# Enriched 11 IB Mathematics A&A HL Y1 2020-21
# The Music of Trigonometry

**M. Hoteit**

Due Date: Thursday, Feb 18 2021 10 PM

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**Reference Declaration**

Complete the Reference Declaration section below in order for your assignment to be graded.

If you used any references beyond the course text and lectures (such as other texts, discussions with classmates or online resources), indicate this information in the space below. If you did not use any aids, state this in the space provided.

Note: Your submitted work must be **your original work**.

Declared References:

1. We spoke to Arthur Huang and Tram-Anh's team for clarifications on our results
2. We spoke to Justin Huang's team for clarifications on our results

**Task 1: Basic Tuning Fork Frequencies.**

For your first task, you are to measure the frequencies of the set of tuning forks provided in class. Using the virtual oscilloscope at https://academo.org/demos/virtual-oscilloscope/, you will record a clear sound wave for each of the tuning forks. Take a screen-shot of the image and approximate the wavelength from the graph. Using the formula $f = \frac{1}{\lambda}$, where $\lambda$ is the wavelength, approximate the frequency of each tuning fork (8 separate computations) and measure the percent error in comparison with the frequency written on the respective tuning forks.

**Please Note:** For each of the graphs in this section, the scale was kept consistent; each division is one millisecond.

1. **C Tuning Fork - 512 Hz:**

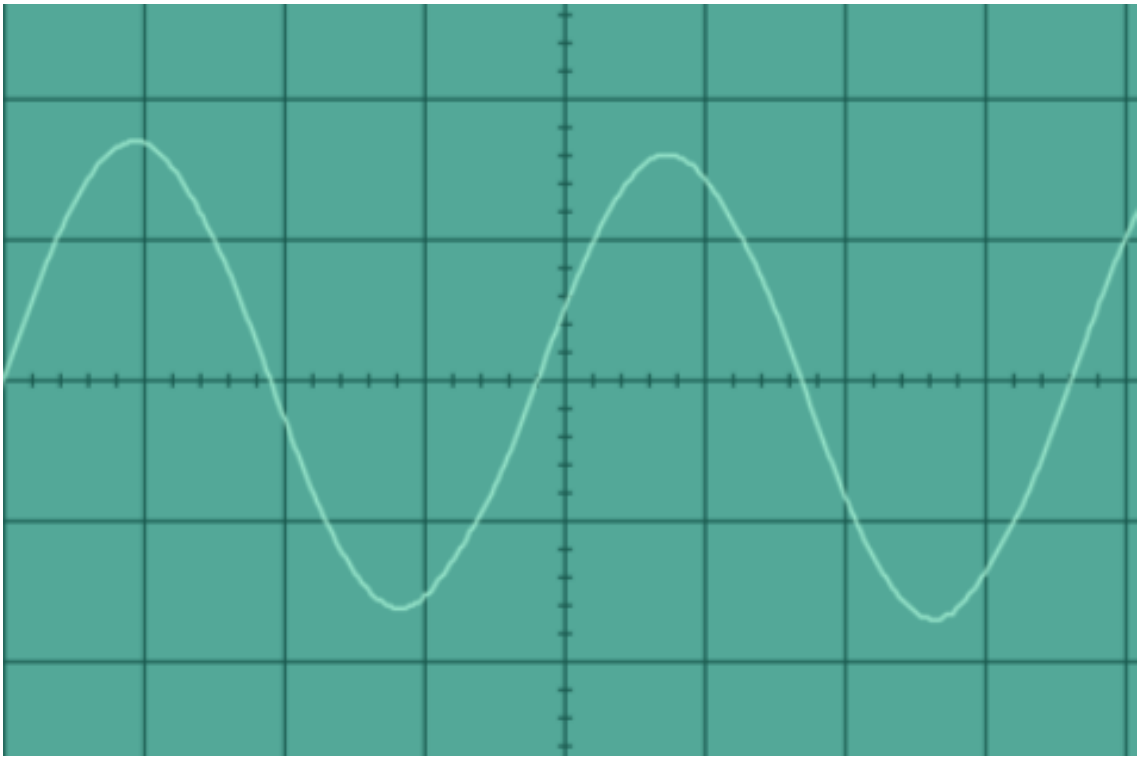

Figure 1: C4 Fork Graph

$$\begin{aligned} \text{freq} &= 0.2 \cdot 19.5 && (1)\\ &= 3.9 && (2)\\ &= \frac{39}{1000} && (3)\\ &= 0.0039 && (4)\\ &= \frac{1}{0.0039} && (5)\\ &= 256.41 && (6) \end{aligned}$$

See Figure 1 for image.

