CIS 471/571(Fall 2020): Introduction to Artificial Intelligence

Lecture 7: Utilities

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Source: http://ai.berkeley.edu/home.html

Reminders

- Project 2: Multi-agent Search
  - Deadline: Oct 27th, 2020

- Homework 2: CSPs
  - Deadline: Oct 24th, 2020

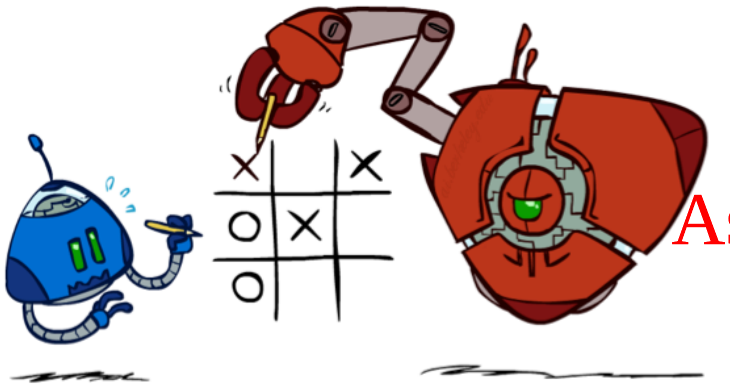
Today

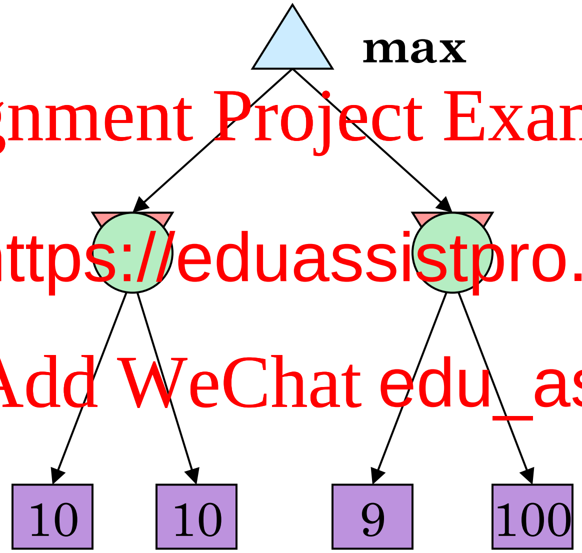
- Expectimax Search

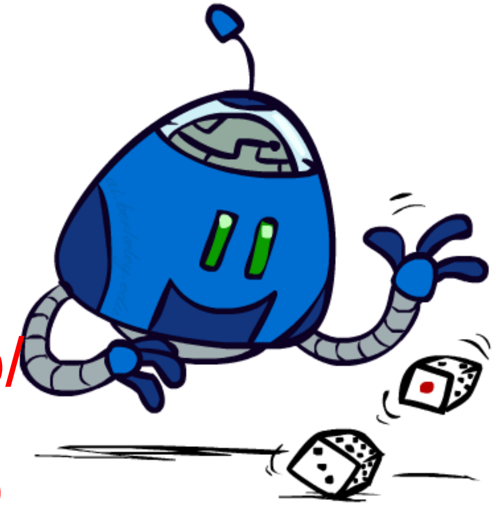
- Utilities

Uncertain Outcomes

Worst-Case vs. Average Case







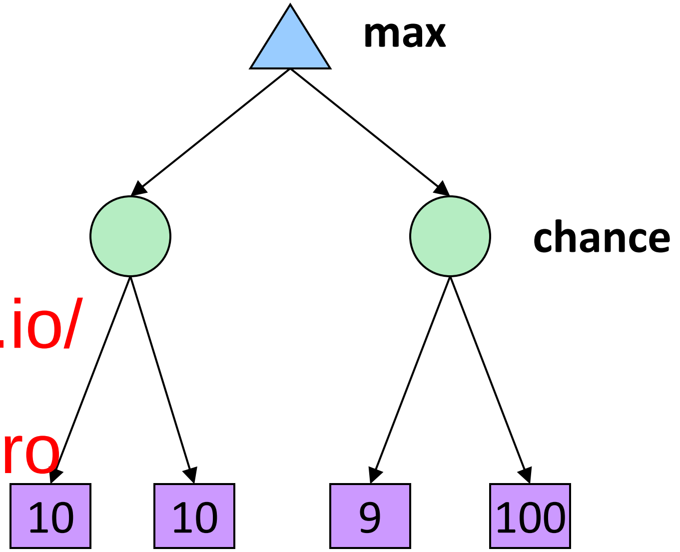
Idea: Uncertain outcomes controlled by chance, not an adversary!

Expectimax Search

- Why wouldn't we know what the result of an action will be?
  - Explicit randomness: rolling dice
  - Unpredictable opponents: the ghosts respond randomly
  - Actions can fail: when moving a robot, wheels might slip

- Values should now reflect aver
  outcomes, not worst-case (mini

- Expectimax search: compute the average score
  optimal play
  - Max nodes as in minimax search
  - Chance nodes are like min nodes but the outcome is uncertain
  - Calculate their expected utilities
  - I.e. take weighted average (expectation) of children



- Later, we'll learn how to formalize the underlying uncertain-result problems as Markov Decision Processes

[Demo: min vs exp (L7D1,2)]

Expectimax Pseudocode

```
def value(state):
    if the state is a terminal state: return the state's utility
    if the next agent is MAX: return max-value(state)
    if the next                              p-value(state)
```

