $\frac{1}{4}$ of the area of the square is shaded, as shown in Table 2.

→ *calculations for $n = 2$*

Each section's hypotenuse is the side length of the newly iterated square, as shown in Figure 5.

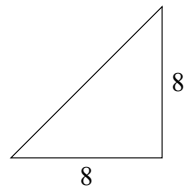


Figure 5: One section

$$\begin{array}{l} c = \sqrt{8^2 + 8^2} \\ c = 8\sqrt{2} \end{array} \quad \left| \begin{array}{l} \text{By Pythagorean Theorem} \end{array} \right.$$

The total area of the smaller rectangle is therefore $(8\sqrt{2})^2$, and the shaded region is one quarter of that area, as shown in Table 2.

→ *calculations for $n = 3$*

Here, we repeat the same calculations, but with a smaller value of $\frac{8\sqrt{2}}{2}$ instead of $\frac{16}{2}$.

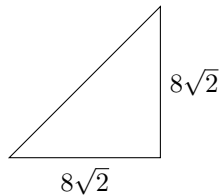


Figure 6: A smaller section

Note: The *shaded region* cell is not the compounded area. It does not take into consideration previous iterations.

n	Shaded Region
1	$\frac{1}{4} \cdot 256 = 64$
2	$\frac{1}{4} \cdot 128 = 32$
3	$\frac{1}{4} \cdot 64 = 16$

Table 2: Calculating the areas of the shaded regions

Notice how each area is essentially the previous divided by two. This is a geometric sequence with $a_0 = 64$ and $r = \frac{1}{2}$.

Step II: Solving for the sum of a geometric sequence

$$S_n = a_0 \frac{1-r^n}{1-r}$$

Use the formula for a geometric series with n terms.

$$S_{10} = 64 \frac{1-r^{\frac{1}{4}}}{1-\frac{1}{4}}$$

Plug all values in

$$S_{10} = 127.875$$

Final Answer

$$S_{10} = 127.875$$

(b) Suppose this process were to continue on forever. What would be the total shaded area?

3.1. Solutions

Step I: Solving for an infinite geometric series

$$S_{\text{inf}} = a_0 \frac{1}{1-r}$$

The sum of an infinite geometric series

$$S_{\text{inf}} = 64 \frac{1}{1-\frac{1}{2}}$$

Sub in all the values

Final Answer

$$S_{\text{inf}} = 128$$

4 Mathematical Induction

4.1. Prove the following statements using mathematical induction $\forall n \in \mathbb{Z}^*$

Required Knowledge

1. Mathematical induction with equalities
2. Mathematical induction with inequalities

4.1. Solutions

Step I: Define the statement

$$p(n) : \sum_{i=1}^n \left(i \frac{1}{2}\right)^{i-1} = 4 - \frac{n+2}{2^{n-1}}; \forall n \in \mathbb{Z}^*$$

Step II: Prove it holds true for a certain base case

$p(1) : 1 = 4 - \frac{1+2}{2^{1-1}}$ $1 = 4 - 3$ $1 = 1$	Plug 1 into $p(x)$ Simplify
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Step III: Assume $p(k)$ is true

$$p(x) : \sum_{i=1}^k i \left(\frac{1}{2}\right)^{i-1} = 4 - \frac{k+2}{2^{k-1}}; \forall k \in \mathbb{Z}^*$$

