Digital Communications - HW4

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We want to implement and evaluate the performances of two modulation schemes in the case of uncoded and coded bits. The first system we consider is a matched filter/DFE receiver with single carrier modulation. The information signal in this case is transmitted through the channel and equalized at the receiver by canceling the ISI due to the postcursors of the total impulse response with feedback, while the ISI due to the precursors is reduced by the feedforward filter c. In the case of OFDM instead we use a multi-carrier approach and we carry out the equalization by using the *cyclic prefix* method.

System setup

The setup of the system is carried out differently for the uncoded and coded cases. The starting point is a sequence of bits b_l on sampling time T_{bit}, produced by a PN sequence with parameter $r = 20$ and length $L = 2^r - 1$ repeated once.

Uncoded

In the uncoded case the sequence b_l directly mapped into a sequence of QPSK symbols a_k at sampling time $T_a = 2T_{bit}$ through a bitmap function that uses Gray coding. At the receiver the detected sequence will pass through an inverse bitmap before the computation of the probability of bit error P_{bit}.

Coded

In the coded case the sequence is first coded using an irregular LDPC code with rate 1/2 and a codelength of $N = 64800$. This operation produces a sequence of bits c_m with double the length of b_l and sampling time T_{cod}. To improve the performances by reducing the effect of burst errors, c_m is passed to an interleaver 43×41 in which we write the input bits by row and read them by column to perform a scramble.
At this point the resulting bit sequence c_p' at $T_a = 2T_{cod}$ is transmitted using the single carrier and OFDM schemes. At the receiver, once the detection has taken place, the reverse operations are carrier out to derive the detected bit sequence $\hat{b}_l$ and compute P_{bit}. In particular a LDPC decoder and a deinterleaver 41×43 are used.

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Single Carrier with DFE

The system takes a sequence of input symbols a_k at sampling time $T = 1$ and applies an upsampling of factor 4, obtaining a'_k at $T/4$. This new sequence is then filtered by q_c as described by the following difference equation:

$$s_c(nT/4) = 0.67s_c((n-1)T/4) + 0.7424a_{n-5} \qquad (1)$$

After the filtering white noise is added. From the following relations we can derive σ_w^2, the variance of the complex valued Gaussian noise:

$$\Gamma = \frac{M_{s_c}}{N_0 \frac{1}{T}} = \frac{\sigma_a^2 E_{q_c}}{\sigma_w^2} \longrightarrow \sigma_w^2 = \frac{\sigma_a^2 E_{q_c}}{\Gamma} = 2\sigma_I^2 \qquad (2)$$

where σ_I^2 is the variance per component. In addition we can also compute the PSD as $N_0 = \sigma_w^2 T_c = \sigma_w^2/4$, because the sampling time T_c at which we add the noise is $T/4$. In figure 1 we plot the impulse response and the frequency response of the filter q_c.

Figure 1: Impulse response of the filter q_c at $T/4$.

At the receiver we have a matched filter g_M (see figure 2), obtained from q_c as $g_M = q_c^*(t_0 - t)$. For simplicity in the last formula we have denoted the filters as is they were defined on continuous time while in the actual simulation they are at $T/4$.

Figure 2: Impulse response of the matched filter g_M for the receiver in point A.

The output of the matched filter is then sampled at T starting from an initial offset called *timing phase* t_0. In our case the choice of t_0 is made easy by the presence of the matched filter, as we can just choose the value $\bar{t}_0$, multiple of $T/4$, that is the index of the peak of the correlation between q_c and g_M, then t_0 will be equal to $\bar{t}_0 T/4$. Following this reasoning we chose $\bar{t}_0 = 16$ (17 with Matlab indexing), equal also to the index of the last sample of g_M (see figure 2).

We equalize with a DFE, that is made of two filters called feedforward and feedback filter denoted by c and b. The feedforward filter has the role of equalizing only the precursors of the overall impulse response, while the ISI due to postcursors will be canceled by filter b positioned on a feedback loop between the output of the threshold detector and its input.

Figure 3: Impulse response of the matched filter g_M for the receiver in point B.

