Cash Flow Optimization in ATM Network Model

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Abstract. The main purpose of this work is to optimize cash flow in case of the encashment process in the ATM network with prediction of ATM refusal. The solution of these problems is based on some modified algorithms for the Vehicle Routing Problem and use statistical methods to compile the requests from the ATM network. A numerical example is considered.

Keywords: ATM network, route optimization, Vehicle Routing Problem, cash flow, statistical methods.

1. Introduction

Nowadays ATM network and credit cards are the essential parts of modern lifestyle, consequently one of the most actual problem in the bank's ATM network is optimization of cash flow and organization of uninterrupted work. For the bank it is important to prevent the rush demand for cash withdrawals by customers, that can be provoked by the delayed loading of ATMs, and reduce the service expenses. Serving the ATMs network is a costly task: it takes employees' time to supervise the network and make decisions about cash management and it involves high operating costs (financial, transport, etc.). At the present time more and more banks are turning their attention to have greater efficiency in how they manage their cash in ATMs.

Through cash management optimization banks can avoid falling into the trap of maintaining too much cash and begin to profit by mobilizing idle cash. Effective cash management and control starts with accurate prediction of ATMs refusal, allowing banks to forecast cash demand for the network, and find an optimal routes, which manage to reduce servicing costs. The increase of transportation and servicing cost can be substantial for banks. Route optimization for the collector teams is allow to reduce bank expenses and to control the encashment process (Simutis et al., 2007).

In this work we consider a problem in which a set of geographically dispersed ATMs with known requirements must be served with a fleet of money collector teams stationed in the depot in such a way as to minimize some distribution objective. This problem is combined with the problem of composition of service requests from the ATM network. We assume that the money collector teams are identical with the equal capacity and must start and finish their routes at the depot. To estimate an average cash amount in each ATM and form the requests for the money collector teams we use following factors:

– ATM location and open hours;
– Irregular number of operations per week;
– Increasing of operation number in holidays and salary days.

Moreover we define the necessity of servicing each ATM and predict the future requests for the collector teams based on the statistical data and restrictions, which are proposed above. The optimal routes to load ATMs depend on the current requests and predictable requests.

The main purpose of this work is to optimize cash flow in case of the encashment process in the ATM network with prediction of ATM refusal. To solve the problem we base on some modified algorithms for the Vehicle Routing Problem and use statistical methods to compile the requests from the ATM network. In the paper a numerical simulation for ATM network of one bank of St. Petersburg is considered.

2. Route optimization for collector teams

In this section we consider a problem in which a set of geographically dispersed ATMs must be served with a fleet of money collector teams stationed in the bank. This problem will be solved based on the Capacitated Vehicle Routing Problem (CVRP) see (Hall, 2003), which is the problem of designing feasible routes for a set of homogeneous vehicles that make up a vehicle fleet. The routes are formed minimizing total travel costs of all of the routes. Each route begins and ends at the depot and contains a subset of the stops requiring service. A solution to this problem is feasible if the vehicle capacity on each route is not exceeded and all stops are assigned to a route. In the simplest statement of the CVRP there are no lower and upper bounds on the duration of each route. As such, there are no route balance considerations.

To describe the basic CVRP we suppose, that a complete undirected network (or graph) $G = (V, E)$ is given where $V = (0, 1, \ldots, n)$ is the set of vertices and E is the set of undirected arcs (edges). A non-negative cost c_{ij} is associated with every edge. $V' = V \setminus \{0\}$ is a set of n vertices, each vertex corresponds to a stop, vertex 0 corresponds to the depot. Henceforth, i will be used interchangeably to refer both to a stop and to its vertex location. Each stop i requires a supply of q_i units from depot 0. A set of M identical vehicles of capacity Q is located at the depot and is used to service the stops; these M vehicles comprise the homogeneous vehicle fleet. It is required that every vehicle route starts and ends at the depot and that the load carried by each vehicle is no greater than Q.

The route cost corresponds to the distance travelled on the route and is computed as the sum of the costs of the edges forming the route.

