

Data Mining and Machine Learning

Assignment Project Exam Help

<https://eduassistpro.github.io/>

Add WeChat edu_assist_pro

Peter Jančovič



Data Mining

- Objective of Data Mining is to find structure and patterns in large, abstract data sets
 - Is the data homogeneous or does it consist of several separately i
 - Are there p
 - If so, do these patterns have interpretation?
 - Are there correlations in the data?
 - Is there redundancy in the data?



Partitioning data into “clusters”

- In this lecture we will start to develop tools to understand the structure of data that can be partitioned into (more or less) distinct subsets
- Can think of **Assignment Project Exam Help** g from distinct “sources” **<https://eduassistpro.github.io/>**
- We will consider three different techniques:
 - Clustering
 - Multi-modal statistical modelling (Gaussian Mixture Models – GMMs)
 - Decision trees



Clustering - Objectives

- To explain the motivation for clustering
 - To introduce the ideas of distance and distortion
 - To describe divisive clustering
 - To explain the relationship between clustering and decision trees
- <https://eduassistpro.github.io/>
Add WeChat edu_assist_pro

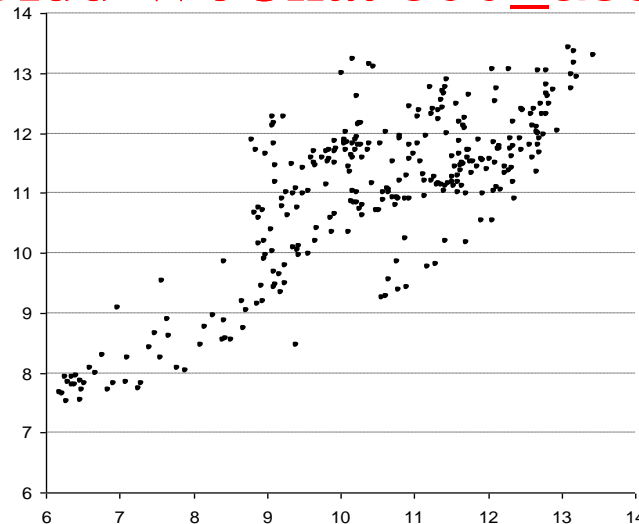


What does the data look like?

Assignment Project Exam Help

<https://eduassistpro.github.io/>

Add WeChat edu_assist_pro



Structure of data

- Typical real data is not uniformly distributed
- It has structure
- Variables may have different sources
- The data may have been generated from different “sources”
- The purpose of cluster analysis is to find this underlying structure automatically



Clusters and centroids

- Assume clusters are spherical - determined by centres
- Cluster centres are called centroids
- Questions: How many centroids do we need? Where should we put them?

Assignment Project Exam Help

<https://eduassistpro.github.io/>

Add WeChat edu_assist_pro



Distance

- A function $d(x,y)$ defined on pairs of points x and y is called a distance or metric if it satisfies:
 - $d(x,y) \geq 0$ and $d(x,y) = 0$ if and only if $x = y$
 - $d(x,y) = d(y,x)$ (symmetry)
 - $d(x,z) \leq d(x,y) + d(y,z)$ if x, y and z (triangle inequality)



Example metrics

- The most common metric is the Euclidean metric
- If $x = [x_1, x_2, \dots, x_N]$ and $y = [y_1, y_2, \dots, y_N]$ then:

$$d(x, y) = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2 + \dots + (x_N - y_N)^2}$$

- This is normal distance in space
- There are lots of others, but focus on this one



The L^p Metrics

- Euclidean distance is sometimes called the L^2 -metric

$$d_2(x, y) = \left[\sum_{n=1}^N (x_n - y_n)^2 \right]^{\frac{1}{2}}$$

Assignment Project Exam Help
<https://eduassistpro.github.io/>

- It is one of a family of metrics
- Add WeChat edu_assist_pro

$$d_p(x, y) = \left[\sum_{n=1}^N (x_n - y_n)^p \right]^{\frac{1}{p}}$$



Special L^p metrics

- $p=1$ – the ‘City Block’ metric

$$d_1(x, y) = \left[\sum_{n=1}^N |x_n - y_n| \right]$$

Assignment Project Exam Help

<https://eduassistpro.github.io/>

Add WeChat edu_assist_pro

- $p=\infty$

$$d_\infty(x, y) = \max_{n=1, \dots, N} |x_n - y_n|$$



Unit sphere

- For a metric d defined on N dimensional space, the unit sphere is the set of vectors \mathbf{x} such that $d(\mathbf{x}, \mathbf{0}) = 1$

