

Weighted Graphs

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More Terminology

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- Each edge in a **weighted graph** has an associated co
- We denote the weight of the edge $\{u, v\}$ by $w(u, v)$

More Terminology

Definition (Tree)

A **tree** is a pair (G, r) where G is a connected, acyclic graph and r is a vertex of G , called the **root**.

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- A **nonrooted tree** is a connected, acyclic graph

More Terminology

Definition (Spanning Tree)

Given a graph $G = (V, E_G)$, a tree $T = (V, E_T)$ such that $E_T \subseteq E_G$ is a **spanning tree** for G .

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Given some network (road, phone, water ...) a **minimum spanning tree** (MST) is an important attribute

- Lowest cost way to connect all points

Minimum Spanning Tree

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Minimum Spanning Tree Problem

Given gra

ST of G .

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Analysis

Question

Given graph $G = (V, E)$, if you were to generate potential solutions for the MST problem, and then test them:

- Wh
- Wh

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Analysis

An MST for G will comprise

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Given this

- If suc

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Analysis

Exercise

Give a case-by-case definition of a function *min_edges* that returns a minimal weight set of n edges selected from array E .

- Wh
- Wh
- Ho

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Analysis

A minimal weight set of n edges, chosen from $E[1, \dots, i]$ is:

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- (Does the problem have optimal substructure
- $\text{min_edges}(|E|, |V| - 1)$ has $|E| \times (|V| - 1)$ subproblems
- Unfortunately, min_edges might not produce MST. Why?

Analysis

Update to *tree_edges* and only return a set of edges that forms a tree

tree_edges(Input: integer *i*, set E_T)

tree_edges(*i*, E) =

$$\begin{cases} E_T & \text{if } i = 1 \\ \min_w \{ E[i] \cup E_T \} & \text{if } i > 1 \text{ and } \{ E[i] \} \cup E_T \text{ is a tree} \\ \text{tree_edges}(i-1, E_T) & \text{otherwise} \end{cases}$$

- Subproblem now completes the tree (using recursion)
- If $\{E[i]\} \cup E_T$ not a tree, do not use $E[i]$
- If $|E_T| + (i-1) < |V| - 1$, insufficient edges
- *tree_edges*($|E|, \emptyset$) still has $|E| \times (|V| - 1)$ subproblems

A New Strategy

Have seen that the problem involves this choice

- Add edge i
- Do n

Have assu

- Maybe this time a **greedy** approach will actually work!
- A greedy algorithm picks the 'obvious' first step
- This is called making a **greedy choice**
- It leaves just one subproblem to solve

So, we identify edge g , the greedy edge, and continue with $E_T \cup \{E[g]\}$

The Greedy Choice

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The Greedy Choice

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What greedy choices are there when computing an MST?

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The Greedy Choice

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- Greedy is only good sometimes
- We have to show that the choice must lead to a correct solution
- (As y

Theorem

Let G be a connected, weighted graph. If e_m is an edge of least weight in G , then e_m is in some minimum spanning tree for

The general method of proving that the greedy choice is

- 1 Suppose you have an optimal solution to the problem
- 2 Show that it is still optimal when the greedy choice is included

Proof

T is some MST for G

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- If e_m is in T the theorem is true
- Suppose e_m is not in T

Let e_m

and let the w

$\{v, x\}$,

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Proof

Now construct T' by removing $\{v, x\}$ from T and adding $\{u, v\}$

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- Since T is a spanning tree, T' is a span
- Since T is an MST and $w_m \leq w$, T' is an MST
- e_m is in T'
- QED

Proof

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Can we keep adding the next least weight edge?

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More Greed

Our greedy choice can be made more general

Theorem

Let $G = (V, E)$ be a connected, weighted graph. Let E_T be a subset of E that is part of a minimum spanning tree. Let $e = \{u, v\}$ be an edge of E that is not in E_T . If e is not in a minimum spanning tree, then e is not in a minimum spanning tree.

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Proof

The proof is similar

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- Let T be the MST that contains E_m
- If e_m is in T , the theorem is true
- Sup

Let e_m
 E_{PQ} be

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T that is in
 and w

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Proof

Now construct T' by removing $\{x, y\}$ from T and adding $\{u, v\}$

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- Since T is a spanning tree, T' is a span
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