An optimal solution for the CVRP is a set of M feasible routes, one for each vehicle, in which all stops are visited, the capacity of each vehicle is not exceeded and the sum of the route costs is minimized.

In the case of street routing the stops are located on a street network and the travel time between stops is computed as the shortest travel time path between stops. If the travel time matrix is symmetric, then we have the symmetric or basic CVRP. In the symmetric CVRP it can be assumed that there are no one-way streets and turn and street crossing difficulties are not considered in setting up the travel time matrix. In this paper we consider one procedure for solving the symmetrical CVRP, using statistical methods to reduce model dimension.

2.1. Integer programming formulation of the CVRP

Consider the presentation of the CVRP, where $V = (0, 1, \ldots, n)$ is the complete set of vertices, subset $V' = V \setminus \{0\}$ is a set of n vertices without depot, each vertex corresponds to a stop, vertex 0 corresponds to the depot. Let x_{ij} be an integer variable representing the number of vehicles traversing the undirected arc (edge) $\{i, j\}$, and let $r(S)$ be the number of vehicles needed to satisfy the demand of ATMs in S, c_{ij} are costs.

Here we present the formalization of the basic CVRP problem:

$$\min \sum_{i<j} c_{ij} x_{ij}, \tag{1}$$

$$\sum_{i<j} x_{ij} + \sum_{i>j} x_{ji} = 2, \quad i \in V', \tag{2}$$

$$\sum_{i \in S} \sum_{j \in V \setminus S} x_{ij} \geq 2r(s), \quad S \subseteq V', \quad |S| > 1, \tag{3}$$

$$\sum_{j \in V'} x_{0j} = 2M, \tag{4}$$

$$x_{ij} \in \{0, 1\}, \quad i \in V', \quad j \in V', \quad i < j, \tag{5}$$

$$x_{0j} \in \{0, 1, 2\}, \quad j \in V'. \tag{6}$$

Constraints (2) are the degree constraints for each ATM. Constraints (3) are the capacity constraints which, for any subset S of ATMs, that does not include the depot, impose that $r(S)$ vehicles enter and leave S, where $r(S)$ is the minimum number of vehicles of capacity Q required for servicing the ATMs in S. Constraints (3) are also called generalized subtour elimination constraints. It is NP-hard to compute $r(S)$, since it corresponds to solve a bin-packing problem where $r(S)$ is the minimum number of bins of capacity Q that are needed for packing the quantities. However, inequalities (3) remain valid if $r(S)$ is replaced by a lower bound to its value, such as $[\sum_{i \in S} q_i / Q]$ where $[y]$ denotes the smallest integer not less than y and q_i is required amount of cash for each ATM. Constraint (4) states that M vehicles must leave and return to the depot while constraints (5) and (6) are the integrality constraints. Finally, $x_{0j} = 2$ corresponds to a route containing only stop j.

This formulation cannot be solved directly by a general purpose integer programming algorithm because constraints (3) are too numerous to be enumerated a priori. However if the dimension of model will be reduced, this formulation will be solved.

At the present time various approaches to the solution of the CVRP have been proposed in different papers. In our work we focused on several papers and handbooks. The main sources is (Hall, 2003), which presents several formulations of Vehicle Routing Problem and it's modification, as well as some ideas and methods of solution are described. Nowadays Vehicle Routing Problem (VRP) is very actual and many authors concentrate on application of different methods and their extensions to solve VRP. This paper is based on some previous works, for instance, (Ralphs et al., 2003), in their work, suggest a decomposition-based separation methodology for the capacity constraints. Meanwhile the paper (Lysgaard et al., 2003) describes a branch-and-cut algorithm based on application of variety of cutting planes.

In current work we offer to reduce dimension of the model and simplify constraints (3) using statistical analysis on the ATM network.

3. Forecasting of Cash Flows in ATM Network

In current section we consider a problem of cash flow forecasting in the ATM network, especially focused on forecasting of cash balance in ATMs and the moment of ATM upload. Moreover we find the moment of each ATM refusal and compile the requests for the bank's processing center (Vasin, 2005). We analyzed cash flows of each ATM, using statistical data, and found that cash withdrawal is heterogeneous process which depends on following factors:

– Paydays, weekends, holidays, etc.;
– ATM location;
– ATM open hours.