Assignment Project Exam Help

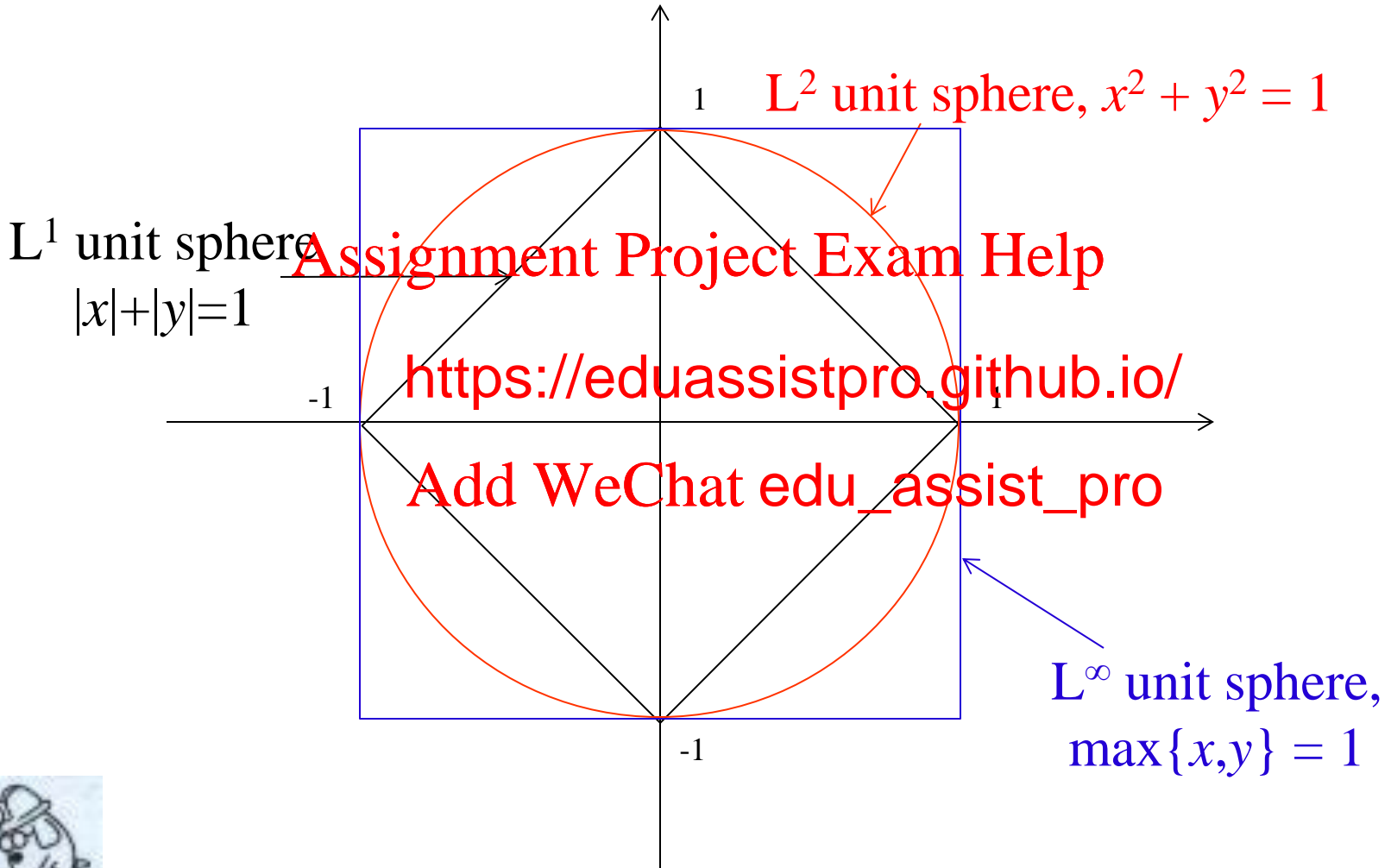
<https://eduassistpro.github.io/>

Add WeChat edu_assist_pro

- What do the unit spheres in 2D look like for these metrics?



Example Unit Spheres (2D)



Distortion

- Distortion is a measure of how well a set of centroids models a set of data
- Suppose we have:
 - data point
 - centroids
- For each data point y_t let $c_{i(t)}$ be its closest centroid
- In other words: $d(y_t, c_{i(t)}) = \min_m d(y_t, c_m)$



Distortion

- The distortion for the centroid set $C = c_1, \dots, c_M$ is defined by:

$$Dist(C) = \sum_{t=1}^T d(y_t, c_{i(t)})$$

<https://eduassistpro.github.io/>

- In other words, the distortion is the sum of distances between each data point and its centroid
- The task of clustering is to find a centroid set C such that the distortion $Dist(C)$ is minimised



Types of Clustering

- We will start with two types of cluster analysis:
 - Agglomerative clustering, or ‘bottom-up’ hierarchical clustering
 - Divisive clustering
- In the next lecture we will discuss more sophisticated clustering methods such as k -means clustering

