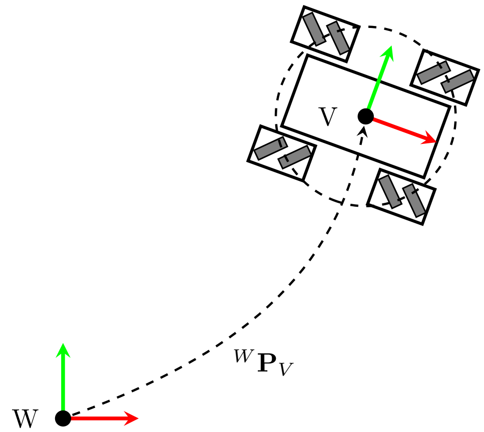

Figure 1: General Overview

1 Kalman Filter Models for Mecanum Drive Kinematic

This document defines the formulas required for a Kalman filter that is used to localize the EduArt robots globally.Several models are defined, each of which has its advantages and disadvantages.

Basically, the movement in the model is linearized, even if an extended Kalman filter is used. This liberalizes around the current state. As a result, a circular path cannot be followed exactly. Instead, an under- or over-tracking occurs.

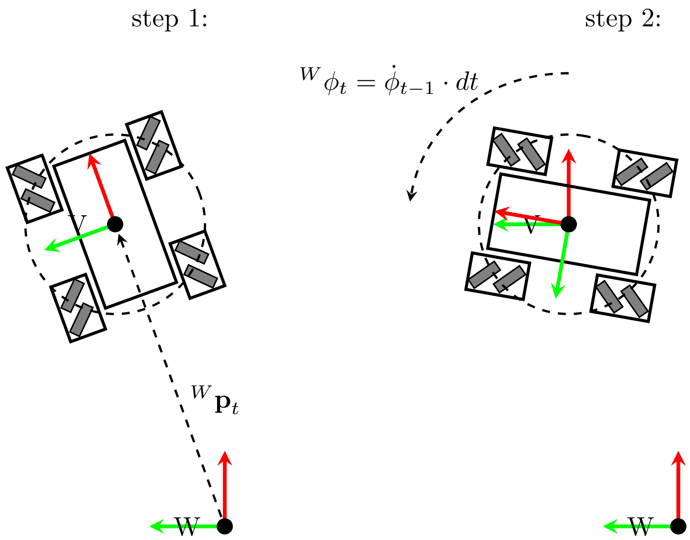

Figure 2: Push and Rotate

2 Model: Push and Rotate

The model defined here first travels a distance based on the velocity and acceleration vectors. The robot is then rotated. We call this Push and Rotate.

2.1 Prediction Model for Eduard with Mecanum

The notation W and V represent the respective coordinate systems. W stands for world and V for vehicle. In the following, all kinematic variables without superscript are in the vehicle coordinate system.

2.1.1 Acceleration

$$\mathbf{a}_{t-1} = \begin{pmatrix} a_{x_{t-1}} \\ a_{y_{t-1}} \end{pmatrix} \tag{1}$$

$$\mathbf{a}_t = \mathbf{a}_{t-1} \tag{2}$$

2.1.2 Velocity

$$\mathbf{v}_{t-1} = \begin{pmatrix} v_{x_{t-1}} \\ v_{y_{t-1}} \end{pmatrix} \tag{3}$$

$$\mathbf{v}_t = \mathbf{v}_{t-1} + \mathbf{a}_{t-1}dt = \begin{pmatrix} a_{x_{t-1}}dt + v_{x_{t-1}} \\ a_{y_{t-1}}dt + v_{y_{t-1}} \end{pmatrix} \tag{4}$$

