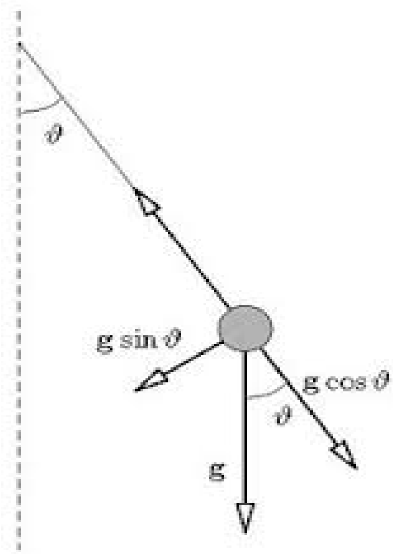
LABORATORIO 10

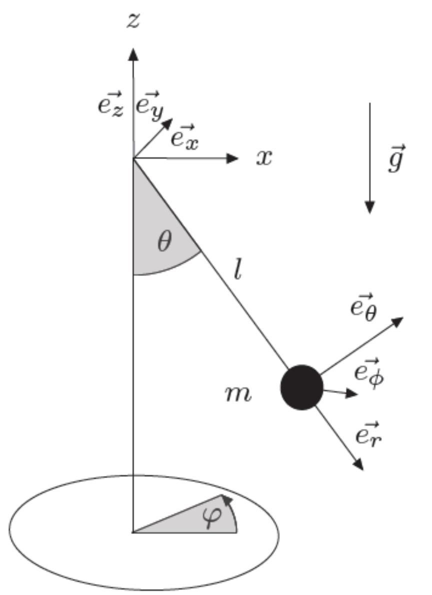
Argomenti: equazioni differenziali ordinarie

1. Risolvere i seguenti problemi di Cauchy, che rappresentano le equazioni del moto di un pendolo semplice di lunghezza l



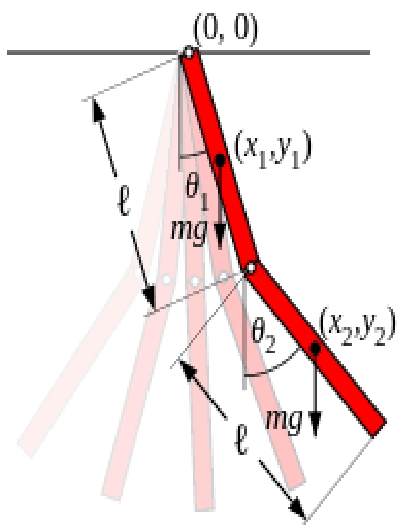
$$\begin{cases} \theta''(t) = -\dfrac{g}{l}\sin\theta(t), \quad t \geq 0, \\ \theta(0) = \theta_0, \\ \theta'(0) = \theta_{10}, \end{cases}$$

di un pendolo sferico



$$\begin{cases} \theta''(t) = \varphi'(t)^{\,2}\, \sin\vartheta(t)\, \cos\vartheta(t) - \dfrac{g}{l}\sin\theta(t), \quad t \geq 0, \\ \varphi''(t) = -2\,\theta'(t)\,\varphi'(t)\, \cot\vartheta(t) \\ \theta(0) = \theta_0, \\ \theta'(0) = \theta_{10}, \\ \varphi(0) = \varphi_0, \\ \varphi'(0) = \varphi_{10}, \end{cases}$$

e di un pendolo doppio che si muove su un piano



$$\begin{cases} \theta_1'(t) = 6\,\dfrac{2p_1(t) - 3\,p_2(t)\cos(\theta_1(t)-\theta_2(t))}{16 - 9\cos^2(\theta_1(t)-\theta_2(t))}, \quad t \geq 0, \\ \theta_2'(t) = 6\,\dfrac{8p_2(t) - 3\,p_1(t)\cos(\theta_1(t)-\theta_2(t))}{16 - 9\cos^2(\theta_1(t)-\theta_2(t))}, \\ p_1'(t) = \dfrac{1}{2}\,\left(\theta_1'(t)\theta_2'(t)\sin(\theta_1(t)-\theta_2(t)) + 3\dfrac{g}{l}\sin(\theta_1(t))\right) \\ p_1'(t) = -\dfrac{1}{2}\,\left(-\theta_1'(t)\theta_2'(t)\sin(\theta_1(t)-\theta_2(t)) + \dfrac{g}{l}\sin(\theta_2(t))\right) \\ \theta_1(0) = \theta_{10}, \\ \theta_2(0) = \theta_{20}, \\ p_1(0) = p_{10}, \\ p_2(0) = p_{20}. \end{cases}$$