**Frequency:** 256.41 Hz

**Expected Frequency: Percent Error:** 0.2%

2. **B Tuning Fork - 480 Hz:**

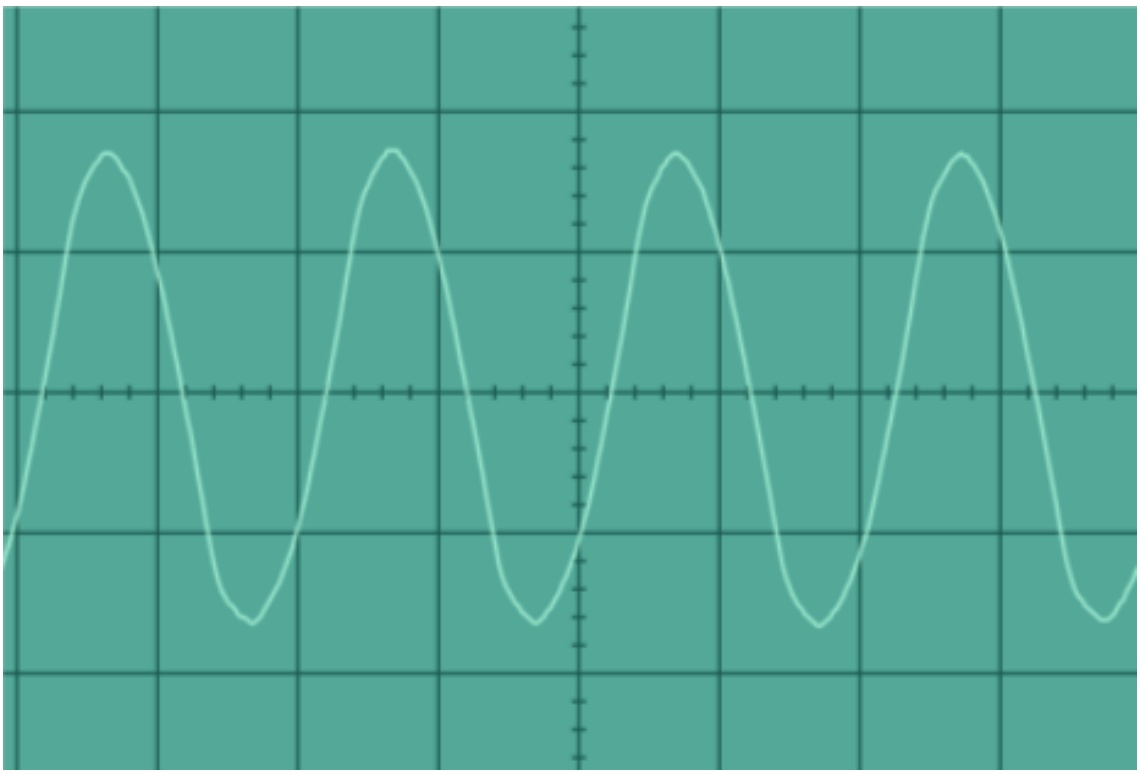

Figure 2: B Tuning Fork Graph

$$\begin{aligned} \text{freq} &= 0.2 \cdot 10 && (7)\\ &= 2 && (8)\\ &= \frac{2}{1000} && (9)\\ &= 0.002 && (10)\\ &= \frac{1}{002} && (11)\\ &= 500 && (12) \end{aligned}$$

See Figure 2 for image.

