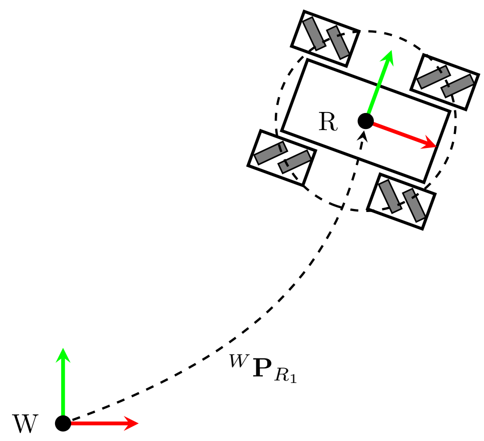

Figure 1: General Overview

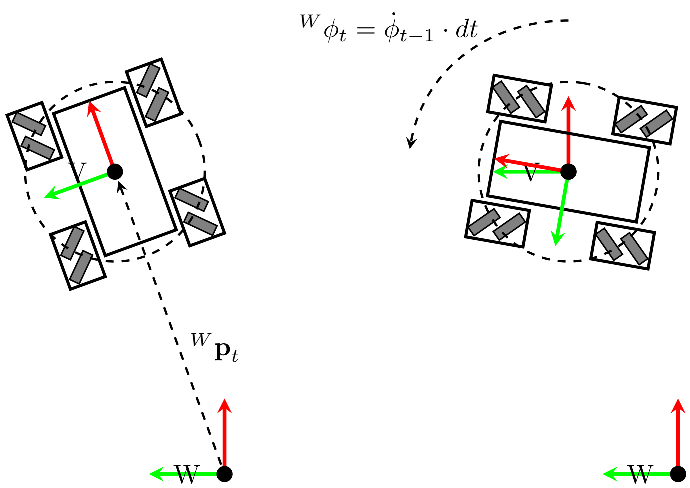

Figure 2: Push and Rotate

1 Kalman Filter Models for Mecanum Drive Kinematic

This document defines the formulas required for a Kalman filter that is used to localize the EduArt robots globally.Several models are defined, each of which has its advantages and disadvantages.

Basically, the movement in the model is linearized, even if an extended Kalman filter is used. This liberalizes around the current state. As a result, a circular path cannot be followed exactly. Instead, an under- or over-tracking occurs.

2 Model Push and Rotate

The model defined here first travels a distance based on the velocity and acceleration vectors. The robot is then rotated. We call this Push and Rotate.

2.1 Prediction Model for Eduard with Mecanum

2.1.1 Acceleration

$$\mathbf{a}_{t-1} = \begin{pmatrix} a_{x_{t-1}} \\ a_{y_{t-1}} \end{pmatrix} \tag{1}$$

$$\mathbf{a}_t = \mathbf{a}_{t-1} \tag{2}$$

2.1.2 Velocity

$$\mathbf{v}_{t-1} = \begin{pmatrix} v_{x_{t-1}} \\ v_{y_{t-1}} \end{pmatrix} \tag{3}$$

$$\mathbf{v}_t = \mathbf{v}_{t-1} + \mathbf{a}_{t-1} dt = \begin{pmatrix} a_{x_{t-1}} dt + v_{x_{t-1}} \\ a_{y_{t-1}} dt + v_{y_{t-1}} \end{pmatrix} \tag{4}$$

2.1.3 Yaw

$$^W\phi_{z_t} =^W \phi_{z_{t-1}} + \dot{\phi}_{z_{t-1}} dt \tag{5}$$

$$\dot{\phi}_{z_t} = \dot{\phi}_{z_{t-1}} \tag{6}$$

2.1.4 Position

$$\cos_\phi = \cos\left(^W\phi_{z_{t-1}}\right) \tag{7}$$

$$\sin_\phi = \sin\left(^W\phi_{z_{t-1}}\right) \tag{8}$$

$$^W\mathbf{R}_{t-1} = \begin{pmatrix} \cos_\phi & -\sin_\phi \\ \sin_\phi & \cos_\phi \end{pmatrix} \tag{9}$$

$$^W\mathbf{p}_{t-1} = \begin{pmatrix} p_{x(t-1)} \\ p_{y(t-1)} \end{pmatrix} \tag{10}$$

$$^W\mathbf{p}_t =^W \mathbf{p}_{t-1} +^W \mathbf{R}_{t-1}\mathbf{v}_{t-1} dt + \frac{1}{2}^W\mathbf{R}_{t-1}\mathbf{a}_{t-1} dt^2 \tag{11}$$

$$= \begin{pmatrix} dt^2\left(0.5a_{x_{t-1}}\cos_\phi - 0.5a_{y_{t-1}}\sin_\phi\right) + dt\left(v_{x_{t-1}}\cos_\phi - v_{y_{t-1}}\sin_\phi\right) + p_{x(t-1)} \\ dt^2\left(0.5a_{x_{t-1}}\sin_\phi + 0.5a_{y_{t-1}}\cos_\phi\right) + dt\left(v_{x_{t-1}}\sin_\phi + v_{y_{t-1}}\cos_\phi\right) + p_{y(t-1)} \end{pmatrix} \tag{12}$$

