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Optimal ATM replenishment policies under demand uncertainty

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Abstract
The use of Automated Teller Machines (ATMs) has become increasingly popular throughout the world due to the widespread adoption of electronic financial transactions and better access to financial services in many countries. As the network of ATMs is becoming denser while the users are accessing them at a greater rate, the current financial institutions are faced with addressing inventory and replenishment optimal policies when managing a large number of ATMs. An excessive ATM replenishment will result in a large holding cost whereas an inadequate cash inventory will increase the frequency of the replenishments and the probability of stock-outs along with customer dissatisfaction. To facilitate informed decisions in ATM cash management, in this paper, we introduce an approach for optimal replenishment amounts to minimize the total costs of money holding and customer dissatisfaction by taking the replenishment costs into account including stock-outs. An important aspect of the replenishment strategy is that the future cash demands are not available at the time of planning. To account for uncertainties in unobserved future cash demands, we use prediction intervals instead of point predictions and solve the cash replenishment-planning problem using robust optimization with linear programming. We illustrate the application of the optimal ATM replenishment policy under future demand uncertainties using data consisting of daily cash withdrawals of 98 ATMs of a bank in Istanbul. We find that the optimization approach introduced in this paper results in significant reductions in costs as compared to common practice strategies.

Keywords Automated teller machines · Replenishment policy · Demand uncertainty

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1 Introduction

The number of ATMs all over the world is approximately 3,000,000 as of 2016 (ATM Industry Association 2016). Specifically for Turkey, the number of ATMs is 48,197 and the number of bank cards is 114,735,756 (The Interbank Card Center 2016). With such a significant widespread use of bank cards and ATMs, the need for optimal ATM replenishment policies is a significant decision to make in cash management across all banks, small or large (Brentnall et al. 2010). The banks need to develop optimal replenishment policies such that ATMs will rarely run out of cash and the cost of borrowing the cash to stock the ATMs will not be higher than the profit resulting from the financial transactions. The aim of this paper is to develop a cash management methodology for ATMs where the optimization decision variable is the amount of money that should be loaded onto the ATMs at the beginning of the planning period under both cost constraints of the bank (e.g., penalty and holding costs) and the demand constraints of the customers.

Optimal replenishment policies involve forecasting the future cash demand. This is because banks have to decide on the amount of money that should be replenished without knowing the actual demand since it is not available at the beginning of the planning period. Obtaining accurate forecasts is needed in ensuring low management expenditures without compromising customer satisfaction. An over-forecast of future demand, i.e., higher cash replenishments than the realized demand, will lead to high opportunity costs. On the other hand, an under-forecast of future demand will result in customer dissatisfaction.

Existing studies focus on either forecasting the future cash demand or optimizing the replenishment policy (except for Baker et al. 2013). Forecasting methods usually provide predictions at the daily level whereas optimization approaches assume that demand is provided at temporal aggregation levels. To the best of our knowledge, there are no existing studies incorporating prediction of future cash demand into optimal replenishment-decision making except for the study of Baker et al. (2013).

Unlike existing studies, demand forecast and optimal replenishment policies are both considered in this study. In addition, we estimate prediction intervals instead of point predictions for demand forecasting to account for uncertainties in the unobserved future cash demands. Using the prediction interval estimates, we solve a robust optimization problem for deriving optimal replenishment cash amounts where the optimization is over all values within the prediction intervals. In the proposed approach, the uncertainty interval for robust optimization problem is chosen as the prediction interval and the robust optimization problem is solved as to minimize the maximum regret (cost).

We illustrate the application of the optimal ATM replenishment policy under future demand uncertainties using data consisting of daily cash withdrawals of 98 ATMs of a bank in Istanbul, Turkey. The comparison results of the costs under the proposed framework (replenishing the optimal amount found by robust optimization) and a classical approach (replenishing the upper bounds of the prediction intervals) show the superiority of the proposed framework. We find that the costs under the optimal replenishment policy (scenario 1: the proposed approach- results can be seen later in this paper in Table 1) are always lower than when using the upper bounds of the prediction intervals for replenishing the ATMs (scenario 2-results can be seen later in this paper in Table 2). In fact, most of the existing studies consider replenishing point predictions (the mean in a prediction interval), however we do not use this approach for comparison since point predictions will result in stock outs in most cases, resulting in an expected high cost. The concept behind prediction intervals has an important role in a problem related to demand forecasting in inventory optimization. One has to forecast total demand over a replenishment lead time. The forecasting task is to determine all the possible values of cumulative demand over this time and their associated probabilities of occurring. For instance, if one calculates a 90% prediction interval for demand, then the upper end of the interval represents the 95th percentile. Setting the reorder point at this level means that there is only a 5% chance of stocking out before replenishment is done.

The paper includes five additional sections. Section 2 summarizes the problem of ATM cash management and gives a summary of the literature review. Section 3 introduces the approach for estimation of future cash demands and for optimal decision making in ATM cash management. Section 4 illustrates the proposed approach using the ATM withdrawals data from the Turkish bank and the last section provides conclusive discussions. Additional details on the forecasting model are deferred to the "Appendix".

2 Literature review: motivating our approach

The aim in ATM cash management is to develop inventory control policy such that ATMs rarely run out of cash and the cost of borrowing the cash to stock the ATMs is not high. Thus, accurate demand forecasting systems and optimal replenishment policy are clearly key ingredients.

The literature on forecasting is very rich and the methods applied to ATM cash demand forecasting are a subset of what can be done in this area (Brockwell et al. 2002; Shumway and Stoffer 2006; Lütkepohl 2005). The ATM cash demand forecasting problem has been extensively researched after the "Forecasting Competition for Artificial Neural Networks and Computational Intelligence" was launched in 2008. The aim of the competition was to forecast the daily cash demands for 111 ATMs of a bank in England. The data included daily ATM cash withdrawals for 2 years and the aim was to forecast the next 56 days' demand (Crone 2008a, b). Andrawis et al. (2011) was ranked first among the machine learning models in the competition. Two important components of their approach consist of model averaging (or combination of forecasts across multiple prediction models) and seasonality modeling. Using the same data, Coyle et al. (2010) applied self-organizing fuzzy neural network (SOFNN); and Taieb et al. (2012) considered the effects of deseasonalization, input variable selection, and forecast combination.

In a different study, Simutis et al. (2007) derived daily forecast using 3 years' data of a bank in Lithuania. They applied the Levenberg–Marquard optimization method for training neural networks (NNs) used in prediction at both the daily and weekly levels.

The existing studies on ATM cash demand forecasting emphasize the need of modeling the trend and seasonality in the historical data. The trend varies across ATMs as it reflects the smooth temporal pattern in cash withdrawal behaviors for each ATM individually. On the other hand, seasonality as well as other cyclical trends and events affect ATM withdrawals in a similar manner. However, the existing approaches primarily perform forecasting for each ATM in isolation of other ATMs' historical data, and thus they do not allow borrowing information across ATMs with similar characteristics, such as demographics of the population surrounding an ATM's location, the urbanicity of the place and the points of interests (such as schools, plazas, other banks' ATMs, shopping centers etc.). "Learning" the seasonal and cyclical patterns from multiple time series derived from proximal locations will increase the predictive power, resulting in more accurate forecasts, as we illustrate in this paper.

Many researchers forecast cash withdrawals from ATMs in a daily basis; however, this is not an easy task to perform. Previous literature shows that the smallest daily-level forecast errors are 20% or larger in terms of mean absolute prediction error (MAPE)[1] (Andrawis et al. 2011). This is not a surprise since the variations in daily demands are large, which makes it difficult to forecast. Even more, targeting small error daily forecasts is not needed since banks rarely load ATMs on a daily basis. Aggregated predictions at weekly levels, for example, could be more meaningful in the context of ATM cash management, which will make it possible to provide more accurate predictions of the cash demand.

Brentnall et al. (2010) considered the development of a system for predicting daily ATM withdrawals for inventory control, using data from 190 ATMs in the United Kingdom over a 2-year period. They argue that density forecasts are more appropriate than point forecasts. A density forecast is the estimated probability distribution of the realization of a random variable at some future time point. Using density forecasts in ATM cash management will require implementation of stochastic optimization, which could be challenging to apply within our framework. Instead, in deriving optimal policies, we propose using prediction intervals, which are an intermediate approach between point forecast and density forecast (Tay and Wallis 2000). A prediction interval is an estimate of an interval in which a future observation will fall. It is a type of confidence interval (CI) used with predictions in regression analysis and it is a range of values that predicts the value of a new observation, based on the existing model. A point forecast is the mean response and is a

[1] We note that this is not a correct scoring rule in evaluating prediction errors. The MAPE can be used when only median predictions are derived. However, for most methods as they are based on L2-norm estimation or least squares estimation, mean square error needs to be used. This fundamental aspect in evaluating predictions is a topic of interest in the statistical literature (Gneiting and Raftery 2007; Gneiting 2011).

forecast of the expected value. Using prediction intervals to address uncertainty in the future cash demand, the resulting cash replenishment-planning problem can be solved using linear programming.

