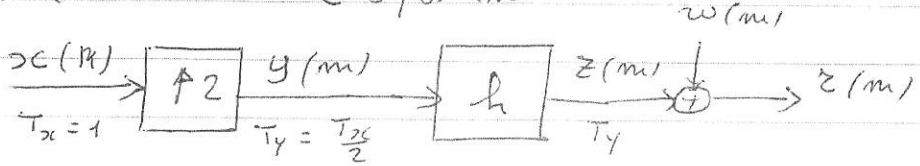
DIGITAL COMMUNICATIONS — APRIL 5, 2018

HW 2 — DUE APRIL 24

PROBLEM 1 (20p)

Given the real value system



where $y(m) = \begin{cases} x(\frac{m}{2}), & m \text{ even} \\ 0, & \text{otherwise} \end{cases}$,

$w(m) \sim N(0, \sigma_w^2)$, i.i.d.

$z(m) = z(m) + w(m)$

$z(m) = -a_1 z(m-1) - a_2 z(m-2) + y(m)$

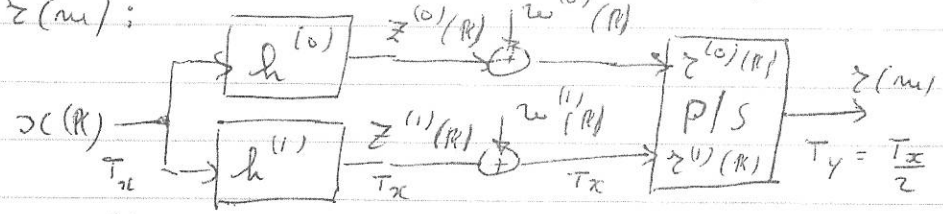
$m = 0, 1, \dots$, $\quad z(-1) = z(-2) = 0$

$a_2 = 0.4642$, $\quad a_1 = -0.9635$

$\sigma_w^2 = -8\ dB$

The receiver knows $\{x(kR)\}$ and a bound on the length of h, N_h: $N_h \leq 20$.

Describe a method to estimate h by a FIR filter with impulse response $\hat{h}_i$, $i = 0, 1, \dots, N-1$, and the noise variance by $\hat{\sigma}_w^2$. Use the following polyphase decomposition of $z(m)$:

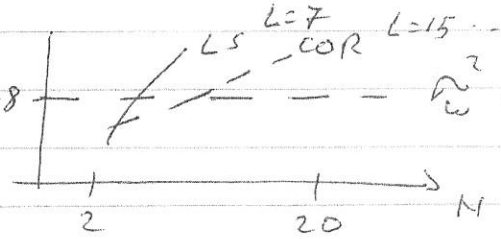


$h^{(0)}$ and $h^{(1)}$ are the two polyphases components of h.

As $\{x(kR)\}$ use a ML sequence of length L, repeated once.

For an estimate use both the correlation (COR) method and the LS method.

Report $\frac{E}{L}$, in dB, vs N and L as parameter for both methods: COR (dashed line), LS (solid line)



Draw the performance bound of $\hat{\sigma}_w^2$ in dashed blue. From the receiver prospective what are suitable values of N and L? Comment your choice. In a table report $\{h_i\}$ and $\{\hat{h}_i\}$, $i = 0, 1, \ldots, N-1$, and $\hat{\sigma}_w^2$ in dB.

PROBLEM 2 (20p)

For a flat fading channel with just one tap $h_0(nT_c)$, assume a Rice factor $K = 2$ dB. Normalize the statistical power of h_0 to one. Moreover assume a 'classical' Doppler spectrum with $f_d T_c = 40\ 10^{-5}$. Simulate h_0 and plot $|h_0|$ vs n for 7500 samples. Remember to remove the transient!

Estimate the pdf of $|h_0| / \sqrt{M_{|h_0|}}$ using a realization of h_0 with 80 000 samples. Plot the estimated pdf with the theoretical curve. Comment the result

Estimate the spectrum of h_0 using the Welch method. Plot the estimate with the theoretical curve. The plot should have a range in the amplitude of 30, 40 dB and a frequency range $(-5 f_d, +5 f_d)$.