BU CS 332 – Theory of Computation

Lecture 9:

- Turing Mac

Reading:

ser Ch 3.1, 3.3

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Turing Machines – Motivation

We've seen finite automata as a restricted model of computation

Finite Automata / Regular Expressions

- Can do simple pattern matching (e.g., substrings), check parity, addition
- Can't perform u
- Can't recognize palindromes

Somewhat more powerful (not in this course):

Pushdown Automata / Context-Free Grammars

- Can count and compare, parse math expressions
- Can't recognize $\{a^n b^n c^n \mid n \geq 0\}$

Turing Machines – Motivation

Goal:

Define a model of computation that is

1) General purp thms that can be implemented nguage.

2) Mathematically simple. We can hope to prove that things are not computable in this model.

A Brief

1900 – Hilbert’s Tenth Problem

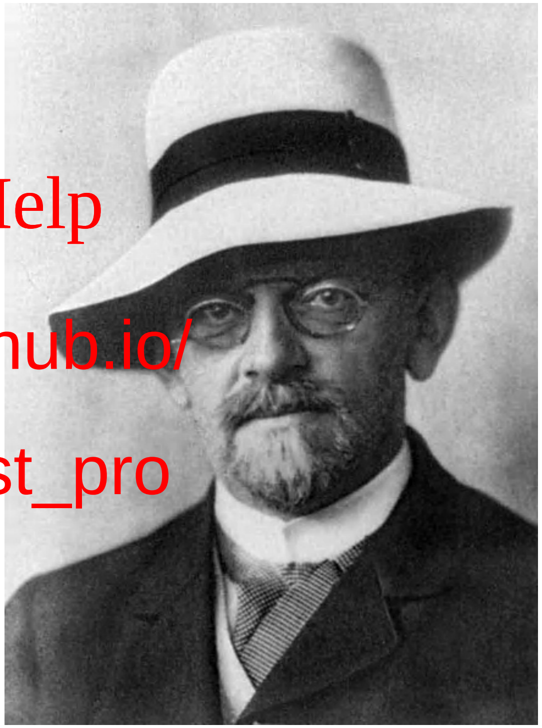
Given a Diophantine e
number of unknown quantities and with
rational integral numerical coefficients: T
devise a process according to which it can
be determined in a finite number of
operations whether the equation is
solvable in rational integers.

David Hilbert 1862-1943

1928 – The *Entscheidungsproblem*

The "Decision Problem"

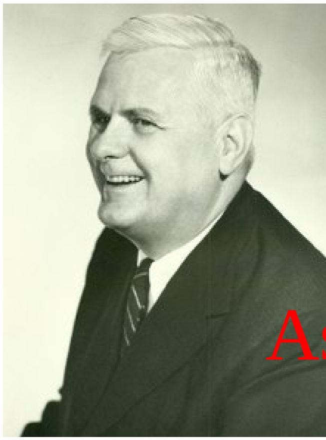
decides whethe
logically valid?



Wilhelm Ackermann 1896-1962

David Hilbert 1862-1943

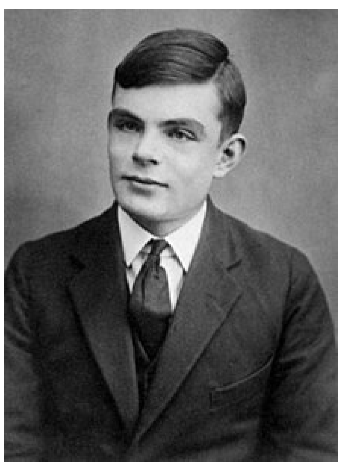
1936 – Solution to the *Entscheidungsproblem*



Alonzo Church 1903-199

"An unsolvable problem of elementary number theory“

Model of computation: λ-calculus (CS 320)



Alan Turing 1912-1954

“On computable numbers, with an application to the *Entscheidungsproblem*”

