Enriched 11 IB Mathematics A&A HL Y1 2020-21
The Music of Trigonometry

M. Hoteit

Due Date: Thursday, Feb 18 2021 10 PM

The Music of the Sines

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Reference Declaration

Complete the Reference Declaration section below in order for your assignment to be graded.

If you used any references beyond the course text and lectures (such as other texts, discussions with classmates or online resources), indicate this information in the space below. If you did not use any aids, state this in the space provided.

Note: Your submitted work must be **your original work**.

Declared References:

1. We spoke to Arthur Huang and Tram-Anh Ngo's team for clarifications on our results
2. We spoke to Justin Huang's team for clarifications on our results

Task 1: Basic Tuning Fork Frequencies.

For your first task, you are to measure the frequencies of the set of tuning forks provided in class. Using the virtual oscilloscope at https://academo.org/demos/virtual-oscilloscope/, you will record a clear sound wave for each of the tuning forks. Take a screen-shot of the image and approximate the wavelength from the graph. Using the formula $f = \frac{1}{\lambda}$, where λ is the wavelength, approximate the frequency of each tuning fork (8 separate computations) and measure the percent error in comparison with the frequency written on the respective tuning forks.

Please Note: For each of the graphs in this section, the scale was kept consistent; each division is one millisecond.

1. **C Tuning Fork - 512 Hz:**

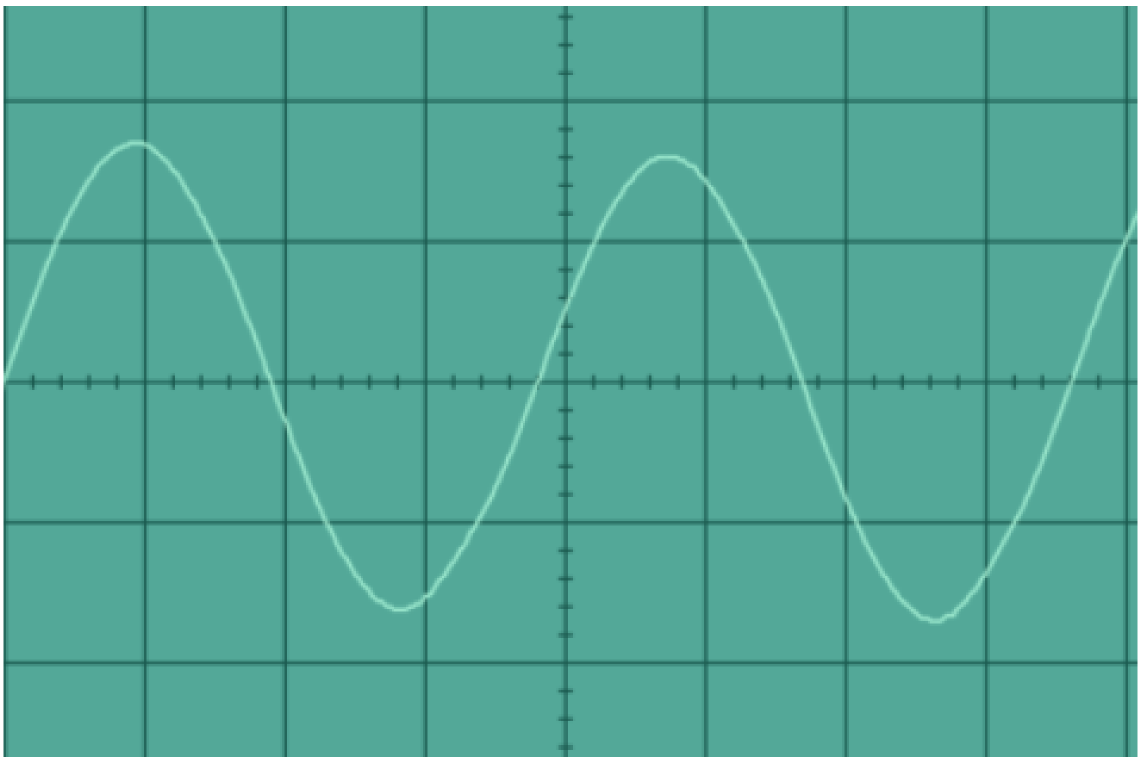

Figure 1: C4 Fork Graph

$$
\begin{aligned}
\text{freq} &= 0.2 \cdot 19.5 &(1)\\
&= 3.9 &(2)\\
&= \frac{39}{1000} &(3)\\
&= 0.0039 &(4)\\
&= \frac{1}{0.0039} &(5)\\
&= 256.41 &(6)
\end{aligned}
$$

See Figure 1 for image.