**Frequency:** 500 Hz

**Expected Frequency:** 480 Hz

**Percent Error:** 4%

3. **A Tuning Fork - 426.6 Hz:**

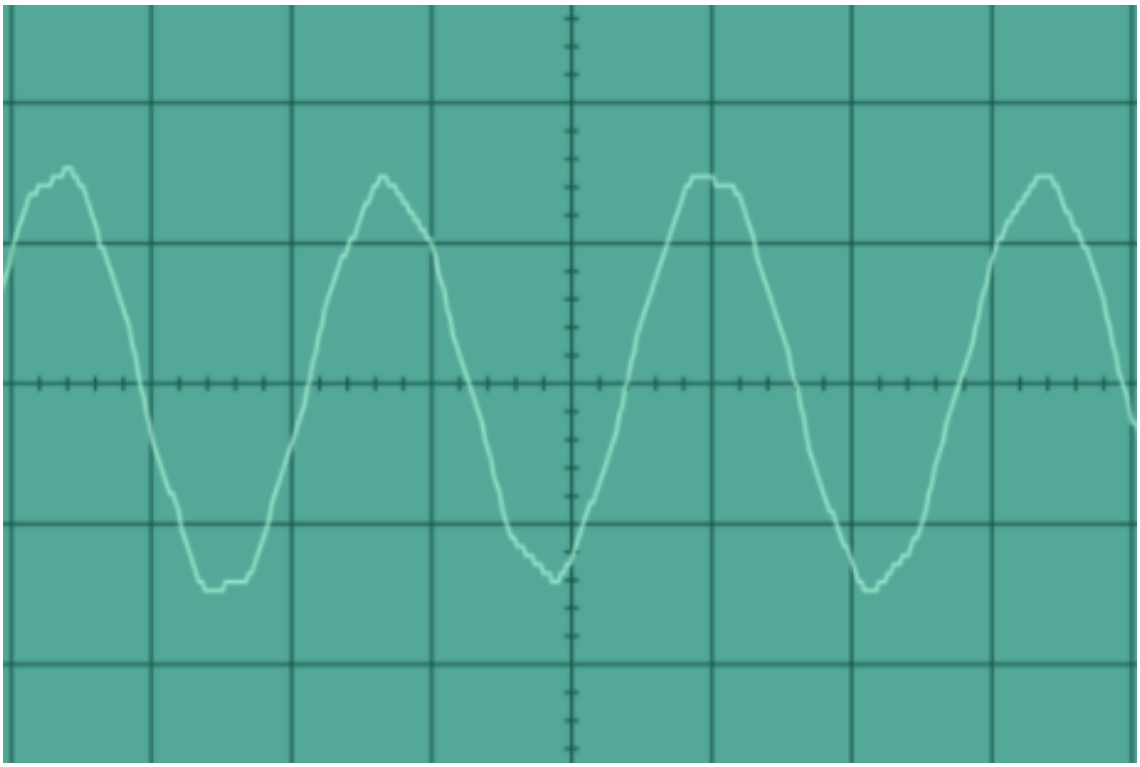

Figure 3: A Tuning Fork Graph

$$\begin{aligned} \text{freq} &= 0.2 \cdot 12 && (13)\\ &= 2.4 && (14)\\ &= \frac{2.4}{1000} && (15)\\ &= 0.0024 && (16)\\ &= \frac{1}{0024} && (17)\\ &= 416.67 && (18) \end{aligned}$$

See Figure 3 for image.