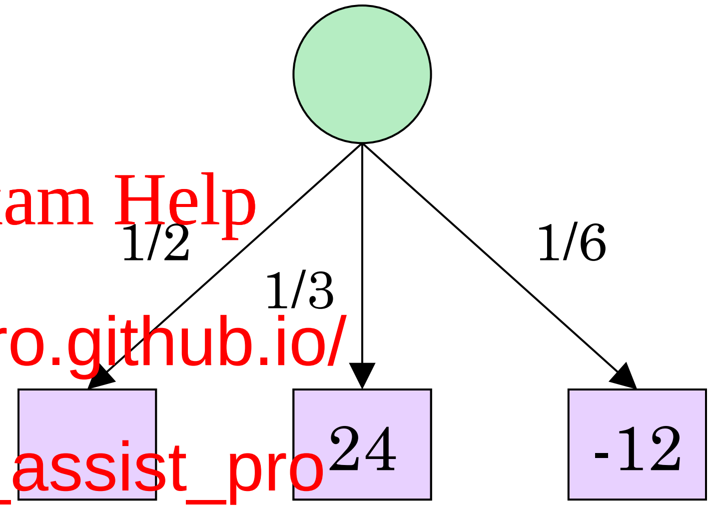


```
def max-value(state):
    initialize v = -∞
    for each successor of state:
        v = max(v, value(successor))
    return v
```

```
def exp-value(state):
    initialize v = 0
    for each successor of state:
        p = probability(successor)
        v += p * value(successor)
    return v
```

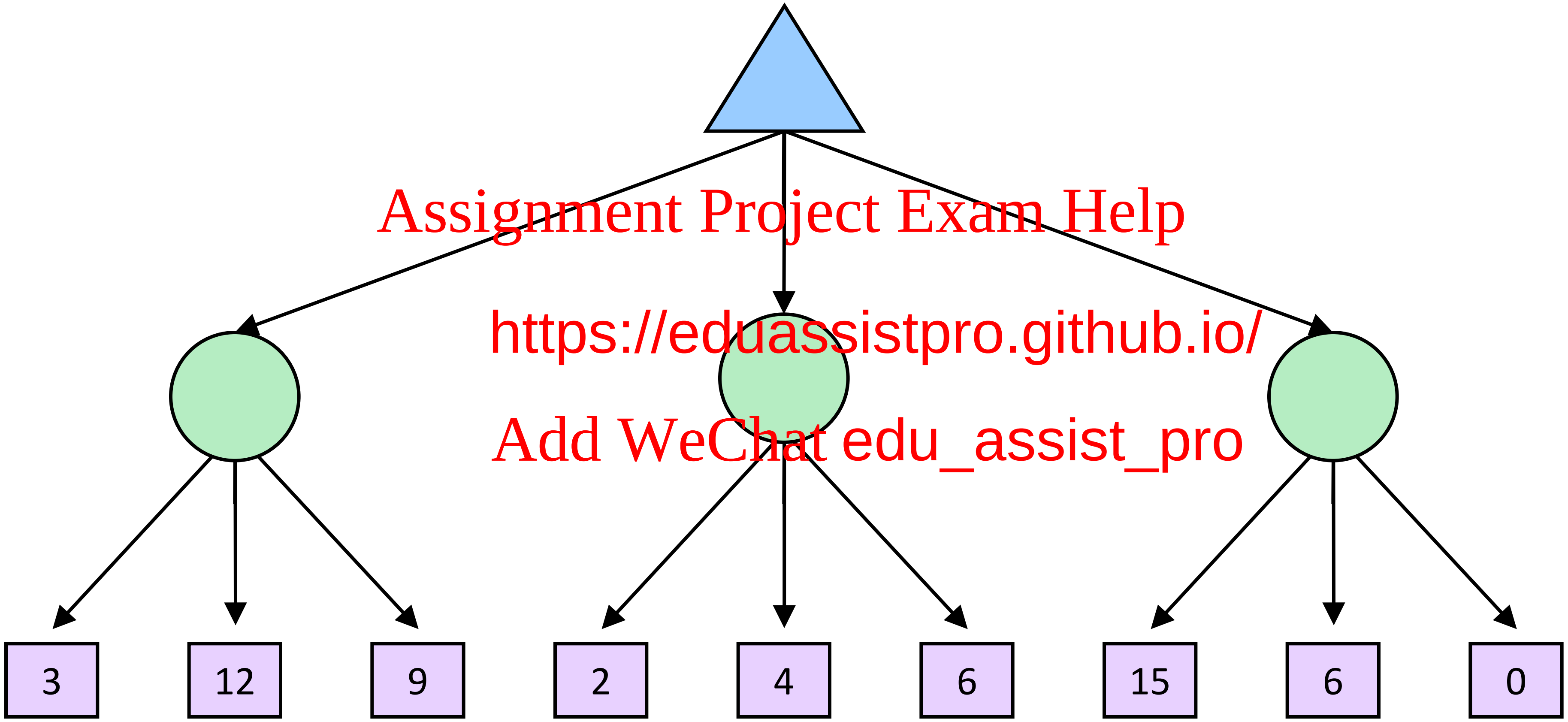
Expectimax Pseudocode

```
def exp-value(state):
    initialize v = 0
    for each successor of state:
        p = probability(suc
        v += p * value(suc
    return v
```

