Imperial College London – Department of Computing

MSc in Computing Science

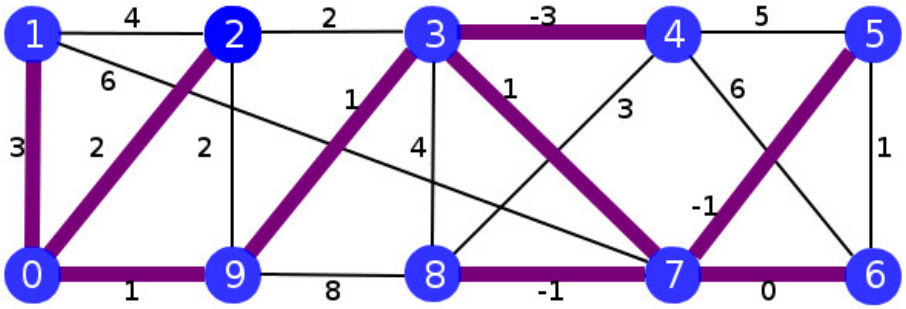
580: Algorithms
Tutorial: Graph Algorithms

1. Compute a minimum spanning tree for the graph below using Kruskal's algorithm. List the edges in the order they are added to the tree, and the weight of the tree after each iteration. Whenever there is a choice of edges, pick the edge $\{u, v\}$ that contains the lowest vertex id.

Answer: The edges added are: $(3, 4)$, $(5, 7)$, $(3, 9)$, $(0, 2)$, $(0, 1)$.

2. Compute a minimum spanning tree for the same graph using Prim's algorithm, starting at vertex 5. List the vertices in the order they are added to the tree, and the weight of the tree after each iteration. Whenever there is a choice of vertices, pick the one with the lowest id.

Answer: The vertices (plus weight of tree) added are: 5(0), 7(-1), 8(-2), 6(-2), 3(-1), 4(-4), 9(-3), 0(-2), 2(0), 1(3).



3. Compute the shortest (lowest weight) path from A to each vertex of the graph below using the Bellman–Ford algorithm. In each pass through the graph, use the order B, C, D, E, A to determine the next vertex u for which all edges (u, v) should be relaxed. List the estimated distance and the parent for each vertex after each pass.

Answer:

Iteration	Distance(parent)				
	A	B	C	D	E
0	0(-)	∞(-)	∞(-)	∞(-)	∞(-)
1	0(-)	8(A)	5(A)	2(A)	∞(-)
2	0(-)	8(A)	3(D)	2(A)	9(B)
3	0(-)	6(C)	3(D)	2(A)	9(B)
4	0(-)	6(C)	3(D)	2(A)	7(B)

4. Compute the shortest (lowest weight) path from A to each vertex of the same graph using Dijkstra's algorithm. List the estimated distance and the parent for each vertex after each iteration of the algorithm.

Answer:

Iteration	Edges relaxed	Distance(parent)				
		A	B	C	D	E
0		0(-)	∞(-)	∞(-)	∞(-)	∞(-)
1	(A, B) (A, C) (A, D)	0(-)	8(A)	5(A)	2(A)	∞(-)
2	(D, C) (D, E)	0(-)	8(A)	3(D)	2(A)	11(D)
3	(C, D) (C, B) (C, E)	0(-)	6(C)	3(D)	2(A)	9(C)
4	(B, E)	0(-)	6(C)	3(D)	2(A)	7(B)