This document defines the formulas required for a Kalman filter that is used to localize the EduArt robots globally.

1 Prediction Model for Eduard with Mecanum

1.1 Acceleration

$$\mathbf{a}_{t-1} = \begin{pmatrix} a_{x_{t-1}} \\ a_{y_{t-1}} \end{pmatrix} \tag{1}$$

$$\mathbf{a}_{t} = \begin{pmatrix} a_{x_{t-1}} \\ a_{y_{t-1}} \end{pmatrix} \tag{2}$$

1.2 Velocity

$$\mathbf{v}_{t-1} = \begin{pmatrix} v_{x_{t-1}} \\ v_{y_{t-1}} \end{pmatrix} \tag{3}$$

$$\mathbf{v}_{t} = \begin{pmatrix} a_{x_{t-1}} dt + v_{x_{t-1}} \\ a_{y_{t-1}} dt + v_{y_{t-1}} \end{pmatrix} \tag{4}$$

1.3 Yaw

$$\phi_{z_t} = \phi_{z_{t-1}} + \frac{d}{dt}\phi_{z_{t-1}} dt \tag{5}$$

$$\frac{d}{dt}\phi_{z_t} = \frac{d}{dt}\phi_{z_{t-1}} \tag{6}$$

1.4 Position

$$\mathbf{R}_{t-1} = \begin{pmatrix} \cos\left(\phi_{z_{t-1}}\right) & -\sin\left(\phi_{z_{t-1}}\right) \\ \sin\left(\phi_{z_{t-1}}\right) & \cos\left(\phi_{z_{t-1}}\right) \end{pmatrix} \tag{7}$$

$$\mathbf{p}_{t-1} = \begin{pmatrix} p_{x(t-1)} \\ p_{y(t-1)} \end{pmatrix} \tag{8}$$

$$\mathbf{p}_t = \mathbf{p}_{t-1} + \mathbf{R}_{t-1}\mathbf{v}_{t-1} dt + \frac{1}{2}\mathbf{R}_{t-1}\mathbf{a}_{t-1} dt^2 \tag{9}$$

$$= \begin{pmatrix} dt^2\left(0.5 a_{x_{t-1}} \cos\left(\phi_{z_{t-1}}\right) - 0.5 a_{y_{t-1}} \sin\left(\phi_{z_{t-1}}\right)\right) + dt\left(v_{x_{t-1}} \cos\left(\phi_{z_{t-1}}\right) - v_{y_{t-1}} \sin\left(\phi_{z_{t-1}}\right)\right) + p_{x(t-1)} \\ dt^2\left(0.5 a_{x_{t-1}} \sin\left(\phi_{z_{t-1}}\right) + 0.5 a_{y_{t-1}} \cos\left(\phi_{z_{t-1}}\right)\right) + dt\left(v_{x_{t-1}} \sin\left(\phi_{z_{t-1}}\right) + v_{y_{t-1}} \cos\left(\phi_{z_{t-1}}\right)\right) + p_{y(t-1)} \end{pmatrix} \tag{10}$$

1.5 Model

$$\mathbf{F}_t = \begin{pmatrix} p_{x_t} \\ p_{y_t} \\ v_{x_t} \\ v_{y_t} \\ a_{x_t} \\ a_{y_t} \\ \phi_t \\ \frac{d}{dt}\phi \end{pmatrix} = \begin{pmatrix} dt^2 \left(0.5 a_{x_{t-1}} \cos\left(\phi_{z_{t-1}}\right) - 0.5 a_{y_{t-1}} \sin\left(\phi_{z_{t-1}}\right)\right) + dt \left(v_{x_{t-1}} \cos\left(\phi_{z_{t-1}}\right) - v_{y_{t-1}} \sin\left(\phi_{z_{t-1}}\right)\right) + p_{x(t-1)} \\ dt^2 \left(0.5 a_{x_{t-1}} \sin\left(\phi_{z_{t-1}}\right) + 0.5 a_{y_{t-1}} \cos\left(\phi_{z_{t-1}}\right)\right) + dt \left(v_{x_{t-1}} \sin\left(\phi_{z_{t-1}}\right) + v_{y_{t-1}} \cos\left(\phi_{z_{t-1}}\right)\right) + p_{y(t-1)} \\ a_{x_{t-1}} dt + v_{x_{t-1}} \\ a_{y_{t-1}} dt + v_{y_{t-1}} \\ a_{x_{t-1}} \\ a_{y_{t-1}} \\ \phi_{z_{t-1}} + \frac{d}{dt}\phi_{z_{t-1}} dt \\ \frac{d}{dt}\phi_{z_{t-1}} \end{pmatrix} \tag{11}$$