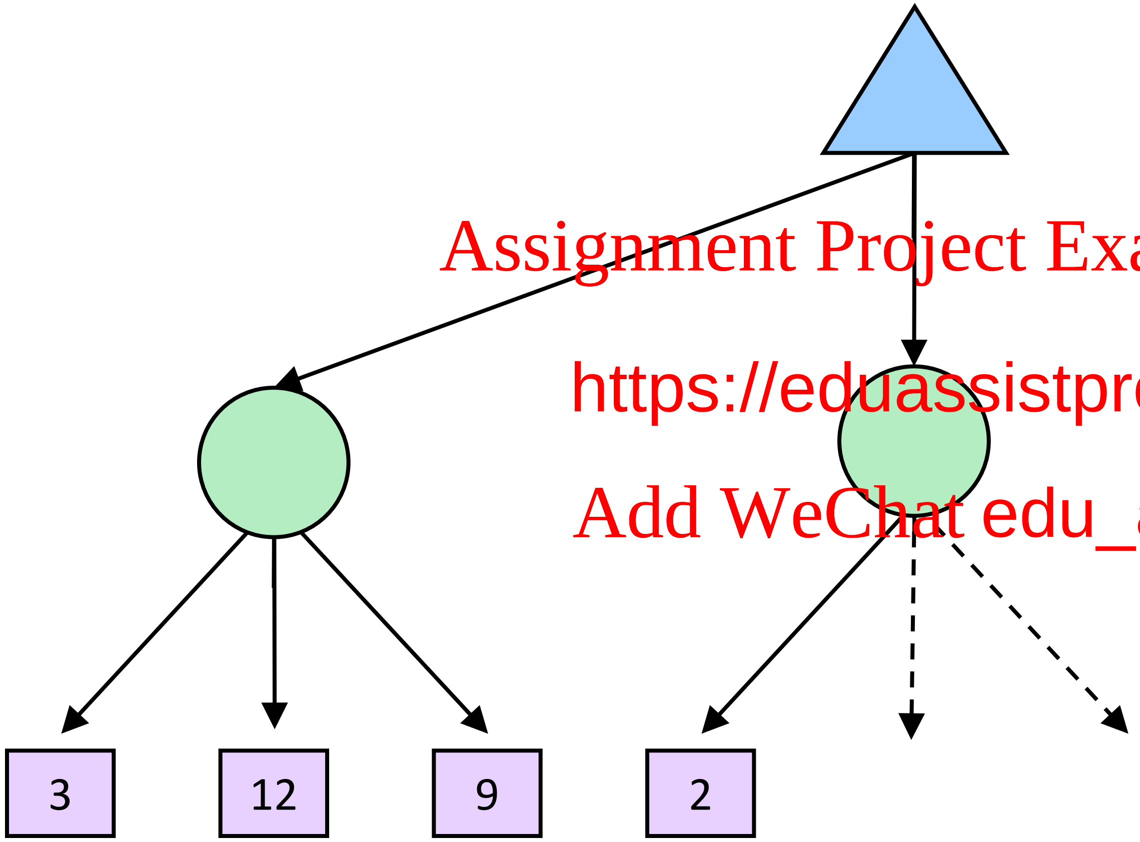



$$v = (1/2)\ (8) + (1/3)\ (24) + (1/6)\ (-12) = 10$$

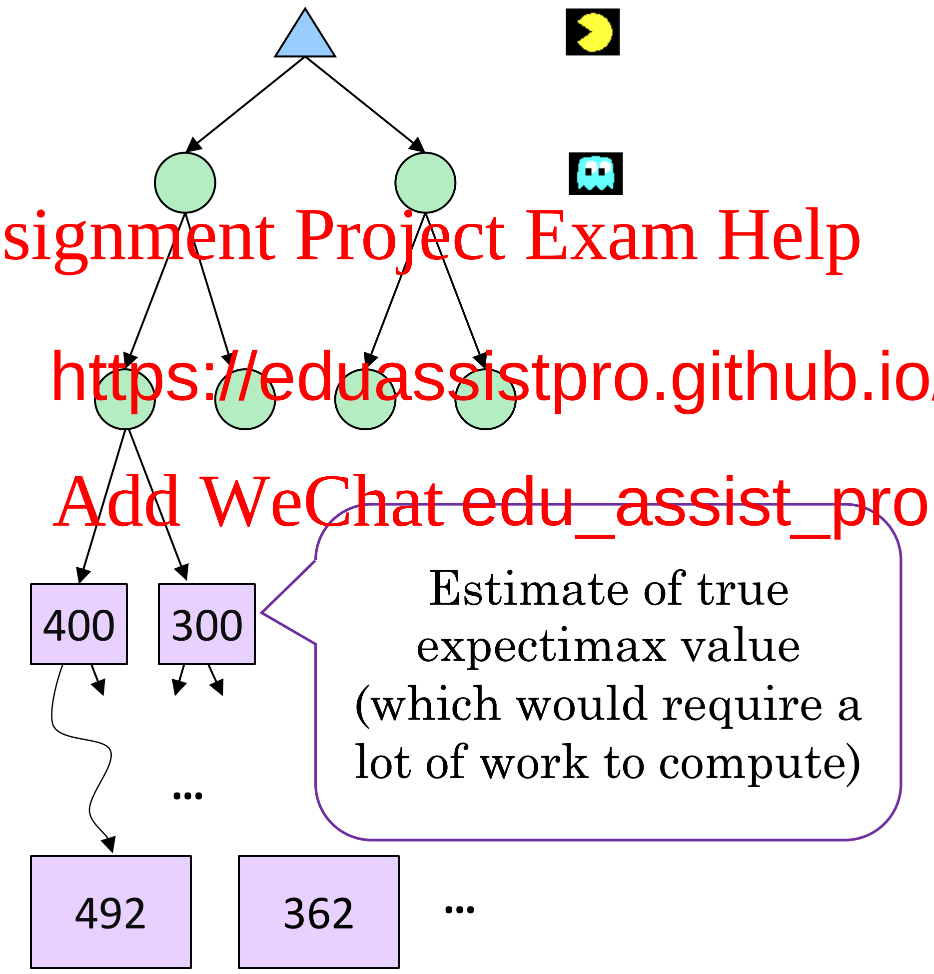
Expectimax Example



Expectimax Pruning?