2.1.3 Yaw

$$^W\phi_{z_t} =^W \phi_{z_{t-1}} + \dot{\phi}_{z_{t-1}}dt \tag{5}$$

$$\dot{\phi}_{z_t} = \dot{\phi}_{z_{t-1}} \tag{6}$$

2.1.4 Position

$$\cos_\phi = \cos\left(^W\phi_{z_{t-1}}\right) \tag{7}$$

$$\sin_\phi = \sin\left(^W\phi_{z_{t-1}}\right) \tag{8}$$

$$^V\mathbf{R}^W_{t-1} = \begin{pmatrix} \cos_\phi & -\sin_\phi \\ \sin_\phi & \cos_\phi \end{pmatrix} \tag{9}$$

$$
{}^{W}\mathbf{p}_{t-1} = \begin{pmatrix} p_{x(t-1)} \\ p_{y(t-1)} \end{pmatrix} \tag{10}
$$

$$
{}^{W}\mathbf{p}_t = {}^{W}\ \mathbf{p}_{t-1} + {}^{V}\ \mathbf{R}^{W}_{t-1}\mathbf{v}_{t-1}dt + \frac{1}{2}\ {}^{V}\mathbf{R}^{W}_{t-1}\mathbf{a}_{t-1}dt^2 \tag{11}
$$

$$
= \begin{pmatrix} dt^2\left(0.5a_{x_{t-1}}\cos_\phi - 0.5a_{y_{t-1}}\sin_\phi\right) + dt\left(v_{x_{t-1}}\cos_\phi - v_{y_{t-1}}\sin_\phi\right) + p_{x(t-1)} \\ dt^2\left(0.5a_{x_{t-1}}\sin_\phi + 0.5a_{y_{t-1}}\cos_\phi\right) + dt\left(v_{x_{t-1}}\sin_\phi + v_{y_{t-1}}\cos_\phi\right) + p_{y(t-1)} \end{pmatrix} \tag{12}
$$

2.1.5 Model

$$
\mathbf{F}_t = \begin{pmatrix} p_{x_t} \\ p_{y_t} \\ v_{x_t} \\ v_{y_t} \\ a_{x_t} \\ a_{y_t} \\ \phi_t \\ \dot{\phi}_t \end{pmatrix} = \begin{pmatrix} dt^2\left(0.5a_{x_{t-1}}\cos_\phi - 0.5a_{y_{t-1}}\sin_\phi\right) + dt\left(v_{x_{t-1}}\cos_\phi - v_{y_{t-1}}\sin_\phi\right) + p_{x(t-1)} \\ dt^2\left(0.5a_{x_{t-1}}\sin_\phi + 0.5a_{y_{t-1}}\cos_\phi\right) + dt\left(v_{x_{t-1}}\sin_\phi + v_{y_{t-1}}\cos_\phi\right) + p_{y(t-1)} \\ a_{x_{t-1}}dt + v_{x_{t-1}} \\ a_{y_{t-1}}dt + v_{y_{t-1}} \\ a_{x_{t-1}} \\ a_{y_{t-1}} \\ \dot{\phi}_{z_{t-1}}dt + \phi_{z_{t-1}} \\ \dot{\phi}_{z_{t-1}} \end{pmatrix} \tag{13}
$$

$$
\mathbf{J}_t = \mathbf{F}\begin{pmatrix} \frac{\partial}{\partial p_x} & \frac{\partial}{\partial p_y} & \frac{\partial}{\partial v_x} & \frac{\partial}{\partial v_y} & \frac{\partial}{\partial a_x} & \frac{\partial}{\partial a_y} & 0 & \frac{\partial}{\partial \dot{\phi}} \end{pmatrix} \tag{14}
$$

$$
= \begin{pmatrix} 1 & 0 & dt\cos_\phi & -dt\sin_\phi & 0.5dt^2\cos_\phi & -0.5dt^2\sin_\phi & 0 & 0 \\ 0 & 1 & dt\sin_\phi & dt\cos_\phi & 0.5dt^2\sin_\phi & 0.5dt^2\cos_\phi & 0 & 0 \\ 0 & 0 & 1 & 0 & dt & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & dt & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & dt \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \tag{15}
$$

3 System Noise Model

3.0.1 Via Acceleration

The system noise is partly determined by the acceleration that the robot can experience.