Step IV: Prove $p(k+1)$ is true

$p(k+1) : \sum_{i=0}^{k+1} i \left(\frac{1}{2}\right)^{i-1} = 4 - \frac{k+3}{2^k}$ $LS = \sum_{i=1}^k i \left(\frac{1}{2}\right)^{i-1} + (k+1) \left(\frac{1}{2}\right)^k = 4$ $LS = 4 - \frac{k+1}{2^{k-1}} + \frac{k+1}{2^k}$ $LS = 4 + \frac{2(k-2)+k+1}{2^k}$ $LS = 4 + \frac{-2k-4+k+1}{2^k}$ $LS = 4 + \frac{-k-3}{2^k}$ $LS = 4 - \frac{k+3}{2^k} = RS$	State $p(k+1)$ Start by looking at the left side. Split the summation. Replace $\sum_{i=1}^k i \left(\frac{1}{2}\right)^{i-1}$ with $p(k)$. Combine with a common denominator of 2^k , by multiplying $\frac{k+2}{2^{k-1}}$ by 2 as 2^{k+1} . $2 = 2^k$. The left side equals the right side after some simplification.
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Step V: Conclusion

By the principle of mathematical induction, $p(x)$ is true.

(a) $(1-a)^n > 1-na$, where $n \geq 2$, $0 < a < 1$

4.1. Solutions

Step I: Define the statement

$$p(n) : (1-a)^n > 1-na; \forall n \in \mathbb{N}, n \geq 2, 0 < a < 1$$

Step II: Prove $p(n)$ holds for the smallest possible value

$$p(2) : (1-a)^2 > 1-2a$$

$$1-2a+a^2 > 1-2a$$

The smallest possible value in this problem, according to the question, is not one but two. Plug this value in $p(n)$.

This is true so long as $a > 0$, as adding something to the same value will make it bigger. For instance, suppose $1-2a$ is -1 (so $a = 1$). $-1 + a^2$ would be greater than -1, so long as $a > 0$.

Step III: Assume $p(k)$ is true

$$p(k) : (1-a)^k > 1-ka; \forall k \in \mathbb{N}^*$$

Step IV: Prove $p(k+1)$ is true

$$p(k+1) : (1-a)^{k+1} \stackrel{?}{>} 1-(k+1)a$$

$$LS = (1-a)^{k+1}$$

$$= (1-a)^k(1-a)$$

$$= (1-ka)(1-a)$$

$$= 1-a-ka+ka^2$$

$$= 1-(k+1)a+ka^2$$

$$1-(k+1)a+ka^2 > 1-(k+1)a$$

Expand the term so that $p(k)$ can be substituted into the inequality.

Substitute $1-ka$ for $(1-a)^k$.

Simplify and rearrange

This is true as ka^2 is being added to essentially the same term on the right side. So long as $a > 0$, this holds.

Step V: Conclusion

By the principle of mathematical induction, $p(n)$ is true.

5 Quadratic Functions

5.1. Solve for x in equation $x^2 - (a + 3b)x + 3ab = 0$ (note that x is in terms of a and b)

Required Knowledge

1. Quadratic formula

5.1. Solutions

$$x^2 - (a + 3b)x + 3ab = 0$$

$$ax^2 + bx + c$$

$$a = 1$$

$$b = -(a + 3b)$$

$$c = 3ab$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-(a+3b)) \pm \sqrt{(a+3b)^2 - 4(1)(3ab)}}{2}$$

$$= \frac{a+3b \pm \sqrt{a^2+6ab+9b^2-12ab}}{2}$$

$$= \frac{a+3b \pm \sqrt{a^2-6ab+9b^2}}{2}$$

$$= \frac{a+3b \pm \sqrt{(a-3b)^2}}{2}$$

$$x = \frac{a+3b \pm (a-3b)}{2}$$

$$x = a$$

$$x = 3b$$

State the equation

The general form of a quadratic function

The quadratic formula

Plug in all values

Some simplification...