Depth-Limited Expectimax



Probabilities

Reminder: Probabilities

- A random variable represents an event whose outcome is unknown
- A probability distribution is an assignment of weights to outcomes

0.25

- Example: Traffic on freeway
  - Random variable: T = whether t
  - Outcomes: T in {none, light, hea
  - Distribution: P(T=none) = 0.25, = 0.25

0.50

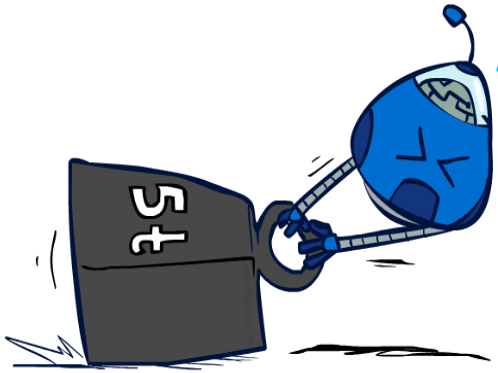
- Some laws of probability (more later):
  - Probabilities are always non-negative
  - Probabilities over all possible outcomes sum to one

0.25

- As we get more evidence, probabilities may change:
  - P(T=heavy) = 0.25, P(T=heavy | Hour=8am) = 0.60
  - We'll talk about methods for reasoning and updating probabilities later

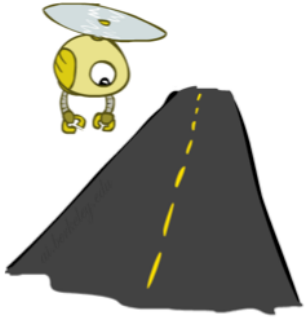
Reminder: Expectations

- The expected value of a function of a random variable is the average, weighted by the probability distribution over outcomes



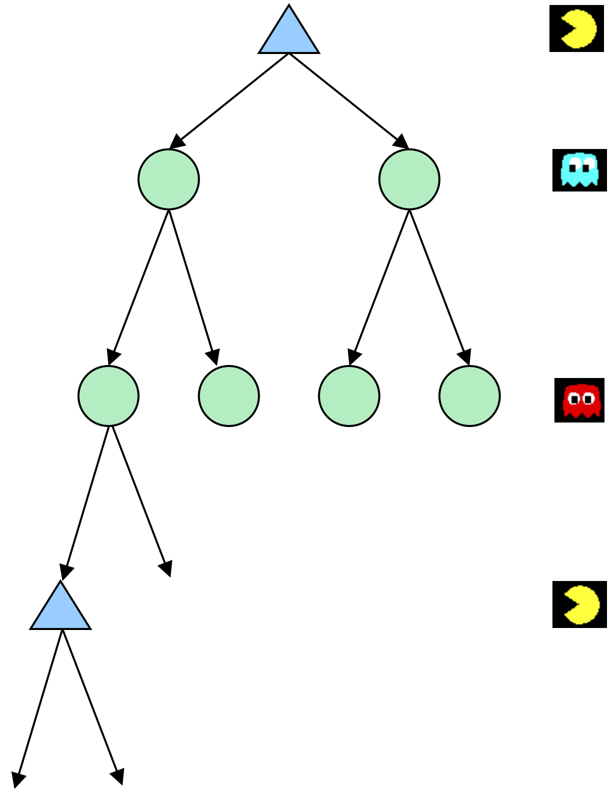
- Example: How long to ge

Time:	20 min		30 min		60 min	
	x	+	x		x	35 min
Probability:	0.25		0.50		0.25	



What Probabilities to Use?

- In expectimax search, we have a probabilistic model of how the opponent (or environment) will behave in any state
  - Model could be a simple uniform distribution (roll a die)
  - Model could be sophisticated a of computation
  - We have a chance node for any outcome out of o control: opponent or environment
  - The model might say that adversarial actions are likely!

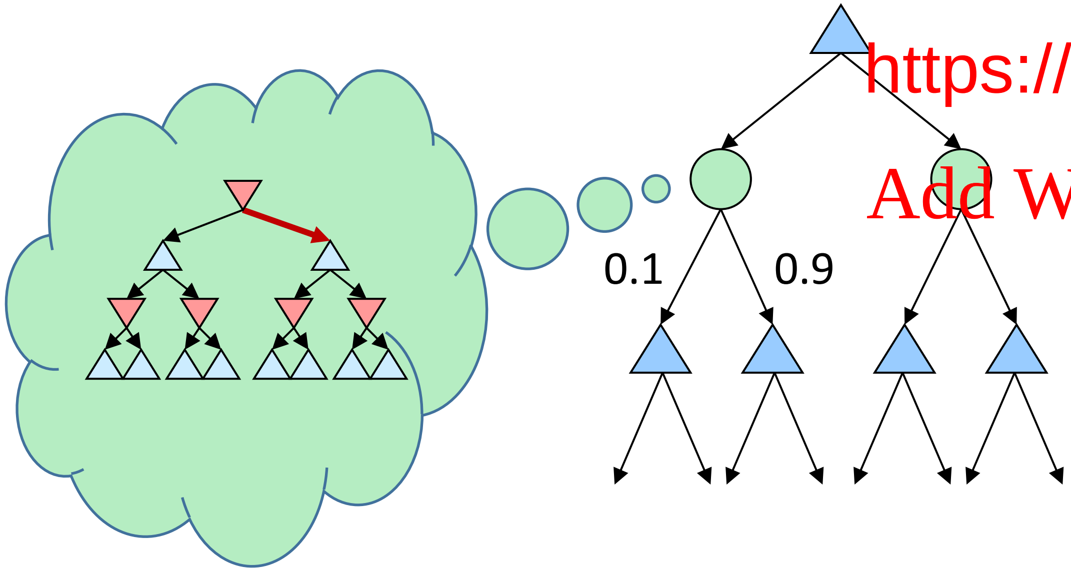


- For now, assume each chance node magically comes along with probabilities that specify the distribution over its outcomes

Having a probabilistic belief about another agent's action does not mean that the agent is flipping any coins!

Quiz: Informed Probabilities

- Let's say you know that your opponent is actually running a depth 2 minimax, using the result 80% of the time, and moving randomly otherwise
- Question: What tree search should you use?



pectimax!