$$
a = a_x = a_y \tag{16}
$$

$$
\mathbf{a} = \begin{pmatrix} a \\ a \end{pmatrix} \tag{17}
$$

$$
\mathbf{v} = \begin{pmatrix} a \cdot dt \\ a \cdot dt \end{pmatrix} \tag{18}
$$

$$
{}^{W}\mathbf{p} = \frac{1}{2}\ {}^{V}\mathbf{R}^{W}_{t-1}\mathbf{a} \cdot dt^2 \tag{19}
$$

$$
= \begin{pmatrix} dt^2\left(-0.5 \cdot a \cdot \sin_\phi + 0.5 \cdot a \cdot \cos_\phi\right) \\ dt^2\left(0.5 \cdot a \cdot \sin_\phi + 0.5 \cdot a \cdot \cos_\phi\right) \end{pmatrix} \tag{20}
$$

$$
\mathbf{a}_{\text{noise}} = \begin{pmatrix} \mathbf{p} \\ \mathbf{v} \\ \mathbf{a} \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0.5\sqrt{2}dt^2\cos\left(\phi_{z_{t-1}} + \frac{\pi}{4}\right) \\ 0.5\sqrt{2}dt^2\sin\left(\phi_{z_{t-1}} + \frac{\pi}{4}\right) \\ dt \\ dt \\ 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} a \tag{21}
$$

$$\mathbf{Q_a} = \sigma_{\mathbf{a}}^2 \cdot \mathbf{a}_{\text{noise}} \cdot \mathbf{a}_{\text{noise}}^T = \sigma_{\mathbf{a}}^2 \begin{pmatrix} \frac{p_x^2}{a^2} & \frac{p_x p_y}{a^2} & \frac{dtp_x}{a} & \frac{dtp_x}{a} & \frac{p_x}{a} & \frac{p_x}{a} & 0 & 0 \\ \frac{p_x p_y}{a^2} & \frac{p_y^2}{a^2} & \frac{dtp_y}{a} & \frac{dtp_y}{a} & \frac{p_y}{a} & \frac{p_y}{a} & 0 & 0 \\ \frac{dtp_x}{a} & \frac{dtp_y}{a} & dt^2 & dt^2 & dt & dt & 0 & 0 \\ \frac{dtp_x}{a} & \frac{dtp_y}{a} & dt^2 & dt^2 & dt & dt & 0 & 0 \\ \frac{p_x}{a} & \frac{p_y}{a} & dt & dt & 1 & 1 & 0 & 0 \\ \frac{p_x}{a} & \frac{p_y}{a} & dt & dt & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{22}$$

3.0.2 Via Yaw Rate

The system noise is partly determined by the yaw rate that the robot can experience.

$$\phi_z = \dot{\phi}_z dt \tag{23}$$

$$\dot{\phi}_{z_{\text{noise}}} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \phi_z \\ \dot{\phi}_z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ dt \\ 1 \end{pmatrix} \dot{\phi}_z \tag{24}$$

$$\mathbf{Q}_{\text{yaw}} = \sigma_{\text{yaw}}^2 \cdot \dot{\phi}_{z_{\text{noise}}} \cdot \dot{\phi}_{z_{\text{noise}}}^T = \sigma_{\text{yaw}}^2 \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & dt^2 & dt \\ 0 & 0 & 0 & 0 & 0 & 0 & dt & 1 \end{pmatrix} \tag{25}$$