Particularly, in ATM cash management, the forecast is an integral part of *inventory optimization*—planning ATM load frequencies, the number of managed stocking locations, and the cash amounts loaded in ATMs among others. Inventory optimization has been widely studied in the operation research literature but less so applied to ATM cash management (Castro 2009; Altunoglu 2010; Baker et al. 2013; Karimi and Davoudpour 2016). Castro (2009) presents three stochastic programming models, two short-term and one mid-term inventory models. The objective is to find the amount of cash to be disposed in an ATM for a certain period of time (e.g., 1 week) using a stochastic programming approach. The short-term model with fixed costs is an integer programming problem which is solved by a fast (i.e., linear running time) algorithm. The short-term model with fixed and staircase costs is solved through its Mixed Integer Linear Programming (MILP) equivalent deterministic formulation. The mid-term model with fixed and staircase costs gives rise to a multi-stage stochastic problem, which is also solved by its MILP deterministic equivalent. Altunoglu (2010) introduces two new centralized inventory policies, which can be adopted by the headquarters such that the branches manage the ATM inventory more efficiently. The first policy reviews the inventory at a given time within the planning period and charges a lump-sum cost for stock-outs (in addition to the inventory holding cost at the end of the period). The second policy also reviews the inventory at a time t before the period ends, but charges a unit stock-out cost if the inventory is below a threshold level. Van Anholt et al. (2016) solve the inventory routing problems with pickups and deliveries arising in the replenishment of ATMs. Diao et al. (2016) propose a cascaded optimization framework which finds the optimal amount to be replenished and the optimal time for the replenishment. The framework is composed of inner loop optimization and outer loop optimization. Batı and Gözüpek (2017) formulate an optimization problem as an integer linear program, which jointly decides on when to visit an ATM, how much money to deliver to which ATM and which road should be followed for the distribution of cash to the ATMs. However, these studies do not to integrate it with the cash demand forecasting model, they assume that the demand is given; which is not realistic since the future cash demand is not realized at the beginning of the planning period. To the best of our knowledge, the study of Baker et al. (2013) is the only study that integrates the cash demand forecast into informed replenishment decision making. This study develops an optimization module that is embedded in a simulation routine to find the cost minimizing target level for a specified service level constraint. They solve the periodic review inventory problem by doing a grid search for the expected total cost minimizing replenishment period, and embedding in this grid search an optimization in simulation to find the expected total cost minimizing order-up-to target. For the forecasting part, they perform rolling horizon time series forecasts; the forecast that has the smallest recent root mean square error is then selected for the next planning horizon. Although the study by Baker et al. (2013) acknowledges the need of using predicted cash demand, it presents similar limitations as discussed in this review; the predictions are performed for each ATM in isolation of the historical data from other ATMs and the forecasts do not account for uncertainty in the predictive system. In order to address these limitations, in this study we allow borrowing information across ATMs with similar characteristics, and instead of finding point predictions of daily withdrawals (which leads to very high errors), we find aggregated predictions; specifically, prediction intervals at weekly levels. Baker et al. (2013) use integer programming in their optimization model since they find point predictions of demand; however, in order to address the uncertainty in the demand predictions; we use robust optimization incorporating the prediction intervals.

In summary, much of the literature is focused on the forecasting of the withdrawals from the ATMs. The research on cash management, which is scarce, is concerned with cash supply optimisation *within a given cash allocation scheme* (Ágoston et al. 2016). The current literature on forecasting do not allow borrowing information across ATMs with similar characteristics, and they try to forecast daily withdrawals, where the errors are usually very high. Moreover, they find point predictions, which is difficult to use in determining the optimal amount for replenishment. In order to fill these gaps in the literature, demand forecast and optimal replenishment policies are both considered in this study. In order to provide more accurate predictions of the cash demand, aggregated predictions at weekly levels are performed. In order to increase the predictive power, we propose a model which learns from the seasonal and cyclical patterns from multiple time series derived from proximal location. We propose using prediction intervals, which are an intermediate approach between point forecast and density forecast. Using the prediction interval estimates, we solve a robust optimization problem which finds the optimal amount to be loaded to the ATMs while minimizing the maximum regret.

3 Model development: proposed approach

The two specific objectives of the proposed approach are:

1. To forecast the demand using prediction intervals; and
2. To optimize the replenishment policy under specific cost assumptions and under decision constraints while accounting for the uncertainty in the future cash demands.

When deciding about the prediction methodology, two scenarios could be considered. When the replenishment period is optimally selected given the trade-off between costs and demand, then forecasting could be performed in aggregation over the selected replenishment period. Alternatively, one could set the replenishment period fixed, for example, weekly, and optimize the replenishment policy. The latter approach is preferable because forecasting over seasonally-meaningful time periods, e.g. week or month, allows averaging out seasonality or/and other cyclical trends, providing higher accuracy estimates of the ATM cash demand with further implications on optimal replenishments. Consequently, in our approach, we obtain weekly forecast of the demand along with prediction interval estimates. We selected *week* as our replenishment period because it is a natural choice given the weekly seasonality of work schedule and life events, and it is also not too short and not too long for planning purposes, for example, banks commonly check once a week whether the off-site ATMs are functioning properly. Our assessment for this replenishment period was validated by the bank, of which data we used to illustrate the application of the proposed method. Since the bank replenishes the ATMs weekly, we decided to take the time period as 1 week. Moreover, in our other study, (Ekinci et al. 2015), the results obtained from several experiments show that the forecasting model built from weekly data does a better job than models built from daily data.

The selection of the modeling approach for obtaining demand forecasts is somewhere dependent on whether weekly rather daily forecasts are needed. Various methods have been studied for demand forecasting in various contexts, not only in ATM cash demand forecasting, as we review in Sect. 2. One important contribution of this work is to build a rather simple regression model for obtaining prediction interval estimates for the weekly cash demand. The model pulls together information across multiple categories of variables. For example, we acknowledge that not only borrowing predictive power across time is important but also across geography; thus, we include a series of location-related and time-related variables in the model jointly. In the next section, we expand on this approach, particularly, the estimation procedure from which we derive the prediction intervals. In the subsequent section, we introduce how to use the interval predictions for cash demand to develop an inventory control policy under the assumption that ATMs will rarely run out of cash and the cost of borrowing the cash to stock the ATM will not be high, while accounting for specific constraints common in the banking industry.

3.1 Forecasting

The most widely used techniques for forecasting are time series modeling and machine learning methods (Gal et al. 2017; Chen et al. 2017). Most time series modeling techniques rely on the stationarity assumption, which does not hold for the ATM cash-withdrawal time series. Machine learning methods allow for nonlinear dynamic relationships between the dependent variable and independent variables, but they have the disadvantage of being less interpretable compared to pure statistical methods. Moreover, it is difficult to estimate prediction intervals, as they are not model-based (Kibekbaev and Duman 2016). Moreover, many approaches in time series analysis and machine learning are univariate prediction methods, in the sense that they only use the information within the time series alone ignoring the behavior of related time series, providing limited predictive power.

This motivates our simple but insightful regression model, which allows borrowing information across time series that are clustered using geographic information. *Borrowing information* within clusters leads to improved prediction accuracy since the demand within proximal geographic locations commonly behaves similarly over time. This follows from the first law of geography, “Everything is related to everything else, but near things are more related than distant things” (Tobler 1970). Using location-related and time-related variables jointly will also allow borrowing information between clusters of ATMs—those that are in areas with similar characteristics will follow a similar cash demand pattern whereas the cash withdrawal pattern in residential areas is different from the pattern in the commercial areas, and these are different from the touristic areas.

Thus our model includes a categorical variable specifying the geographic cluster of each ATM along with detailed location data. The demographics information together with the urbanicity of the place where the ATM is located, and the points of interests (such as schools, plazas, other banks' ATMs, shopping centers etc.) in the ATM's vicinity are some examples of location-related variables. In addition, we include time related variables such as festivals, holidays and special events to address the time-varying effects on cash demand. Generally, in order to determine a comprehensive list of location related variables, Geographical Information Systems (GIS) are used together with expert opinion.