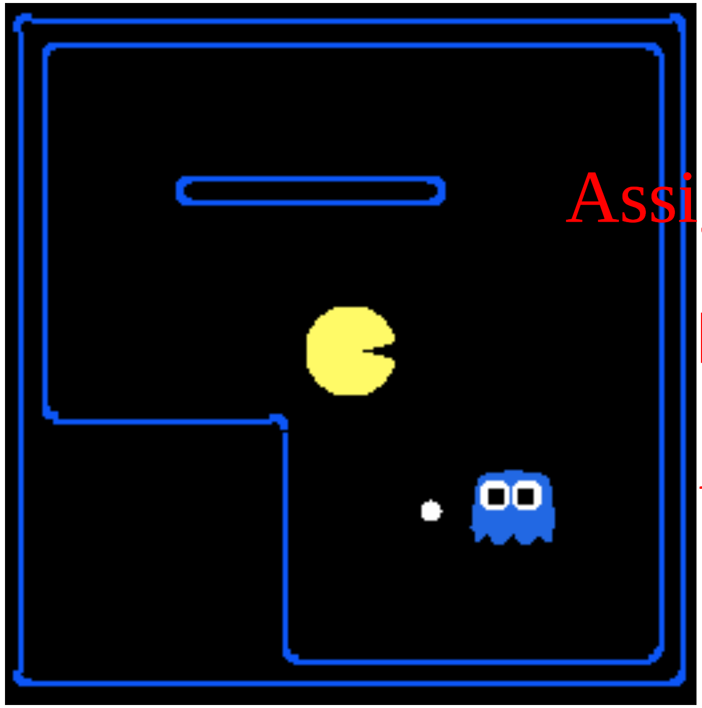
- t EACH chance node's s, you have to run a simulation of nt
- This kind of thing gets very slow very quickly
- Even worse if you have to simulate your opponent simulating you…
- … except for minimax, which has the nice property that it all collapses into one game tree

Modeling Assumptions

The Dangers of Optimism and Pessimism

Dangerous Optimism	Dangerous Pessimism
Assuming chance when the world is adversarial	Assuming the worst case when it's not likely

Assumptions vs. Reality

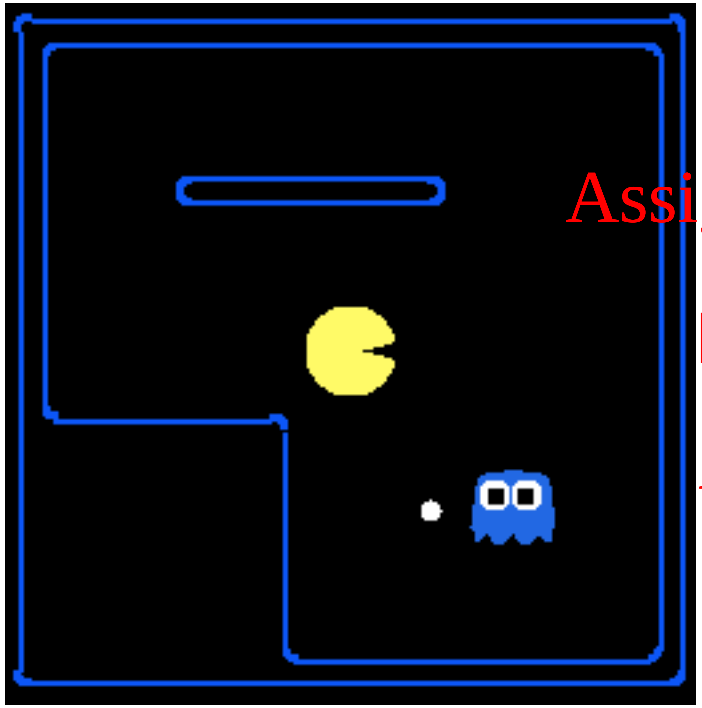


	Adversarial Ghost	Random Ghost
Expectimax Pacman		

Results from playing 5 games

Pacman used depth 4 search with an eval function that avoids trouble
Ghost used depth 2 search with an eval function that seeks Pacman

Assumptions vs. Reality



	Adversarial Ghost	Random Ghost
[illegible]	Won 5/5 Avg. Score: 483	Won 5/5 Avg. Score: 493
Expectimax Pacman	Won 1/5 Avg. Score: -303	Won 5/5 Avg. Score: 503