3.0.3 Final System Noise Matrix

$$\mathbf{Q} = \mathbf{Q_j} + \mathbf{Q}_{\text{yaw}} = \begin{pmatrix} \frac{\sigma_{\mathbf{a}}^2 p_x^2}{a^2} & \frac{\sigma_{\mathbf{a}}^2 p_x p_y}{a^2} & \frac{\sigma_{\mathbf{a}}^2 dtp_x}{a} & \frac{\sigma_{\mathbf{a}}^2 dtp_x}{a} & \frac{\sigma_{\mathbf{a}}^2 p_x}{a} & \frac{\sigma_{\mathbf{a}}^2 p_x}{a} & 0 & 0 \\ \frac{\sigma_{\mathbf{a}}^2 p_x p_y}{a^2} & \frac{\sigma_{\mathbf{a}}^2 p_y^2}{a^2} & \frac{\sigma_{\mathbf{a}}^2 dtp_y}{a} & \frac{\sigma_{\mathbf{a}}^2 dtp_y}{a} & \frac{\sigma_{\mathbf{a}}^2 p_y}{a} & \frac{\sigma_{\mathbf{a}}^2 p_y}{a} & 0 & 0 \\ \frac{\sigma_{\mathbf{a}}^2 dtp_x}{a} & \frac{\sigma_{\mathbf{a}}^2 dtp_y}{a} & \sigma_{\mathbf{a}}^2 dt^2 & \sigma_{\mathbf{a}}^2 dt^2 & \sigma_{\mathbf{a}}^2 dt & \sigma_{\mathbf{a}}^2 dt & 0 & 0 \\ \frac{\sigma_{\mathbf{a}}^2 dtp_x}{a} & \frac{\sigma_{\mathbf{a}}^2 dtp_y}{a} & \sigma_{\mathbf{a}}^2 dt^2 & \sigma_{\mathbf{a}}^2 dt^2 & \sigma_{\mathbf{a}}^2 dt & \sigma_{\mathbf{a}}^2 dt & 0 & 0 \\ \frac{\sigma_{\mathbf{a}}^2 p_x}{a} & \frac{\sigma_{\mathbf{a}}^2 p_y}{a} & \sigma_{\mathbf{a}}^2 dt & \sigma_{\mathbf{a}}^2 dt & \sigma_{\mathbf{a}}^2 & \sigma_{\mathbf{a}}^2 & 0 & 0 \\ \frac{\sigma_{\mathbf{a}}^2 p_x}{a} & \frac{\sigma_{\mathbf{a}}^2 p_y}{a} & \sigma_{\mathbf{a}}^2 dt & \sigma_{\mathbf{a}}^2 dt & \sigma_{\mathbf{a}}^2 & \sigma_{\mathbf{a}}^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \sigma_{\dot{\phi}_z}^2 dt^2 & \sigma_{\dot{\phi}_z}^2 dt \\ 0 & 0 & 0 & 0 & 0 & 0 & \sigma_{\dot{\phi}_z}^2 dt & \sigma_{\dot{\phi}_z}^2 \end{pmatrix} \tag{26}$$

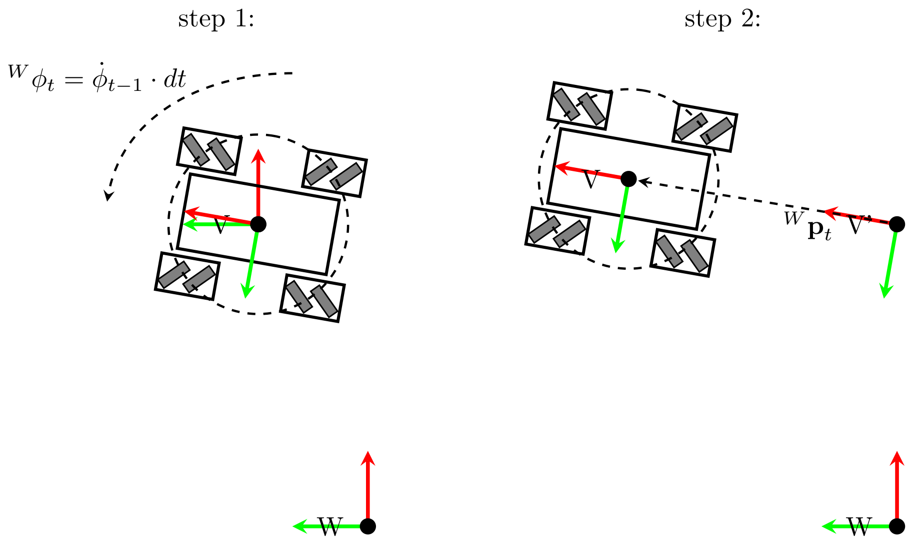

Figure 3: Push and Rotate

4 Model: Rotate and Push

The model defined here first rotates the robot first. Then the robot travels a distance based on the velocity and acceleration that are rotated, too first. We call Rotate and Push.