We denote the withdrawn amount from an ATM Y_{iw} where i is the ATM's index and w is the time of withdrawal, where $i=1,\ldots,n$ (n is the number of ATMs) and $w=u_i,\ldots,W$ with u_i being the first observation time point of the ith ATM and W is the last recorded time point for all ATMs. In our implementation, w indexes the week of the withdrawn amount. We seek to predict cash demand for m lags/weeks ahead, $Y_{iW+1}, \ldots, Y_{iW+m}$. To do so, we detrend the data using non-parametric regression and then, we apply a linear regression model to the de-trended data. Hence, the trend estimation step is followed by the application of the regression model on the de-trended data. Using the two sets of predictions, we obtain a point forecast at a future time t, where t is a future time/week (in our study, $t=W+1, \ldots, W+4$ weeks). For the linear regression model, some categorical and some numeric variables are used in order to explain the variability in the ATM withdrawn amounts across multiple ATMs.

Using more advanced statistical models could improve the prediction accuracy if daily forecasts are needed, but a regression model with a well-thought set of predictors will often suffice when focusing on weekly predictions. Moreover, one would prefer simpler models when considering the estimation of prediction intervals over point predictions.

One has to bear in mind that the time-varying predictors, which will be used in the regression model, should specify deterministic events (e.g., holidays) that do not need to be predicted. Since special days and holidays such as festival days, weekends, mothers' day etc. are known on the calendar, we do not have to predict the time they are going to occur. Other time-varying predictors (e.g., temperature, rainfall) could lead to variations in cash demands, however they will need to be predicted also, thus adding another level of error in the prediction of the cash demand. One approach to fully capturing time variations is to model the trend in the data using nonparametric regression applied to the withdrawal data of ATM separately.

Using this model we further derive prediction intervals to account for uncertainties in unobserved future cash demands. It is better to remind here that we seek to predict cash demand for m lags/weeks ahead, $Y_{iW+1}, \ldots, Y_{iW+m}$. The general formula $\hat{\mu}_{Yit} +/- z_{\alpha/2}\hat{\sigma}_{Yit}$ gives the prediction interval where $\hat{\mu}_{Yit}$ is the predicted mean of the withdrawn amounts at a future time t ($t=W+1, \ldots, W+m$), $\hat{\sigma}_{Yit}$ is the estimated standard error of the predicted mean, and $z_{\alpha/2}$ is the normal quantile. The skewness and kurtosis values of the forecast errors are calculated as 0.62 and 2.86; respectively. These values indicate that we can assume a normal distribution. While there are many approaches used to obtain the predicted mean as described in Sect. 2, the estimation of the variance $\hat{\sigma}^2_{Yit}$ is not commonly explored. In modeling ATM cash withdrawal time series, the variance of the predicted mean will depend on the strength of the serial correlation within each time series and on the variance of the model errors. Serial correlation estimation is dependent on the assumed model whereas the error variance can be estimated depending on the heteroskedastic behavior of the time series. For example, for ATM cash withdrawals, the variance is constant within a short period of time while varying across longer time intervals. This suggests that we can compute the variance based on recent observed cash withdrawal values, where the number of past observations to be used can be determined such that the prediction intervals will be both reliable and accurate.

We will expand on the implementation of this approach in Sect. 3, where we illustrate it with a specific case study.

3.2 Optimization

Demand prediction alone is a simple forecasting exercise; it should be integrated within informed decision making and strategic planning (Lau and Nakandala 2012; Wang et al. 2012). In this study, the overarching goal in decision making is to derive an optimal ATM cash replenishment policy, balancing the costs that will be incurred and service-level objectives while taking demand and supply volatility into account. This translates into low inventory levels while meeting customers' demand. In the ATM cash management problem, the inventory consists of the cash stocked in the ATMs at a given time. As the realized cash demand is not available at the planning time, we propose to integrate interval forecasts to account for demand uncertainties in the replenishment optimization. The resulting replenishment policy decides on the amount of money that should be loaded to each ATM based on the forecast intervals. Specifically, we devise an objective function and its constraints that will provide the optimal value, where the optimization is over all values within the prediction interval as we describe next. The resulting optimization problem is thus ILP, a particular application of robust optimization, which is commonly used to find robust solutions under data perturbation. There are two basic approaches to solve interval linear programming. The first one involves robust optimization and fuzzy programming among others; in this approach we resign on bounds that cover all possible results (Wan et al. 2015). In the second approach, one computes optimal interval solutions for all possible scenarios or values in the interval. The methods used in the second approach are multi-parametric programming, perturbation theory, and interval arithmetic (Hladik 2012). Among these techniques, *robust optimization* seeks for a solution that is robust (stable) under some data perturbation, being more powerful than the scenario-based approaches (Boukani et al. 2016). More importantly, the concept of robust optimization is consistent with the risk-averse approach, in which the ATM cash management systems are operated (Bertsimas et al. 2011; Hahn and Kuhn 2012; Bertsimas and Zhao 2013).

Denote Y_{it} be the amount of cash demand from ATM i at a future time t ($t = W+1, \ldots, W+m$). We consider here a robust optimization (Ben-Tal and Nemirovski 1998) problem in which the decision variable (x_{it}) is the optimal amount that should be replenished to ATM i at time t. Using this notation, we have two possible outcomes:

Excess in the ATM loading: $E_{it} = (x_{it} - Y_{it})_+$ (where '+' means that this value is either the difference when positive or 0 when negative).

Excess in the demand for cash at the same ATM: $D_{it} = (Y_{\mathrm{it}} - x_{it})_+$ (where '+' means that this value is either the difference when positive or 0 when negative).

The two outcomes have also associated costs. For an excess in the ATM loading, the associated cost is the interest rate per dollar, defined herein by c. Thus the total cost for an excess in the ATM loading is $c\,E_{it}$. For an excess in the demand for cash, the associated cost consists of two components. One component is the fixed penalty cost; denote this cost h. The second cost component is the customer dissatisfaction, which in principle should depend on the demand amount that was not satisfied, i.e. the higher this excess is the more customers are dissatisfied although it doesn't have to be linear; denote the dissatisfaction cost per dollar g. Therefore, the total cost due to an excess in the demand for cash becomes $h + g\,D_{it}$. The cost function for ATM i at time t is given by

$$C_{it} = \begin{cases} cE_{it} & \mathit{if} \quad x_{it} > Y_{it}, \\ h + g\,D_{it} & \mathit{if} \quad x_{it} < Y_{it} \end{cases} \tag{1}$$

This function is very similar to the classical single-item newsboy modeling approach except that it maximizes the profit, whereas we are minimizing the cost (see Silver and Peterson 1985; Vairaktarakis 2000). We first introduce a robust formulation that is independent of any distribution assumptions. Then we present a solution methodology that can determine a decision for every ATM loading that is robust against any perturbations in demand.

The first step is to construct an uncertainty interval for future demand. Unlike the approaches based on stochastic assumptions that uses a probability distribution (i.e., stochastic optimization), *the uncertainty model in a robust optimization formulation is in the form of a deterministic set*. This deterministic set is defined in our paper as U and defined within the interval $[L_{it}, U_{it}]$. In this formulation L_{it} and U_{it} corresponds to the lower and upper limits for demand in ATM i at time t, respectively. In other words, there is no assumption on probability distribution of uncertainty in the proposed robust optimization model. The set U would be greater or smaller, depending on the level of risk the firms would be willing to take. Our robust optimization model obtains the worst case within the set U, and identifies the best solution for the worst case scenario within the deterministic set U. The uncertainty set U is application specific and determined through expert opinion or statistical analysis as presented in the previous section. Specifically, the limits are chosen as points that represent significant variations in the system. As this interval increases, the robust solution might be more costly, but would protect the system against higher degrees of uncertainty in demand (Gabriel et al. 2014; Chiou 2015; Ramezani et al. 2014).

We next introduce the corresponding robust optimization model for our planning problem:

$$\begin{aligned} &min\, f\left(x_{it}\right) \\ &G(x_{it}, Y_{it}) \in K, \\ &\forall Y_{it} \in U \end{aligned} \tag{2}$$

where f is the objective function, G and K are the structural elements of the constraints that are assumed to be certain, and Y_{it} are the uncertain parameters of the problem (Tutuncu and Cornuejols 2007).