Frequency: 256.41 Hz

Expected Frequency: 256 Hz

Percent Error: 0.2%

2. **B Tuning Fork - 480 Hz:**

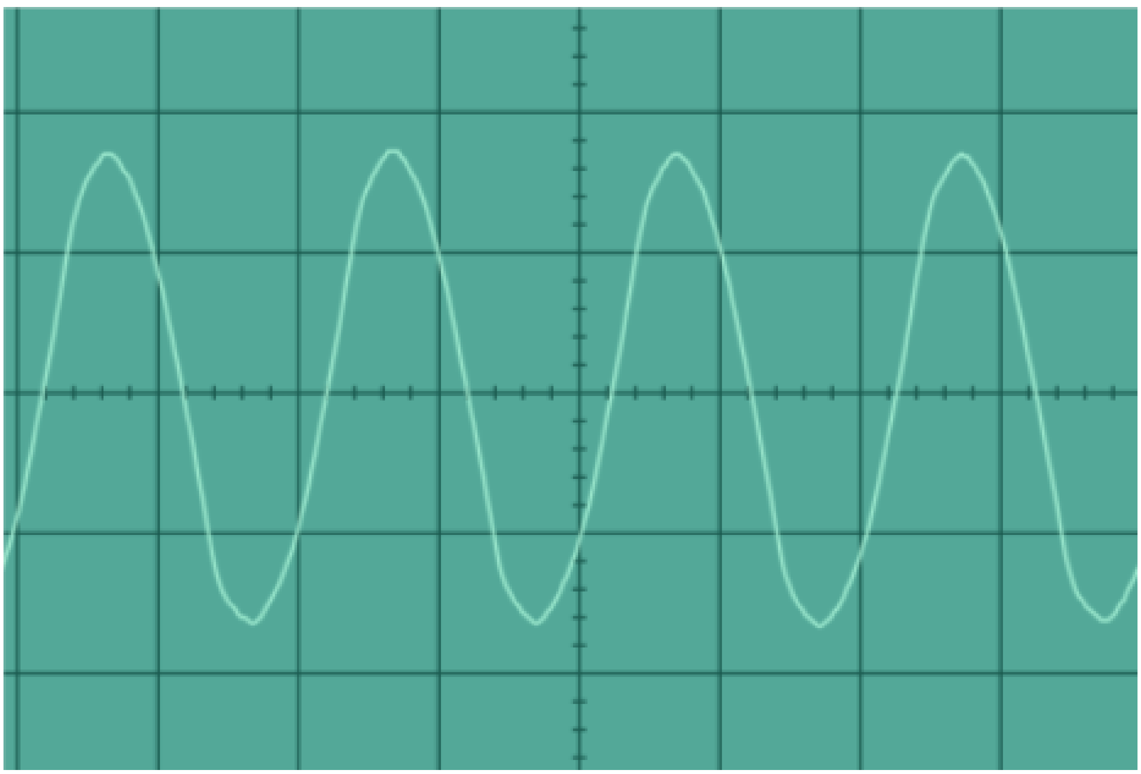

Figure 2: B Tuning Fork Graph

$$
\begin{aligned}
\text{freq} &= 0.2 \cdot 10 && (7)\\
&= 2 && (8)\\
&= \frac{2}{1000} && (9)\\
&= 0.002 && (10)\\
&= \frac{1}{002} && (11)\\
&= 500 && (12)
\end{aligned}
$$

See Figure 2 for image.

Frequency: 500 Hz

Expected Frequency: 480 Hz

Percent Error: 4%

3. **A Tuning Fork - 426.6 Hz:**

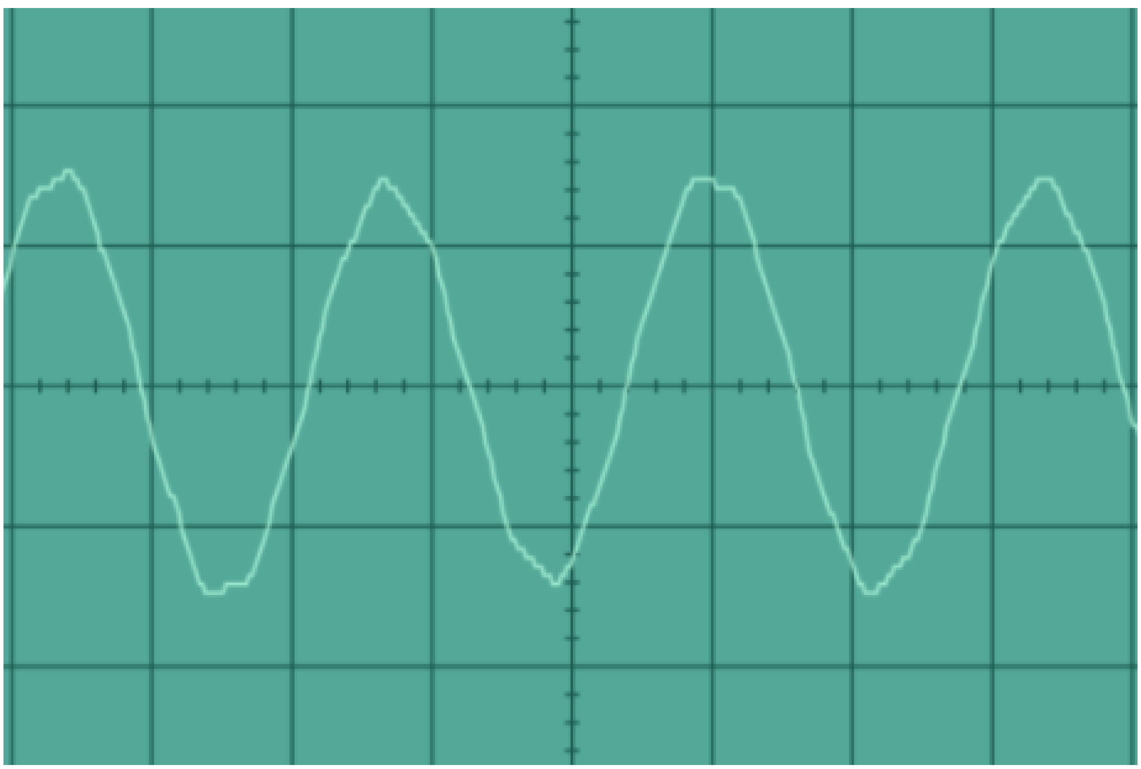

Figure 3: A Tuning Fork Graph

$$
\begin{aligned}
\text{freq} &= 0.2 \cdot 12 && (13)\\
&= 2.4 && (14)\\
&= \frac{2.4}{1000} && (15)\\
&= 0.0024 && (16)\\
&= \frac{1}{0024} && (17)\\
&= 416.67 && (18)
\end{aligned}
$$

See Figure 3 for image.