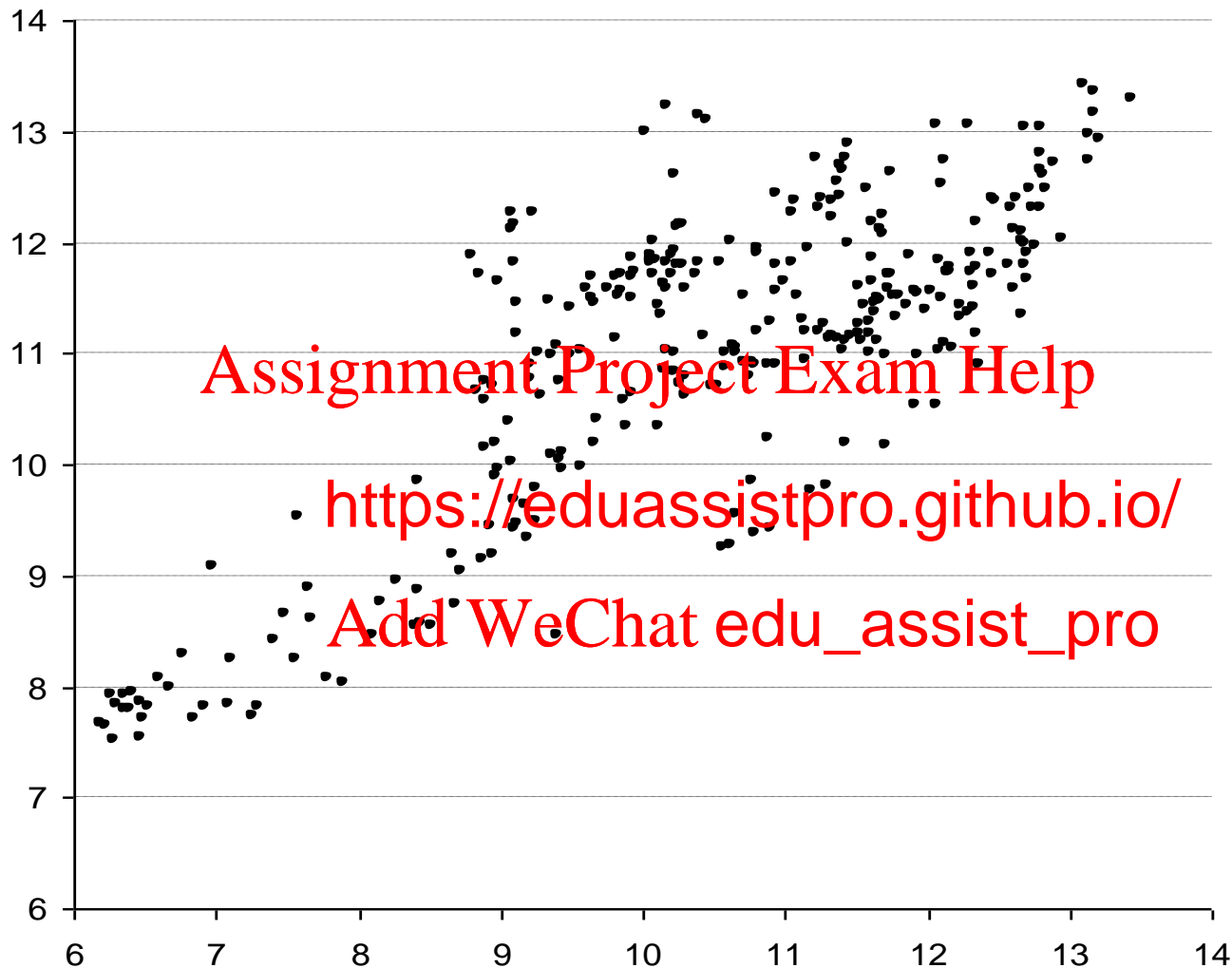


Agglomerative clustering

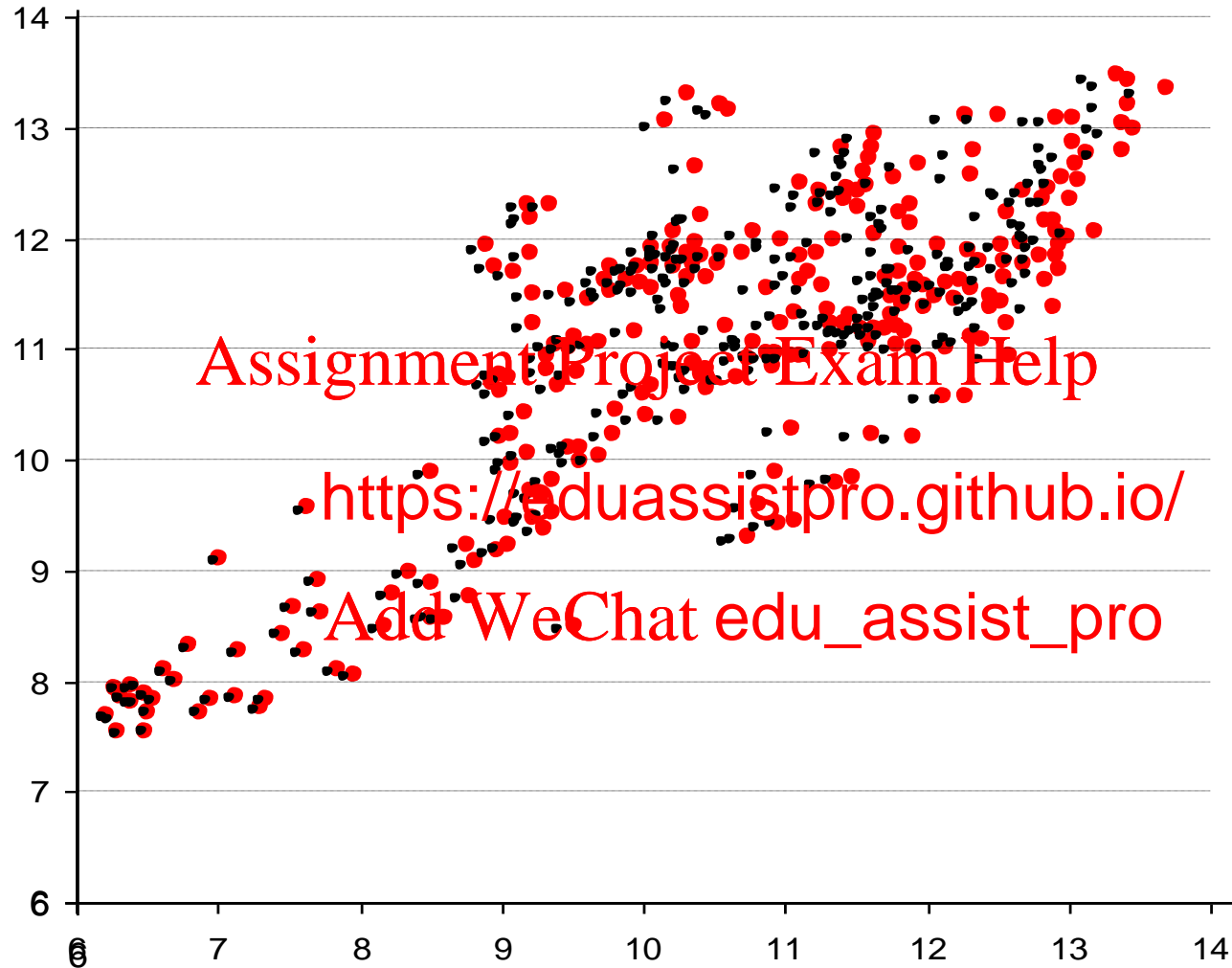
- Agglomerative clustering begins by assuming that each data point belongs to its own, unique, 1 point cluster — each point is a centroid
- Clusters are merged until a required number of clusters is reached
- The simplest agglomerative clustering algorithm is one which, at each stage, combines the two closest centroids into a single centroid