For our specific problem, for any x_{it} decision, increasing E_{it} corresponds to a higher cost of $c\ E_{it}$. Similarly, increasing D_{it} corresponds to a higher cost of $h+g\ D_{it}$. Since robust optimization depends on the idea of minimizing the worst case cost, we find the points where the maximum cost values occur. They occur at $Y_{it}=L_{it}$ and $Y_{it}=U_{it}$ for the case of $x_{it}> Y_{it}$ and for the case of $x_{it}< Y_{it}$ respectively. We can now formulate the problem as a robust linear optimization problem by minimizing the C_{it}, which is the maximum cost incurred across all possible outcomes measured by the excess in the ATM loading or excess in the demand, and solve for the optimal amount that will be loaded to the ATM i at a t future week. The robust optimization problem becomes:

$$\begin{aligned} &minimize\ C_{it} \quad \forall i, t \\ &subject\ to \\ &C_{it} > c\,(x_{it} - L_{it}) \\ &C_{it} > h + g\,(U_{it} - x_{it}) \\ &x_{it} \in [L_{it,} U_{it}] \end{aligned} \tag{3}$$

In the above formulation, the derivation of the cost is simple because of the linearity of our cost function. The resulting optimization problem is easy to solve using linear programming. For every fixed $x_{it} \in [L_{it}, U_{it}]$, the first constraint is a linear increasing function of x_{it} while the second is linear decreasing. Hence, the decision variable x_{it} takes its optimal value (x_{it}^*) in the intersection point of the two lines, i.e., the solution is:

$$x_{it}^* = (h + g\ U_{it} + L_{it}c)/(c + g) \tag{4}$$

Note that x_{it}^* is a convex combination of $h/(c+g)$, U_{it} and L_{it} where U_{it} and L_{it} are weighted by $g/(c+g)$ and $c/(c+g)$, respectively. On the other hand, there may be some cases when there is no intersection point of the two lines in the $[L_{it}, U_{it}]$ region. In such rare cases, we take $x_{it}^* = U_{it}$. For a specific example, take $L_{it}=3$, $U_{it}=5$, $c=0.3$, $h=1$ and $g=0.1$. In this small example, the two cost lines intersect at $x=6$, however, it is out of the [3, 5] region, hence $x^*=5$.

In order to show the performance of the proposed model, we benchmark the optimal policy derived from this model to the current policy that the bank uses. We compare the costs resulting from the implementation of the optimal policy (defined herein scenario 1) using Eq. 4 and from the replenishment policy when the demand is set to be equal to the upper bound of the interval estimate (defined herein scenario 2). This comparison is motivated by the fact that in practice banks usually prefer not to be stock out. The cost functions used for each scenario are given by the following two formulae:

1. If the replenished amount is greater than the realized demand then

$$Cost_{it} = c\,(x_{it} - Y_{it})$$

2. If the replenished amount is lower than the realized demand then

$$Cost_{it} = h + g\,(Y_{it} - x_{it})$$

Note that Y_{it} is the realized demand in the above formulae. The resulting costs for each scenario are compared while making a decision on the selected policy to implement. For this, the sensitivity analysis on the customer dissatisfaction rate (g) plays an important role.

4 Empirical study: ATM data

The proposed approach is illustrated using the data acquired from a bank in Turkey. The data include the daily cash withdrawals of 98 ATMs in Istanbul (thus in our notation i=1, 2, …, 98). The demand varies significantly between ATMs and across time. One difficulty is that the observation time period varies across ATMs, with a maximum period length of 2.5 years. The aim in this empirical study is to forecast the demand and optimize the replenishment amounts for the next 4 weeks because the bank generally decides the replenishment plan at the beginning of each month.

4.1 Forecasting

Early in this paper, we highlighted the importance of borrowing information across multiple time series that have similar characteristics. This motivates the clustering technique implemented in this case study. We begin our discussion by exploring this aspect.

Figure 1 presents the cash withdrawals for ATMs in different locations. ATM1 and ATM2 are located in the same county, and they were launched on consecutive months: 10/9/2009 and 11/11/2009. ATM3 is located in a different county and it was launched on 4/7/2011. The patterns of the cash withdrawals of the ATMs in the same county are similar while the pattern for the ATM in the other county (not in close vicinity) is dissimilar. Moreover, the amounts of withdrawals are also different across the two counties since the values for ATM3 are quite smaller compared to the other two ATMs. The highest value for ATM3 is approximately \$15,000 while it is more than \$35,000 for ATM1 and ATM2. Moreover, the variability in the cash withdrawals for ATM3 is smaller than for the other two ATMs.

This insight is the motivation for using location-related clustering to borrow information across ATMs with similar characteristics. We inform the clustering of

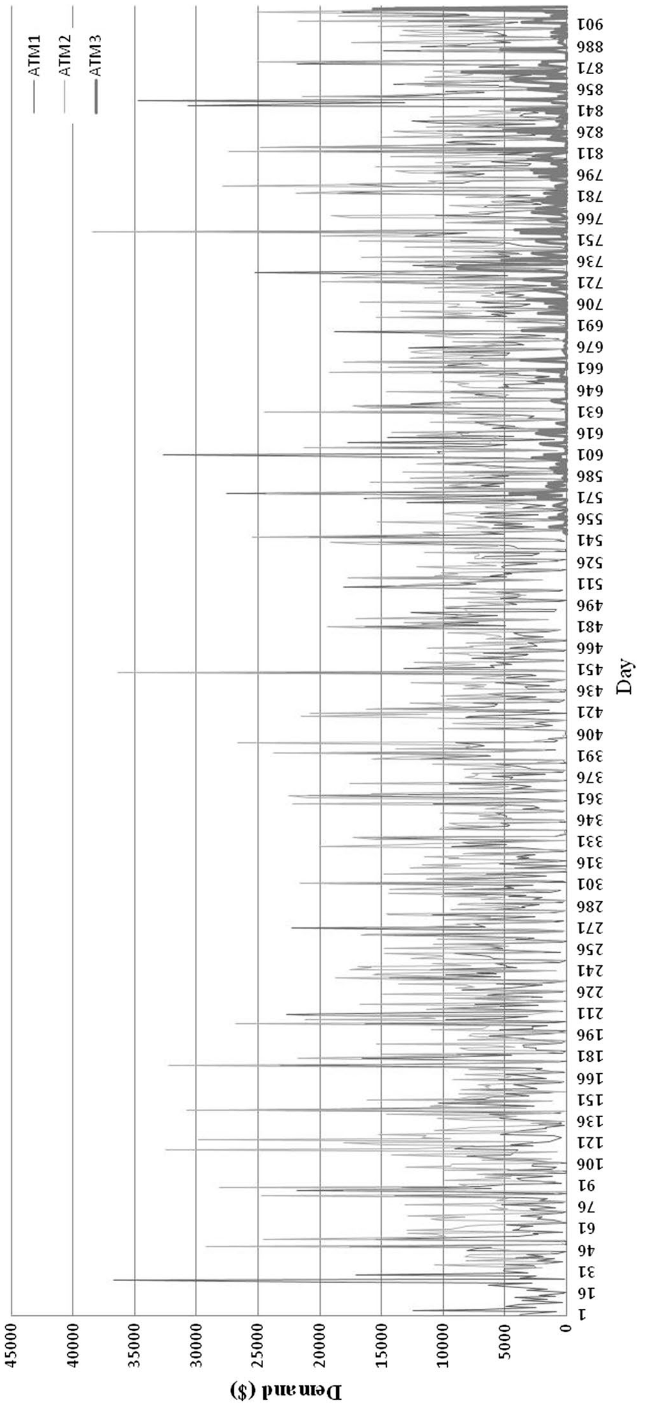

Fig. 1 Examples of the time series plot of three ATMs

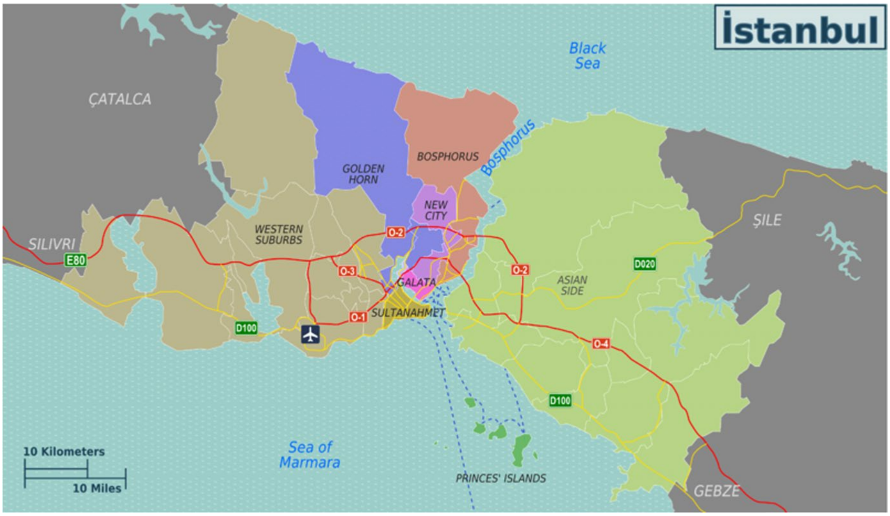

Fig. 2 Istanbul district map (Wikipedia 2013)