Frequency: 416.67 Hz

Expected Frequency: 426.6 Hz

Percent Error: 2.33%

4. **G Tuning Fork - 384 Hz:**

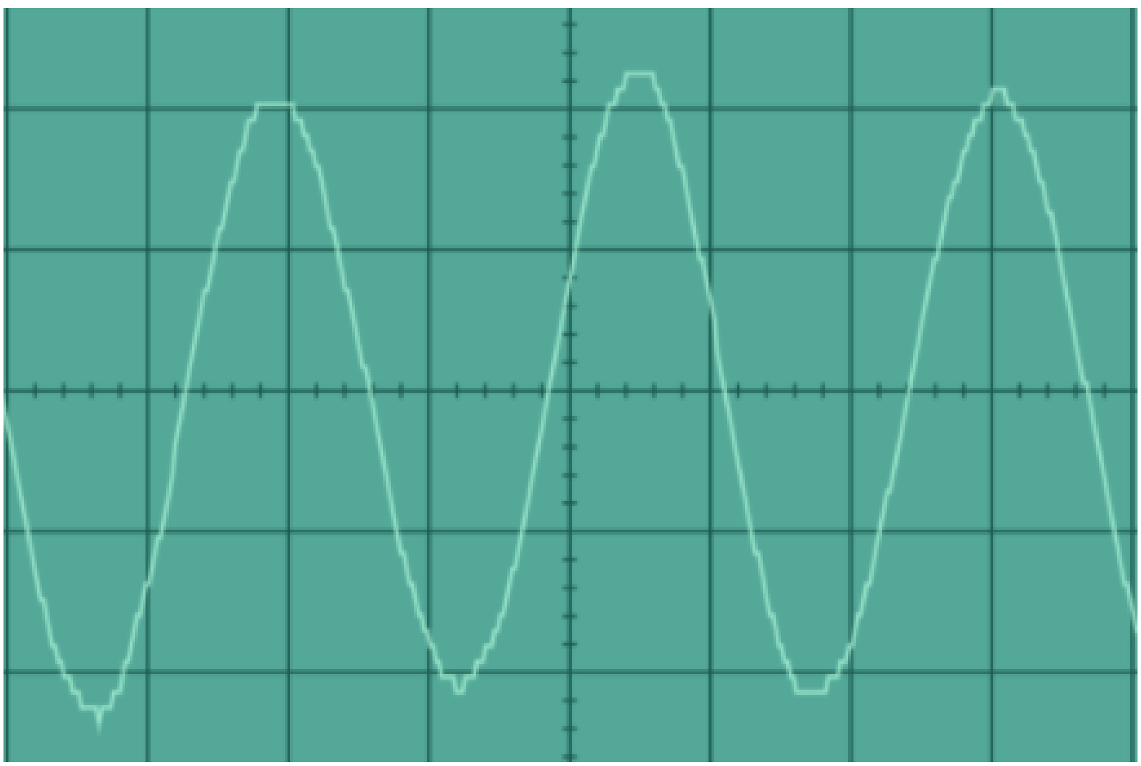

Figure 4: G4 Tuning Fork Graph

$$
\begin{aligned}
\text{freq} &= 0.2 \cdot 12.5 && (19)\\
&= 2.5 && (20)\\
&= \frac{2.5}{1000} && (21)\\
&= 0.0025 && (22)\\
&= \frac{1}{0.0025} && (23)\\
&= 400 && (24)
\end{aligned}
$$

See Figure 4 for image.

Frequency: 400 Hz

Expected Frequency: 384 Hz

Percent Error: 4%

5. **F Tuning Fork - 341.3 Hz:**

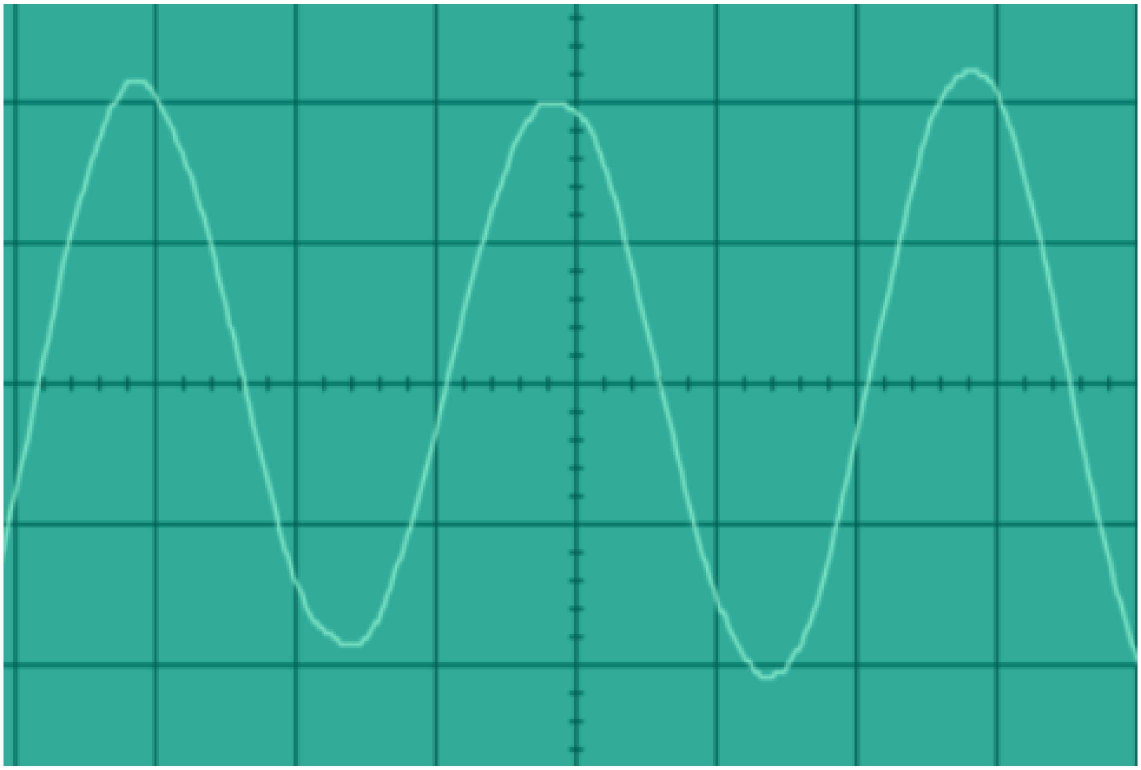

Figure 5: F Tuning Fork Graph

$$
\begin{aligned}
\text{freq} &= 0.2 \cdot 15 && (25)\\
&= 3 && (26)\\
&= \frac{3}{1000} && (27)\\
&= 0.003 && (28)\\
&= \frac{1}{0.003} && (29)\\
&= 333.33 && (30)
\end{aligned}
$$

See Figure 5 for image.

