Midterm

Name: ______________________________

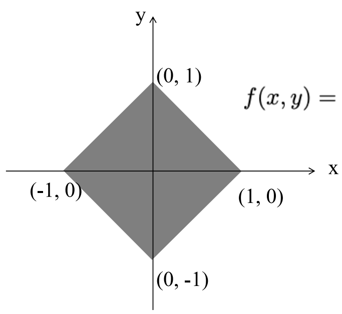
Andrew ID: ______________________________

Problem	Score	Max
1		5
2		5
[illegible]		
4		
Total		

Notes:

- Open notes exam. But no electronic equipments to be used (this includes cellphones, calculators, computers, tablets etc.)
- Precision and thoroughness are both appreciated. **Reaching the correct answer without the appropriate justification or incorrect reasoning will be penalized.**
- We have proof-read the problem set multiple times to ensure there are no bugs. Some details are however left out intentionally and are for you to figure out. So, if you really think some piece of information is missing and need to make an assumption to solve the problem — please go ahead; there is no need to run it by the instructors. Do not forget to mention the assumption made for solving the problem. Needless to say, unreasonable assumptions are, by definition, unreasonable.

1. **(5 pts)** Derive an expression for the 2D Fourier transform of the image shown below.



$$f(x,y) = \begin{cases} 1 & \text{inside the gray region} \\ 0 & \text{otherwise} \end{cases}$$

Hint: Useful Fourier pair

$$b(t) = \begin{cases} 1 & |t| \leq T_0 \\ 0 & \text{otherwise} \end{cases} \xleftrightarrow{1DFT} 2T_0 \ \text{sinc}(2T_0 f) = \frac{\sin 2\pi T_0 f}{\pi f}$$

2. **(5 pts)** Suppose that an image $i_1(x, y)$ has a radon transform $r_1(\alpha, \theta)$.
   Derive an expression for the radon transform for $i_2(x, y)$ defined as follows:

$$i_2(x, y) = i_1(x - x_0, y - y_0).$$

3. **(5 pts)** Consider the linear operator $\mathcal{A}$ defined as follows.

$$\mathcal{A}(f) = g \implies g(x,y) = f(x,y)a(x,y),$$

where $a(x,y)$ is a complex valued.

Derive an expression for the adjoint operator $\mathcal{A}^*$.

4. **(5 pts)** The Continuous Wavelet Transform (CWT) of a 1D signal $i_1(t)$ is given as

$$W_1(\tau, s) = \int_t i_1(t)\psi^*_{s,\tau}(t)dt,$$

where

$$\psi_{s,\tau}(t) = \frac{1}{\sqrt{s}}\psi_0\left(\frac{t-\tau}{s}\right).$$

Derive expressions for the CWT of the following signal in terms of $W_1(\tau, s)$ and other relevant variables.

- $i_2(t) = i_1(a(t - t_0))$, where a is a positive real-valued scalar.