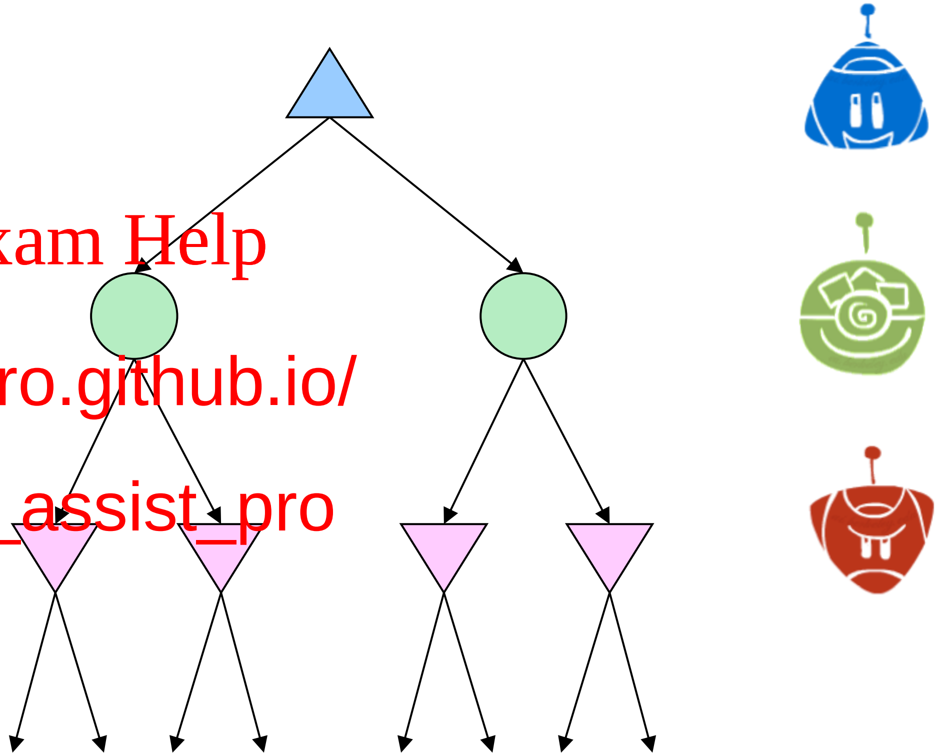
Results from playing 5 games

Pacman used depth 4 search with an eval function that avoids trouble
Ghost used depth 2 search with an eval function that seeks Pacman

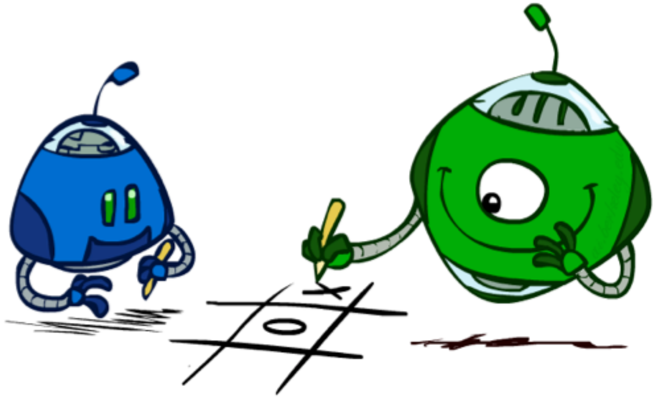
Other Game Types

Mixed Layer Types

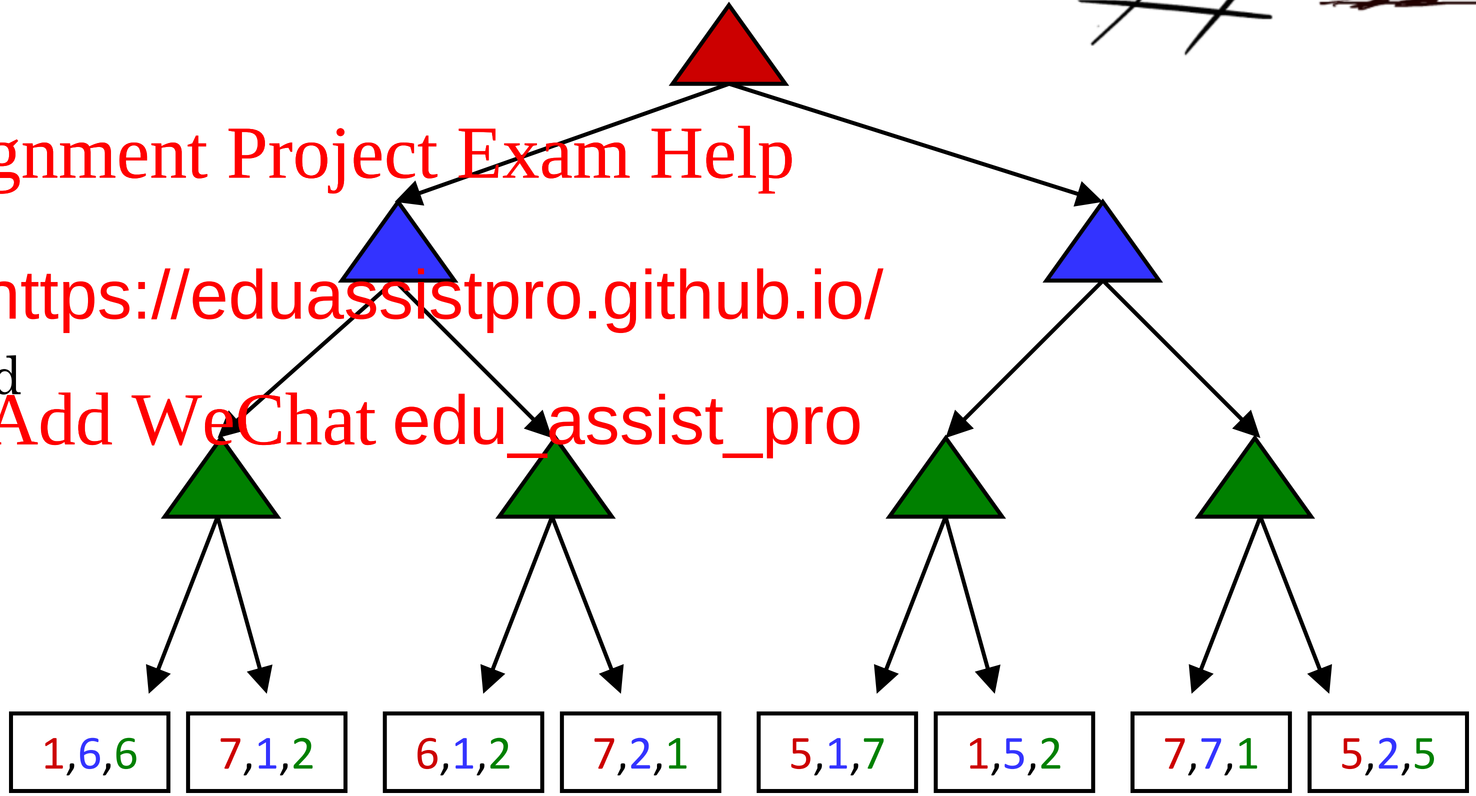
- E.g. Backgammon
- Expectiminimax
  - Environment is an extra "random agent" player that moves after each min/max agent
  - Each node computes the appropriate combination of its children