2.1.5 Model

$$\mathbf{F}_t = \begin{pmatrix} p_{x_t} \\ p_{y_t} \\ v_{x_t} \\ v_{y_t} \\ a_{x_t} \\ a_{y_t} \\ \phi_t \\ \dot{\phi}_t \end{pmatrix} = \begin{pmatrix} dt^2\left(0.5a_{x_{t-1}}\cos_\phi - 0.5a_{y_{t-1}}\sin_\phi\right) + dt\left(v_{x_{t-1}}\cos_\phi - v_{y_{t-1}}\sin_\phi\right) + p_{x(t-1)} \\ dt^2\left(0.5a_{x_{t-1}}\sin_\phi + 0.5a_{y_{t-1}}\cos_\phi\right) + dt\left(v_{x_{t-1}}\sin_\phi + v_{y_{t-1}}\cos_\phi\right) + p_{y(t-1)} \\ a_{x_{t-1}} dt + v_{x_{t-1}} \\ a_{y_{t-1}} dt + v_{y_{t-1}} \\ a_{x_{t-1}} \\ a_{y_{t-1}} \\ \dot{\phi}_{z_{t-1}} dt + \phi_{z_{t-1}} \\ \dot{\phi}_{z_{t-1}} \end{pmatrix} \tag{13}$$

$$\mathbf{J}_t = \mathbf{F}\begin{pmatrix} \frac{\partial}{\partial p_x} & \frac{\partial}{\partial p_y} & \frac{\partial}{\partial v_x} & \frac{\partial}{\partial v_y} & \frac{\partial}{\partial a_x} & \frac{\partial}{\partial a_y} & 0 & \frac{\partial}{\partial \phi} \end{pmatrix} \tag{14}$$

$$= \begin{pmatrix} 1 & 0 & dt\cos_\phi & -dt\sin_\phi & 0.5dt^2\cos_\phi & -0.5dt^2\sin_\phi & 0 & 0 \\ 0 & 1 & dt\sin_\phi & dt\cos_\phi & 0.5dt^2\sin_\phi & 0.5dt^2\cos_\phi & 0 & 0 \\ 0 & 0 & 1 & 0 & dt & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & dt & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & dt \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \tag{15}$$

3 System Noise Model

3.0.1 Via Acceleration

The system noise is partly determined by the acceleration that the robot can experience.

$$a = a_x = a_y \tag{16}$$

$$\mathbf{a} = \begin{pmatrix} a \\ a \end{pmatrix} \tag{17}$$

$$\mathbf{v} = \begin{pmatrix} a \cdot dt \\ a \cdot dt \end{pmatrix} \tag{18}$$

$$\mathbf{p} = \frac{1}{2}\mathbf{R}_{t-1}\mathbf{a} \cdot dt^2 \tag{19}$$

$$= \begin{pmatrix} dt^2 \left(-0.5 \cdot a \cdot \sin_\phi + 0.5 \cdot a \cdot \cos_\phi\right) \\ dt^2 \left(0.5 \cdot a \cdot \sin_\phi + 0.5 \cdot a \cdot \cos_\phi\right) \end{pmatrix} \tag{20}$$

$$\mathbf{a}_{\text{noise}} = \begin{pmatrix} \mathbf{p} \\ \mathbf{v} \\ \mathbf{a} \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0.5\sqrt{2}dt^2 \cos\left(\phi_{z_{t-1}} + \frac{\pi}{4}\right) \\ 0.5\sqrt{2}dt^2 \sin\left(\phi_{z_{t-1}} + \frac{\pi}{4}\right) \\ dt \\ dt \\ 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} a \tag{21}$$

