15-351 / 15-650 / 02-613 Homework #6

Due: Friday, Dec. 4 by 11:59pm

You may discuss these problems with your current classmates, but you must write up your solutions independently, without using common notes or worksheets. You must indicate at the top of your homework who you worked with. Your write up should be clear and concise. You are trying to convince a skeptical reader that your answers are correct. Avoid pseudocode if possible. Your homework should be submitted via **GradeScope** as a typeset PDF. A LaTeX tutorial and template are available on the class website if you choose to use that system to typeset.

1. "**Thom's Problem.**" On graduate school visit days, professors need to meet with visiting students to explain to them how great CMU is. V
and professors have certain visi
various fixed time slots, and som
is to find an assignment of studen
possible.

https://eduassistpro.github.io/

Assignment Project Exam Help

More formally, we have:

- A set of visiting students $S = \{s_1, \ldots, s_m\}$.
- A set of professors $P = \{p_1, \ldots, t_n\}$.
- A set of time slots $T = \{t_1, \ldots, t_k\}$.
- A collection of sets $A_{p_i} \subseteq T$, where A_{p_i} gives the time slots t ilable.
- A collection of sets s_i is available.
- A collection of sets son p_i is interested in meeting.
- A collection of sets $M_{s_i} \subseteq P$, where M_{s_i} gives the professors that visiting student s_i wants to meet.

Add WeChat edu_assist_pro
Assignment Project
https://eduassistpro.github.io/
Add WeChat edu_assist_pro

Professor p_i will only meet with students on her list meet with professors on their list M_{s_i}. (We'll relax these requirements later.) A meetin t_i if both the student and professor are available. At any time, each student can each professor can meet with at most 1 student.

Example input:

Professor Meeting Preferences:

	Steve	Tom	Uri	Wei	Xin	Yang	Zhang
Prof. Abe	x			x		x	
Prof. Bob	x	x	x				
Prof. Carl	x			x		x	
Prof. Dave	x	x	x	x	x	x	x

S (columns); $= M_{Abe}$ (row Prof. Abe); P (rows)

Student Meeting Preferences:

	Prof. Abe	Prof. Bob	Prof. Carl	Prof. Dave
Steve			x	
Tom		x	x	
Uri	x	x		x
Wei			x	
Xin			x	
Yang			x	
Zhang			x	

$= M_{Steve}$ (row Steve)

Everyone's availability:

	2:00-2:30	2:30-3:00	3:00-3:30	3:30-4:00	4:00-4:30
Prof. Abe	x			x	
Prof. Bob	x	x	x		
Prof. Carl	x			x	
Prof. Dave		x		x	x
Steve	x	x		x	x
Tom		x	x	x	x
Uri	x		x	x	x
Wei	x	x		x	x
Xin	x		x	x	x
Yang	x	x	x	x	x
Zhang	x	x	x	x	x

T (columns); $= A_{Abe}$ (row Prof. Abe); $= A_{Steve}$ (row Steve)

(a) Write an integer linear program that finds maximum the number of (prof, student, time) meetings that can happen.

(b) Explain how to get the actual meeting schedule from your integer linear program.

(c) Suppose you are now also given, for each professor, a "bigshot score" b_{p_i} that is a positive number that says how important it is to satisfy that professor's demands. For every meeting that professor p_i takes with a student in her preference list M_{p_i}, you get b_{p_i} points. Describe how to modify your integer linear program to find a schedule that maximizes the number of bigshot points your schedule gets.

For the following NP-completeness proof problems, you can only assume that VERTEX COVER, SET COVER, INDEPENDENT SET, 3-SAT, HA G SALESMAN PROBLEM, which we discussed in class, are NP-co

https://eduassistpro.github.io/

2. Let INTEGER LINEAR PROG ization integer linear program has a solution of objective value $\geq$ a given k. Prove INTEGER LINEAR PROGRAMMING is NP-complete.

Assignment Project Exam Help

3. A double-Hamiltonian traversal in an undirected graph G is a closed walk that goes through every vertex in G exactly *twice*.

   Prove that the problem of testing whether a graph has a double-Hamiltonian

Assignment Project
Add WeChat edu_assist_pro

4. Suppose $G = (V, E)$ is an undirected graph. A *strongly independent set* is a subset S of vertices such that for any two vertices nd v. Consider the following STRONGLY INDEPEN E) and an integer k, does G have a strongly indep

https://eduassistpro.github.io/

   Prove that the STRONGLY INDEPENDENT SET problem is NP-complete.

Add WeChat edu_assist_pro

5. The SUBGRAPH-ISOMORPHISM PROBLEM takes two undir and asks whether G_1 is a subgraph of G_2. In other words, we ask whether there is an injective f f to map the vertices of G_1 to the vertices of G_2 such that there is an edge $\{u, v\}$ in G_1 exactly when $\{f(u), f(v)\}$ is in G_2.

   Prove that SUBGRAPH-ISOMORPHISM PROBLEM is NP-complete.

6. The REQUIRED PATHS SUBGRAPH problem takes as input an undirected graph G with nodes $v_1, \ldots, v_n$, an $n \times n$ symmetric matrix R of natural numbers, and an integer b, and asks "Is there a set S of b edges of G with the following property: between every pair of nodes v_i and v_j, $i \neq j$, there are at least R_{ij} disjoint paths (that is, paths sharing no other node except for the endpoints) using edges in S."

   Prove that this problem is NP-complete.