Consider the set of geographically dispersed ATMs and a bank, defined in Section 2. Here $V' = V \setminus \{0\}$ is a subset of n ATMs, without bank, as in previous section vertex 0 corresponds to the bank. According to the factors mentioned above the whole set of ATMs V' is divided into m subsets $S_k \subseteq V'$, $S_k \cap S_l = \emptyset$, $k \neq l$, $k, l \in \{1, \ldots, m\}$. Lets define y_{it} as the amount of cash withdrawal from ATM i at day t.

Consider each subset S_k consisting of n_k ATMs separately. Using statistical cash withdrawal data for each ATM over N days, we describe an average amount of cash withdrawal per day for subset S_k by following expression:

$$\bar{y}_{kt} = \frac{1}{n_k} \sum_{i \in S_k} y_{it}, \quad t = 1, \ldots, N, \quad k = 1, \ldots, m. \tag{7}$$

Thus we obtain cash withdrawal time series for each subset S_k: $(\bar{y}_{k1}, \bar{y}_{k2}, \ldots, \bar{y}_{kN})$. Estimate trend and periodic component of the time series, for that purpose choose linear trend and find equation of linear regression (Bure, Evseev, 2004):

$$tr_{kt} = a_k + b_k t, \quad t = 1, \ldots, N, \quad k = 1, \ldots, m, \tag{8}$$

where the evaluations of unknown parameters a_k, b_k are calculated with the least-squares procedure.

The periodic component is estimated by the the following formula, removing trend from the time series:

$$P_{kt} = \frac{1}{z_k} \sum_{j=0}^{z_k - 1} (\bar{y}_{k,t+j\cdot T_k} - tr_{k,t+j\cdot T_k}), \quad t = 1, \ldots, T_k, \quad k = 1, \ldots, m, \tag{9}$$

where N – number of observations in subset S_k, T_k – period length, $z_k = \frac{N}{T_k}$ – integer number of periods.

Thereby we define cash withdrawal time series for each subset S_k:

$$\bar{y}_{kt} = tr_{kt} + P_{kt}, \quad t = 1, \ldots, N, \quad k = 1, \ldots, m. \tag{10}$$

The cash withdrawal forecast for two weeks is evaluated with the formula (10):

$$Y_{kt} = tr_{kt} + P_{kt}, \quad t = N+1, \ldots, N+14, \quad k = 1, \ldots, m. \tag{11}$$

Proceed the forecasting of cash balance in ATMs network and extend this procedure for all subgroups S_k. Assume B_{it} is the cash balance in ATM i at day t. If we know the cash balance for exact day, we can make the cash balance forecast using cash withdrawal forecast calculated by formula (11), i.e. find B_{it} when $t = N+1, \ldots, N+14$. Now we can detect the moment of each ATM refusal, i.e. when the cash balance approaches zero, and generate requests for the money collector teams.

Our purpose is to form the requests for service senter for each day of the forecasting period. Every request should contain the information about ATMs needed to be serviced and the quantity of cartridges (Diebold). We do not take into account the nominal of banknotes.

The algorithm for cash balance forecast was simulated in the MATLAB system and for each date of the forecasting period we received the requests for ATM service with numbers of ATMs, their addresses and number of cartridges. Requests are transmitted to the encashment department and can be used to reduce CVRP dimension and respectively the number on constraints (3), while constructing optimal encashment routes.

3.1. Numerical simulation

Consider one bank of St. Petersburg with ATM network consisting of 49 items and apply the procedure offered above. Divide subset V' into several subsets S_k which are presented in the Table 1:

Table 1. Subsets of ATM network

Subset S_k	Name (location)	Quantity of ATMs	Open hours
S_1	Subway	21	Open daily
S_2	Shop	8	Open daily
S_3	Shopping Mall	9	Open daily
S_4	Educational Organization	2	Mo–Fr
S_5	Enterprise	2	Mo–Fr
S_6	Bank	7	Mo–Fr

Consider each group separately and analyze cash withdrawal time series using formulas (7)-(9). For example, examine the subset S_1, which is the group of ATMs situated at the subway. We construct time series for the period from 10 may 2010 to 20 june 2010. The diagram of the average cash withdrawal time series for the subset S_1 is presented in Fig.1. It shows that on Friday and Saturday the amount of cash withdrawal increases rapidly.