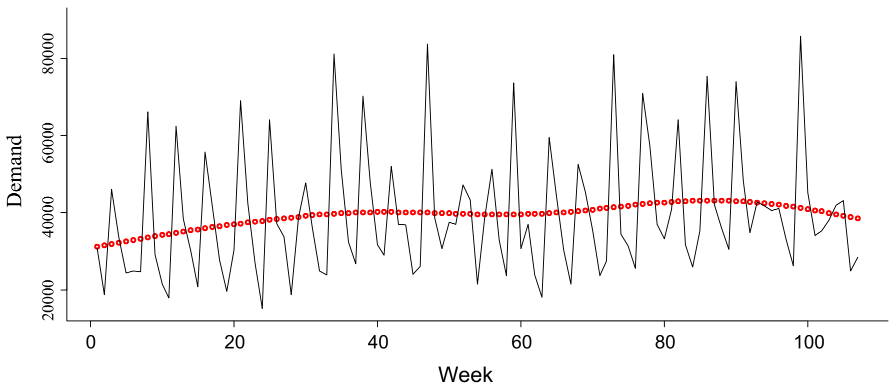

Fig. 3 Time series plot of an ATM along with estimated trend in pointed line

the ATMs using the district map of Istanbul (see Fig. 2). The first cluster consists of New City and Bosphorus, the second cluster consists of Galata, Golden Horn and Sultanahmet, and the third cluster includes Western Suburbs. There are 31 ATMs in the first cluster, 5 ATMs in the second cluster and 62 ATMs in the third cluster. The first cluster includes main business district, it is home to many modern shopping malls, and has a higher-income population. In the region corresponding to the second cluster, there are many nightlife venues, touristic and historical places including the Old City. The third cluster is the western part of the European side of Istanbul,

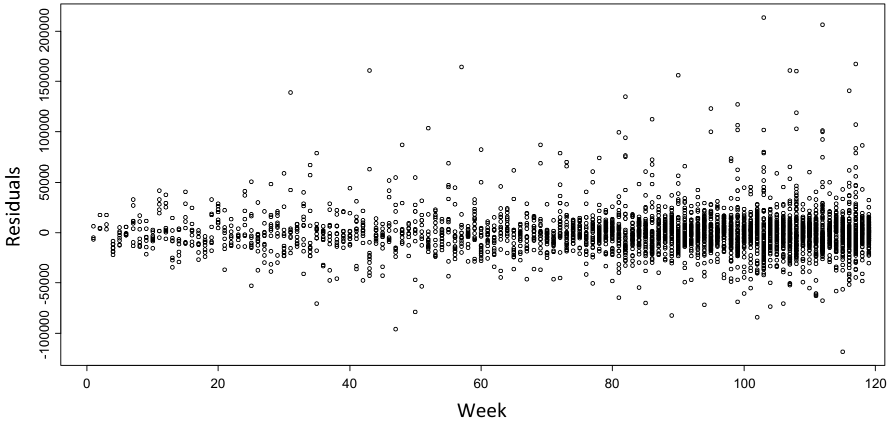

Fig. 4 Residual plot of the linear regression model applied to de-trended data

and it is mostly a residential area. The cluster membership will be used as one of the independent variables in the prediction model.

Figure 3 shows the cash withdrawal amounts for one ATM. With this example, we illustrate the presence of a time-varying trend that could be explained by changes in the withdrawal behavior over time rather than local variations or cyclical trends. Thus the first step is to estimate the trend corresponding to each ATM time series and apply the regression analysis on the de-trended data.

In order to model the trend in the data, non-parametric regression (Wasserman 2006) is used *and* $\hat{\mu}_{iw}=\hat{r}_i(w)$ is derived as a function of time/week for each *i*-th ATM. We apply thin plate splines regression to estimate the time-varying trend using the *gam* library for non-parametric regression in the R statistical software. We extrapolate the estimated function to predict the trend beyond the observation domain and obtain the mean prediction of the trend, $\hat{\mu}_{\mathrm{it}}=\hat{\mathrm{r}}_{\mathrm{i}}(\mathrm{t})$ where t is a future time/week (in our study, $t=W+1, \ldots, W+4$ weeks).

In the second step, we apply linear regression and mixed effects model with the predictors described in "Appendix 1" to the de-trended data $Z_{iw}=Y_{iw}-\hat{\mu}_{iw}$ jointly for all ATMs using the *lm or lme* function in the R statistical software. The regression model includes both time and location related variables. These variables are listed in "Appendix 1"; the dependent variable is the weekly demand of the ATMs. The resulting predicted values based on this model are denoted by $\hat{Z}_{it}$. Using the two sets of predictions, we obtain a point forecast at a future time t, $\hat{Y}_{it}=\hat{Z}_{it}+\hat{\mu}_{it}$ where t is a future time/week (in our study, $t=W+1, \ldots, W+4$ weeks).

We compare the goodness-of-fit for the linear regression model and the mixed effects model (which adds a random effect for capturing within-location variability) and we found that the additional random effect does not result in an improved model. The R^2 of the linear regression model is sufficiently large, 0.76, with a reduction in the R^2 for the mixed effects model.

The residual plot of this model is given in Fig. 4. An important observation from this plot is that although there is no trend, the variance of the residuals is non-constant, that is, $V[Y_{iw}|X_{iw}] = \sigma_i$. While the presence of heteroskedasticity is not important in deriving accurate point estimates, making inference on these estimates relies on the estimation of the error variance and thus needs to be correctly accounted for. Disregarding the presence of heteroskedasticity will result in inaccurate prediction intervals, thus providing too wide ranges of possible values of future cash withdrawals to be optimized over in planning cash replenishment.

Because the variability in the residuals as provided in Fig. 4 varies little within small intervals of time, we assume constant variance within a period of consecutive weeks. We thus evaluate the reliability and accuracy of the prediction intervals with the variance estimated based on varying numbers of last weeks of observations starting with 4 up to 18 weeks; we thus obtain 15 different prediction intervals for each time lag and for each ATM. We also compare multiple robust variance estimators including IQR (inter-quartile-range) and 20% trimmed-sample variance. The reliability of the prediction intervals is computed by counting the number of intervals that contain the observed value, also called coverage in statistical terms, and the accuracy is measured by the length of the intervals. Thus, we seek to find intervals which are not too wide but have high coverage.

Across all combinations of robust variance estimators and time-periods considered, we find that the coverage of the intervals generated by estimating the 20% trimmed-sample variance using the last 8 weeks of cash withdrawal data from all ATMs jointly is the highest. The coverage is approximately equal to 0.97 (comparative to the significance level of 0.95) and the total width of the intervals is 82,492.5, reasonably small in comparison to the total widths of other intervals. The prediction intervals calculated using this approach are reported in "Appendix 2".

We also investigate the model without de-trending the data (first step), applying the regression model only. The best R^2 found from different combination of selected variables is 0.32, significantly smaller than for the model based on de-trended data. The resulting prediction intervals with a total length equal to 91,735.36 for a coverage ~ 0.97. Therefore, we conclude that de-trending the data prior to the application of the regression model leads to significantly more accurate prediction intervals.

4.2 Optimization

In this section, we implement the replenishment policy across a time span of 4 weeks providing the optimal amounts that should be loaded to each ATM for each week. The parameters entering the optimization model for the replenishment policy are informed by the bank that provided the data. Specifically, the penalty cost (h) is set to \$10 while the parameter c (interest cost rate for 1 week) is set to 0.001.

Because the dissatisfaction rate (g) can be subjective, we use the range of 0.5–1% (i.e., 0.5%, 0.6%, 0.7%, 0.8%, 0.9%, 1%) as suggested by the bank and perform sensitivity analysis for multiple parameter values in this range. A 0.5–1% dissatisfaction rate is reasonable because a customer will incur a cost of a base $2 plus one percent of the withdrawn amount (limited to a maximum of $6) if uses the ATM of another bank. Equation 4 is used to solve the problem for each ATM *i* and future time *t* using different dissatisfaction rates.

Since the decision about the replenishment policy is made at the beginning of the month, the actual future demand is not known. To assess the performance of our model, we use the data without the last 4 weeks of cash withdrawal data, and apply the statistical modeling for prediction and the optimization model for deriving optimal replenishment amounts to obtain the realized costs for each ATM and for all 4 weeks.