The computation of the optimal filters c and b is carried out using the Wiener filter approach. The relation between the input random process and the output is:

$$y_k = x_{FF,k} + x_{FB,k}$$
$$= \sum_{i=0}^{M_1-1} c_i x_{k-i} + \sum_{j=1}^{M_2} b_j a_{k-D-j} \qquad (3)$$

where M_1 is the order of the feedforward filter, M_2 is the order of the feedback filter and a_{k-D} are the already detected past symbols fed back through b. Defining postcursors and precursors as in point A, we have that we can apply the Wiener-Hopf equations on the process:

$$y_k = \sum_{i=0}^{M_1-1} c_i \left(x_{k-i} - \sum_{j=1}^{M_2} h_{j+D-i} a_{k-j-D} \right) \qquad (4)$$

The result can be easily computed as $c_{opt} = \boldsymbol{R}^{-1} \boldsymbol{p}$ once we find the autocorrelation matrix $\boldsymbol{R}$ and the correlation vector $\boldsymbol{p}$, expressed as [1]:

$$[\mathbf{R}]_{p,q} = \sigma_a^2 \left(\sum_{j=-N_1}^{N_2} h_j h_{j-(p-q)}^* - \sum_{j=1}^{M_2} h_{j+D-q} h_{j+d-p}^* \right) + r_{\tilde{w}}(p-q) \qquad (5)$$

$$[\mathbf{p}]_p = \sigma_a^2 h_{D-p}^* \qquad\qquad p = 0, 1, \ldots, M_1 - 1 \qquad (6)$$

where for a QPSK scheme $\sigma_a^2 = 2$ because it is the sum of two orthogonal components each with power 1. The values of $r_{\tilde{w}}$ are the result of the autocorrelation of the noise after being filtered by g_M, so being the noise white we have $r_{\tilde{w}}(n) = N_0 r_{g_M}(nT)$. At this point we can define the overall impulse response up to the threshold detector $\psi = h * c_{opt}$ and derive the optimal coefficients for filter b as $b_i = -\psi_{i+D}$ for $i = 1, \ldots, M_2$.

The value of the cost function J_{min} obtained using these the optimal filters is :

$$J_{min} = \sigma_a^2 \left(1 - \sum_{l=0}^{M_1-1} c_{opt,l} h_{D-l} \right) \qquad (7)$$

Again the parameters to choose are the order of filter c, M_1, and the delay introduced D. This is because the order of b can be chosen in such a way that all the postcursors are canceled by the feedback: $M_2 = N_2 + M_1 - D - 1$, and also the expression of the autocorrelation matrix significantly simplifies. The choice is carried out by selecting the values that minimize the functional J_{min}, this time being $M_1 = 5$ and $D = 4$, and consequently $M_2 = 4$ because $N_2 = 4$. Once we noticed that for a certain value of D the functional could only decrease, we let M_1 vary to choose the optimal couple of parameters. The following figure shows the knee of the curve at $M_1 = 5$ with D fixed at 4:

Figure 4: Behavior of J_{min} for $D = 4$ varying M_1.

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In figure 5, 6 and 7 we plot the resulting filters c, ψ and b at sampling time T.

Figure 5: Magnitude of the impulse response of the filter c (feedforward filter) for the receiver in point B.

Figure 6: Magnitude of the impulse response of the system ψ in point B.

Figure 7: Magnitude of the impulse response of the filter b (feedback filter) in point B.

OFDM with cyclic prefix

The system considered in this case is represented in figure 8:
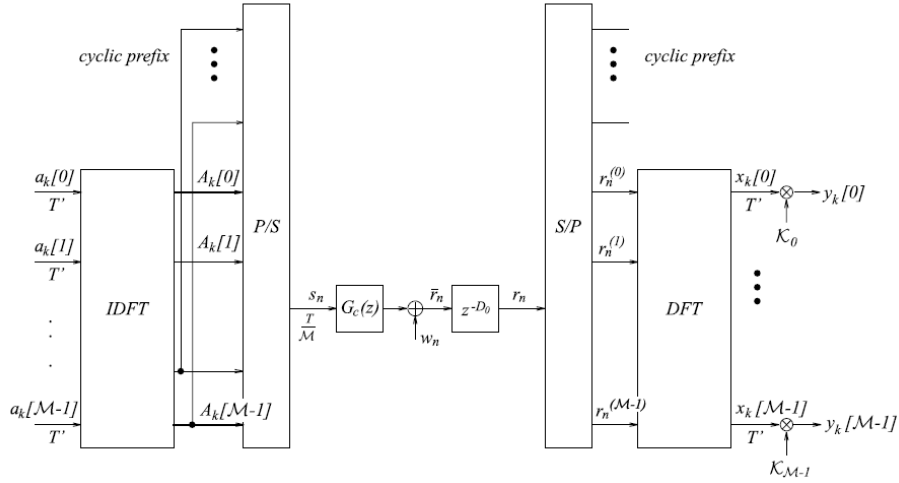