4.1 Prediction Model for Eduard with Mecanum

The notation W and V represent the respective coordinate systems. W stands for world and V for vehicle. In the following, all kinematic variables without superscript are in the vehicle coordinate system.

4.1.1 Acceleration

$$\mathbf{a}_{t-1} = \begin{pmatrix} a_{x_{t-1}} \\ a_{y_{t-1}} \end{pmatrix} \tag{27}$$

$$\mathbf{a}_t = \mathbf{a}_{t-1} \tag{28}$$

4.1.2 Velocity

$$\mathbf{v}_{t-1} = \begin{pmatrix} v_{x_{t-1}} \\ v_{y_{t-1}} \end{pmatrix} \tag{29}$$

$$\mathbf{v}_t = \mathbf{v}_{t-1} + \mathbf{a}_{t-1}dt = \begin{pmatrix} a_{x_{t-1}}dt + v_{x_{t-1}} \\ a_{y_{t-1}}dt + v_{y_{t-1}} \end{pmatrix} \tag{30}$$

4.1.3 Yaw

$$^W\phi_{z_t} =^W \phi_{z_{t-1}} + \dot{\phi}_{z_{t-1}}dt \tag{31}$$

$$\dot{\phi}_{z_t} = \dot{\phi}_{z_{t-1}} \tag{32}$$

4.1.4 Position

The $\dot{\phi}_{z_{t-1}}dt$ term is added to the $\phi_{z_{t-1}}$ before the position $^W\mathbf{p}_t$ is being calculated.

$$\cos_\phi = \cos\left(\dot{\phi}_{z_{t-1}}dt + \phi_{z_{t-1}}\right) \tag{33}$$

$$\sin_\phi = \sin\left(\dot{\phi}_{z_{t-1}} dt + \phi_{z_{t-1}}\right) \tag{34}$$

$$^{V}\mathbf{R}_{t-1}^{W} = \begin{pmatrix} \cos_\phi & -\sin_\phi \\ \sin_\phi & \cos_\phi \end{pmatrix} \tag{35}$$

$$^{W}\mathbf{p}_{t-1} = \begin{pmatrix} p_{x(t-1)} \\ p_{y(t-1)} \end{pmatrix} \tag{36}$$

$$^{W}\mathbf{p}_t = ^{W}\mathbf{p}_{t-1} + ^{V}\mathbf{R}_{t-1}^{W}\mathbf{v}_{t-1}dt + \frac{1}{2}\ ^{V}\mathbf{R}_{t-1}^{W}\mathbf{a}_{t-1}dt^2 \tag{37}$$

$$= \begin{pmatrix} dt^2\left(0.5a_{x_{t-1}}\cos_\phi - 0.5a_{y_{t-1}}\sin_\phi\right) + dt\left(v_{x_{t-1}}\cos_\phi - v_{y_{t-1}}\sin_\phi\right) + p_{x(t-1)} \\ dt^2\left(0.5a_{x_{t-1}}\sin_\phi + 0.5a_{y_{t-1}}\cos_\phi\right) + dt\left(v_{x_{t-1}}\sin_\phi + v_{y_{t-1}}\cos_\phi\right) + p_{y(t-1)} \end{pmatrix} \tag{38}$$