Model of computation: Turing Machine

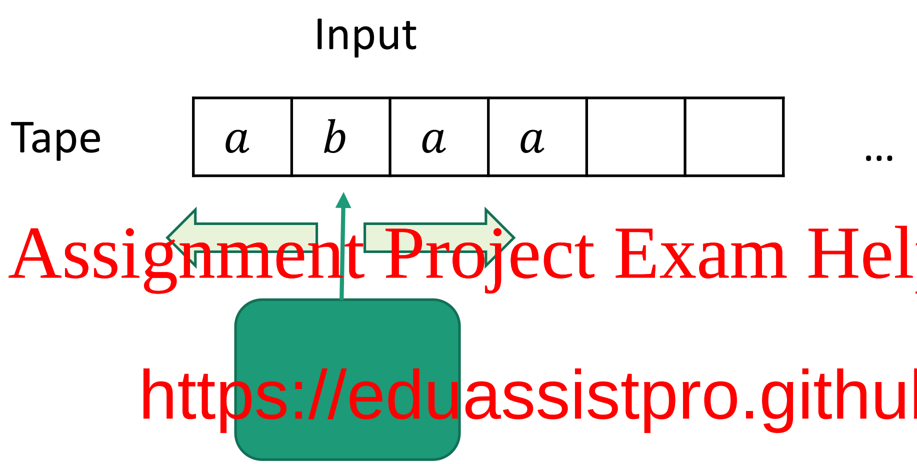
Turing

Assignment Project Exam Help

https://eduassistpro.github.io/

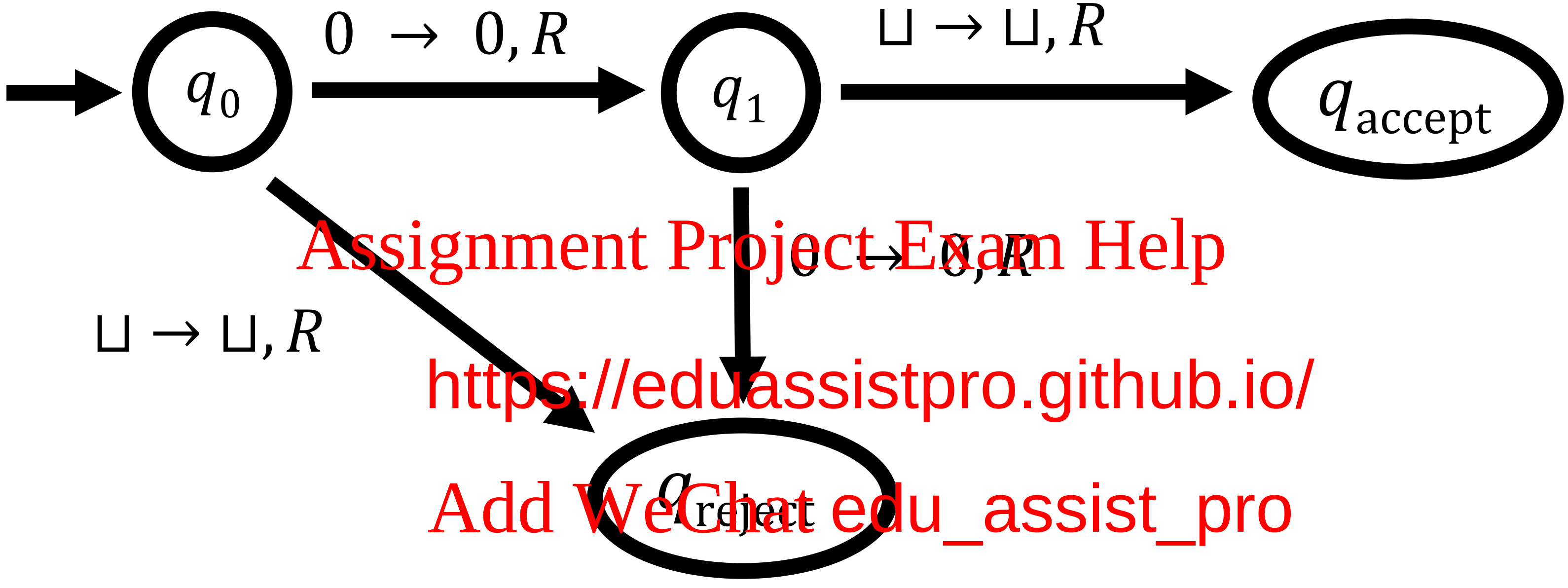
Add WeChat edu_assist_pro

The Basic Turing Machine (TM)