Frequency: 333.33 Hz

Expected Frequency: 341.3 Hz

Percent Error: 2%

6. **E Tuning Fork - 320 Hz:**

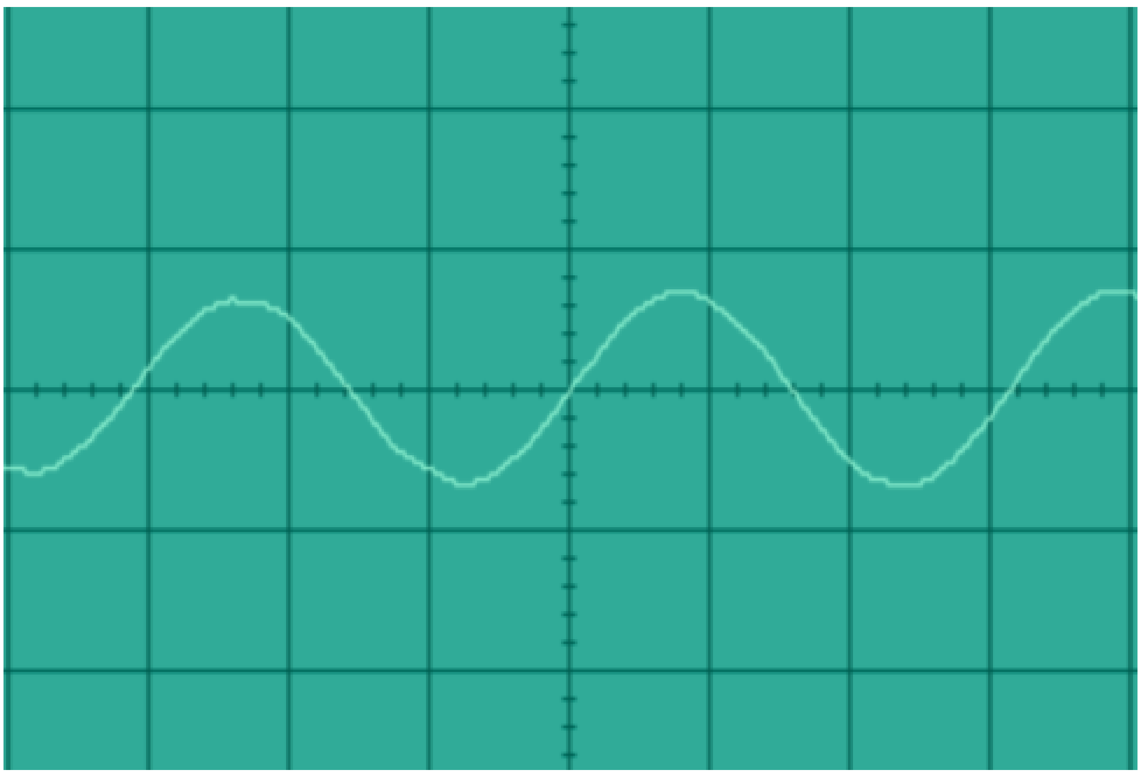

Figure 6: E Tuning Fork Graph

$$\begin{aligned}
\text{freq} &= 0.2 \cdot 16 && (31)\\
&= 3.2 && (32)\\
&= \frac{3.2}{1000} && (33)\\
&= 0.0032 && (34)\\
&= \frac{1}{0.0032} && (35)\\
&= 312.5 && (36)
\end{aligned}$$

See Figure 6 for image.

Frequency: 312.5 Hz

Expected Frequency: 320 Hz

Percent Error: 2%

7. **D Tuning Fork - 288 Hz:**

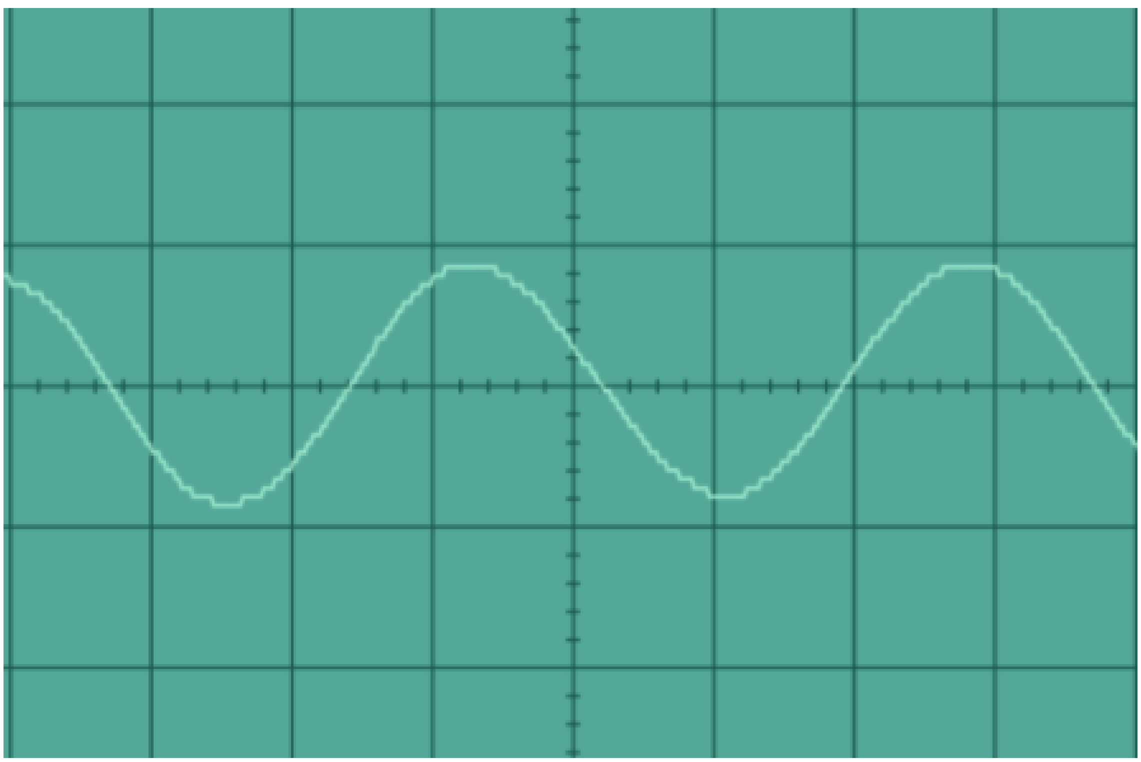

Figure 7: D Tuning Fork Graph

$$
\begin{aligned}
\text{freq} &= 0.2 \cdot 17.5 && (37)\\
&= 3.5 && (38)\\
&= \frac{3.5}{1000} && (39)\\
&= 0.0035 && (40)\\
&= \frac{1}{0.0035} && (41)\\
&= 285.71 && (42)
\end{aligned}
$$

See Figure 7 for image.