Table 1 presents the realized costs after replenishing the optimal cash amounts (scenario 1-the proposed approach). We randomly select only 10 ATMs out of the total of 98 to show these results. The cells in boldface correspond to cases when the replenished amount is less than the actual demand. When the replenished amount is less than the actual demand, the cost is calculated by $Cost_{it} = h + g\ (Y_{it} - X_{it})$, whereas when the replenished amount is more than the actual demand, the cost is calculated by $Cost_{it} = c\ (X_{it} - Y_{it})$. We note that the cost values indicated in bold are high when compared to the other cells. Moreover, when the dissatisfaction cost rate increases, they increase also.

Table 2 presents the realized costs after replenishing the upper limits of the prediction intervals (scenario 2-classical approach). Under this setting, there are only two cells for which the replenishment amount does not meet the actual demand: Week1-ATM9 and Week3-ATM8 (indicated in boldface). Therefore, only the costs of these two cells change when the dissatisfaction cost rate changes.

Comparing the realized costs from Tables 1 and 2, we find that the costs under the optimal replenishment policy (scenario 1) are always lower than when using the upper bounds of the prediction intervals for replenishing the ATMs (scenario 2). Table 3 shows the comparison of the total costs in the two scenarios and presents the reduction in the cost when the proposed model (scenario 1) is employed. The interest costs are high under scenario 2 because the replenished amount is significantly greater than the required demand. Although, the costs under having excess demand (bold cells) are higher in Table 2 as compared to Table 1, the total costs are lower under scenario 1. These results show that the resulting replenishing optimal policy from the robust-optimization model performs well.

Table 1 Realized cost after replenishing the optimal values found from the proposed model (scenario 1)

	Cost of optimal replenishment under 0.5% dis-satisfaction cost				Cost of optimal replenishment under 0.6% dis-satisfaction cost				Cost of optimal replenishment under 0.7% dis-satisfaction cost			
	Week 1	Week 2	Week 3	Week 4	Week 1	Week 2	Week 3	Week 4	Week 1	Week 2	Week 3	Week 4
ATM 1	49	35	28	23	51	36	29	24	52	37	30	24
ATM 2	50	31	27	30	51	32	28	31	52	32	29	31
ATM 3	0	29	38	28	2	30	39	29	3	31	40	30
ATM 4	23	27	19	11	25	29	21	12	26	30	22	14
ATM 5	23	27	39	36	25	29	41	38	26	30	42	39
ATM 6	**23**	16	40	60	**15**	18	42	62	0	19	43	63
ATM 7	60	78	84	92	62	80	86	94	63	82	87	95
ATM 8	53	57	**165**	7	54	59	**186**	9	56	60	**206**	10
ATM 9	**665**	55	60	90	**785**	57	61	92	**906**	58	63	93
ATM 10	45	46	28	2	47	48	30	4	48	49	31	5
Total cost	2301				2491				2659			
	Cost of optimal replenishment under 0.8% dis-satisfaction cost				Cost of optimal replenishment under 0.9% dis-satisfaction cost				Cost of optimal replenishment under 1% dis-satisfaction cost			
	Week 1	Week 2	Week 3	Week 4	Week 1	Week 2	Week 3	Week 4	Week 1	Week 2	Week 3	Week 4
ATM 1	53	37	30	25	53	38	31	26	54	38	31	26
ATM 2	53	33	29	32	53	33	29	32	54	34	30	33
ATM 3	4	32	40	31	5	33	41	32	6	33	41	32
ATM 4	28	31	23	15	28	32	24	15	29	33	25	16
ATM 5	27	31	43	40	28	32	44	41	29	32	44	41
ATM 6	1	20	44	64	2	21	45	65	3	22	46	66
ATM 7	64	83	88	96	65	83	89	97	66	84	90	98

Table 1 (continued)

	Cost of optimal replenishment under 0.8% dis-satisfaction cost				Cost of optimal replenishment under 0.9% dis-satisfaction cost				Cost of optimal replenishment under 1% dis-satisfaction cost			
	Week 1	Week 2	Week 3	Week 4	Week 1	Week 2	Week 3	Week 4	Week 1	Week 2	Week 3	Week 4
ATM 8	57	61	**226**	11	57	62	**246**	12	58	62	**265**	13
ATM 9	**1026**	59	64	94	**1145**	60	64	95	**1265**	61	65	95
ATM 10	49	50	32	6	50	51	33	7	50	51	34	7
Total cost	2833				3000				3161			

Table 2 Realized cost after replenishing using the upper bounds of the prediction intervals (scenario 2)

	Cost of replenishing upper bound under 0.5% dissatisfaction cost				Cost of replenishing upper bound under 0.6% dissatisfaction cost				Cost of replenishing upper bound under 0.7% dissatisfaction cost			
	Week 1	Week 2	Week 3	Week 4	Week 1	Week 2	Week 3	Week 4	Week 1	Week 2	Week 3	Week 4
ATM 1	59	42	34	30	59	42	34	30	59	42	34	30
ATM 2	59	37	32	36	59	37	32	36	59	37	32	36
ATM 3	12	38	46	37	12	38	46	37	12	38	46	37
ATM 4	36	39	31	23	36	39	31	23	36	39	31	23
ATM 5	36	39	50	48	36	39	50	48	36	39	50	48
ATM 6	10	28	52	72	10	28	52	72	10	28	52	72
ATM 7	72	91	96	104	72	91	96	104	72	91	96	104
ATM 8	65	69	**105**	19	65	69	**124**	19	65	69	**143**	19
ATM 9	**604**	67	72	102	**723**	67	72	102	**842**	67	72	102
ATM 10	57	58	40	14	57	58	40	14	57	58	40	14
Total cost	2562				2700				2838			
	Cost of replenishing upper bound under 0.8% dissatisfaction cost				Cost of replenishing upper bound under 0.9% dissatisfaction cost				Cost of replenishing upper bound under 1% dissatisfaction cost			
	Week 1	Week 2	Week 3	Week 4	Week 1	Week 2	Week 3	Week 4	Week 1	Week 2	Week 3	Week 4
ATM 1	59	42	34	30	59	42	34	30	59	42	34	30
ATM 2	59	37	32	36	59	37	32	36	59	37	32	36
ATM 3	12	38	46	37	12	38	46	37	12	38	46	37
ATM 4	36	39	31	23	36	39	31	23	36	39	31	23
ATM 5	36	39	50	48	36	39	50	48	36	39	50	48
ATM 6	10	28	52	72	10	28	52	72	10	28	52	72
ATM 7	72	91	96	104	72	91	96	104	72	91	96	104

Table 2 (continued)

	Cost of replenishing upper bound under 0.8% dissatisfaction cost				Cost of replenishing upper bound under 0.9% dissatisfaction cost				Cost of replenishing upper bound under 1% dissatisfaction cost			
	Week 1	Week 2	Week 3	Week 4	Week 1	Week 2	Week 3	Week 4	Week 1	Week 2	Week 3	Week 4
ATM 8	65	69	**162**	19	65	69	**180**	19	65	69	**199**	19
ATM 9	**961**	67	72	102	**1080**	67	72	102	**1199**	67	72	102
ATM 10	57	58	40	14	57	58	40	14	57	58	40	14
Total cost	2976				3114				3252			

Table 3 Comparison of Scenario 1 and Scenario 2

	0.5% Dissatisfaction cost	0.6% Dissatisfaction cost	0.7% Dissatisfaction cost	0.8% Dissatisfaction cost	0.9% Dissatisfaction cost	1% Dissatisfaction cost
Total cost—scenario 1	2301	2491	2659	2833	3000	3161
Total cost—scenario 2	2562	2700	2838	2976	3114	3252
Total cost improvement with the proposed model (i.e., scenario 1) (%)	10.19%	7.74%	6.31%	4.81%	3.66%	2.80%

5 Discussion

In this study we provide an approach for deriving optimal replenishment strategies in ATM cash management. Within this problem, the decisions compound to ‘when to visit each ATM’ and ‘how much to replenish at each visit’. Since most ATMs are off-site (ATMs that are not located in a branch of the bank), our partner bank suggested that a weekly replenishment policy is most practical. Thus, the problem reduces to decisions on the replenishment amounts, taking into account two possible cases: (i) when the replenishment amount will be greater than the realized demand, the bank will face cash holding cost of the excess amount, (ii) when the realized demand will be greater than the replenishment amount, the bank will face a penalty cost (fixed cost) plus the cost of dissatisfied customers whose demand cannot be met due to the lack of cash in the ATM. Banks typically do not apply optimization models in place to control the costs under these two cases. In current practice, most banks use point predictions, replenishing a forecast of the future cash demand plus an arbitrary-selected additional safety amount to avoid stock-outs.