Final Answer

$$x \in \{a, 3b\}$$

5.2. Find the value of c such that the vertex of the below parabola is $(\frac{4}{3}, \frac{-1}{3})$:

$$3x^2 - 8x + c$$

Required Knowledge

1. Formulae for n and k of the vertex of a parabola.

5.2. Solutions

Step I: State what is known

$$\begin{aligned}a &= 3 \\b &= -8 \\c &= c\end{aligned}$$

Step II: Calculate the k of the parabola

$k = \frac{-\Delta}{4a}$	Formula for k
$= \frac{4ac-b^2}{4a}$	
$= \frac{4(3)c-(-8)^2}{4(3)}$	
$= \frac{12c-64}{12}$	
$= \frac{3c-16}{3}$	
	Simplify

Step III: Solve for c

$\frac{3c-16}{3} = \frac{-1}{3}$	Equate the formula for the k value in terms of c to the formula given in the question.
$3c - 16 = -1$	
$c = 5$	
	Simplify

Final Answer

$$c = 5$$

Checking Your Answer

There is an alternate way of solving for k , which is of doing $f(h)$.

Calculate h of the parabola

$$h = \frac{-b}{2a}$$

The formula for h .

$\frac{4}{3}$ Simplify

Find k in terms of c

$$f\left(\frac{4}{3}\right) = \frac{-1}{3}$$

Plug h into the function to find the corresponding y value

$$3\left(\frac{4}{3}\right)^2 - 8\left(\frac{4}{3}\right) + c$$

Expand

$$\frac{16}{3} - \frac{32}{3} + c$$

$$\frac{-16}{3} + c$$

Equate the value of k in terms of c with the value given in the question.

Note: Although we could keep on going here, it is not necessary, as we know that the value of k we have obtained is identical to the one we got via the other method.

$$\frac{3c - 16}{3} = c - \frac{16}{3}$$

✓ The solution has been verified and is correct.

5.3. The quadratic function $f(x)$ has the following properties:

1. It passes through point (2,4)
2. It has a maximum value of 6 when $x = 4$
3. It has a zero of $x = -4 + 2\sqrt{3}$

If the function is of the form $ax^2 + bx + c$, find the values of a , b , and c .

Required Knowledge

1. Vertex form of a parabola
2. Expanding vertex form into general form

5.3. Solutions

Step I: Find the Vertex Form

$$f(x) = a(x - h)^2 + k$$

The generalized vertex form of a parabola.

$$= a(x - 4)^2 + 6$$

It is said in the question, albeit with different wording, that the vertex of the parabola is at (4, 6).

$$4 = a(2 - a)^2 + 6$$

It is said that the parabola passes through point (2, 4)

$$4 = 4a + 6$$

$$-2 = 4a$$

$$a = \frac{-1}{2}$$

Step II: Expand Vertex Form into Standard Form

$$\frac{-1}{2}(x - 4)^2 + 6$$

The vertex form from the previous step

$$\frac{-1}{2}(x^2 - 8x + 16) + 6$$

$$\frac{-1}{2}x^2 + 4x - 2$$

Simplify

Final Answer

$$a = \frac{-1}{2}$$

$$b = 4$$

$$c = -2$$

Checking Your Answer

To solve this problem, we did not need to know the third piece of information that was given to us. That said, knowing one of the roots does help us check our answers.

Find Roots of The Polynomial

$$\frac{-1}{2}x^2 + 4x - 2 = 0$$

Our polynomial from before

$$x = -4 \pm \sqrt{12}$$

These are both roots

$$x \in \{-4 - \sqrt{12}, -4 + \sqrt{12}\}$$

The root $-4 + \sqrt{12}$ corresponds to the root that was given to us, being $-4 + 2\sqrt{3}$
 ✓ The solution has been verified and is correct.

- 5.4.** Consider the function $p(x) = mx^2 - 2(m+2)x + m + 2$
(a) Find all values m such that $p(x)$ has two real roots.

5.4. Solutions

This is a discriminant question, as it asks for the *number* of roots.

$$\Delta \geq 0$$

$$b^2 - 4ac \geq 0$$

$$(-2m - 4)^2 - 4(m)(m + 2) \geq 0$$

$$4m^2 + 16m + 16 - 4m^2 - 8m \geq 0$$

$$8m + 16 > 0$$

$$m > -2$$

For $p(x)$ to have two real roots, its discriminant must be greater than or equal to 0. The question never says it can't be the same zero twice.