4.1.5 Model

$$\mathbf{F}_t = \begin{pmatrix} p_{x_t} \\ p_{y_t} \\ v_{x_t} \\ v_{y_t} \\ a_{x_t} \\ a_{y_t} \\ \phi_t \\ \dot{\phi}_t \end{pmatrix} = \begin{pmatrix} dt^2\cdot\left(0.5a_{x_{t-1}}\cos_\phi - 0.5a_{y_{t-1}}\sin_\phi\right) + dt\left(v_{x_{t-1}}\cos_\phi - v_{y_{t-1}}\sin_\phi\right) + p_{x(t-1)} \\ dt^2\cdot\left(0.5a_{x_{t-1}}\sin_\phi + 0.5a_{y_{t-1}}\cos_\phi\right) + dt\left(v_{x_{t-1}}\sin_\phi + v_{y_{t-1}}\cos_\phi\right) + p_{y(t-1)} \\ a_{x_{t-1}}dt + v_{x_{t-1}} \\ a_{y_{t-1}}dt + v_{y_{t-1}} \\ a_{x_{t-1}} \\ a_{y_{t-1}} \\ \dot{\phi}_{z_{t-1}}dt + \phi_{z_{t-1}} \\ \dot{\phi}_{z_{t-1}} \end{pmatrix} \tag{39}$$

$$\mathbf{J}_t = \mathbf{F}\begin{pmatrix} \frac{\partial}{\partial p_x} & \frac{\partial}{\partial p_y} & \frac{\partial}{\partial v_x} & \frac{\partial}{\partial v_y} & \frac{\partial}{\partial a_x} & \frac{\partial}{\partial a_y} & 0 & \frac{\partial}{\partial \dot{\phi}} \end{pmatrix} \tag{40}$$

$$= \begin{pmatrix} 1 & 0 & dt\cos_\phi & -dt\sin_\phi & 0.5dt^2\cos_\phi & -0.5dt^2\sin_\phi & 0 & dt^2\left(-0.5dt\left(a_{x_{t-1}}\sin_\phi + a_{y_{t-1}}\cos_\phi\right) - v_{x_{t-1}}\sin_\phi - v_{y_{t-1}}\cos_\phi\right) \\ 0 & 1 & dt\sin_\phi & dt\cos_\phi & 0.5dt^2\sin_\phi & 0.5dt^2\cos_\phi & 0 & dt^2\cdot\left(0.5dt\left(a_{x_{t-1}}\cos_\phi - a_{y_{t-1}}\sin_\phi\right) + v_{x_{t-1}}\cos_\phi - v_{y_{t-1}}\sin_\phi\right) \\ 0 & 0 & 1 & 0 & dt & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & dt & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & dt \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \tag{41}$$

5 System Noise Model

5.0.1 Via Acceleration

The system noise is partly determined by the acceleration that the robot can experience.

$$a = a_x = a_y \tag{42}$$

$$\mathbf{a} = \begin{pmatrix} a \\ a \end{pmatrix} \tag{43}$$

$$\mathbf{v} = \begin{pmatrix} a\cdot dt \\ a\cdot dt \end{pmatrix} \tag{44}$$

$$^{W}\mathbf{p} = \frac{1}{2}\ ^{V}\mathbf{R}_{t-1}^{W}\mathbf{a}\cdot dt^2 \tag{45}$$

$$= \begin{pmatrix} dt^2\left(-0.5\cdot a\cdot\sin_\phi + 0.5\cdot a\cdot\cos_\phi\right) \\ dt^2\left(0.5\cdot a\cdot\sin_\phi + 0.5\cdot a\cdot\cos_\phi\right) \end{pmatrix} \tag{46}$$

$$\mathbf{a}_{\text{noise}} = \begin{pmatrix} \mathbf{p} \\ \mathbf{v} \\ \mathbf{a} \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{dt^2(-0.5 \cdot a \cdot \sin_\phi + 0.5 \cdot a \cdot \cos_\phi)}{a} \\ \frac{dt^2 \cdot (0.5 \cdot a \cdot \sin_\phi + 0.5 \cdot a \cdot \cos_\phi)}{a} \\ dt \\ dt \\ 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} a \tag{47}$$