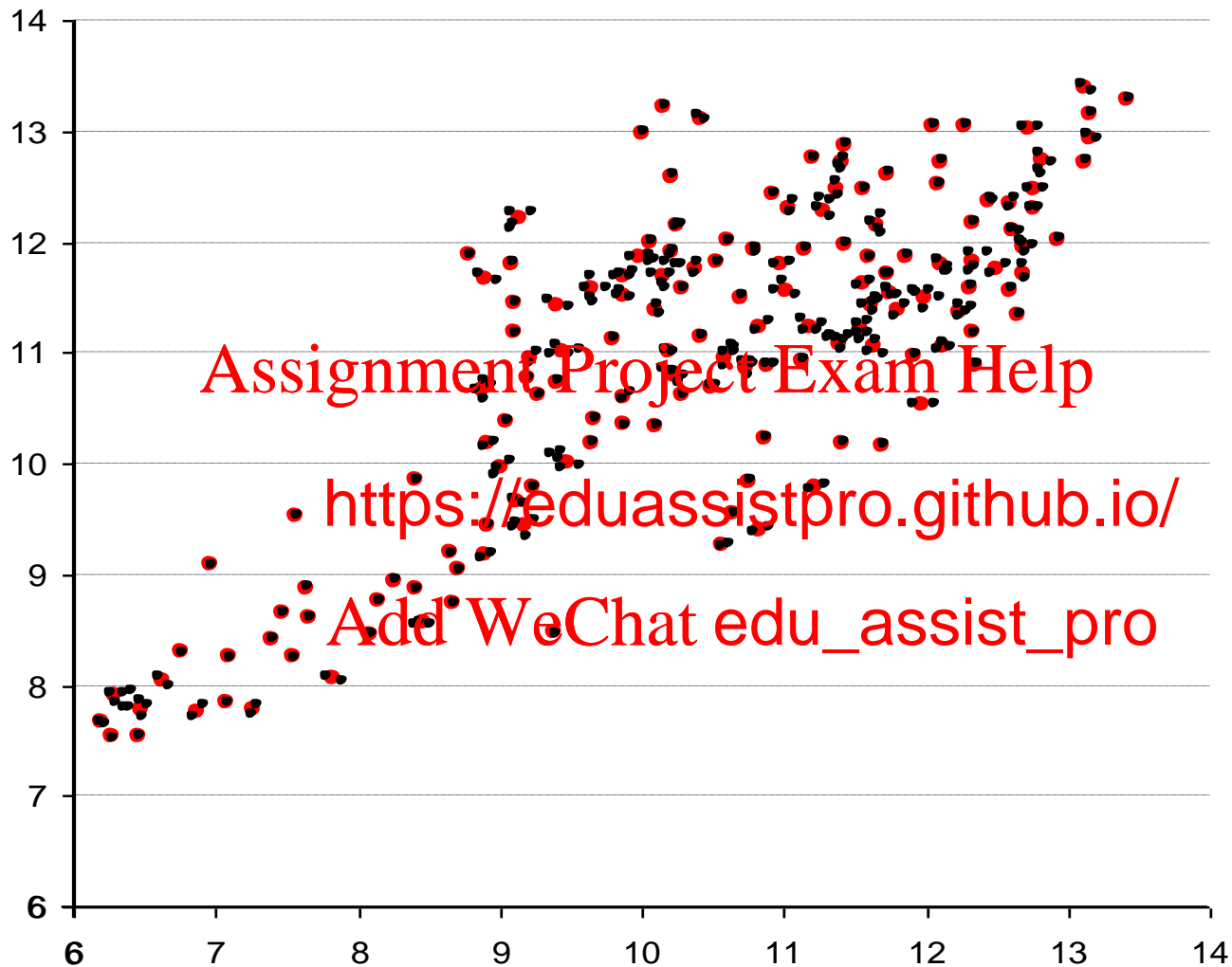
Original data (302 points)