Expand Δ

Plug in the proper values

Simplify...

Final Answer

$$m > -2; m \neq 0$$

Checking Your Answer

To check the answer, we test for a value of m less than 2, a value of m equal to 2, and a value of m greater than 2.

$$m = -3$$

$$mx^2 - 2(m+2)x + m + 2 = 0$$

$$-3x^2 - 2(-1)x - 1 = 0$$

$$-3x^2 + 2x - 1 = 0$$

$$\Delta = (-3)^2 - 4(-3)(-1)$$

$$\Delta = -3$$

State the question

Plug in $m = -3$.

Simplify

State the discriminant

Simplify.

m has no roots at $m = -3$

$$m = -2$$

$mx^2 - 2(m+2)x + m + 2 = 0$	State the question
$-2x^2$	Plug in $m = -2$
$\Delta = 0 - 4(-2)(0)$	Find the discriminant

m has one root ($\sqrt{0}$) at $m = -2$.

$$m = -1$$

$mx^2 - 2(m+2)x + m + 2 = 0$	State the question
$-x^2 - 2(1)x - 1 + 2 = 0$	Substitute -1 as the value of m .
$-x^2 - 2x + 1 = 0$	Simplify
$\Delta = 4 - 4(-1)(1)$	Find the discriminant

m has more than one root at $m = -1$
 ✓ The solution has been verified and is correct.

(b) Has two positive roots.

5.4. Solutions

Step I: Find the roots of m

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	The quadratic formula represents both roots
$a = m, b = -2(m+2), c = m+2$	State a, b , and c .
$x = \frac{2m+4 \pm \sqrt{4m^2+16m+16-4(m)(m+2)}}{2m}$	Plug in a, b , and c .

(c) Has one positive and one negative root.

6 Linear Inequalities

6.1. What values of x solve the following inequality?

$$\frac{3x-2}{5} + 3 \geq \frac{4x-1}{3}$$

6.1. Solutions

$$\frac{3x-2+15}{5} \geq \frac{4x-1}{3}$$

$$\frac{3x+13}{5} \geq \frac{4x-1}{3}$$

$$15 \cdot \frac{3x+13}{5} \geq \frac{4x-1}{3} \cdot 15$$

$$9x + 39 \geq 20x - 5$$

$$44 \geq 11x$$

$$4 \geq x$$

$$x \leq 4$$

Combine the left side under a common denominator of 5.

Multiply both sides by 15, as it is the $lcm(5, 3)$. The inequality remains unchanged as these values are both greater than 0.

Some simplification allows us to get the solution.

Final Answer

$$x \leq 4$$

Checking Your Answer

Check at 4

$$\frac{3(4)-2}{5} + 3 \stackrel{?}{\geq} \frac{4(4)-1}{3}$$
$$5 \stackrel{?}{\geq} 5$$

Plug 4 as x .

Simplify.

TRUE at 4 ✓

Check at 3

$$\frac{3(3)-2}{5} + 3 \stackrel{?}{\geq} \frac{4(3)-1}{3}$$

Plug 3 as x

$$\frac{22}{5} \stackrel{?}{\geq} \frac{11}{3}$$

Simplify ...

$$\frac{66}{15} \stackrel{?}{\geq} \frac{55}{15}$$

Common denominator

TRUE at 3 ✓

Check 5

$$\frac{3(5)-2}{5} + 3 \stackrel{?}{\geq} \frac{4(5)-1}{3}$$

Plug 5 in as x

$$\frac{28}{5} \stackrel{?}{\geq} \frac{19}{3}$$

Simplify

$$\frac{84}{15} \stackrel{?}{\geq} \frac{95}{15}$$

Common denominator

FALSE at 5 ✓

✓ The solution has been verified and is correct.