This document defines the formulas required for a Kalman filter that is used to localize the EduArt robots globally.

1 Prediction Model for Eduard with Mecanum

1.1 Acceleration

$$\mathbf{a}_{t-1} = \begin{pmatrix} a_{x_{t-1}} \\ a_{y_{t-1}} \end{pmatrix} \tag{1}$$

$$\mathbf{a}_{t} = \begin{pmatrix} a_{x_{t-1}} \\ a_{y_{t-1}} \end{pmatrix} \tag{2}$$

1.2 Velocity

$$\mathbf{v}_{t-1} = \begin{pmatrix} v_{x_{t-1}} \\ v_{y_{t-1}} \end{pmatrix} \tag{3}$$

$$\mathbf{v}_{t} = \begin{pmatrix} a_{x_{t-1}} dt + v_{x_{t-1}} \\ a_{y_{t-1}} dt + v_{y_{t-1}} \end{pmatrix} \tag{4}$$

1.3 Yaw

$$\phi_{z_t} = \phi_{z_{t-1}} + \frac{d}{dt}\phi_{z_{t-1}} dt \tag{5}$$

$$\frac{d}{dt}\phi_{z_t} = \frac{d}{dt}\phi_{z_{t-1}} \tag{6}$$

1.4 Position

$$\mathbf{R}_{t-1} = \begin{pmatrix} \cos\left(\phi_{z_{t-1}}\right) & -\sin\left(\phi_{z_{t-1}}\right) \\ \sin\left(\phi_{z_{t-1}}\right) & \cos\left(\phi_{z_{t-1}}\right) \end{pmatrix} \tag{7}$$

$$\mathbf{p}_{t-1} = \begin{pmatrix} p_{x(t-1)} \\ p_{y(t-1)} \end{pmatrix} \tag{8}$$

$$\mathbf{p}_t = \mathbf{p}_{t-1} + \mathbf{R}_{t-1}\mathbf{v}_{t-1} dt + \frac{1}{2}\mathbf{R}_{t-1}\mathbf{a}_{t-1} dt^2 \tag{9}$$

$$= \begin{pmatrix} dt^2\left(0.5 a_{x_{t-1}} \cos\left(\phi_{z_{t-1}}\right) - 0.5 a_{y_{t-1}} \sin\left(\phi_{z_{t-1}}\right)\right) + dt\left(v_{x_{t-1}} \cos\left(\phi_{z_{t-1}}\right) - v_{y_{t-1}} \sin\left(\phi_{z_{t-1}}\right)\right) + p_{x(t-1)} \\ dt^2\left(0.5 a_{x_{t-1}} \sin\left(\phi_{z_{t-1}}\right) + 0.5 a_{y_{t-1}} \cos\left(\phi_{z_{t-1}}\right)\right) + dt\left(v_{x_{t-1}} \sin\left(\phi_{z_{t-1}}\right) + v_{y_{t-1}} \cos\left(\phi_{z_{t-1}}\right)\right) + p_{y(t-1)} \end{pmatrix} \tag{10}$$

1.5 Model

$$\mathbf{F}_t = \begin{pmatrix} p_{x_t} \\ p_{y_t} \\ v_{x_t} \\ v_{y_t} \\ a_{x_t} \\ a_{y_t} \\ \phi_t \\ \frac{d}{dt}\phi \end{pmatrix} = \begin{pmatrix} dt^2\left(0.5a_{x_{t-1}}\cos\left(\phi_{z_{t-1}}\right) - 0.5a_{y_{t-1}}\sin\left(\phi_{z_{t-1}}\right)\right) + dt\left(v_{x_{t-1}}\cos\left(\phi_{z_{t-1}}\right) - v_{y_{t-1}}\sin\left(\phi_{z_{t-1}}\right)\right) + p_{x(t-1)} \\ dt^2\left(0.5a_{x_{t-1}}\sin\left(\phi_{z_{t-1}}\right) + 0.5a_{y_{t-1}}\cos\left(\phi_{z_{t-1}}\right)\right) + dt\left(v_{x_{t-1}}\sin\left(\phi_{z_{t-1}}\right) + v_{y_{t-1}}\cos\left(\phi_{z_{t-1}}\right)\right) + p_{y(t-1)} \\ a_{x_{t-1}}dt + v_{x_{t-1}} \\ a_{y_{t-1}}dt + v_{y_{t-1}} \\ a_{x_{t-1}} \\ a_{y_{t-1}} \\ \phi_{z_{t-1}} + \frac{d}{dt}\phi_{z_{t-1}}dt \\ \frac{d}{dt}\phi_{z_{t-1}} \end{pmatrix} \tag{11}$$

$$\mathbf{J}_t = \mathbf{F}\begin{pmatrix} \frac{\partial}{\partial p_x} & \frac{\partial}{\partial p_y} & \frac{\partial}{\partial v_x} & \frac{\partial}{\partial v_y} & \frac{\partial}{\partial a_x} & \frac{\partial}{\partial a_y} & \frac{\partial}{\partial \phi} & \frac{\partial^2}{\partial^2 \phi} \end{pmatrix} \tag{12}$$

