Team Assignment 2 [63 Points]

This assignment is to be prepared in the team you have been assigned to. Upload a text report written in managerial style, of at most 12 pages via Canvas (11 pt font). Next to the text report, submit the zipped Excel and CPLEX files used for obtaining the answers.

Give your files the following name: **[Team Id] SCDA TeamAssg2**, where "Team Id" is your team code. Mention your names and student numbers also in the report itself.

Here are some writing guidelines for the assignment report:

1. When you are asked for a mathematical formulation, please present the mathematical model with properly defined notation. Examples of mathematical models can be found in lecture slides. You may provide necessary explanations for your objective function and constraints. In grading, the correctness, clarity and conciseness will be taken into account.

2. In general, be considerate of your audience with clear format and structure, highlighting important messages, etc.

3. Add detailed comments in your CPLEX models (for each variable, parameter, constraint, and the objective function).

4. When you are asked to fin   simply provide a screenshot of the CPLEX output.   t. For example, do not sim   need to be produced in the optimal solution.

Deadline and submission: Friday, 14 October 2022, 22:00 CET. Su   **Canvas**. Assignments submitted via e-mail are considered as invalid submissions. All assignments are checked on **plagiarism**.

Problem 1 Theoretical questions [6 Points]

Consider the following statements about any IP problem (in maximization form) and its LP relaxation. Label each of the statements as True or False, and then justify your answer:

1. The feasible region for the LP relaxation is a subset of the feasible region for the IP problem [2 points].

2. If an optimal solution for the LP relaxation is an integer solution, then the optimal value of the objective function is the same for both problems [2 points].

3. If a noninteger solution is feasible for the LP relaxation, then the nearest integer solution (rounding each variable to the nearest integer) is a feasible solution for the IP problem [2 points].

Problem 2 Branch and Bound [15 Points]

The goal of this exercise is to solve the integer programming problem defined below using the branch and bound algorithm explained in class. To do so, use LP relaxations to produce lower bounds. The advantage of using upper bounds for a minimization problem is explained later in Section 2.1. You may choose whether or not to use upper bounds in your tree, but taking advantage of them may lead to solving the problem faster. In order to find the solution to each LP relaxation, you should use Excel solver (do **not** use CPLEX for this exercise). Note that the

$$\text{Assignment Project Exam Help} \tag{1a}$$

$$\text{subject to:} \quad 4x_3 \geq 3 \tag{1b}$$

$$\text{https://eduassistpro.github.io/} \tag{1c}$$

$$4x_3 + 5x_5 \geq 7 \tag{1d}$$

$$\text{Add WeChat edu\_assist\_pro} \tag{1c}$$

$$6x_2 + 3x_4 \geq 8 \tag{1e}$$

$$x_i \geq 0 \text{ and integer,} \quad i \in \{1, 2, 3, 4, 5\} \tag{1f}$$

To create the branch and bound tree, make use of the following rules:

A) Branch on the **variable** with the highest, non-integer value in your current node;

B) To select which **node** should be branched next, pick the one with the lowest lower bound.

In your report, include your branch and bound tree [3 Points], and provide the Excel file used to compute your solutions (make sure to have one different tab for each LP problem you solve) [7 Points]. Moreover, include your answer to the following questions:

i) Why does solving an LP relaxation provide a lower bound [1 Points]?

ii) Which nodes did you prune (fathom) and why (pruning or fathoming a node means choosing **not** to further explore that node) [2 Points]?

iii) What is the optimal objective function value? What are the optimal values for the decision variables [2 Points]?

2.1   Upper bounds in a minimization problem

In this exercise it is helpful to also use an upper bound while constructing the branch and bound tree. While using lower bounds is essential, upper bounds is optional but potentially advantageous. Using the upper bound has the advantage that it may give you a reason to prune (or fathom) a node in the tree. Thus, avoiding the exploration of certain nodes.

We illustrate now how to use upper bounds, and later explain how to compute them. Consider the tree in Figure 1. Assume that tree is the result of the first steps of the Branch and Bound algorithm: node 1 is the root node, then after branching on one variable we obtain nodes 2 and 3. In the Figure, we reported two figures next to the leaf nodes: the top one is an upper bound we found for that problem, while the bottom one is a lower bound. We observe that the upper bound in node 2 is strictly lower than the lower bound in node 3. This implies that when we would continue to branch on node 3, we will never find a solution to the minimization problem that is lower than the one we will find when we branch further on node 2. This means that we can prune/fathom node 3 in this case. We call this operation *pruning by bound*.