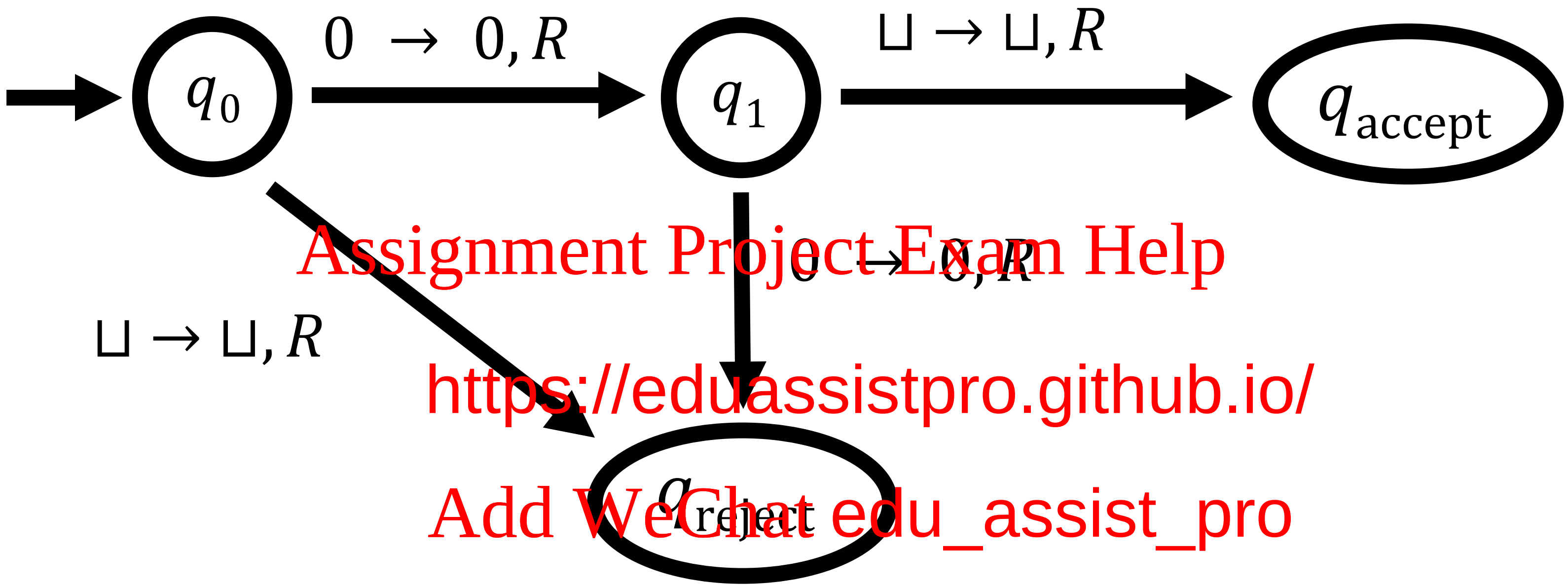
- Input is written on an infin tape
- Head can both read and write, and move in both directions
- Computation halts as soon as control reaches "accept" or "reject" state

Example



q_0
$0 \to 0, R$
q_1
$\sqcup \to \sqcup, R$
q_{accept}
$0 \to 0, R$
$\sqcup \to \sqcup, R$
q_{reject}

