Homework 3

Submission instructions.

- Submissions are due on Thursday 09/24 at 10.00pm ET
- Please upload scans of your solution in GradeScope (via Canvas)

Instructions

- Please solve all non-MATLAB problems using **only** paper and pen, without resorting to a computer.
- Please show all necessary steps to get the final answer. However, there is no need to be overly elaborate. Crisp and complete answers.
- For all MATLAB problems, include all code written to generate solutions.
- Please post all questions on the discussion board on the Piazza course website.
- If you feel some information is missing, you are welcome to make reasonable assumptions and proceed. Sometimes the omissions are intentional. Needless to say, only reasonable assumptions will be accepted.

1. *(Some elementary radon transforms and properties)* Consider an image $f(x, y)$ and its radon transform $r(\theta, \alpha)$.

   a) If $f(x, y)$ is rotationally-invariant, show that $r(\theta, \alpha) = r(\alpha)$, i.e., the radon transform is not a function of the projection angle θ.

   b) Let $r(\alpha) = \frac{1}{\sqrt{2\pi}} e^{-\frac{\alpha^2}{2}}$. Derive $f(x, y)$.

2. *(Scaling and Radon Transforms)* Let the Radon transform of the image $i_1(x, y)$ be $r_1(\alpha, \theta)$. Let $i_2(x, y) = i_1(ax, ay)$, be a scaled image.

   Find an expression for $r_2(\alpha, \theta)$, the Radon transform of $i_2(x, y)$, in terms of r_1.

3. *(Translations and Radon Transforms)* Let the Radon transform of an image $i_1(x, y)$ be $r_1(\alpha, \theta)$. Let $i_2(x, y) = i_1(x - a, y - b)$.

   Find an expression for $r_2(\alpha, \theta)$, the Radon transform of $i_2(x, y)$, in terms of r_1 and other quantities.

4. *(Implement filtered backprojection)* In `hw03.mat`, you are given radon transform measurements in the variable `rad`, corresponding to values in `alpha` and `theta`. Implement filtered backprojection.

   Notes:

(a) You are restricted to basic commands like `fft, fft2, conv, conv2, meshgrid, interp2`, and other basic commands. Specifically, you cannot use commands that do radon inversion like `iradon`.

(b) Follow the steps laid out in the lecture: (1) 1D Fourier transform of line integral, (2) Ramp filtering, (3) Inverse FT, and finally (4) Backprojection.

(c) Attend recitation

Problems below wont be graded. We wont release solutions as well. We are happy to verify yours, if you post on piazza, and discuss them

5. Suppose that image $f(x, y)$ has radon transform $r_f(\alpha, \theta)$, and the image $g(x, y)$ has radon transform $r_g(\alpha, \theta)$. Suppose that $r_g(\alpha, \theta) = \frac{d}{d\theta} r_f(\alpha, \theta)$. Derive an expression relating $g(x, y)$ to $f(x, y)$.

6. Suppose that image $f(x, y)$ has radon transform $r_f(\alpha, \theta)$, and the image $g(x, y)$ has radon transform $r_g(\alpha, \theta)$. Suppose that $g(x, y) = \frac{d}{dx} f(x, y)$. Derive an expression relating r_g in terms of r_f.

7. You are given the radon transform of an image. Derive an analytical expression for the image. It is ok if the values in your expression are approximate.