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Figure 1: Example of the use of an upper bound for a minimization problem. W
lower bound of node 3 is strictly larger than the upper bound of node 2.

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Next, we explain how to compute such an upper bound. Consider the setting that we have a problem with two decision variables x_1 and x_2 with objective function coefficients 2 and 3 respectively. Assume that we found a solution to the LP relaxation where the values for the optimal solution are $1, 3$ and $6, 6$ respectively and we found a lower bound of 21. To find the upper bound, we only need to round all non-integer solution values to the next integer solution. In this case, we set $x_1 = 2$ and $x_2 = 7$. Then, the upper bound can be calculated as: $2 * 2 + 7 * 3 = 25$.

To summarize, for a given node, we compute the upper bound by rounding up all non-integer solutions that we found when computing lower bound (always round up, never down). Finally, for the nodes with an integer solution, we note that the lower bound is identical to the upper bound.

Problem 3   Purchasing at Antwerp Airlines [6 points]

Antwerp Airlines is considering the purchase of new long-, medium-, and short-range jet passenger airplanes. The purchase price would be $335 million for each long-range plane, $250 million for each medium-range plane, and $175 million for each short-range plane. The board of directors has authorized a maximum commitment of $7.5 billion for these purchases. Regardless of which airplanes are purchased, air travel of all distances is expected to be sufficiently large that these planes would be utilized at essentially maximum capacity. It is estimated that the net annual profit would be $21 million per long-range plane, $15 million per medium-range plane, and $11.5 million per short-range plane.

It is predicted that enough trained pilots will be available to the company to crew 30 new airplanes. If only short-range planes were purchased, the maintenance facilities would be able to handle 40 new planes. However, each medium-range plane is equivalent to 1.33 short-range planes, and each long-range plane is equivalent to 1.67 short-range planes in terms of their use of the maintenance facilities.

Using the preceding data, management wishes to know how many planes of each type should be purchased to maximize profit.

3.1   Formulation [4 Points]

Formulate an integer progr   cision
variable and constraint. Ke

3.2   Solution [2 Points]

Calculate the maximum amount of profit received and the optimal solution by solving the IP described in question 3.1 using CPLEX. Interpret your solution.

Problem 4   Production scheduling at SleepingBaby [**9 Points**]

4.1   Production scheduling at SleepingBaby [**4 Points**]

SleepingBaby runs a day and a night shift. No matter how many units are produced, the only production cost during a shift is a setup cost. It costs $8000 to run the day shift and $5500 to run the night shift. Demand for the next two days is as follows: day 1, 1000; night 1, 3800; day 2, 1000; night 2, 3800. It costs $3 per unit to hold a unit in inventory for a shift and there are no initial inventories. Formulate an IP to determine a production schedule that minimizes the sum of setup and inventory costs. All demand must be met on time and backlogging is not allowed. Keep the notation as general and as compact as possible.

4.2   Solve the model with CPLEX [**3 Points**]

Calculate the minimum cost for the production scheduling problem. Report the objective function value and the optimal solution. Interpret your solution.

4.3   Increasing energy prices [**2 Points**]

Experts predict that the per- sing energy prices. Determine a b . What happens to the average inventory level? Explain your answer.

Problem 5   Capacity planning at CheapEnergy [10 Points]

5.1   Capacity planning [4 Points]

CheapEnergy believes it will need the generating capacity shown in Table 1 during the next five years. The company has a choice of building (and then operating) power plants with the specifications shown in Table 2. If the company decides to build a plant, the plant can generate power starting from year 1. In each year, the company can choose whether or not to operate the plant without facing any additional (re-)opening or shutdown cost (only the annual operating cost is relevant if the plant operates). Formulate an IP to minimize the total costs of meeting the generating capacity requirements of the next five years. Keep the notation as general and as compact as possible.

Year	Generating Capacity (Million kWh)
1	100
2	130
3	150
4	170
5	190

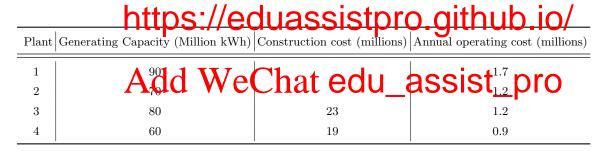| Plant	Generating Capacity (Million kWh)	Construction cost (millions)	Annual operating cost (millions)
1	90		1.7
2	70		1.2
3	80	23	1.2
4	60	19	0.9

Table 2: Power plants with specifications

5.2   Solve the model with CPLEX [3 Points]

Calculate the minimum cost for the capacity planning problem. Report the objective function value and the optimal solution. Interpret your solution.