Frequency: 285.71 Hz

Expected Frequency: 288 Hz

Percent Error: 1%

8. **C Tuning Fork - 256 Hz:**

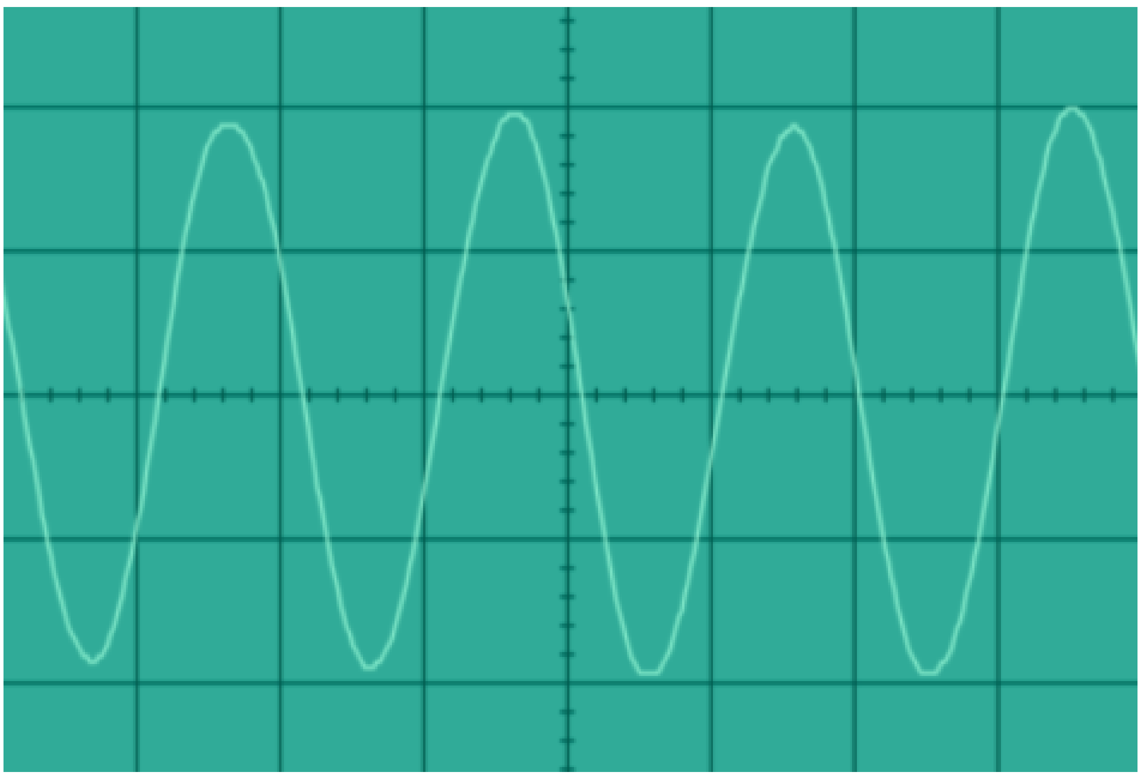

Figure 8: C5 Tuning Fork Graph

$$
\begin{aligned}
\text{freq} &= 0.2 \cdot 9.5 && (43)\\
&= 1.9 && (44)\\
&= \frac{1.9}{1000} && (45)\\
&= 0.0019 && (46)\\
&= \frac{1}{0.0019} && (47)\\
&= 526.32 && (48)
\end{aligned}
$$

See Figure 8 for image.

Frequency: 526.32 Hz

Expected Frequency: 512 Hz

Percent Error: 3%

Task 2: Combining Sinusoidals.

For your second task, you will be playing two tuning forks at once. Ideally, these should be played at amplitudes (loudness) that are as similar as possible. Using the formula:

$$\sin(2\pi f_1 t) + \sin(2\pi f_2 t) = 2\cos\left(2\pi \cdot \frac{f_1 - f_2}{2} \cdot t\right) \sin\left(2\pi \cdot \frac{f_1 + f_2}{2} \cdot t\right)$$

where f_1 and f_2 are the frequencies, you will be computing the theoretical function formed by playing two tuning forks simultaneously and comparing this with your experimental results. To do so, you will need to take screen-shots of the resultant sound waves, making sure you capture the fluctuations in amplitude as well. Then, using a dynamic geometry software such as GeoGebra or Desmos, you will graph the theoretical function you obtained from the identity above and then layer over it the screen-shot you took to see how well they match. This will require you to make the layered image opaque (increase its transparency) and will require some horizontal and vertical scaling of the image to match the function. Once you have successfully layered these together, take a screen-shot of the two waves <u>and</u> comment on how well matched your results are to the expected values, including discussing the possible sources of error in your computations. You will need to perform this task on three different combinations of tuning forks (ex. A & B, B & F, D & G). Clearly indicate which grouping you are using in your submission.

Note: In the interest of keeping this report succinct, let the basic form of the function $C(x)$ be $2\cos\left(2\pi \cdot \frac{f_1 - f_2}{2} \cdot x\right) \sin\left(2\pi \cdot \frac{f_1 + f_2}{2} \cdot x\right)$, with f_1 and f_2 being the frequencies of the two different notes being summed up.

1. **<u>First combination (state clearly which notes and their frequencies):</u>**

   First Note: F4 (341.1 Hz)

   Second Note: B4 (480 Hz)