$$\mathbf{Q}_{\mathbf{a}} = \sigma_{\mathbf{a}}^2 \cdot \mathbf{a}_{\text{noise}} \cdot \mathbf{a}_{\text{noise}}^T = \sigma_{\mathbf{a}}^2 \begin{pmatrix} \frac{p_x^2}{a^2} & \frac{p_x p_y}{a^2} & \frac{dt p_x}{a} & \frac{dt p_x}{a} & \frac{p_x}{a} & \frac{p_x}{a} & 0 & 0 \\ \frac{p_x p_y}{a^2} & \frac{p_y^2}{a^2} & \frac{dt p_y}{a} & \frac{dt p_y}{a} & \frac{p_y}{a} & \frac{p_y}{a} & 0 & 0 \\ \frac{dt p_x}{a} & \frac{dt p_y}{a} & dt^2 & dt^2 & dt & dt & 0 & 0 \\ \frac{dt p_x}{a} & \frac{dt p_y}{a} & dt^2 & dt^2 & dt & dt & 0 & 0 \\ \frac{p_x}{a} & \frac{p_y}{a} & dt & dt & 1 & 1 & 0 & 0 \\ \frac{p_x}{a} & \frac{p_y}{a} & dt & dt & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{22}$$

3.0.2 Via Yaw Rate

The system noise is partly determined by the yaw rate that the robot can experience.

$$\phi_z = \dot{\phi}_z dt \tag{23}$$

$$\dot{\phi}_{z_{\text{noise}}} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \phi_z \\ \dot{\phi}_z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ dt \\ 1 \end{pmatrix} \dot{\phi}_z \tag{24}$$

$$\mathbf{Q}_{\text{yaw}} = \sigma_{\text{yaw}}^2 \cdot \dot{\phi}_{z_{\text{noise}}} \cdot \dot{\phi}_{z_{\text{noise}}}^T = \sigma_{\text{yaw}}^2 \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & dt^2 & dt \\ 0 & 0 & 0 & 0 & 0 & 0 & dt & 1 \end{pmatrix} \tag{25}$$

3.0.3 Final System Noise Matrix

$$\mathbf{Q} = \mathbf{Q_j} + \mathbf{Q}_{\text{yaw}} = \begin{pmatrix} \frac{\sigma_{\mathbf{a}}^2 p_x^2}{a^2} & \frac{\sigma_{\mathbf{a}}^2 p_x p_y}{a^2} & \frac{\sigma_{\mathbf{a}}^2 dt p_x}{a} & \frac{\sigma_{\mathbf{a}}^2 dt p_x}{a} & \frac{\sigma_{\mathbf{a}}^2 p_x}{a} & \frac{\sigma_{\mathbf{a}}^2 p_x}{a} & 0 & 0 \\ \frac{\sigma_{\mathbf{a}}^2 p_x p_y}{a^2} & \frac{\sigma_{\mathbf{a}}^2 p_y^2}{a^2} & \frac{\sigma_{\mathbf{a}}^2 dt p_y}{a} & \frac{\sigma_{\mathbf{a}}^2 dt p_y}{a} & \frac{\sigma_{\mathbf{a}}^2 p_y}{a} & \frac{\sigma_{\mathbf{a}}^2 p_y}{a} & 0 & 0 \\ \frac{\sigma_{\mathbf{a}}^2 dt p_x}{a} & \frac{\sigma_{\mathbf{a}}^2 dt p_y}{a} & \sigma_{\mathbf{a}}^2 dt^2 & \sigma_{\mathbf{a}}^2 dt^2 & \sigma_{\mathbf{a}}^2 dt & \sigma_{\mathbf{a}}^2 dt & 0 & 0 \\ \frac{\sigma_{\mathbf{a}}^2 dt p_x}{a} & \frac{\sigma_{\mathbf{a}}^2 dt p_y}{a} & \sigma_{\mathbf{a}}^2 dt^2 & \sigma_{\mathbf{a}}^2 dt^2 & \sigma_{\mathbf{a}}^2 dt & \sigma_{\mathbf{a}}^2 dt & 0 & 0 \\ \frac{\sigma_{\mathbf{a}}^2 p_x}{a} & \frac{\sigma_{\mathbf{a}}^2 p_y}{a} & \sigma_{\mathbf{a}}^2 dt & \sigma_{\mathbf{a}}^2 dt & \sigma_{\mathbf{a}}^2 & \sigma_{\mathbf{a}}^2 & 0 & 0 \\ \frac{\sigma_{\mathbf{a}}^2 p_x}{a} & \frac{\sigma_{\mathbf{a}}^2 p_y}{a} & \sigma_{\mathbf{a}}^2 dt & \sigma_{\mathbf{a}}^2 dt & \sigma_{\mathbf{a}}^2 & \sigma_{\mathbf{a}}^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \sigma_{\dot{\phi}_z}^2 dt^2 & \sigma_{\dot{\phi}_z}^2 dt \\ 0 & 0 & 0 & 0 & 0 & 0 & \sigma_{\dot{\phi}_z}^2 dt & \sigma_{\dot{\phi}_z}^2 \end{pmatrix} \tag{26}$$