The approach in this paper decides on the optimal replenishment amounts for multiple ATMs of the same bank using robust optimization, accounting for uncertainties in the predicted future demands using prediction intervals. The robust optimization problem is solved as to minimize the maximum regret (total cost).

We evaluate the performance of the optimal replenishment approach, using retrospective ATM cash withdrawal data. The approach in this paper provides the optimal replenishment amounts that result in lower costs than those incurred if the current practice approach would have been implemented. The benchmarking is tested on different levels of customer dissatisfactions levels and across all cases it outperforms current practice.

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Appendix 1: Independent variables considered in the regression model

See the Table 4.

Table 4 Independent variables considered in the regression model

Code	Type (categorical/numerical)	Explanation of the variable
A	Categorical	Variable which denotes the location cluster
B	Categorical	Variable specifying whether the week is before a religious festival
C	Categorical	Variable specifying whether the week is before a holiday
D	Categorical	Variable specifying whether the week is before a special day, such as Valentine's day, mothers' day, fathers' day etc.
E	Categorical	Variable specifying whether the week includes a special day, such as Valentine's day, mothers' day, fathers' day etc.
F	Categorical	Variable specifying whether the week includes a religious festival or national festival
G	Categorical	Variable specifying whether the week includes the first day of the month (This is important since this day is the salary day in Turkey)
H	Categorical	Variable specifying whether the week includes the fifteenth day of the month (This is important since this day is the salary day in Turkey)
I	Numerical	Variable which denotes the week number (there are 52 weeks in a year and this variable is important to capture the seasonality)
J	Numerical	Variable which denotes the number of points of interests (school, restaurant, shopping center, airport, plaza etc.) in 500 m vicinity of the ATM
K	Numerical	Variable which denotes the number of points of interests (school, restaurant, shopping center, plaza etc.) in 1000 m vicinity of the ATM
L	Numerical	Variable which denotes the number of other ATMs in the street
M	Numerical	Variable which denotes the number of social buildings in the street (such as restaurant, shop, cafeteria etc.)
N	Numerical	Variable which denotes the number of work related buildings in the street (such as plaza, governmental building, school, etc.)
O	Numerical	Variable which denotes the number of commercial buildings in the street (such as airport, port, bus terminal)
P	Numerical	Variable which denotes the total number of points of interests in the street
Q	Numerical	Variable which denotes the population in the street
R	Numerical	Variable which denotes the number of people who are university graduate in the street

Table 4 (continued)

Code	Type (categorical/numerical)	Explanation of the variable
S	Numerical	Variable which denotes the number of people who are at the age of between 15 and 19
T	Numerical	Variable which denotes the number of people who are at the age of between 20 and 44
U	Numerical	Variable which denotes the number of people who are at the age of between 45 and 54
V	Numerical	Variable which denotes the number of people who are older than 55

Appendix 2: Prediction plots

See the Figs. 5, 6, 7 and 8.