We construct equation of linear regression with estimated trend and periodic component: $tr_t = 369,5t + 167022$. We have 42 observations, then the length of time period $T_1 = 7$ and hence the integer number of periods is $z_1 = \frac{n}{T_1} = 6$. Make the cash withdrawal forecast for two weeks. The diagram of the time series estimation and the cash withdrawal forecast for the period from 21 june 2010 to 4 july 2010 is presented in Fig.2.

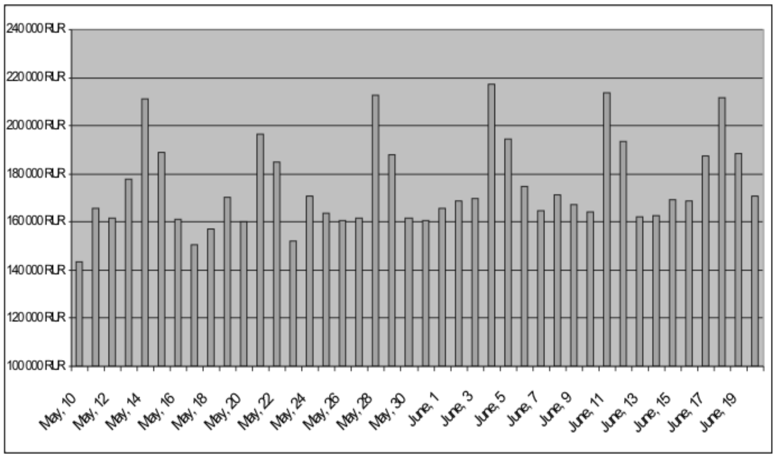

Fig. 1. Cash withdrawal time series for the subset S_1

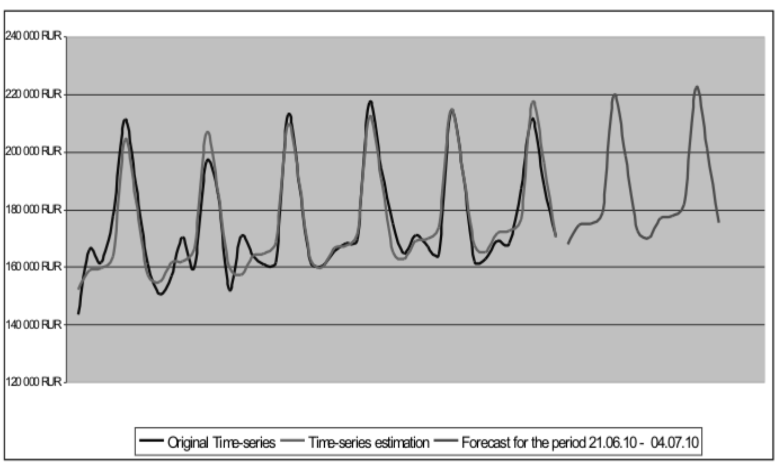

Fig. 2. Cash withdrawal time series, time series estimation and forecast for the subset S_1

Analogous, the cash withdrawal time series were analyzed and the forecast of cash withdrawal were done for each subset of ATMs. Cash withdrawal forecast for all groups is presented in Table 2.