Example



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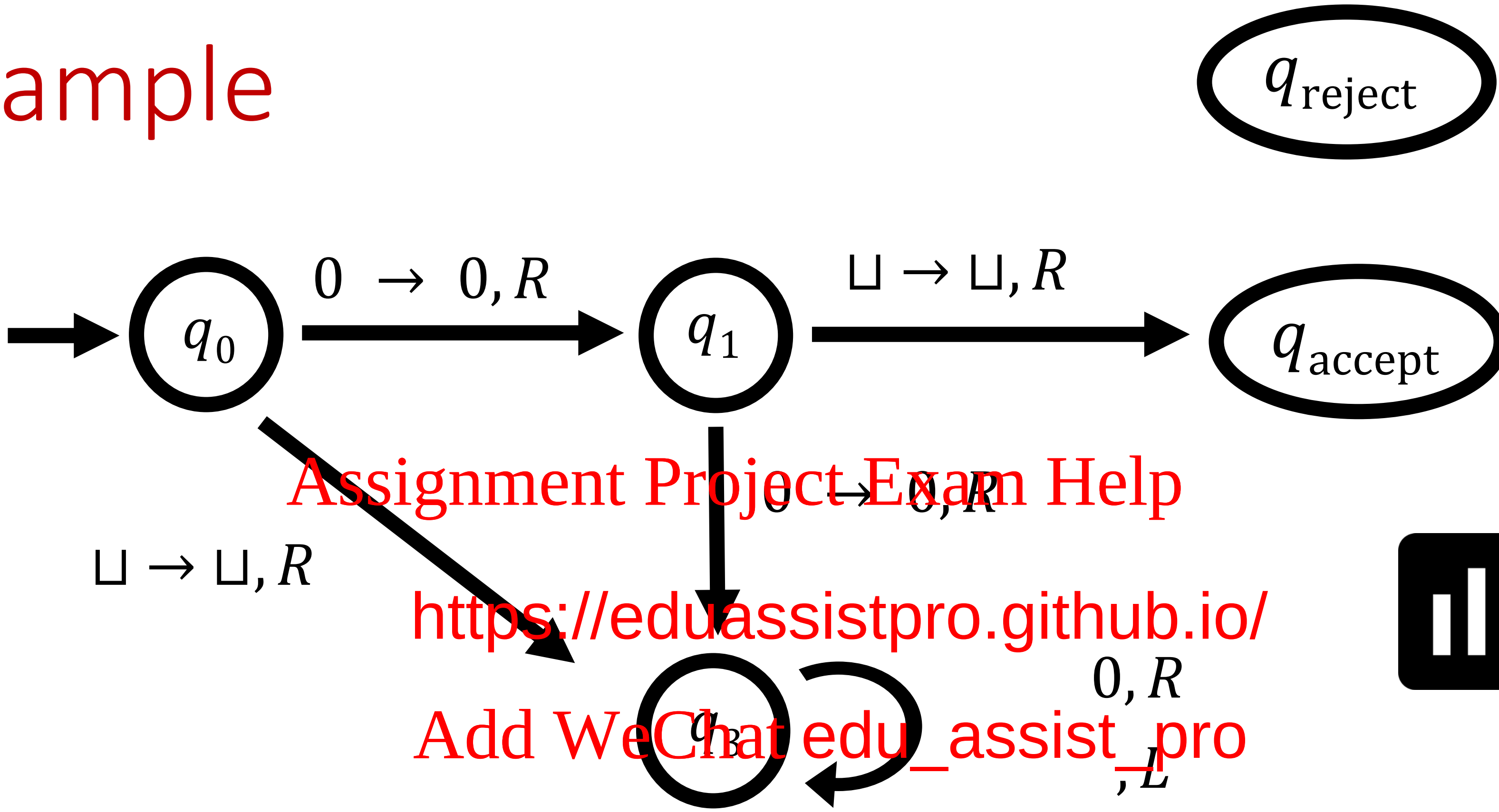
q_{reject}

Example



What does this TM do on input 000?

a) Halt and accept
b) Halt and reject
c) Halt in state q_3
d) Loop forever without halting

Three Levels of Abstraction

High-Level Description

An algorithm (like CS 330)

Implementation-Level Description

Describe (in Engli a TM

- How to move the head
- What to write on the tape

Low-Level Description

State diagram or formal specification

Example

Determine if a string $w \in A = \left\{0^{2^n} \,\middle|\, n \geq 0\right\}$

High-Level Description

Repeat the follow

- If there is exactly one 0 in
- If there is an odd number of 0s in w (> 1), reject
- Delete half of the 0s in w