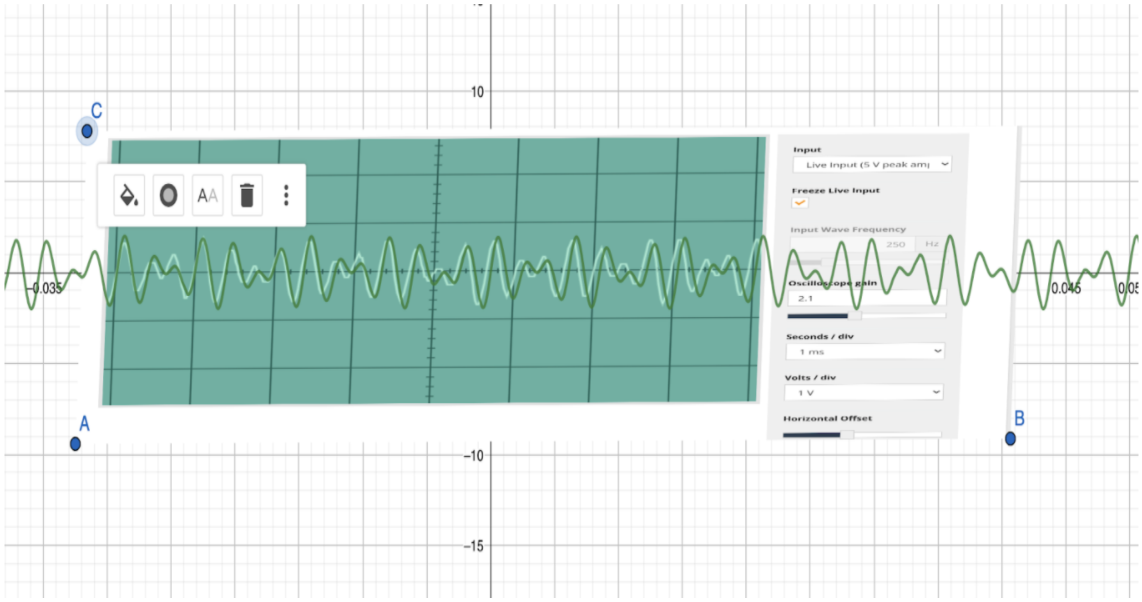

Figure 9: First Combination

Calculations:

$$\begin{aligned}
A(x) &= \sin(2\pi \cdot 341.1 \cdot x) && (49)\\
B(x) &= \sin(2\pi \cdot 480 \cdot x) && (50)\\
C(x) &= A(x) + B(x) && (51)\\
&= 2\cos\left(2\pi \cdot \frac{480 - 341.1}{2} \cdot x\right)\sin\left(2\pi \cdot \frac{480 + 341.1}{2} \cdot x\right) && (52)\\
&= 2\cos\left(2\pi \cdot 69.45 \cdot x\right)\sin\left(2\pi \cdot 410.55 \cdot x\right) && (53)
\end{aligned}$$

2. **Second combination (state clearly which notes and their frequencies):**

First Note: F4 (341.1 Hz)

Second Note: G4 (384 Hz)

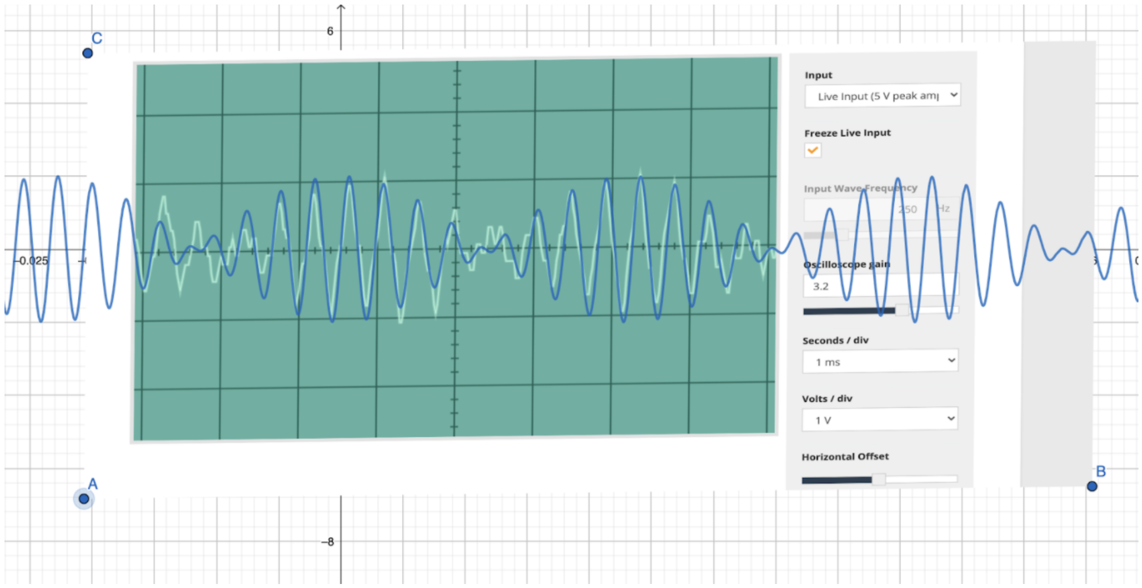

Figure 10: Second Beat

Calculations:

$$
\begin{aligned}
A(x) &= \sin(2\pi \cdot 341.1 \cdot x) && (54)\\
B(x) &= \sin(2\pi \cdot 384 \cdot x) && (55)\\
C(x) &= A(x) + B(x) && (56)\\
&= 2\cos\left(2\pi \cdot \frac{384 - 341.1}{2} \cdot x\right) \sin\left(2\pi \cdot \frac{384 - 341.1}{2} \cdot x\right) && (57)\\
&= 2\cos\left(2\pi \cdot 21.45 \cdot x\right) \sin\left(2\pi \cdot 362.9 \cdot x\right) && (58)
\end{aligned}
$$

3. **Third combination (state clearly which notes and their frequencies):**

First Note: C5 (512 Hz)

Second Note: G4 (384 Hz)