Table 2. Forecast of withdrawals cash for the period 21.06.10 – 04.07.10, RUR

Date	Subway	Shop	Shopping Mall	Educational Organization	Enterprize	Bank
21.06	167900	90800	94400	27000	48200	180000
22.06	174500	88200	92600	25900	88200	169100
23.06	175200	89200	96800	27300	89200	171200
24.06	179200	91000	96000	27000	49500	171500
25.06	219700	118400	122100	46400	90100	208500
26.06	198700	161300	174400	94300	0	0
27.06	172900	133900	151600	0	0	0
28.06	170500	92400	98100	108900	49400	180800
29.06	177100	89900	96300	97400	174100	169900
30.06	177800	90800	100400	135000	255400	171900
01.07	181800	92700	99700	102200	135000	172200
02.07	222300	120100	125700	47600	91300	209200
03.07	201300	163000	178100	53100	0	0
04.07	175500	135500	155300	0	0	0

With the data from the Table 2 and knowing the cash balance in each ATM on 20 june 2010 we compile the cash balance forecast for the period from 21 june 2010 to 4 july 2010. The service requests for the money collector teams were generated in MATLAB system. All requests contain the information about the number of ATMs to service, their location, cash amount, etc. Now we can reduce the CVRP dimension and generate the routes for the money collector teams using the concrete request.

For instance, let we receive request for 3 July 2010 to service 9 ATMs located at the different subway stations of St.Petersburg: 2 – Tekhnologicheskiy Institut, 3 – Moskovskie Vorota, 4 – Lomonosovskaya, 5 – Vasileostrovskaya, 6 – Prospekt Bol'shevikov, 7 – Ploschad' Lenina, 8 – Narvskaja, 9 – Chkalovskaja and 10 – Sennaja Ploschad'. Construct optimal routes for the current request. Firstly we assume that the bank has three collector teams with equal vehicle capacity $Q = 12$ cartridges and each ATM requires $q_i = 3$ cartridges. Distances between ATMs and the Bank are given in the Table 3.

We received the solution for this example in Maple system. Routes, which were constructed are represented in the Figure 3.

The optimal solution in the current model consists of three routes, one for each collector team. The first team drives through ATMs 4-6 (subway stations: Lomonosovskaya, Vasileostrovskaya, Prospekt Bol'shevikov), the second team goes through ATMs 2-3-8-10 (subway stations: Tekhnologicheskiy Institut, Moskovskie Vorota, Narvskaja, Sennaja Ploschad') and the third team goes through ATMs 5-9-7 (subway stations: Vasileostrovskaya, Ploschad' Lenina, Chkalovskaja). Every route begins and ends at the bank, vehicle capacity on each route is not exceeded, all ATMs are assigned to a route and total travel costs are minimized. Thus, we got optimal routes for the current request. The distance travelled on this optimal route

Table 3. Distances between ATMs and bank, m

	Bank	?2	?3	?4	?5	?6	?7	?8	?9	?10
Bank	0	3250	6530	9000	5005	10007	6680	7810	7650	3940
?2	3250	0	2930	10000	4870	13500	5480	3860	6770	1280
?3	6530	2930	0	10120	7940	13070	10610	5410	9180	4050
?4	9000	10000	10120	0	13690	6000	11900	14500	15100	10540
?5	5005	4870	7940	13690	0	15300	5990	5970	2750	4030
?6	10007	13500	13070	6000	15300	0	11100	14560	14600	10480
?7	6680	5480	10610	11900	5990	11100	0	9070	4690	6500
?8	7810	3860	5410	14500	5970	14560	9070	0	8300	4670
?9	7650	6770	9180	15100	2750	14600	4690	8300	0	5010
?10	3940	1280	4050	10540	4030	10480	6500	4670	5010	0

corresponds to 64 332 meters, this is a minimal length of all possible routes for the money collector teams.

