This document defines the formulas required for a Kalman filter that is used to localize the EduArt robots globally.

1 Prediction Model for Eduard with Mecanum

1.1 Acceleration

$$\mathbf{a}_{t-1} = \begin{pmatrix} a_{x_{t-1}} \\ a_{y_{t-1}} \end{pmatrix} \tag{1}$$

$$\mathbf{a}_{t} = \begin{pmatrix} a_{x_{t-1}} \\ a_{y_{t-1}} \end{pmatrix} \tag{2}$$

1.2 Velocity

$$\mathbf{v}_{t-1} = \begin{pmatrix} v_{x_{t-1}} \\ v_{y_{t-1}} \end{pmatrix} \tag{3}$$

$$\mathbf{v}_{t} = \begin{pmatrix} a_{x_{t-1}} dt + v_{x_{t-1}} \\ a_{y_{t-1}} dt + v_{y_{t-1}} \end{pmatrix} \tag{4}$$

1.3 Yaw

$$\phi_{z_t} = \phi_{z_{t-1}} + \phi_{z_{t-1}} \frac{d}{dt} dt \tag{5}$$

$$\phi_{z_t} \frac{d}{dt} = \phi_{z_{t-1}} \frac{d}{dt} \tag{6}$$

1.4 Position

$$\mathbf{R}_{t-1} = \begin{pmatrix} \cos\left(\phi_{z_{t-1}}\right) & -\sin\left(\phi_{z_{t-1}}\right) \\ \sin\left(\phi_{z_{t-1}}\right) & \cos\left(\phi_{z_{t-1}}\right) \end{pmatrix} \tag{7}$$

$$\mathbf{p}_{t-1} = \begin{pmatrix} p_{x(t-1)} \\ p_{y(t-1)} \end{pmatrix} \tag{8}$$

$$\mathbf{p}_t = \mathbf{p}_{t-1} + \mathbf{R}_{t-1}\mathbf{v}_{t-1} dt + \frac{1}{2}\mathbf{R}_{t-1}\mathbf{a}_{t-1} dt^2 \tag{9}$$

$$= \begin{pmatrix} dt^2 \left(0.5 a_{x_{t-1}} \cos\left(\phi_{z_{t-1}}\right) - 0.5 a_{y_{t-1}} \sin\left(\phi_{z_{t-1}}\right)\right) + dt\left(v_{x_{t-1}} \cos\left(\phi_{z_{t-1}}\right) - v_{y_{t-1}} \sin\left(\phi_{z_{t-1}}\right)\right) + p_{x(t-1)} \\ dt^2 \left(0.5 a_{x_{t-1}} \sin\left(\phi_{z_{t-1}}\right) + 0.5 a_{y_{t-1}} \cos\left(\phi_{z_{t-1}}\right)\right) + dt\left(v_{x_{t-1}} \sin\left(\phi_{z_{t-1}}\right) + v_{y_{t-1}} \cos\left(\phi_{z_{t-1}}\right)\right) + p_{y(t-1)} \end{pmatrix} \tag{10}$$

1.5 Model

$$\mathbf{F}_t = \begin{pmatrix} p_{x_t} \\ p_{y_t} \\ v_{x_t} \\ v_{y_t} \\ a_{x_t} \\ a_{y_t} \\ \phi_t \\ \phi \frac{d}{dt} \end{pmatrix} = \mathbf{F} = \begin{pmatrix} dt^2 \left(0.5 a_{x_{t-1}} \cos\left(\phi_{z_{t-1}}\right) - 0.5 a_{y_{t-1}} \sin\left(\phi_{z_{t-1}}\right)\right) + dt \left(v_{x_{t-1}} \cos\left(\phi_{z_{t-1}}\right) - v_{y_{t-1}} \sin\left(\phi_{z_{t-1}}\right)\right) + p_{x(t-1)} \\ dt^2 \left(0.5 a_{x_{t-1}} \sin\left(\phi_{z_{t-1}}\right) + 0.5 a_{y_{t-1}} \cos\left(\phi_{z_{t-1}}\right)\right) + dt \left(v_{x_{t-1}} \sin\left(\phi_{z_{t-1}}\right) + v_{y_{t-1}} \cos\left(\phi_{z_{t-1}}\right)\right) + p_{y(t-1)} \\ a_{x_{t-1}} dt + v_{x_{t-1}} \\ a_{y_{t-1}} dt + v_{y_{t-1}} \\ a_{x_{t-1}} \\ a_{y_{t-1}} \\ \phi_{z_{t-1}} + \phi_{z_{t-1}} \frac{d}{dt} dt \\ \phi_{z_{t-1}} \frac{d}{dt} \end{pmatrix} \tag{11}$$

$$\mathbf{J}_t = \mathbf{F} \begin{pmatrix} \frac{\partial}{\partial p_x} & \frac{\partial}{\partial p_y} & \frac{\partial}{\partial v_x} & \frac{\partial}{\partial v_y} & \frac{\partial}{\partial a_x} & \frac{\partial}{\partial a_y} & \frac{\partial}{\partial \phi} & \frac{\partial^2}{\partial^2 \phi} \end{pmatrix} \tag{12}$$

$$= \begin{pmatrix} 1 & 0 & dt \cos\left(\phi_{z_{t-1}}\right) & -dt \sin\left(\phi_{z_{t-1}}\right) & 0.5 dt^2 \cos\left(\phi_{z_{t-1}}\right) & -0.5 dt^2 \sin\left(\phi_{z_{t-1}}\right) & dt^2 \left(-0.5 a_{x_{t-1}} \sin\left(\phi_{z_{t-1}}\right) - 0.5 a_{y_{t-1}} \cos\left(\phi_{z_{t-1}}\right)\right) + dt \left(-v_{x_{t-1}} \sin\left(\phi_{z_{t-1}}\right) - v_{y_{t-1}} \cos\left(\phi_{z_{t-1}}\right)\right) & 0 \\ 0 & 1 & dt \sin\left(\phi_{z_{t-1}}\right) & dt \cos\left(\phi_{z_{t-1}}\right) & 0.5 dt^2 \sin\left(\phi_{z_{t-1}}\right) & 0.5 dt^2 \cos\left(\phi_{z_{t-1}}\right) & dt^2 \left(0.5 a_{x_{t-1}} \cos\left(\phi_{z_{t-1}}\right) - 0.5 a_{y_{t-1}} \sin\left(\phi_{z_{t-1}}\right)\right) + dt \left(v_{x_{t-1}} \cos\left(\phi_{z_{t-1}}\right) - v_{y_{t-1}} \sin\left(\phi_{z_{t-1}}\right)\right) & 0 \\ 0 & 0 & 1 & 0 & dt & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & dt & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & dt \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \tag{13}$$