Figure 8: Block diagram of the OFDM system with the cyclic prefix method.

The input sequence of symbols a_k is arranged in a matrix with $\mathcal{M} = 512$ rows and on these blocks of length $\mathcal{M}$ we compute the IDFT to obtain the sequence $A_k[i]$ for $i = 0, \ldots, \mathcal{M} - 1$. At this point we used the cyclic prefix method to perform equalization: a prefix of length N_{px} taken from the end of the block obtained form IDFT is repeated on top of the same block. In this way, choosing $N_{px} \geq N_c - 1$ (where N_c is the length of the channel impulse response) the data of

the next block will be separated from the present one by N_c-1 or more samples, that is enough to fill the channel and not have any ICI between consecutive blocks. At the receiver the additional $A_k[i]$ with $i = \mathcal{M} - 1 - N_{px}, \ldots, \mathcal{M} - 1$ can be just discarded.

OFDM channel

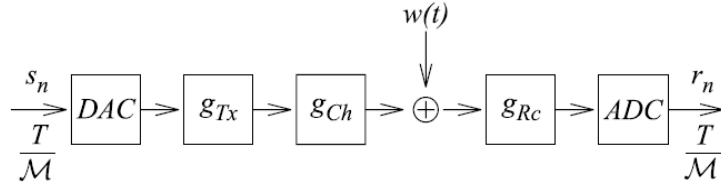The channel, represented as a single block in figure 8 contains all blocks depicted in figure 9

Figure 9: Channel + transmitter and receiver filters in OFDM.

where in our case g_{tx} and g_{Rc} are taken both as square root raised cosine filters, and the channel impulse response is the q_c used also for the single carrier channel:

$$s_c(nT/4) = 0.67 s_c((n-1)T/4) + 0.7424 a_{n-5} \tag{8}$$

The system takes a sequence of input symbols s_k at sampling time $T_{OFDM} = T_{block}/(M + N_{px})$ and applies an upsampling of factor 4, obtaining a'_k at $T_c = T_{OFDM}/4$. This new sequence is then filtered by $g_{\sqrt{rcos}}$ where the impulse response is described by the following equation:

$$g_{\sqrt{rcos}}(n) = \frac{\sin\left[\pi\left(1-\rho\right)\frac{n}{T_c}\right] + 4\rho\frac{n}{T_c}\cos\left[\pi\left(1-\rho\right)\frac{n}{T_c}\right]}{\pi\left[1 - \left(4\rho\frac{n}{T_c}\right)^2\right]\frac{n}{T_c}} \tag{9}$$

After the convolution with the transmit filter, we send the data through the channel with the impulse response given by (8) and white noise is added. From the following relations we can derive σ_w^2, the variance of the complex valued Gaussian noise:

$$\Gamma = \frac{M_{s_c}}{N_0\frac{1}{T}} = \frac{\sigma_a^2 E_{g_{\sqrt{rcos}}*q_c}}{\sigma_w^2} \longrightarrow \sigma_w^2 = \frac{\sigma_a^2 E_{g_{\sqrt{rcos}}*q_c}}{\Gamma} = 2\sigma_I^2 \tag{10}$$