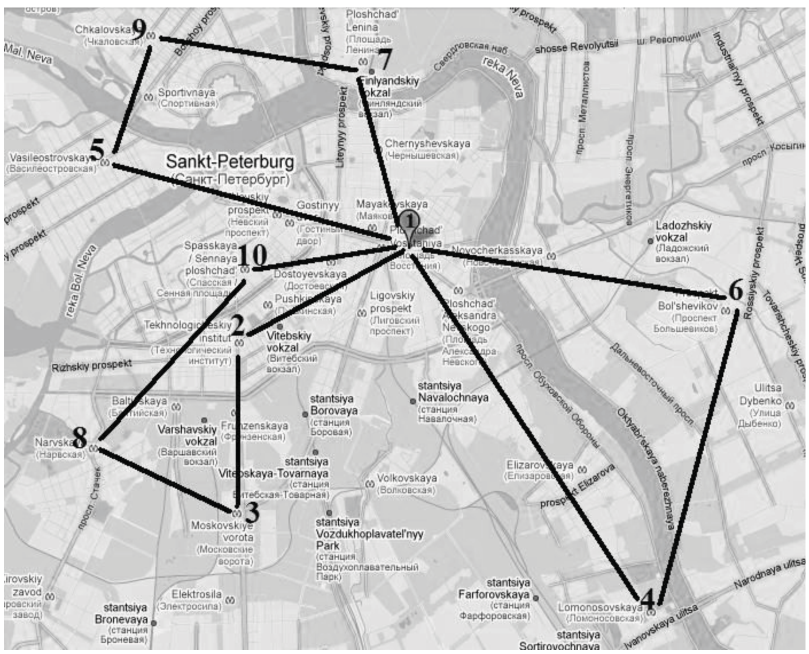

Fig. 3. Routes of the money collector teams.

4. Discussions

This paper's main contribution is in using general results from the theory of CVRP and statistical analysis to simulate the optimal bank strategy for the encashment process and predict the ATM failure in the network.

In discussion section we would like to consider one application of the optimization procedure for the encashment process, in particular case, in two different situations and compare the average costs of servicing the ATM network in these situations. The **situation 1** is: the upload amount of cash is small, this involves the bank to increase the currency of ATM network service so the transportation costs arise and can be substantial. In **situation 2** the uploaded cash is rather big, it means that the bank can incur losses related to the opportunity cost of cash uploading.

Consider average service costs as the average sum of the following costs:

– route costs in cash equivalent;
– uploaded cash;
– opportunity cost of cash uploading.

It should be noticed that route costs are minimized during the CVRP optimization. Also, route costs RC, particularly the distances c_{ij} between ATMs in optimal route $\hat{R}$, can be converted to cash equivalent with the following formula:

$$RC = f \cdot p \cdot \sum_{i,j \in \hat{R}} c_{ij}, \tag{12}$$

where f is the average fuel rate, measured in liter per meter, p is the fuel price. For example, in numerical simulation we got the optimal route corresponds to 64 332 m. In cash equivalent it is 139 RUR under the assumption that the average price of gasoline is 27 RUR for liter and the vehicle's fuel rate comes up to 8 liters per 100 km (in average).

Consider our two situations and assume that the uploaded cash can possess two values: 2 mln or 4,25 mln rubles. In the **situation 1** bank upload 2 mln rubles to the ATM network and in the **situation 2** – 4,25 mln rubles, respectively. The calculation of average total costs of servicing the whole ATM network were simulated in MATLAB subject to different upload values and presented in the Table 4.

Table 4. Average service costs, RUR

Upload value (one ATM)	Route costs	Cash uploading (ATM network)	Opportunity cost of cash uploading	Average total costs
2 mln	7 243 050	174 mln	24 467 090	205 710 140
4,25 mln	4 991 400	225,25 mln	19 757 714	249 999 114

Thus we can conclude that in this particular case, if uploaded amount of cash in the ATM network is small then servicing expenses (Route costs) arise for the bank and equal to 7 243 050 RUR, but negative profit from the idle (nonused) cash amount increases because Average Total Costs are equal to 205 710 140 RUR. In the same time if uploaded amount of cash in the ATM network is large, in our case 4,25 mln RUR, then servicing expenses are going down and Route Costs are 4 991 400 RUR. Hence bank store large amount of cash in his ATM network then it can fail into the trap of maintaining too much idle (unused) cash and so Average Total Costs are equal to 249 999 114 RUR.

So we can see that optimization of cash distribution through ATM network could be a very actual problem. Therefore in the future work we plan to define the optimal cash amount to store in ATM network, which will guarantee optimal servicing, including the reducing expenses of the bank. Also we plan to improve our simulation procedures.

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