5.3   Reopening and shutting down plants [3 Points]

Suppose that at the beginning of year 1, all power plants have been constructed and are in operation. At the beginning of each year, CheapEnergy may shut down a plant that is operating or reopen a shut-down plant. The costs associated with reopening or shutting down a plant are shown in Table 3. Formulate an IP to minimize the total cost of meeting demands of the next five years. Keep the notation as general and as compact as possible. You can use the following decision variables:

- $a_{it} = \begin{cases} 1 & \text{if plant } i \text{ is operated during year } t \\ 0 & \text{otherwise} \end{cases}$

- $b_{it} = \begin{cases} 1 & \text{if plant } i \text{ is shut down at the end of year } t \\ 0 & \text{otherwise} \end{cases}$

- $c_{it} = \begin{cases} 1 & \text{if plant } i \text{ is reopened at the beginning of year } t \\ 0 & \text{otherwise} \end{cases}$

You must ensure that if $a_{it} = 1$ and $a_{i,t+1} = 0$, then $b_{it} = 1$. You must also ensure that if $a_{i,t-1} = 0$ and $a_{it} = 1$, then $c_{it} = 1$. The model does **not** need to be solved with CPLEX.

Plant	Reopening cost (millions)	Shutdown cost (millions)
1	1.9	1.7
2	1.5	1.2
3	1.6	1.3
4	1.1	0.8

Table 3: Reopening cost and shutdown cost for all power plants

Problem 6   Project assignment at ConsultantX [17 Points]

ConsultantX is a small but fast growing company that has to decide on a day-to-day basis which one-day projects to undertake. Currently, ConsultantX has m employees and each employee is scheduled to work on such projects for a_i hours per day $(i = 1, 2, ..., m)$. Each of the one-day projects has a duration of d_j hours to complete and only one employee at a time can carry out the whole project $(j = 1, 2, ..., n)$. If project j is carried out, ConsultantX receives a profit of p_j. Clearly, the company is interested in undertaking the projects for which the total amount of profit received is maximal.

The following data is provided by ConsultantX:

	Employee 1	Employee 2	Employee 3	Employee 4	Employee 5	Employee 6
Work duration a_i (hours)	6	5	7	9	5	7
Expert person?	Yes	Yes	No	No	No	Yes

(a) Employees

	Project 1	Project 2	Project 3	Project 4	Project 5	Project 6	Project 7	Project 8	Project 9	Project 10
Duration d_j (hours)	3	3	3	2	5	4	2	2	5	6
Profit p_j (×\$100)	7	5	3	3	3	8	4	3	2	5
Difficult project?	No	Yes	No	Yes	No	Yes	Yes	Yes	No	Yes

(b) Projects

Table 4: Data Project Select

6.1   Formulation [6 Points]

Formulate an integer programming (IP) model for this problem and give a short description of each decision variable and constraint. Keep the notation as general and as compact as possible.

6.2   Solution [3 Points]

Calculate the maximum amount of profit received and the optimal solution by solving the IP described in previous subquestion using CPLEX. Interpret your solution.

6.3   Impact of Skill Level [2 Points]

Assume that ConsultantX has expert employees that can carry out any project and junior employees that can only carry out easy projects. Reformulate your IP model from question 6.1 and calculate the maximum amount of profit received in this case and provide and interpret the new optimal solution.

6.4   Logical Constraints [6 Points]

Please formulate the following requirements with proper logical constraints. In answering this question, you do not need to resolve the model; your answers will be graded based only on the correctness of the logical constraints.

(a) Projects 1 and 4 cannot be carried out by the same employee [1 Point].

(b) If an employee undertakes both Projects 4 and 5, then he or she must not undertake Project 7 [1 Point].

(c) If an employee undertakes none of Projects 1, 2, 3, then he or she must not undertake Project 5 [1 Point].

(d) If an employee undertakes none of Projects 1, 2, 3, then he or she must not undertake both Projects 5 and 6 [1 Point].

(e) If Employee 6 undertakes five projects or more, project 8 must not be assigned to him/her. This constraint can be formulated as a big-M constraint, but it is **not** mandatory to do so. In case you formulate a big-M constraint, you have to specify for which value (or range of values) for M that the constraint would be valid [2 Points].