**Frequency:** 416.67 Hz

**Expected Frequency:** 426.6 Hz

**Percent Error:** 2.33%

4. **G Tuning Fork - 384 Hz:**

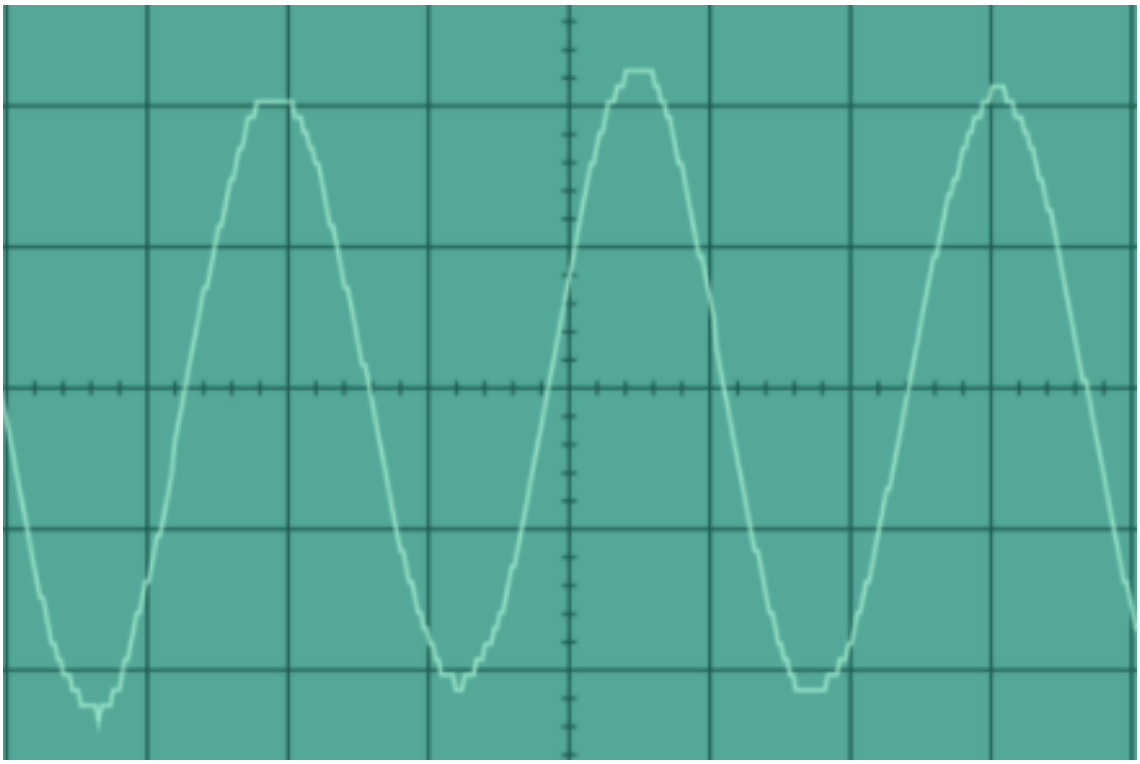

Figure 4: G4 Tuning Fork Graph

$$\begin{aligned} \text{freq} &= 0.2 \cdot 12.5 && (19)\\ &= 2.5 && (20)\\ &= \frac{2.5}{1000} && (21)\\ &= 0.0025 && (22)\\ &= \frac{1}{0.0025} && (23)\\ &= 400 && (24) \end{aligned}$$

See Figure 4 for image.

**Frequency:** 400 Hz

**Expected Frequency:** 384 Hz

**Percent Error:** 4%

5. **F Tuning Fork - 341.3 Hz:**

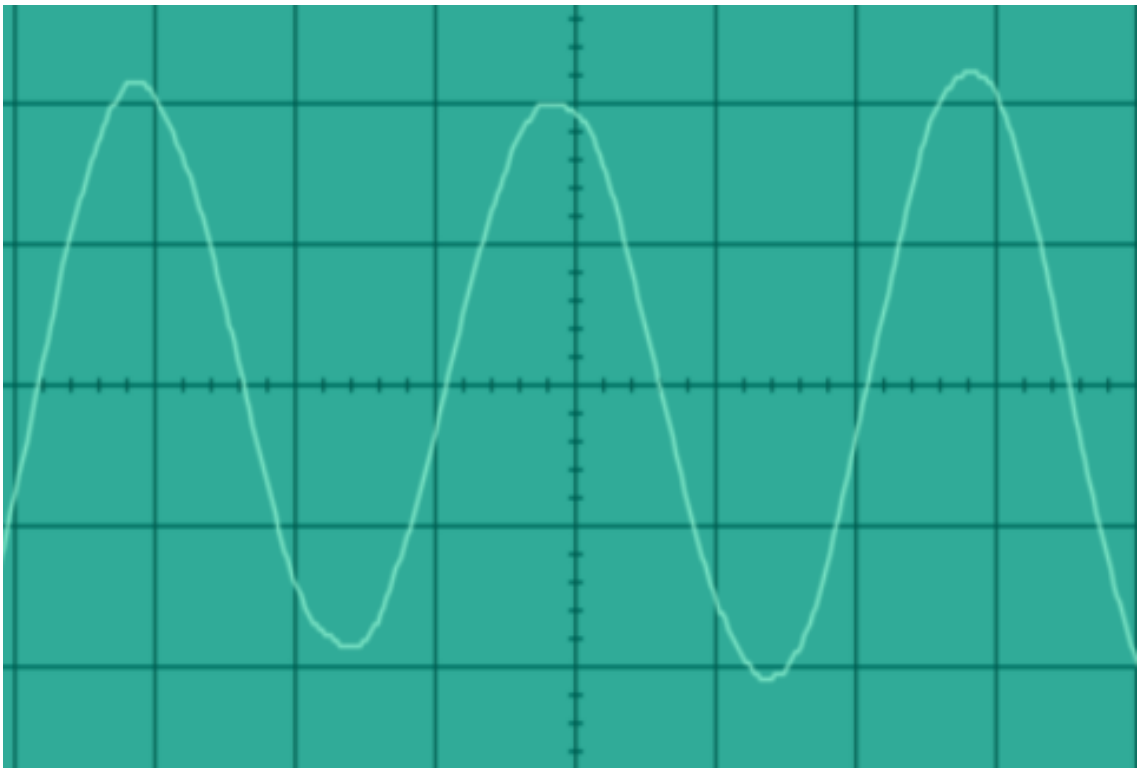

Figure 5: F Tuning Fork Graph

$$\begin{aligned} \text{freq} &= 0.2 \cdot 15 && (25)\\ &= 3 && (26)\\ &= \frac{3}{1000} && (27)\\ &= 0.003 && (28)\\ &= \frac{1}{0.003} && (29)\\ &= 333.33 && (30) \end{aligned}$$

See Figure 5 for image.