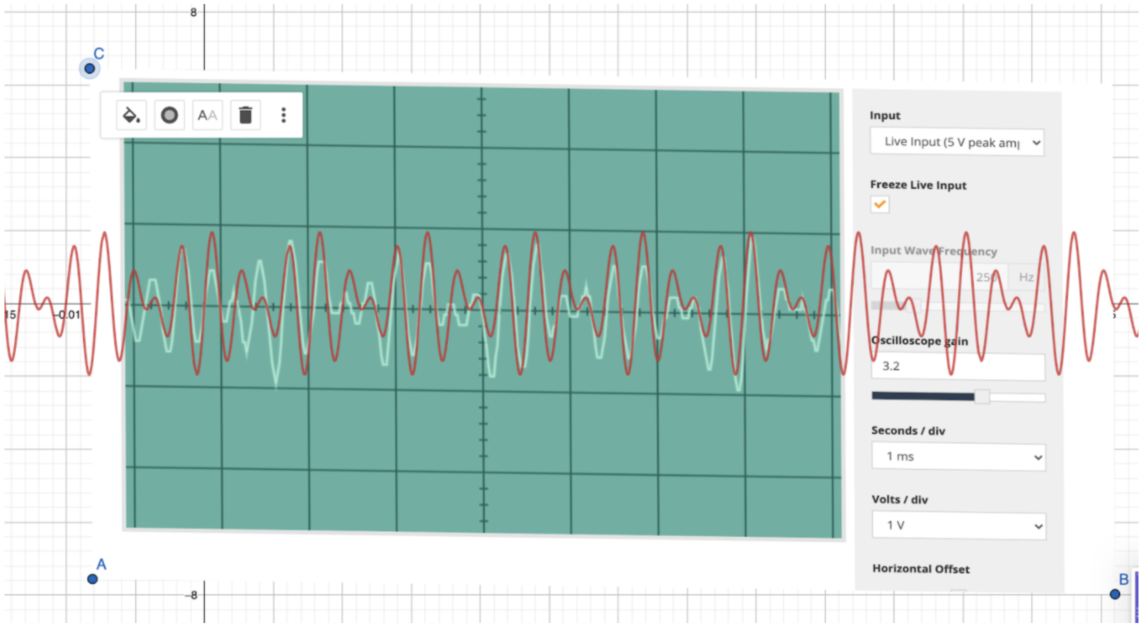

Figure 11: Third Beat

Calculations:

$$
\begin{aligned}
A(x) &= \sin(2\pi \cdot 512 \cdot x) && (59)\\
B(x) &= \sin(2\pi \cdot 384 \cdot x) && (60)\\
C(x) &= A(x) + B(x) && (61)\\
&= 2\cos\left(2\pi \frac{512-384}{2} \cdot x\right) \sin\left(2\pi \cdot \frac{512+384}{2} \cdot x\right) && (62)\\
&= 2\cos\left(2\pi \cdot 64 \cdot x\right) \sin\left(2\pi \cdot 448 \cdot x\right) && (63)
\end{aligned}
$$

Comments: As per the instructions, comments should be made for the results of this experiment, though we were unsure whether a separate set of comments should be produced for each computation. Given that the computations were rather repetitive, we have made the decision to only include one set of comments for all trials at the end of this section.

We have identified several sources of error, which we define in the below list. We strongly believe that these sources of error are the reasons why the composite waves we obtained were not exactly identical to those produced by our graphing software. Indeed, in all cases, the waves were close, though they were far from identical (see Figures 9, 10, and 11).

1. The imprecision of the tuning forks
2. The imprecision of the computer's microphone
3. The imprecision of the oscilloscope
4. Constructive interference due to the surrounding environment

Further elaboration on each of these sources of error can be found below.

The imprecision of the tuning forks

We suspect it does not come as much of a surprise that the tuning forks employed in this experiment were not very precise. Take the note F4, for instance. A quick google search reveals that the accepted frequency of F4 is 349.23 Hz. That said, the frequency inscribed on the F4 tuning fork was 341.1. This is a percent error of 2.33% right off the bat.

In addition, some of the tuning forks were strangely identical, which suggests that the the value was written on the each of the forks was produced without any tests having been conducted to support it.

We would like to share a quick test that can be conducted in order to support this argument; take a tuning fork in a quiet environment and make it vibrate. Consult an oscilloscope and take note of the frequency. Then, use an electronic device capable of producing a pure sine wave (i.e. a phone) and play the same note into the oscilloscope in order to record its wave. You will notice that the frequencies are different. Even in a perfectly quiet room, the fork is unable to produce a pure sine wave.

Finally, in hindsight, the fact that the forks were repeatedly banged against the wall probably did not aid their precision. By the end of the experiment, many of them no longer had perfect edges. Although we recognize our lack of expertise in the domain, we suspect that dents in the forks must have introduced some sort of imprecision.

The Imprecision of the Computer's Microphone

The mac we used was an A1708. Now, it is very difficult to obtain technical specifications about it's microphone, apart from the fact that it has one (thank you so much Apple), but a quick Google search reveals that it is far from being adequate to recording anything above the quality of a FaceTime call. In other words, the Mac microphone is great for speaking with a friend, but for high-precision experiments, it is so subpar that Apple has decided not to even post its specifications. In light of this, we suspect that it may have been unable to faithfully record the waves we created.

The Imprecision of the Osilloscope

Although the oscilloscope app we used had undeniable potential as an educational tool, its quality was extremely questionable. For instance, when we were calculating the wavelength, we counted the divisions and approximated. Because each tick on the graph was 0.2 ms, the precision of the oscilloscope itself was about ± 0.1ms, and this does not include the imprecision of the microphone, which may have been much lower. The wavelength of middle C (C4) is about 3.9 milliseconds (in theory), so the percent of the oscilloscope alone is about 3%. Moreover, changing the parameters of the oscilloscope seemed to produce illogical results (though this may also have been due to the surrounding environment), which suggests that there was perhaps a bug in the underlying javascript code of the application.

The Constructive interference due to the Surrounding Environment

The room we conducted our tests in was far from silent despite it being a library; we conducted our tests amid a symphony of chatter, of banging tuning forks, and of shuffling students This undoubtedly produced some interferance, which changed our results. We suspect that this was the factor that contributed the most to the descrepancy between the theoretical wave and the actual wave we obtained. A simple experiment reveals this imprecision; measure the sine wave of the tuning fork in a quiet room, and it takes two seconds and only one try to get an acceptable wave (the precision of the forks themselves, however, is another story). Do the same in a noisy room, and it can take several tries to obtain the same results. Ambient noise has a significant part to play when it comes to precision.

Task 3: Drop a Beat!

For this task, you will creating **<u>three different beats</u>** using the beat generator at Academo.org, which you can find at https://academo.org/demos/wave-interference-beat-frequency/. To do so, you will need to choose two frequencies that are close together, turn the sound on, and take a screen-shot of your set-up <u>as well as</u> an audio recording of the beat created. You will then need to either include the audio directly into your assignment or include a link to a Google Drive recording to hear the beat. The screen-shot taken must clearly show the frequencies of the initial waves and the resulting beats (3-4 envelopes, as shown in class).

