This document defines the formulas required for a Kalman filter that is used to localize the EduArt robots globally.

1 Prediction Model for Eduard with Skid

1.1 Acceleration

$$a_{t-1} : \text{previous scalar acceleration value in vehicle direction} \tag{1}$$

$$a_t = a_{t-1} \tag{2}$$

1.2 Velocity

$$v_{t-1} : \text{previous scalar velocity value in vehicle direction} \tag{3}$$

$$v_t = a_{t-1}dt + v_{t-1} \tag{4}$$

1.3 Yaw

$$\phi_{z_t} = \phi_{z_{t-1}} + \phi_{z_{t-1}}\frac{d}{dt}dt \tag{5}$$

$$\phi_{z_t}\frac{d}{dt} = \phi_{z_{t-1}}\frac{d}{dt} \tag{6}$$

1.4 Position

$$\mathbf{R}_{t-1} = \begin{pmatrix} \cos\left(\phi_{z_{t-1}}\right) & -\sin\left(\phi_{z_{t-1}}\right) \\ \sin\left(\phi_{z_{t-1}}\right) & \cos\left(\phi_{z_{t-1}}\right) \end{pmatrix} \tag{7}$$

$$\mathbf{p}_{t-1} = \begin{pmatrix} p_{x(t-1)} \\ p_{y(t-1)} \end{pmatrix} \tag{8}$$

$$\mathbf{p}_t = \mathbf{p}_{t-1} + \mathbf{R}_{t-1}\mathbf{e}_x v_{t-1}dt + \frac{1}{2}\mathbf{R}_{t-1}\mathbf{e}_x a_{t-1}dt^2 \tag{9}$$

$$= \begin{pmatrix} 0.5a_{t-1}dt^2\cos\left(\phi_{z_{t-1}}\right) + dtv_{t-1}\cos\left(\phi_{z_{t-1}}\right) + p_{x(t-1)} \\ 0.5a_{t-1}dt^2\sin\left(\phi_{z_{t-1}}\right) + dtv_{t-1}\sin\left(\phi_{z_{t-1}}\right) + p_{y(t-1)} \end{pmatrix} \tag{10}$$

1.5 Model

$$\mathbf{F}_t = \begin{pmatrix} p_{x_t} \\ p_{y_t} \\ v_t \\ a_t \\ \phi_{z_t} \\ \phi_{z_t}\frac{d}{dt} \end{pmatrix} = \begin{pmatrix} 0.5a_{t-1}dt^2\cos\left(\phi_{z_{t-1}}\right) + dtv_{t-1}\cos\left(\phi_{z_{t-1}}\right) + p_{x(t-1)} \\ 0.5a_{t-1}dt^2\sin\left(\phi_{z_{t-1}}\right) + dtv_{t-1}\sin\left(\phi_{z_{t-1}}\right) + p_{y(t-1)} \\ a_{t-1}dt + v_{t-1} \\ a_{t-1} \\ \phi_{z_{t-1}} + \phi_{z_{t-1}}\frac{d}{dt}dt \\ \phi_{z_{t-1}}\frac{d}{dt} \end{pmatrix} \tag{11}$$

$$\mathbf{J}_t = \mathbf{F}\begin{pmatrix} \frac{\partial}{\partial p_x} & \frac{\partial}{\partial p_y} & \frac{\partial}{\partial v_x} & \frac{\partial}{\partial v_y} & \frac{\partial}{\partial a_x} & \frac{\partial}{\partial a_y} & \frac{\partial}{\partial \phi}\frac{\partial^2}{\partial^2\phi} \end{pmatrix} \tag{12}$$

$$= \begin{pmatrix} 1 & 0 & dt\cos\left(\phi_{z_{t-1}}\right) & 0.5dt^2\cos\left(\phi_{z_{t-1}}\right) & -0.5a_{t-1}dt^2\sin\left(\phi_{z_{t-1}}\right) - dtv_{t-1}\sin\left(\phi_{z_{t-1}}\right) & 0 \\ 0 & 1 & dt\sin\left(\phi_{z_{t-1}}\right) & 0.5dt^2\sin\left(\phi_{z_{t-1}}\right) & 0.5a_{t-1}dt^2\cos\left(\phi_{z_{t-1}}\right) + dtv_{t-1}\cos\left(\phi_{z_{t-1}}\right) & 0 \\ 0 & 0 & 1 & dt & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & dt \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \tag{13}$$