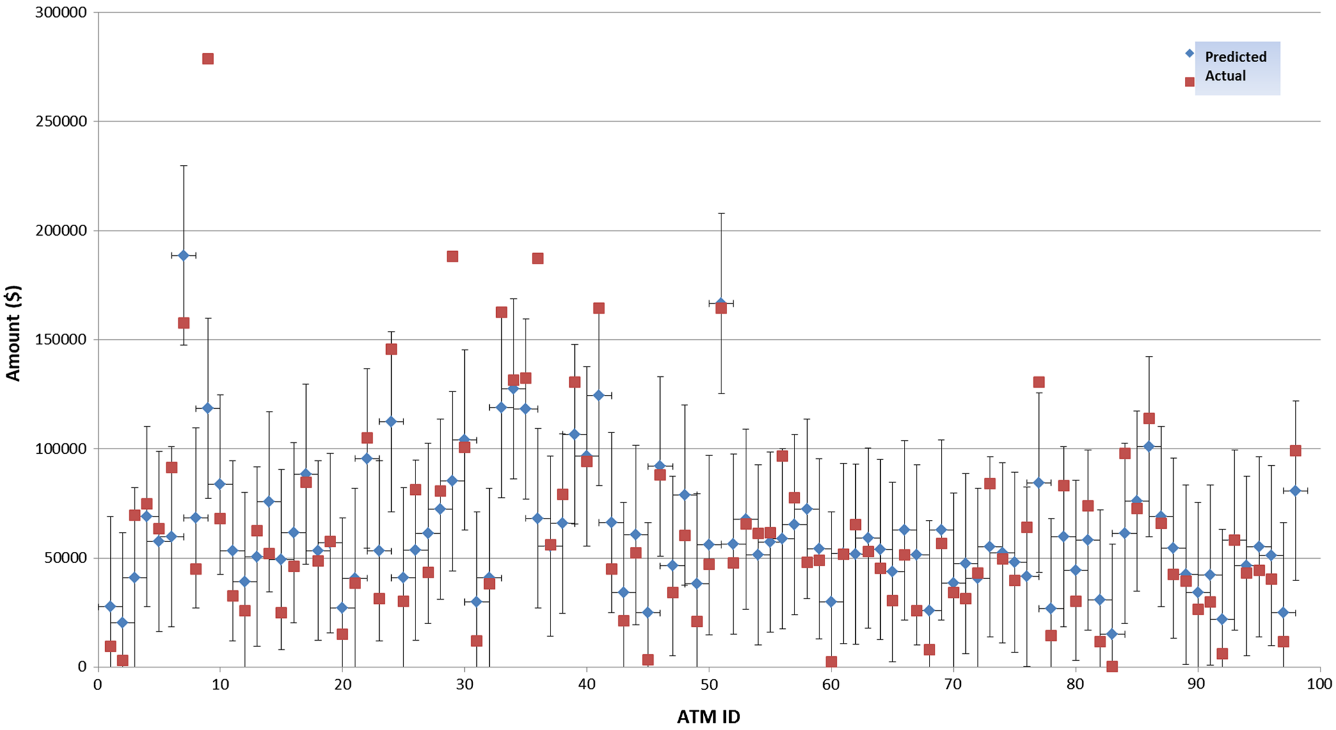

Fig. 5 Prediction intervals of the first week

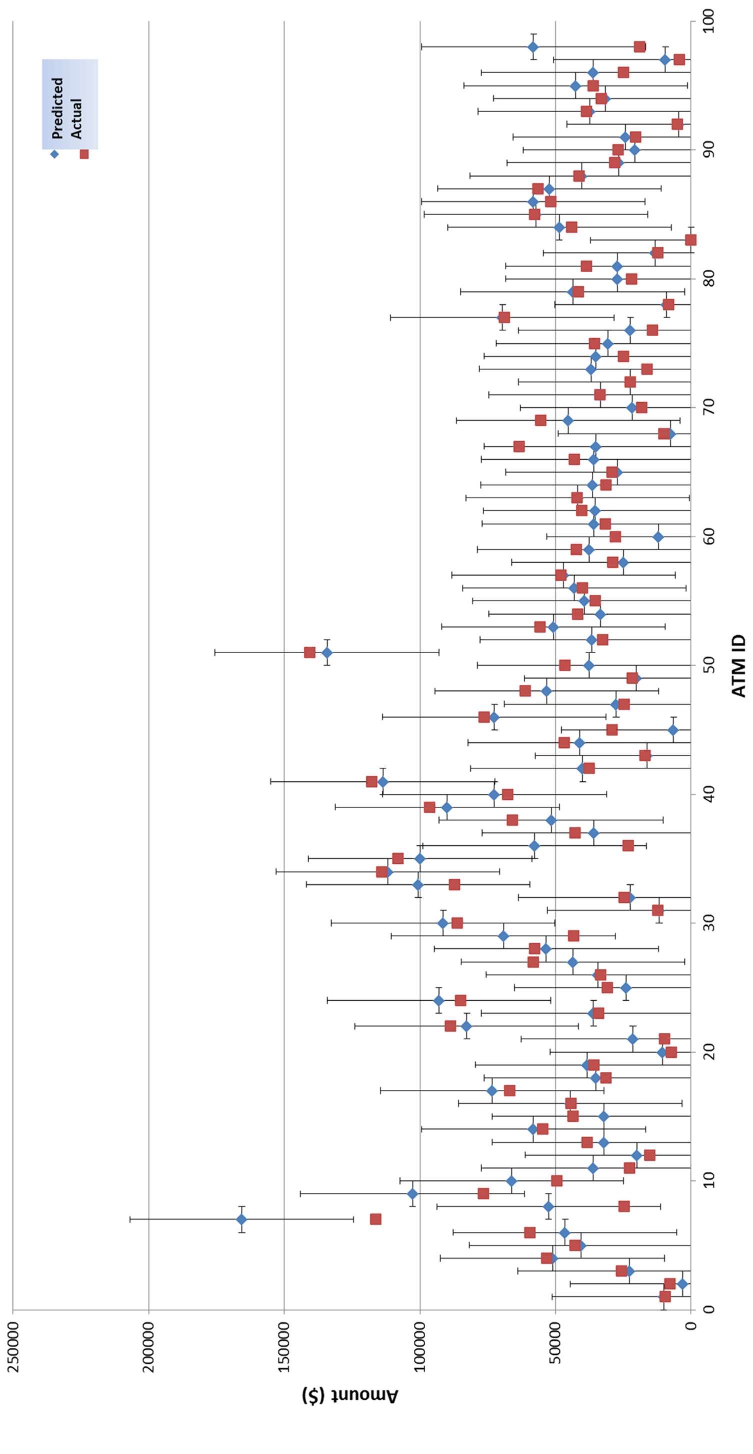

Fig. 6 Prediction intervals of the second week

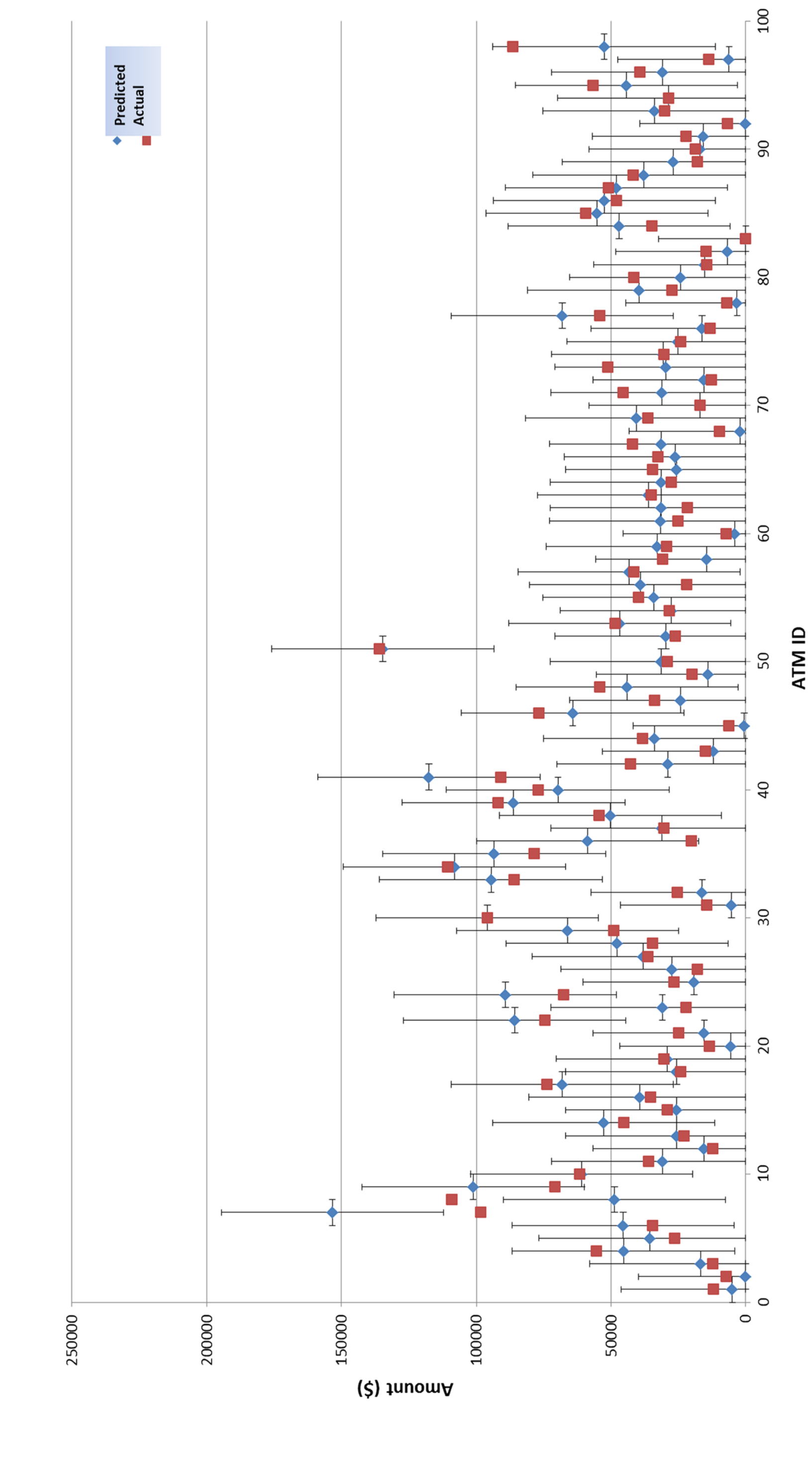

Fig. 7 Prediction intervals of the third week

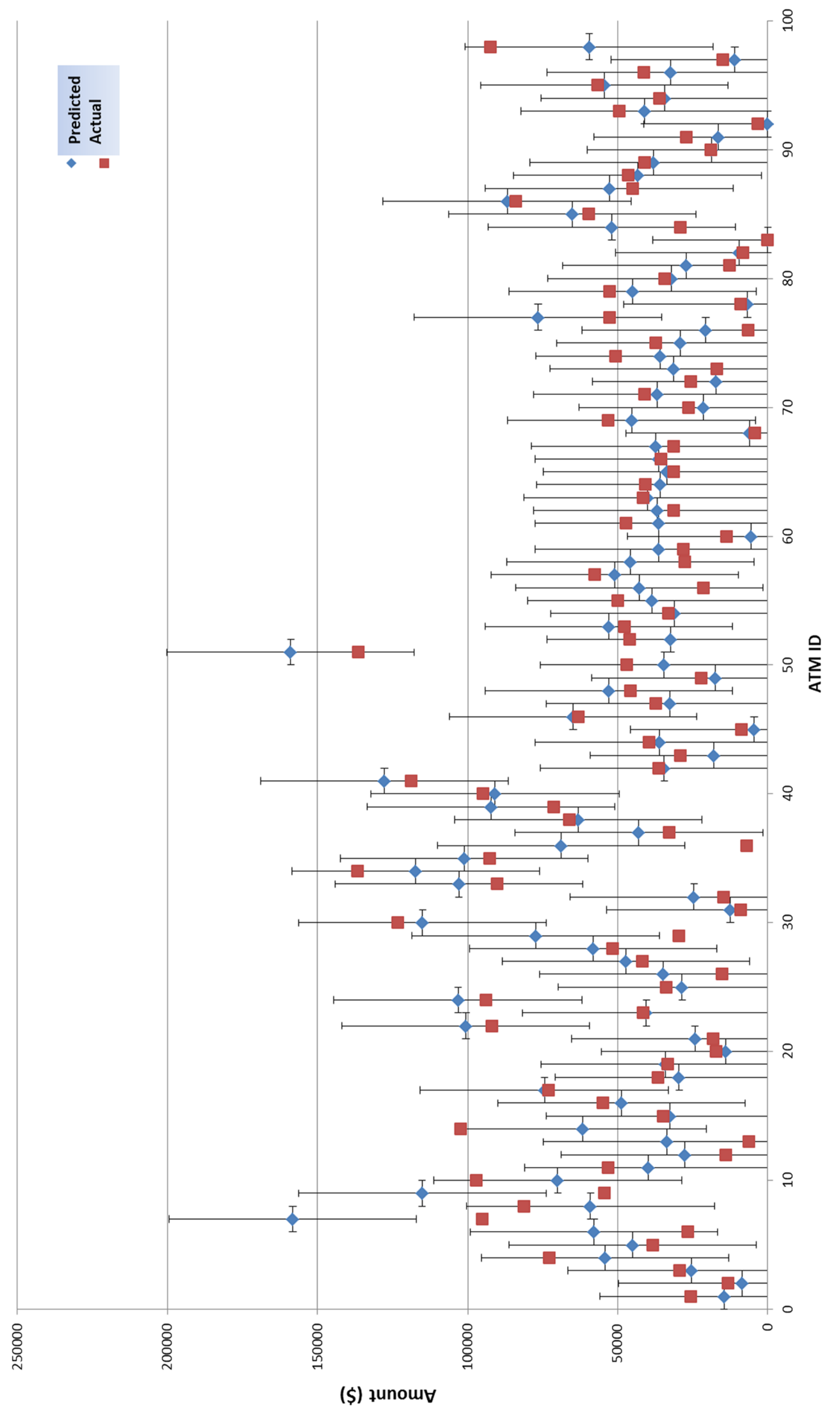

Fig. 8 Prediction intervals of the fourth week

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