**Frequency:** 333.33 Hz

**Expected Frequency:** 341.3 Hz

**Percent Error:** 2%

6. **E Tuning Fork - 320 Hz:**

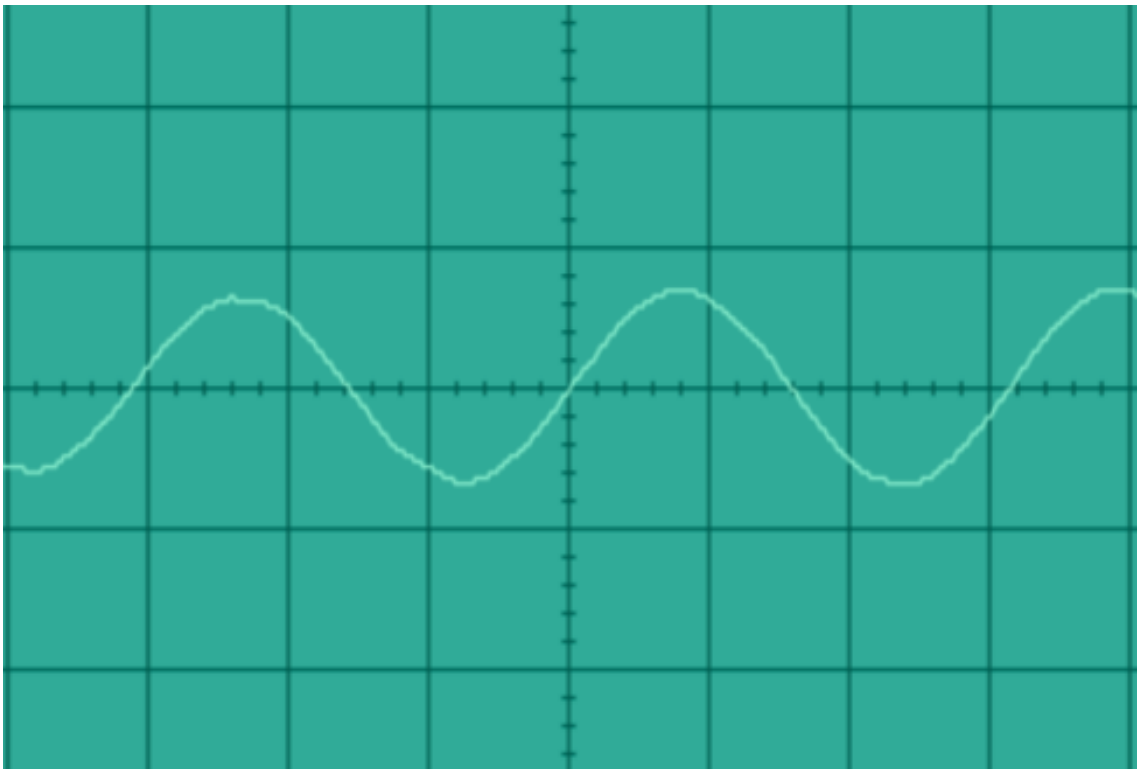

Figure 6: E Tuning Fork Graph

$$\begin{aligned} \text{freq} &= 0.2 \cdot 16 && (31)\\ &= 3.2 && (32)\\ &= \frac{3.2}{1000} && (33)\\ &= 0.0032 && (34)\\ &= \frac{1}{0.0032} && (35)\\ &= 312.5 && (36) \end{aligned}$$

See Figure 6 for image.