$$\mathbf{Q}_{\mathbf{a}} = \sigma_{\mathbf{a}}^2 \cdot \mathbf{a}_{\text{noise}} \cdot \mathbf{a}_{\text{noise}}^T = \sigma_{\mathbf{a}}^2 \begin{pmatrix} \frac{p_x^2}{a^2} & \frac{p_x p_y}{a^2} & \frac{dt p_x}{a} & \frac{dt p_x}{a} & \frac{p_x}{a} & \frac{p_x}{a} & 0 & 0 \\ \frac{p_x p_y}{a^2} & \frac{p_y^2}{a^2} & \frac{dt p_y}{a} & \frac{dt p_y}{a} & \frac{p_y}{a} & \frac{p_y}{a} & 0 & 0 \\ \frac{dt p_x}{a} & \frac{dt p_y}{a} & dt^2 & dt^2 & dt & dt & 0 & 0 \\ \frac{dt p_x}{a} & \frac{dt p_y}{a} & dt^2 & dt^2 & dt & dt & 0 & 0 \\ \frac{p_x}{a} & \frac{p_y}{a} & dt & dt & 1 & 1 & 0 & 0 \\ \frac{p_x}{a} & \frac{p_y}{a} & dt & dt & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{48}$$

5.0.2 Via Yaw Rate

needs to be redone! Velocity is missing in equations!

The system noise is partly determined by the yaw rate that the robot can experience.

$$\phi_z = \dot{\phi}_z dt \tag{49}$$

$$\dot{\phi}_{z_{\text{noise}}} = \begin{bmatrix} dt^2\left(-0.5a\sin_\phi + 0.5a\cos_\phi\right) \\ dt^2 \cdot \left(0.5a\sin_\phi + 0.5a\cos_\phi\right) \\ 0 \\ 0 \\ 0 \\ 0 \\ dt \\ 1 \end{bmatrix} \tag{50}$$

$$\mathbf{Q}_{\text{yaw}} = \sigma_{\text{yaw}}^2 \cdot \dot{\phi}_{z_{\text{noise}}} \cdot \dot{\phi}_{z_{\text{noise}}}^T \tag{51}$$

∞

$$= \sigma_{\text{yaw}}^2 \begin{pmatrix} 0.25dt^4\left(-a\sin_\phi + a\cos_\phi\right)^2 & dt^4\left(-0.5a\sin_\phi + 0.5a\cos_\phi\right)\left(0.5a\sin_\phi + 0.5a\cos_\phi\right) & 0 & 0 & 0 & 0 & dt^3\left(-0.5a\sin_\phi + 0.5a\cos_\phi\right) & dt^2\left(-0.5a\sin_\phi + 0.5a\cos_\phi\right) \\ dt^4\left(-0.5a\sin_\phi + 0.5a\cos_\phi\right)\left(0.5a\sin_\phi + 0.5a\cos_\phi\right) & 0.25dt^4\left(a\sin_\phi + a\cos_\phi\right)^2 & 0 & 0 & 0 & 0 & dt^3 \cdot \left(0.5a\sin_\phi + 0.5a\cos_\phi\right) & dt^2 \cdot \left(0.5a\sin_\phi + 0.5a\cos_\phi\right) \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ dt^3\left(-0.5a\sin_\phi + 0.5a\cos_\phi\right) & dt^3 \cdot \left(0.5a\sin_\phi + 0.5a\cos_\phi\right) & 0 & 0 & 0 & 0 & dt^2 & dt \\ dt^2\left(-0.5a\sin_\phi + 0.5a\cos_\phi\right) & dt^2 \cdot \left(0.5a\sin_\phi + 0.5a\cos_\phi\right) & 0 & 0 & 0 & 0 & dt & 1 \end{pmatrix} \tag{52}$$

5.0.3 Final System Noise Matrix

$$\mathbf{Q} = \mathbf{Q_j} + \mathbf{Q}_{\text{yaw}} \tag{53}$$