$$\mathbf{J}_t = \mathbf{F} \begin{pmatrix} \frac{\partial}{\partial p_x} & \frac{\partial}{\partial p_y} & \frac{\partial}{\partial v_x} & \frac{\partial}{\partial v_y} & \frac{\partial}{\partial a_x} & \frac{\partial}{\partial a_y} & \frac{\partial}{\partial \phi} \, \frac{\partial^2}{\partial^2 \phi} \end{pmatrix} \tag{12}$$

$$= \begin{pmatrix} 1 & 0 & dt\cos\left(\phi_{z_{t-1}}\right) & -dt\sin\left(\phi_{z_{t-1}}\right) & 0.5dt^2\cos\left(\phi_{z_{t-1}}\right) & -0.5dt^2\sin\left(\phi_{z_{t-1}}\right) & dt^2 \left(-0.5 a_{x_{t-1}} \sin\left(\phi_{z_{t-1}}\right) - 0.5 a_{y_{t-1}} \cos\left(\phi_{z_{t-1}}\right)\right) + dt \left(-v_{x_{t-1}} \sin\left(\phi_{z_{t-1}}\right) - v_{y_{t-1}} \cos\left(\phi_{z_{t-1}}\right)\right) & 0 \\ 0 & 1 & dt\sin\left(\phi_{z_{t-1}}\right) & dt\cos\left(\phi_{z_{t-1}}\right) & 0.5dt^2\sin\left(\phi_{z_{t-1}}\right) & 0.5dt^2\cos\left(\phi_{z_{t-1}}\right) & dt^2 \left(0.5 a_{x_{t-1}} \cos\left(\phi_{z_{t-1}}\right) - 0.5 a_{y_{t-1}} \sin\left(\phi_{z_{t-1}}\right)\right) + dt \left(v_{x_{t-1}} \cos\left(\phi_{z_{t-1}}\right) - v_{y_{t-1}} \sin\left(\phi_{z_{t-1}}\right)\right) & 0 \\ 0 & 0 & 1 & 0 & dt & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & dt & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & dt \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \tag{13}$$

2 System Noise Model

2.1 Via Acceleration

The system noise is partly determined by the acceleration that the robot can experience.

$$
\begin{aligned}
a &= a_x = a_y && (14)\\
\mathbf{a} &= \begin{pmatrix} a \\ a \end{pmatrix} && (15)\\
\mathbf{v} &= \begin{pmatrix} a \cdot dt \\ a \cdot dt \end{pmatrix} && (16)\\
\mathbf{p} &= \frac{1}{2}\mathbf{R}_{t-1}\mathbf{a} \cdot dt^2 && (17)\\
&= \begin{pmatrix} dt^2 \left(-0.5a \sin\left(\phi_{z_{t-1}}\right) + 0.5a \cos\left(\phi_{z_{t-1}}\right)\right) \\ dt^2 \left(0.5a \sin\left(\phi_{z_{t-1}}\right) + 0.5a \cos\left(\phi_{z_{t-1}}\right)\right) \end{pmatrix} && (18)
\end{aligned}
$$

$$
\mathbf{a}_{\text{noise}} = \begin{pmatrix} \mathbf{p} \\ \mathbf{v} \\ \mathbf{a} \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0.5\sqrt{2}dt^2 \cos\left(\phi_{z_{t-1}} + \frac{\pi}{4}\right) \\ 0.5\sqrt{2}dt^2 \sin\left(\phi_{z_{t-1}} + \frac{\pi}{4}\right) \\ dt \\ dt \\ 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} a \tag{19}
$$

$$
\mathbf{Q_a} = \sigma_{\mathbf{a}}^2 \cdot \mathbf{a}_{\text{noise}} \cdot \mathbf{a}_{\text{noise}}^T = \sigma_{\mathbf{a}}^2 \begin{pmatrix} \frac{p_x^2}{a^2} & \frac{p_x p_y}{a^2} & \frac{dt p_x}{a} & \frac{dt p_x}{a} & \frac{p_x}{a} & \frac{p_x}{a} & 0 & 0 \\ \frac{p_x p_y}{a^2} & \frac{p_y^2}{a^2} & \frac{dt p_y}{a} & \frac{dt p_y}{a} & \frac{p_y}{a} & \frac{p_y}{a} & 0 & 0 \\ \frac{dt p_x}{a} & \frac{dt p_y}{a} & dt^2 & dt^2 & dt & dt & 0 & 0 \\ \frac{dt p_x}{a} & \frac{dt p_y}{a} & dt^2 & dt^2 & dt & dt & 0 & 0 \\ \frac{p_x}{a} & \frac{p_y}{a} & dt & dt & 1 & 1 & 0 & 0 \\ \frac{p_x}{a} & \frac{p_y}{a} & dt & dt & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{20}
$$