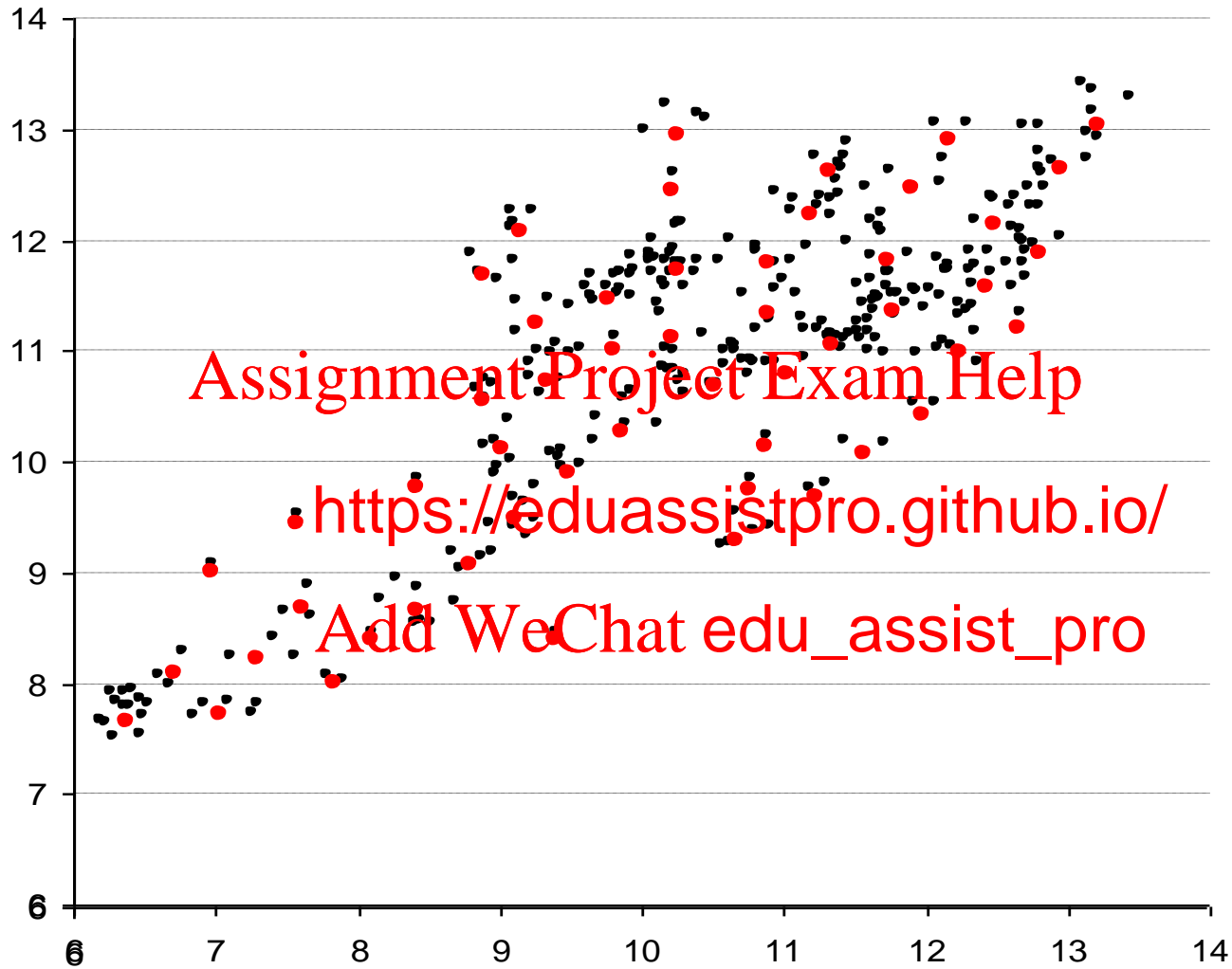
252 centroids



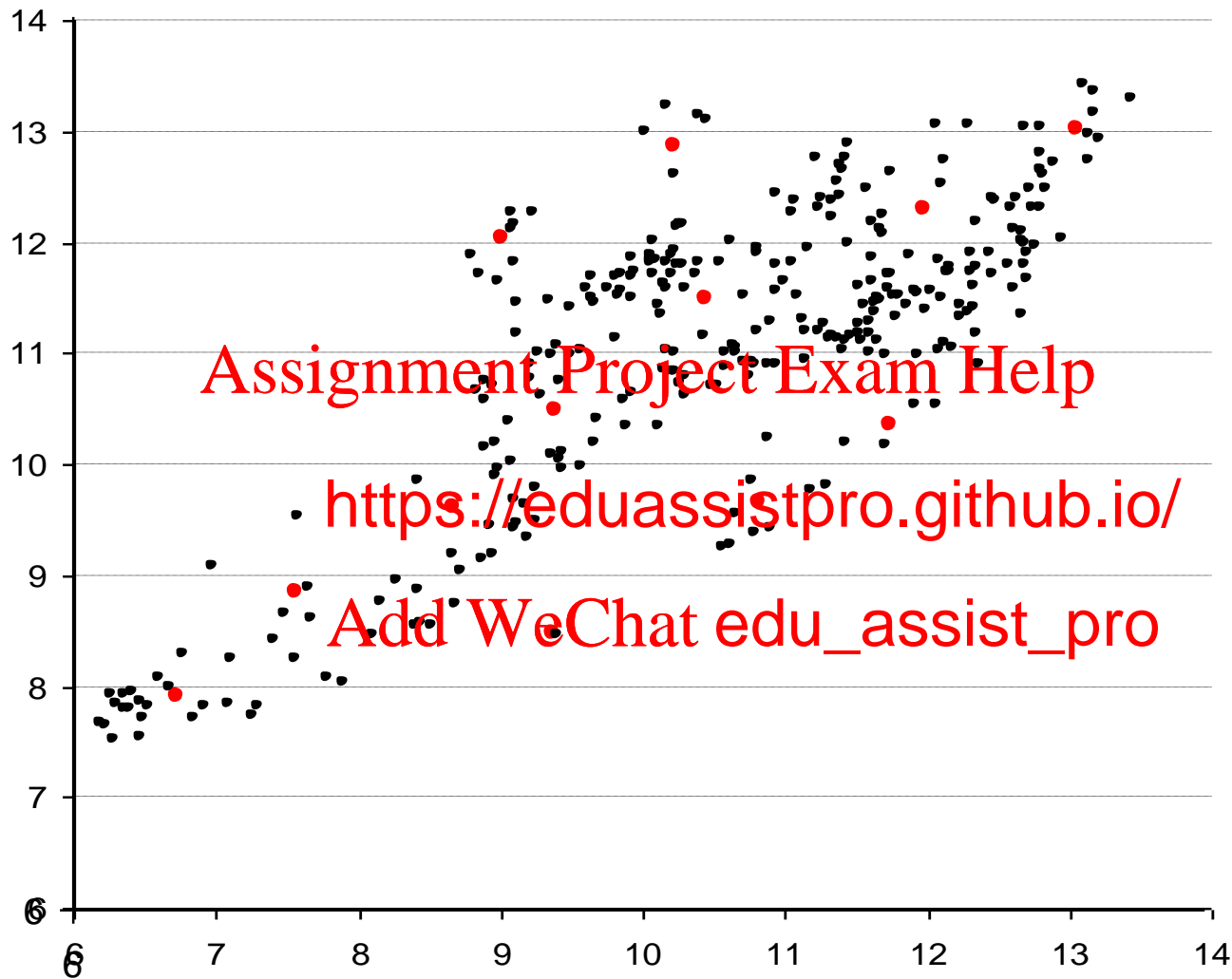
152 centroids



52 centroids



12 centroids



Optimality of agglomerative clustering

- The result of agglomerative clustering is not optimal
- Generally it does not result in a set of centroids C such that