Example

Determine if a string $w \in A = \{0^{2^n} \mid n \geq 0\}$

Implementation-Level Description

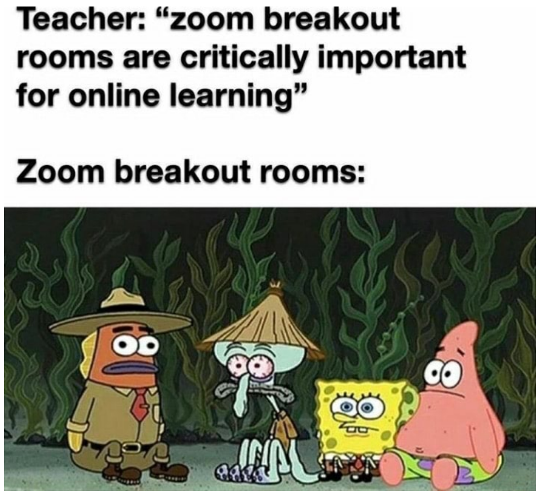
1. While moving right:
   a) Cross off every other 0
   b) If there is exactly one 0 whe he right end of the tape, accept
   c) If there is an odd number of 0s when we reach the right end of the tape, reject
2. Return the head to the left end of the tape
3. Go back to step 1

Example

Determine if a string $w \in A = \{0^{2^n} \mid n \geq 0\}$

Low-Level Description

TMs vs. Finite Automata



Formal Definition of a TM

A TM is a 7-tuple $M = (Q, \Sigma, \Gamma, \delta, q_0, q_{\text{accept}}, q_{\text{reject}})$

- Q is a finite set of states
- Σ is the input alphabet (does **not** include $\sqcup$)
- Γ is the tape al Σ)
- δ is the transiti

...mor ter

- $q_0 \in Q$ is the start state
- $q_{\text{accept}} \in Q$ is the accept state
- $q_{\text{reject}} \in Q$ is the reject state ($q_{\text{reject}} \neq q_{\text{accept}}$)

TM Transition Function

$$\delta : Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R\}$$

L means "move left" and R means "move right"

$\delta(p, a) = (q, b, R)$ means:

- Replace a with
- Transition fro

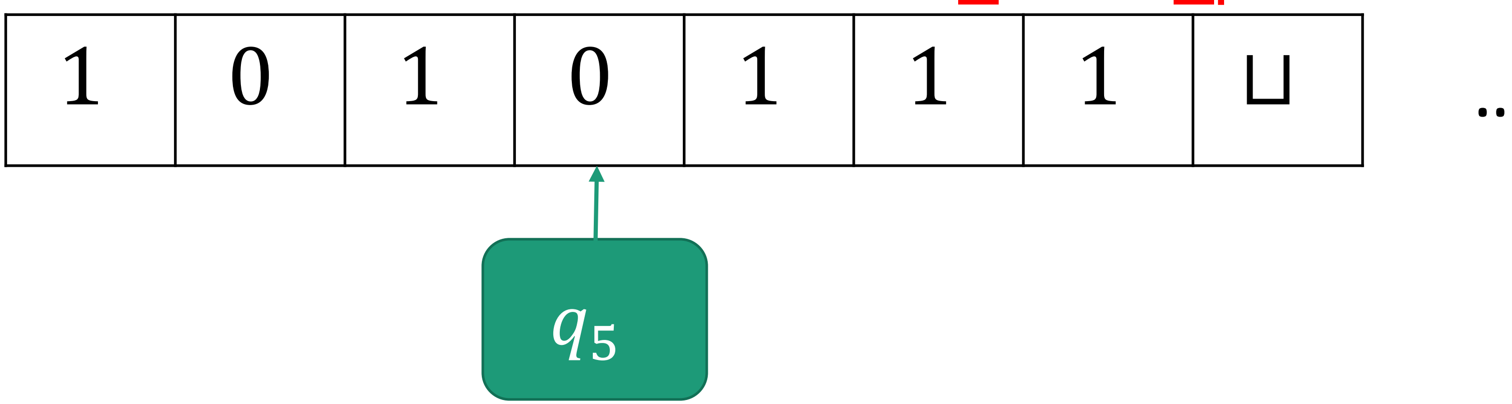
- Move tape head right

$\delta(p, a) = (q, b, L)$ means:

- Replace a with b in current cell
- Transition from state p to state q
- Move tape head left UNLESS we are at left end of tape, in which case don't move