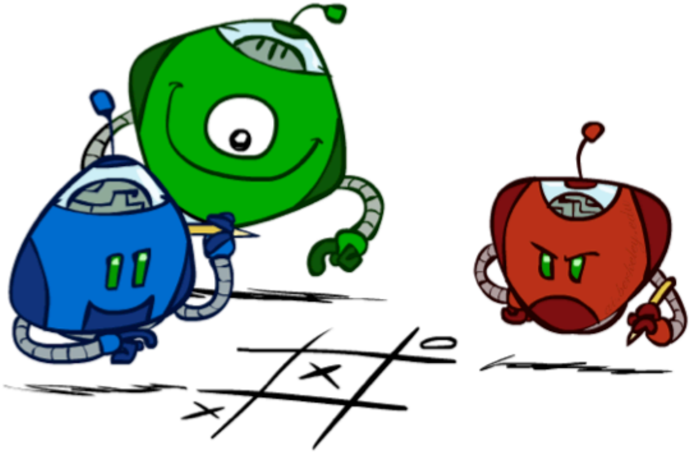


Multi-Agent Utilities



- What if the game is not zero-sum, or has multiple players?
- Generalization of minimax:
  - Terminals have utility tuples
  - Node values are also utility tu
  - Each player maximizes its ow
  - Can give rise to cooperation and competition dynamically…





Utilities

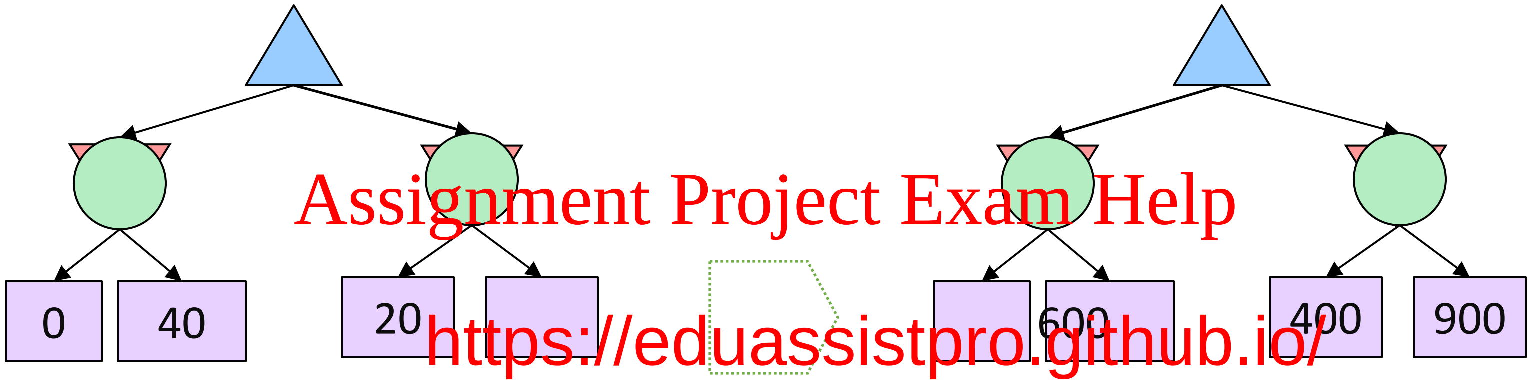
Maximum Expected Utility

- Why should we average utilities? Why not minimax?

- Principle of maximum expected utility:
  - A rational agent should chos its expected utility, given its kn

- Questions:
  - Where do utilities come from?
  - How do we know such utilities even exist?
  - How do we know that averaging even makes sense?
  - What if our behavior (preferences) can't be described by utilities?

What Utilities to Use?