<https://eduassistpro.github.io/>

- For example, Add WeChat edu_assist_pro
 - Outliers may be given their own centroids
 - Dense clusters may be given too few centroids

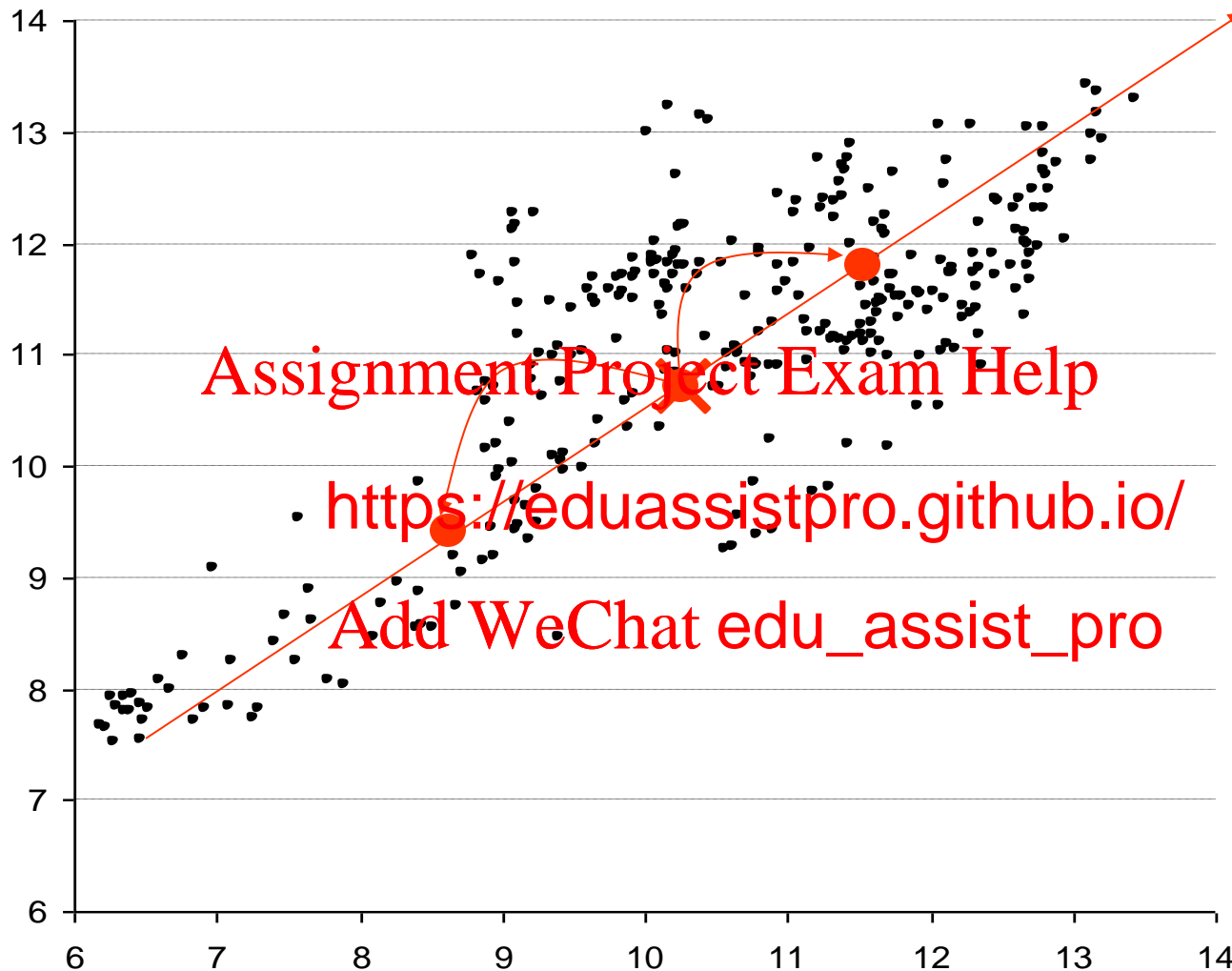


Divisive Clustering

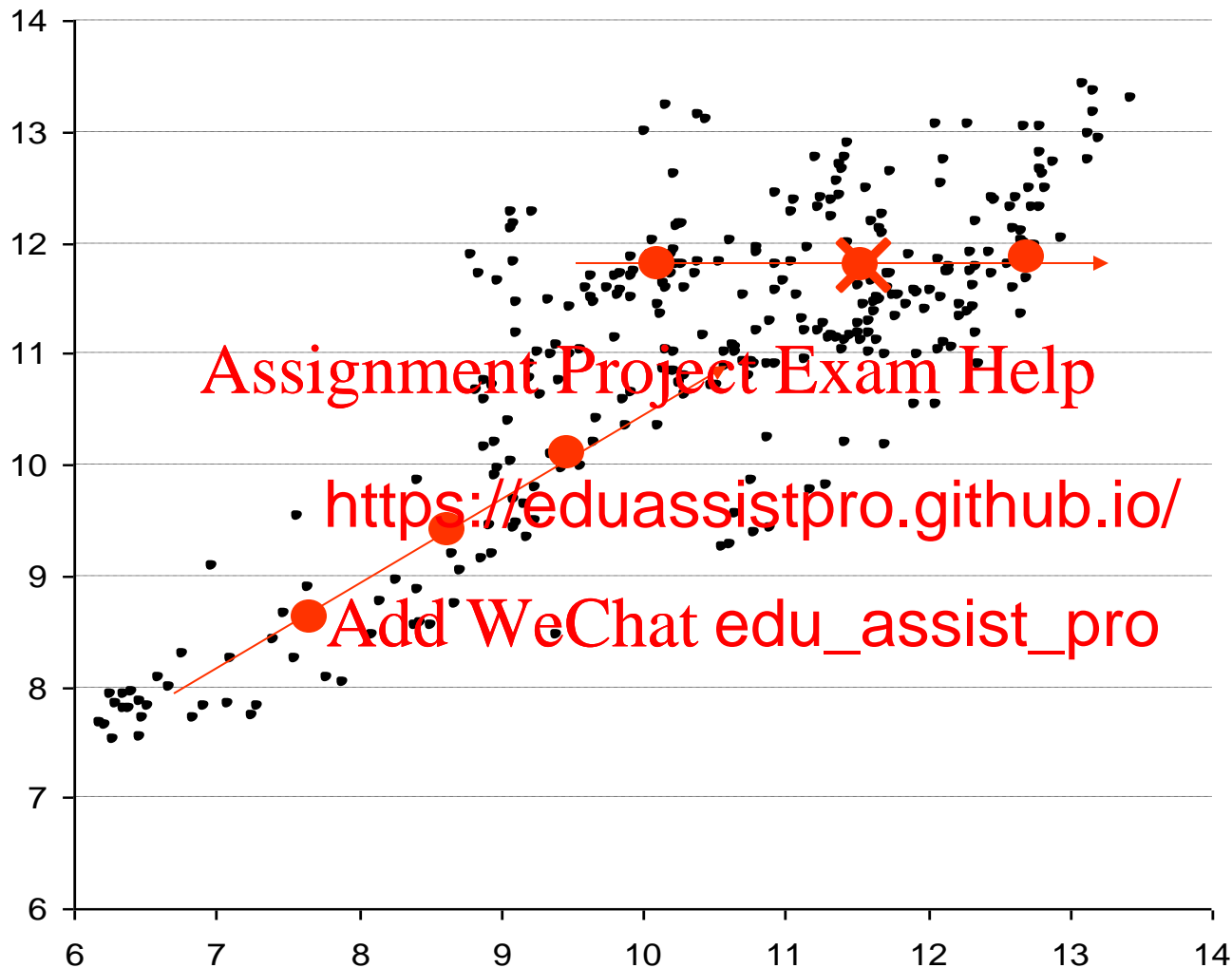
- Divisive clustering begins by assuming that there is just one centroid – typically in the centre of the set of data points
- That point is
- Then each of these is replaced by new centroids
- ...



Original data (302 points)



Original data (302 points)



Optimality of divisive clustering

- The result of agglomerative clustering is not optimal
- Generally it does not result in a set of centroids C such that

Assignment Project Exam Help

<https://eduassistpro.github.io/>

- Sequential decision making is generally suboptimal
 - Decisions are not reversible
 - If a point goes to a particular half of a partition it will never be re-allocated to the other half
 - Probably not how a human would do it



Decision tree interpretation

Top down
clustering -
divisive

Single centroid - whole set

Assignment Project Exam Help

<https://eduassistpro.github.io/>

Add WeChat edu_assist_pro

Multiple centroids – one per data point

Bottom up
clustering -
agglomerative



Optimality

- An ‘optimal’ set of centroids is one which minimises the distortion
- In general, neither method gives optimal sets of centroids <https://eduassistpro.github.io/>
- A more principled approach is to think of distortion as a function of the centroid set and minimize it



Notation and method

- N dimensional space
- T data points $X = \{x_1, \dots, x_T\}$
- K centroids
- Calculate <https://eduassistpro.github.io/>

Add WeChat edu_assist_pro

$$\frac{d}{dc_k^n} \text{Dist}(C)$$

for each k and n , set to zero and solve



Summary

- Distance metrics and distortion
 - Agglomerative clustering
 - Divisive clu
 - Decision tre
- Assignment Project Exam Help**
<https://eduassistpro.github.io/>
Add WeChat edu_assist_pro