Configuration of a TM

A string with captures the state of a TM together with the contents of the tape

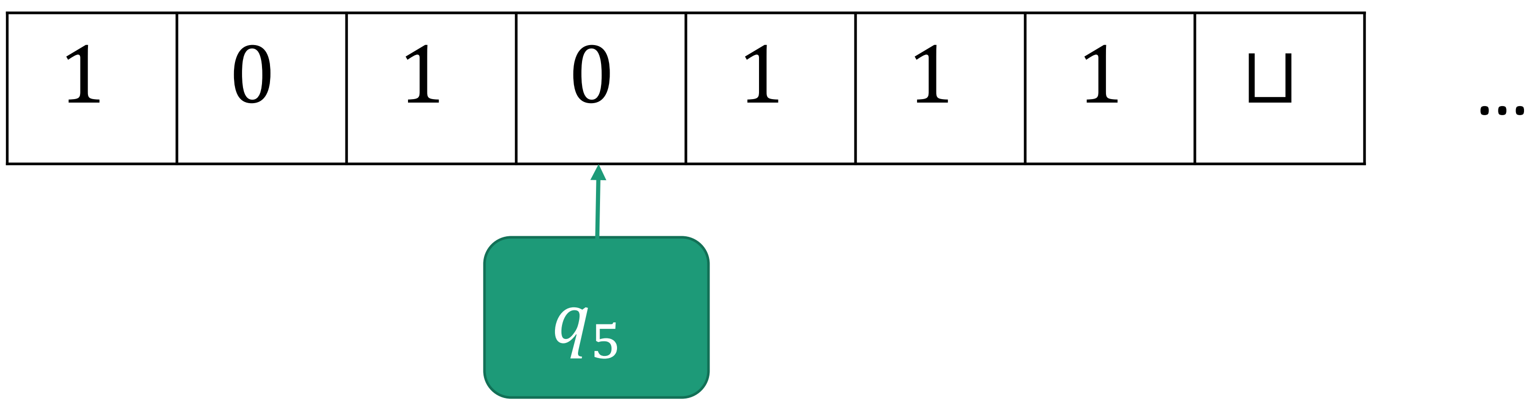


Configuration of a TM: Formally

A configuration is a string uqv where $q \in Q$ and $u, v \in \Gamma^*$

- Tape contents = uv (followed by blanks $\sqcup$)
- Current state = q
- Tape head on fi

Example: $101\, q_5\, 0111$



How a TM Computes

Start configuration: $q_0 w$

One step of computation:

- $ua\ q\ bv$ yields $uac\ q'\ v$ if $\delta(q, b) = (q', c, R)$
- $ua\ q\ bv$ yields ( ) $q', c, L)$
- If we are at the left end of th configuration $q\ bv$, what configuration do we rea ( ) $= (q', c, L)$?

How a TM Computes

Start configuration: $q_0 w$

One step of computation:

- $ua\ q\ bv$ yields $uac\ q'\ v$ if $\delta(q, b) = (q', c, R)$
- $ua\ q\ bv$ yields $(\quad)$ $q', c, L)$

- $q\ bv$ yields $q'\ cv$ if $\delta(q, b)$

Accepting configuration: q = q_{accept}

Rejecting configuration: q = q_{reject}

How a TM Computes

M accepts input w if there is a sequence of configurations $C_1, \ldots, C_k$ such that:

- C_1 = $q_0 w$
- C_i yields C_{i+1} for every i
- C_k is an accepti

$L(M)$ $=$ the set of all strings w accepts

A is Turing-recognizable if $A = L(M)$ for some TM M:

- $w \in A \implies M$ halts on w in state q_{accept}
- $w \notin A \implies M$ halts on w in state q_{reject} OR
  M runs forever

Recognizers vs. Deciders

$L(M) \; =$ the set of all strings w which M accepts

A is Turing-recognizable if $A = L(M)$ for some TM M:

- $w \in A \implies M$ halts on w in state q_{accept}
- $w \notin A \implies M$ hal
  M ru

A is (Turing-)decidable if $A = L(M)$ for some TM M which halts on every input

- $w \in A \implies M$ halts on w in state q_{accept}
- $w \notin A \implies M$ halts on w in state q_{reject}

Back to Hilbert's Tenth Problem

Computational Problem: Given a Diophantine equation, does it have a solution over the integers?

$L =$

- L is Turing-recognizable

- L is not decidable (1949-70)