**Frequency:** 312.5 Hz

**Expected Frequency:** 320 Hz

**Percent Error:** 2%

7. **D Tuning Fork - 288 Hz:**

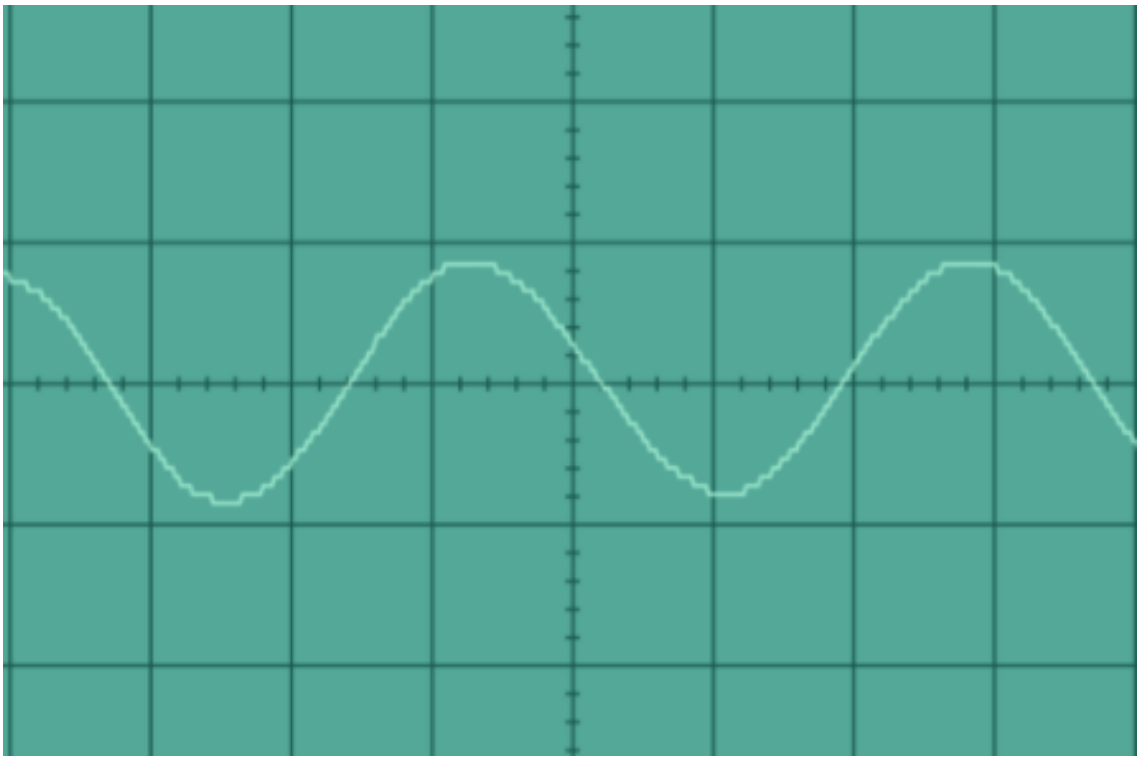

Figure 7: D Tuning Fork Graph

$$\begin{aligned} \text{freq} &= 0.2 \cdot 17.5 && (37)\\ &= 3.5 && (38)\\ &= \frac{3.5}{1000} && (39)\\ &= 0.0035 && (40)\\ &= \frac{1}{0.0035} && (41)\\ &= 285.71 && (42) \end{aligned}$$

See Figure 7 for image.

**Frequency:** 285.71 Hz

**Expected Frequency:** 288 Hz

**Percent Error:** 1%

8. **C Tuning Fork - 256 Hz:**

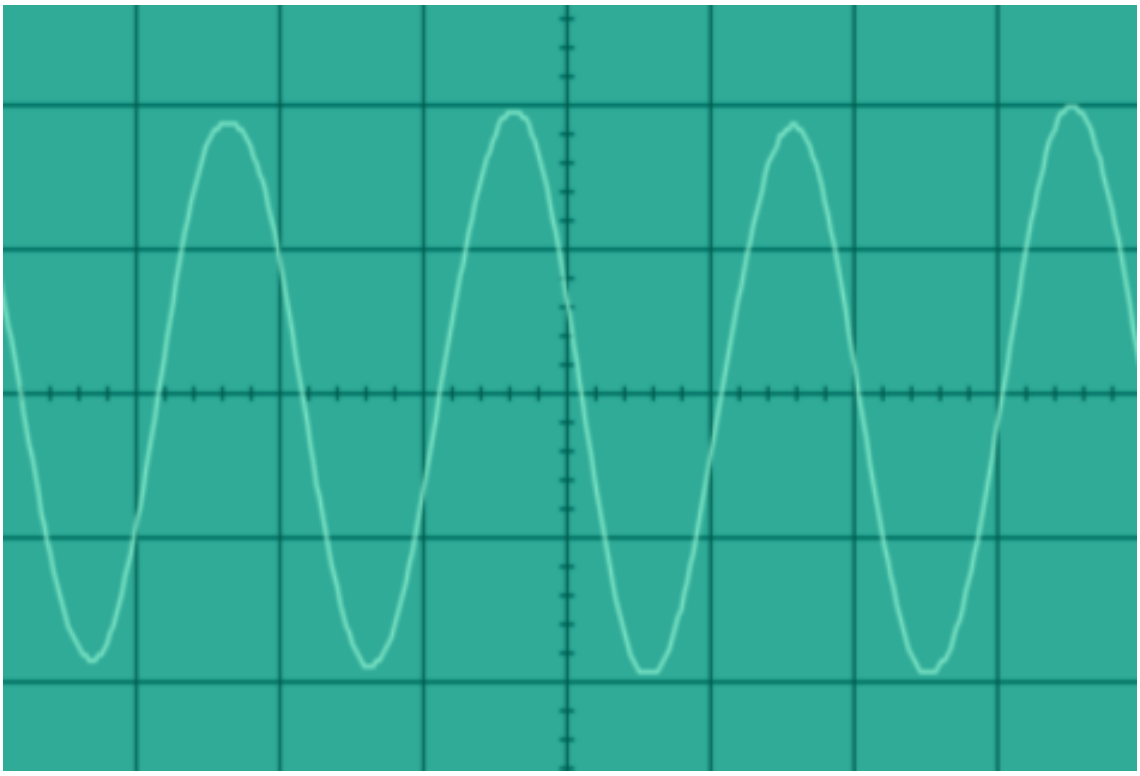

Figure 8: C5 Tuning Fork Graph

$$\begin{aligned}
\text{freq} &= 0.2 \cdot 9.5 && (43)\\
&= 1.9 && (44)\\
&= \frac{1.9}{1000} && (45)\\
&= 0.0019 && (46)\\
&= \frac{1}{0.0019} && (47)\\
&= 526.32 && (48)
\end{aligned}$$

See Figure 8 for image.