At the receiver we have the matched filter of the transmission filter which is another $g_{\sqrt{rcos}}$ with the impulse response given by (9). The output of the matched filter is then sampled at T_{OFDM} starting from an initial offset called *timing phase* t_0. We can just choose the value $\bar{t}_0$, multiple of $T_{OFDM}/4$ and we have chosen $\bar{t}_0 = 90$ (91 with Matlab indexing) "because this is the value with the best performances"
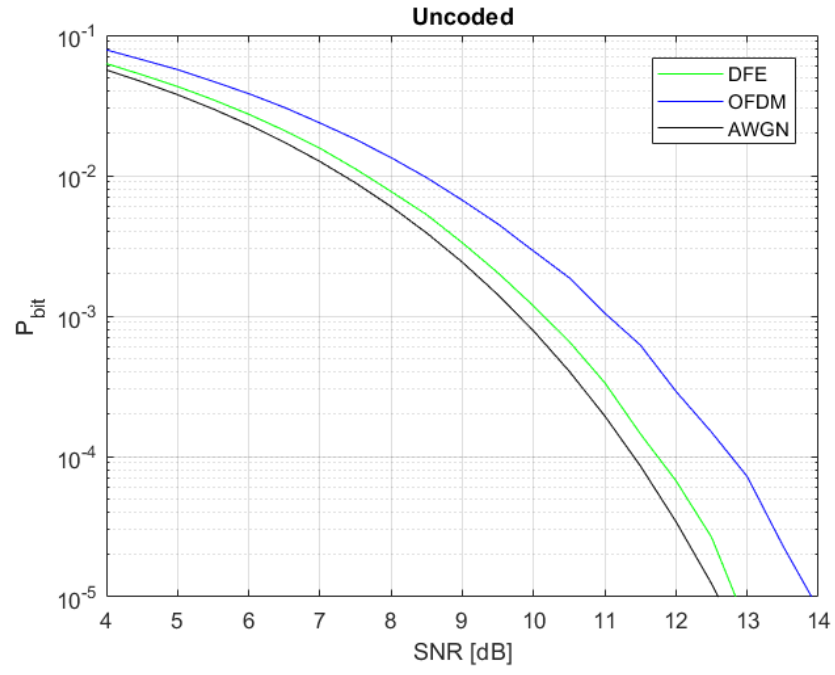
Figure 10: P_{bit} obtained for various values of the SNR with DFE equalization and OFDM, no channel coding used.
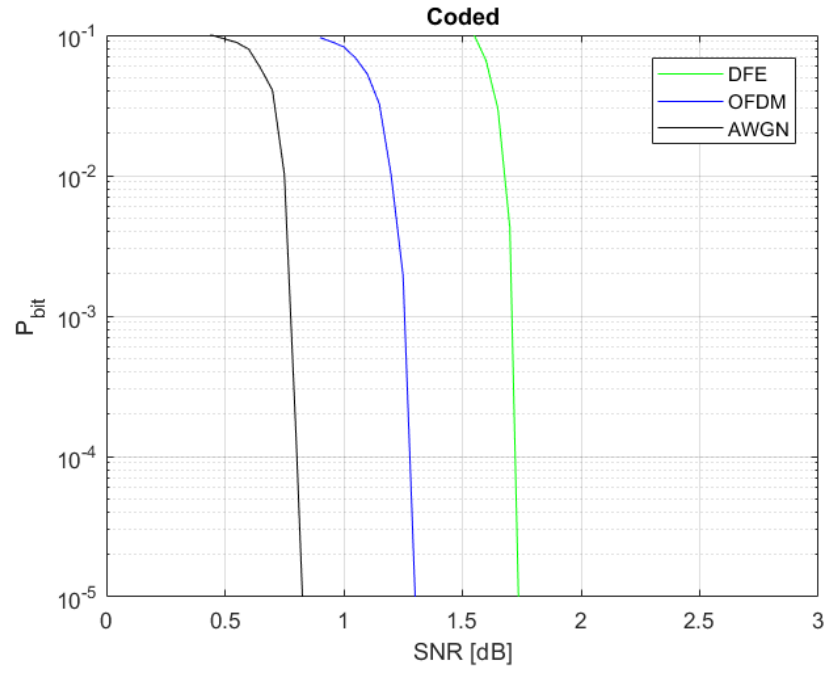
Figure 11: P_{bit} obtained for various values of the SNR with DFE equalization and OFDM, LDPC coding has been used.

References

[1] Nevio Benvenuto, Giovanni Cherubini, *Algorithms for Communication Systems and their Applications.* Wiley, 2002.