1. **<u>First beat:</u>**

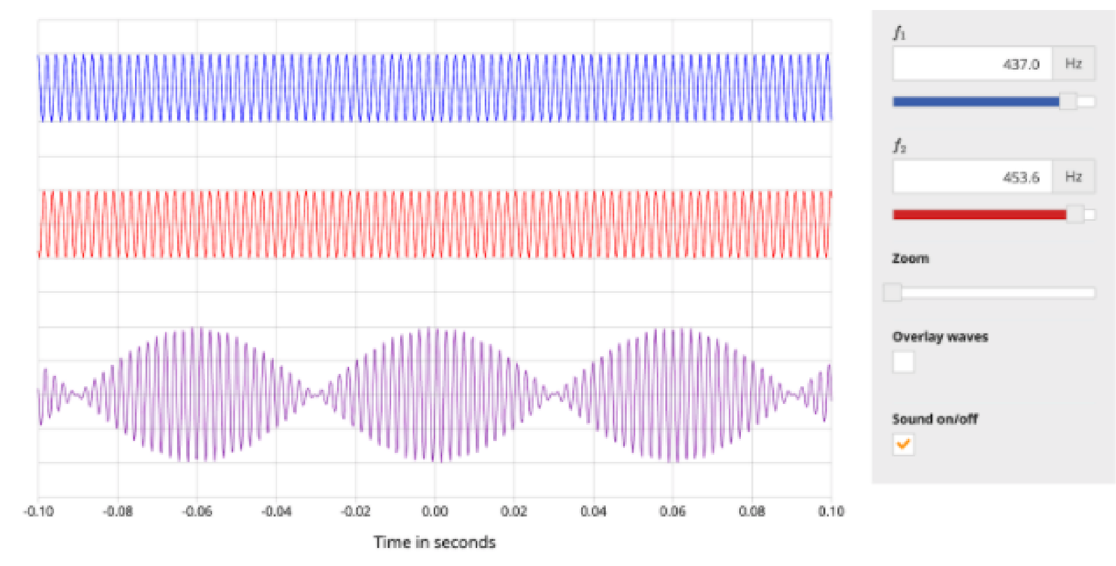

Figure 12: Graph of the First Beat

Frequency 1: 437 Hz

Frequency 2: 453.6 Hz

Click here to listen to the recording

2. **<u>Second beat:</u>**

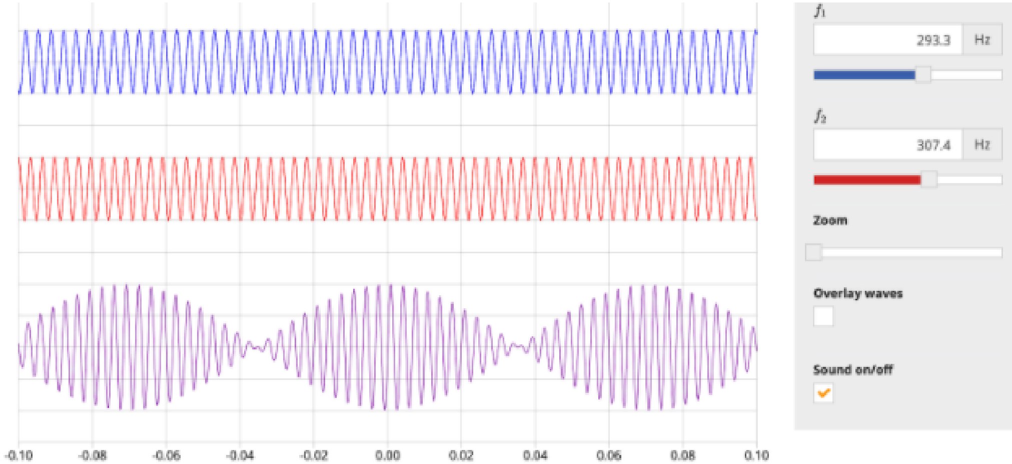

Figure 13: Graph of the Second Beat

Frequency 1: 293.3 Hz

Frequency 2: 307.4 Hz

Click here to listen to the recording

3. **Third beat:**

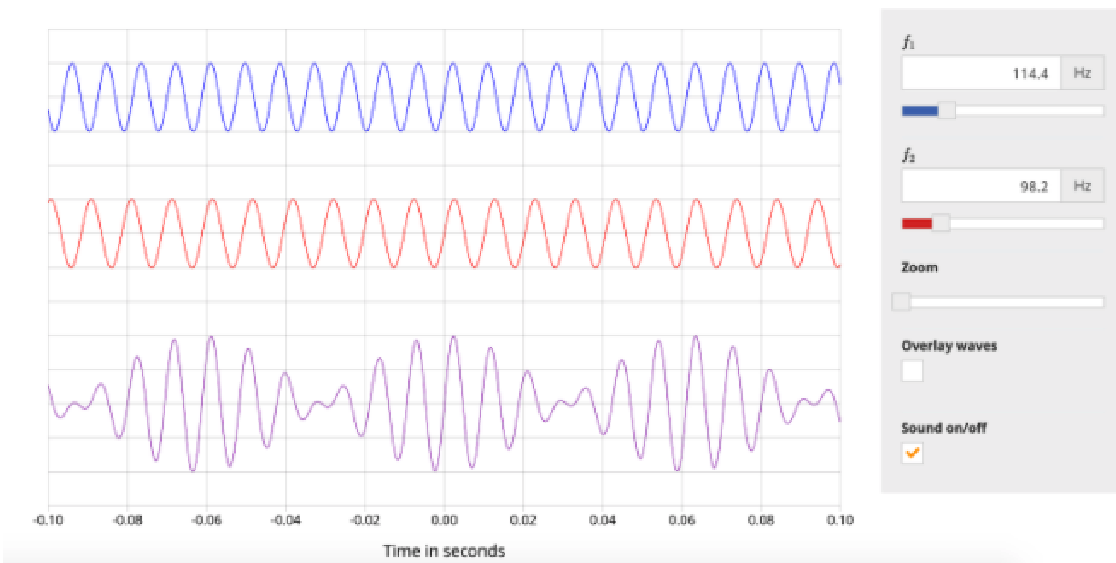

Figure 14: Graph of the Third Beat

Frequency 1: 114.4 Hz

Frequency 2: 98.2 Hz

Click here to listen to the recording