**Frequency:** 526.32 Hz

**Expected Frequency:** 512 Hz

**Percent Error:** 3%

**Task 2: Combining Sinusoidals.**

For your second task, you will be playing two tuning forks at once. Ideally, these should be played at amplitudes (loudness) that are as similar as possible. Using the formula:

$$\sin(2\pi f_1 t) + \sin(2\pi f_2 t) = 2\cos\left(2\pi \cdot \frac{f_1 - f_2}{2} \cdot t\right)\sin\left(2\pi \cdot \frac{f_1 + f_2}{2} \cdot t\right)$$

where $f_1$ and $f_2$ are the frequencies, you will be computing the theoretical function formed by playing two tuning forks simultaneously and comparing this with your experimental results. To do so, you will need to take screen-shots of the resultant sound waves, making sure you capture the fluctuations in amplitude as well. Then, using a dynamic geometry software such as GeoGebra or Desmos, you will graph the theoretical function you obtained from the identity above and then layer over it the screen-shot you took to see how well they match. This will require you to make the layered image opaque (increase its transparency) and will require some horizontal and vertical scaling of the image to match the function. Once you have successfully layered these together, take a screen-shot of the two waves and comment on how well matched your results are to the expected values, including discussing the possible sources of error in your computations. You will need to perform this task on three different combinations of tuning forks (ex. A & B, B & F, D & G). Clearly indicate which grouping you are using in your submission.

1. **First combination (state clearly which notes and their frequencies):**

   **First Note:** F4 (341 Hz)

   **Second Note:** B4 (480 Hz)

2. **Second combination (state clearly which notes and their frequencies):**

   **First Note:** F4 (341 Hz)

   **Second Note:** G4 (384 Hz)

3. **Third combination (state clearly which notes and their frequencies):**

   **First Note:** C5 (512 Hz)

   **Second Note:** G4 (384 Hz)

**Task 3: Drop a Beat!**

For this task, you will creating **three different beats** using the beat generator at Academo.org, which you can find at https://academo.org/demos/wave-interference-beat-frequency/. To do so, you will need to choose two frequencies that are close together, turn the sound on, and take a screen-shot of your set-up as well as an audio recording of the beat created. You will then need to either include the audio directly into your assignment or include a link to a Google Drive recording to hear the beat. The screen-shot taken must clearly show the frequencies of the initial waves and the resulting beats (3-4 envelopes, as shown in class).