$$= \begin{pmatrix} 1 & 0 & dt\cos\left(\phi_{z_{t-1}}\right) & -dt\sin\left(\phi_{z_{t-1}}\right) & 0.5dt^2\cos\left(\phi_{z_{t-1}}\right) & -0.5dt^2\sin\left(\phi_{z_{t-1}}\right) & dt^2\left(-0.5a_{x_{t-1}}\sin\left(\phi_{z_{t-1}}\right) - 0.5a_{y_{t-1}}\cos\left(\phi_{z_{t-1}}\right)\right) + dt\left(-v_{x_{t-1}}\sin\left(\phi_{z_{t-1}}\right) - v_{y_{t-1}}\cos\left(\phi_{z_{t-1}}\right)\right) & 0 \\ 0 & 1 & dt\sin\left(\phi_{z_{t-1}}\right) & dt\cos\left(\phi_{z_{t-1}}\right) & 0.5dt^2\sin\left(\phi_{z_{t-1}}\right) & 0.5dt^2\cos\left(\phi_{z_{t-1}}\right) & dt^2\left(0.5a_{x_{t-1}}\cos\left(\phi_{z_{t-1}}\right) - 0.5a_{y_{t-1}}\sin\left(\phi_{z_{t-1}}\right)\right) + dt\left(v_{x_{t-1}}\cos\left(\phi_{z_{t-1}}\right) - v_{y_{t-1}}\sin\left(\phi_{z_{t-1}}\right)\right) & 0 \\ 0 & 0 & 1 & 0 & dt & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & dt & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & dt \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \tag{13}$$

2 System Noise Model

2.1 Via Jerk

The system noise is partly determined by the jerk that the robot can experience. The jerk is not directly observed and is well suited for describing the disturbance.

$$\mathbf{j} = \begin{pmatrix} j_x \\ j_y \end{pmatrix} \tag{14}$$

$$\mathbf{a} = \begin{pmatrix} dtj_x \\ dtj_y \end{pmatrix} \tag{15}$$

$$\mathbf{v} = \begin{pmatrix} dt^2 j_x \\ dt^2 j_y \end{pmatrix} \tag{16}$$

$$\mathbf{p} = \begin{pmatrix} dt\left(dt^2 j_x \cos\left(\phi_{z_{t-1}}\right) - dt^2 j_y \sin\left(\phi_{z_{t-1}}\right)\right) \\ dt\left(dt^2 j_x \sin\left(\phi_{z_{t-1}}\right) + dt^2 j_y \cos\left(\phi_{z_{t-1}}\right)\right) \end{pmatrix} \tag{17}$$

$$\mathbf{j}_{\text{noise}} = \begin{pmatrix} \mathbf{p} \\ \mathbf{v} \\ \mathbf{a} \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} dt\left(dt^2 j_x \cos\left(\phi_{z_{t-1}}\right) - dt^2 j_y \sin\left(\phi_{z_{t-1}}\right)\right) \\ dt\left(dt^2 j_x \sin\left(\phi_{z_{t-1}}\right) + dt^2 j_y \cos\left(\phi_{z_{t-1}}\right)\right) \\ dt^2 j_x \\ dt^2 j_y \\ dtj_x \\ dtj_y \\ 0 \\ 0 \end{pmatrix} \tag{18}$$

$$\mathbf{Q}_{\mathbf{j}} = \sigma_{\mathbf{j}}^2 \cdot \mathbf{j}_{\text{noise}} \cdot \mathbf{j}_{\text{noise}}^T \tag{19}$$

2.2 Via Yaw Acceleration

The system noise is partly determined by the yaw acceleration that the robot can experience. The yaw acceleration is not directly observed and is well suited for describing the disturbance.

$$\frac{d}{dt}\phi_z = \frac{d^2}{dt^2}\phi_z dt \tag{20}$$

$$\phi_z = \frac{d^2}{dt^2}\phi_z dt^2 \tag{21}$$

$$\frac{d^2}{dt^2}\phi_{z_{\text{noise}}} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{d^2}{dt^2}\phi_z dt^2 \\ \frac{d^2}{dt^2}\phi_z dt \end{pmatrix} \tag{22}$$

$$\tag{23}$$

$$\mathbf{Q}_{\text{yaw}} = \sigma_{\text{yaw}}^2 \cdot \frac{d^2}{dt^2}\phi_{z_{\text{noise}}} \cdot \frac{d^2}{dt^2}\phi_{z_{\text{noise}}}^T \tag{24}$$

2.3 Final System Noise Matrix

$$\mathbf{Q} = \mathbf{Q}_{\mathbf{j}} + \mathbf{Q}_{\text{yaw}} \tag{25}$$