2.2 Via Yaw Rate

The system noise is partly determined by the yaw rate that the robot can experience.

$$
\phi_z = \dot{\phi}_z dt \tag{21}
$$

$$
\dot{\phi}_{z_{\text{noise}}} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \phi_z \\ \dot{\phi}_z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ dt \\ 1 \end{pmatrix} \dot{\phi}_z \tag{22}
$$

$$\mathbf{Q}_{\text{yaw}} = \sigma_{\text{yaw}}^2 \cdot \dot{\phi}_{z_{\text{noise}}} \cdot \dot{\phi}_{z_{\text{noise}}}^T = \sigma_{\text{yaw}}^2 \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & dt^2 & dt \\ 0 & 0 & 0 & 0 & 0 & 0 & dt & 1 \end{pmatrix} \tag{23}$$

2.3 Final System Noise Matrix

$$\mathbf{Q} = \mathbf{Q_j} + \mathbf{Q}_{\text{yaw}} = \begin{pmatrix} \frac{\sigma_{\mathbf{a}}^2 p_x^2}{a^2} & \frac{\sigma_{\mathbf{a}}^2 p_x p_y}{a^2} & \frac{\sigma_{\mathbf{a}}^2 dt p_x}{a} & \frac{\sigma_{\mathbf{a}}^2 dt p_x}{a} & \frac{\sigma_{\mathbf{a}}^2 p_x}{a} & \frac{\sigma_{\mathbf{a}}^2 p_x}{a} & 0 & 0 \\ \frac{\sigma_{\mathbf{a}}^2 p_x p_y}{a^2} & \frac{\sigma_{\mathbf{a}}^2 p_y^2}{a^2} & \frac{\sigma_{\mathbf{a}}^2 dt p_y}{a} & \frac{\sigma_{\mathbf{a}}^2 dt p_y}{a} & \frac{\sigma_{\mathbf{a}}^2 p_y}{a} & \frac{\sigma_{\mathbf{a}}^2 p_y}{a} & 0 & 0 \\ \frac{\sigma_{\mathbf{a}}^2 dt p_x}{a} & \frac{\sigma_{\mathbf{a}}^2 dt p_y}{a} & \sigma_{\mathbf{a}}^2 dt^2 & \sigma_{\mathbf{a}}^2 dt^2 & \sigma_{\mathbf{a}}^2 dt & \sigma_{\mathbf{a}}^2 dt & 0 & 0 \\ \frac{\sigma_{\mathbf{a}}^2 dt p_x}{a} & \frac{\sigma_{\mathbf{a}}^2 dt p_y}{a} & \sigma_{\mathbf{a}}^2 dt^2 & \sigma_{\mathbf{a}}^2 dt^2 & \sigma_{\mathbf{a}}^2 dt & \sigma_{\mathbf{a}}^2 dt & 0 & 0 \\ \frac{\sigma_{\mathbf{a}}^2 p_x}{a} & \frac{\sigma_{\mathbf{a}}^2 p_y}{a} & \sigma_{\mathbf{a}}^2 dt & \sigma_{\mathbf{a}}^2 dt & \sigma_{\mathbf{a}}^2 & \sigma_{\mathbf{a}}^2 & 0 & 0 \\ \frac{\sigma_{\mathbf{a}}^2 p_x}{a} & \frac{\sigma_{\mathbf{a}}^2 p_y}{a} & \sigma_{\mathbf{a}}^2 dt & \sigma_{\mathbf{a}}^2 dt & \sigma_{\mathbf{a}}^2 & \sigma_{\mathbf{a}}^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \sigma_{\dot{\phi}_z}^2 dt^2 & \sigma_{\dot{\phi}_z}^2 dt \\ 0 & 0 & 0 & 0 & 0 & 0 & \sigma_{\dot{\phi}_z}^2 dt & \sigma_{\dot{\phi}_z}^2 \end{pmatrix} \tag{24}$$