1. **First beat:**

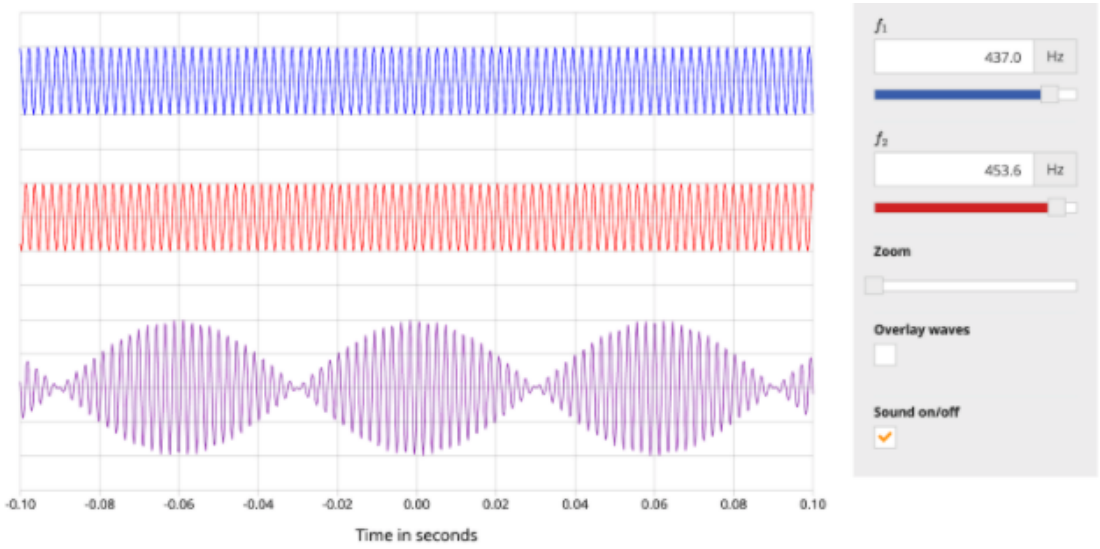


Figure 9: Graph of the First Beat

**Frequency 1:** 437 Hz

**Frequency 2:** 453.6 Hz

Click here to listen to the recording

2. **Second beat:**

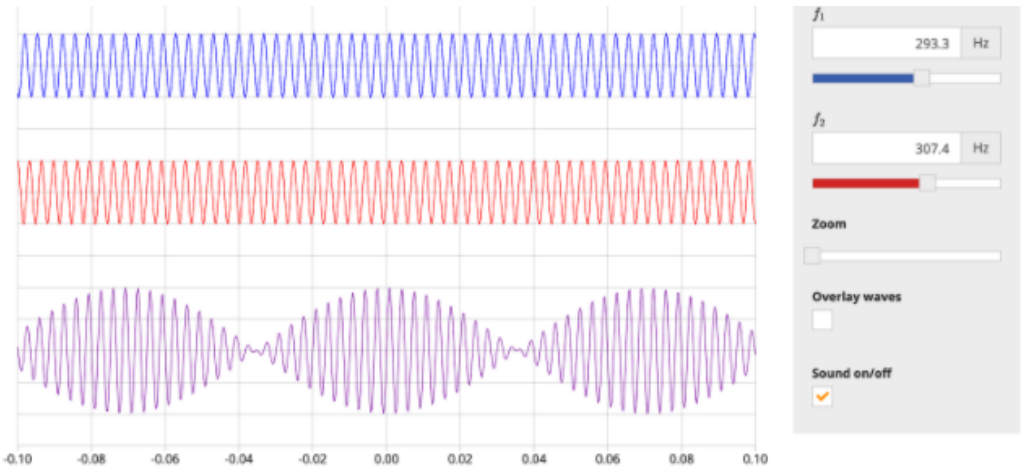


Figure 10: Graph of the Second Beat

**Frequency 1:** 293.3 Hz

**Frequency 2:** 307.4 Hz

Click here to listen to the recording

3. **Third beat:**

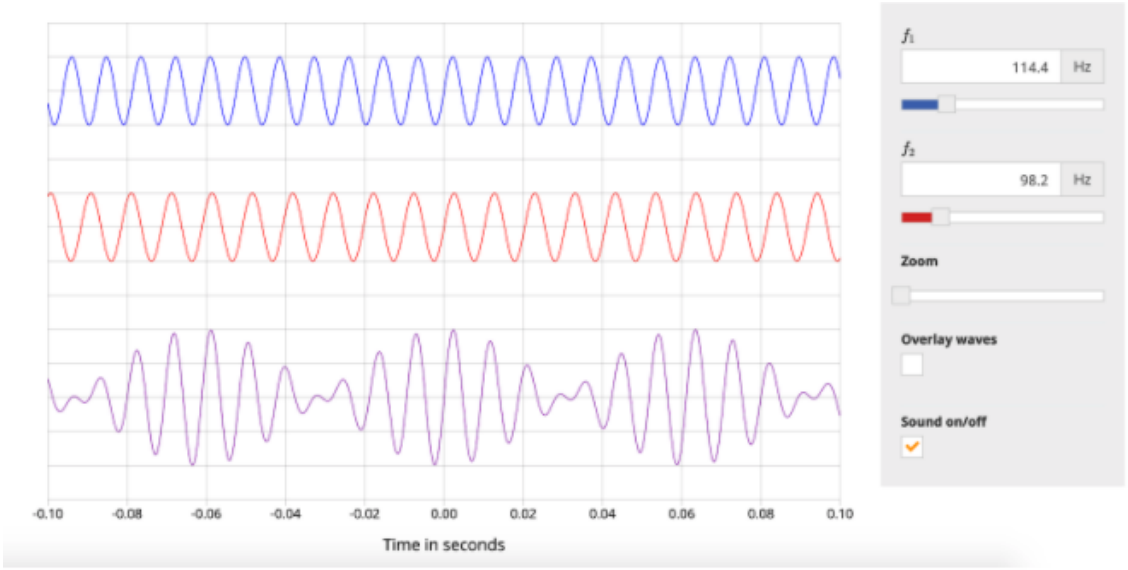


Figure 11: Graph of the Third Beat

**Frequency 1:** 114.4 Hz

**Frequency 2:** 98.2 Hz

Click here to listen to the recording