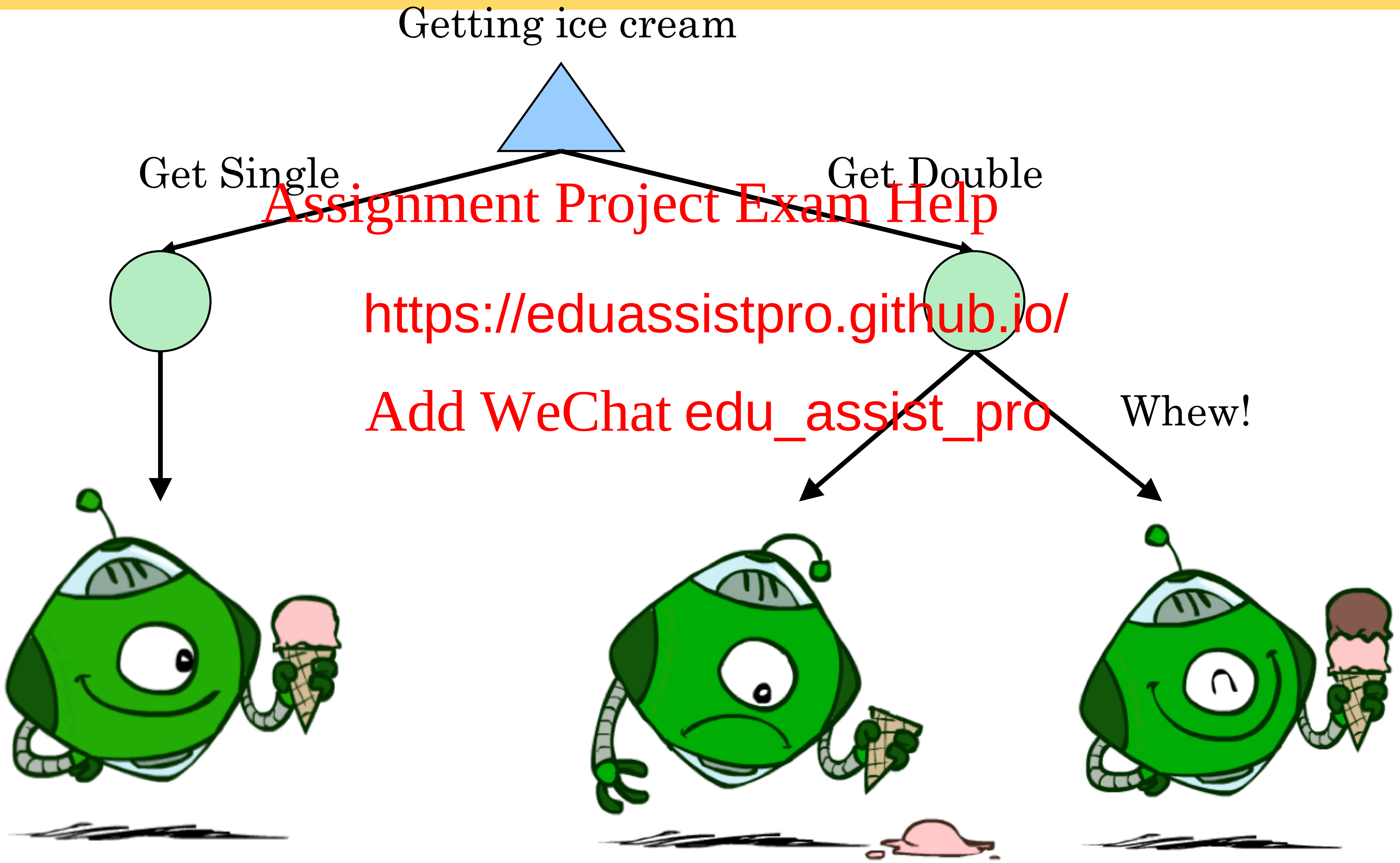
- For worst-case minimax reasoning, terminal function scale doesn't matter
  - We just want better states to have higher evaluations (get the ordering right)
  - We call this insensitivity to monotonic transformations
- For average-case expectimax reasoning, we need *magnitudes* to be meaningful

Utilities

- Utilities are functions from outcomes (states of the world) to real numbers that describe an agent's preferences

- Where do utilities come from?
  - In a game, may be simple (+1/-1)
  - Utilities summarize the agent's goals
  - Theorem: any "rational" preferences can be summarized as a utility function

Utilities: Uncertain Outcomes



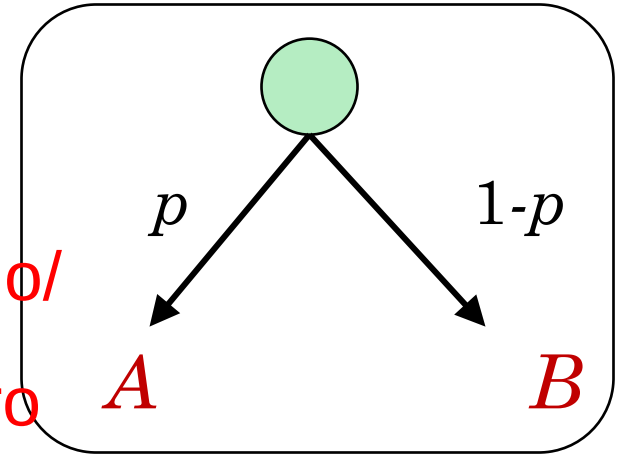
Preferences

- An agent must have preferences among:
  - Prizes: *A, B*, etc.
  - Lotteries: situations with uncertain prizes

A Prize

A

A Lottery



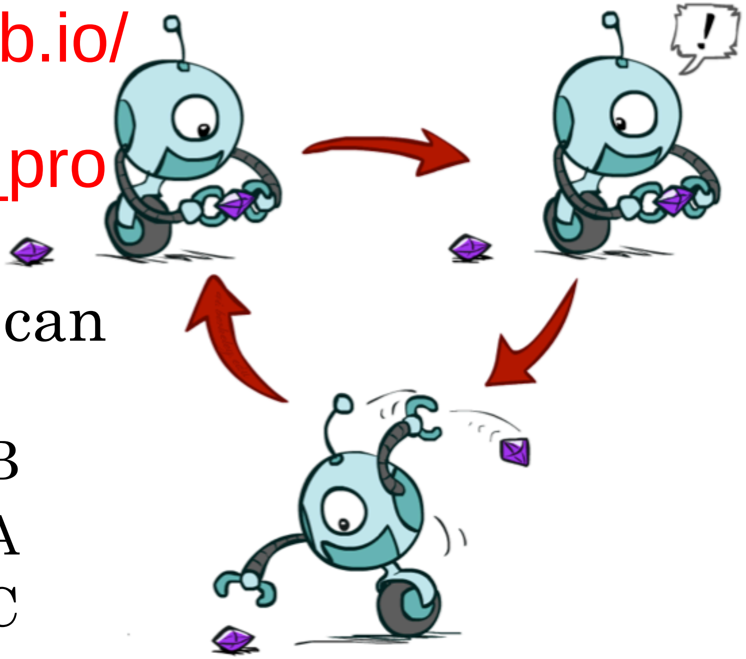
- Notation:
  - Preference: $A \succ B$
  - Indifference: $A \sim B$

Rationality

Rational Preferences

- We want some constraints on preferences before we call them rational, such as:

$$\text{Axiom of Transitivity: } (A \succ B) \wedge (B \succ C) \Rightarrow (A \succ C)$$



- For example: an agent with intransitive preferences can be induced to give away all of its money
  - If B > C, then an agent with C would pay (say) 1 cent to get B
  - If A > B, then an agent with B would pay (say) 1 cent to get A
  - If C > A, then an agent with A would pay (say) 1 cent to get C

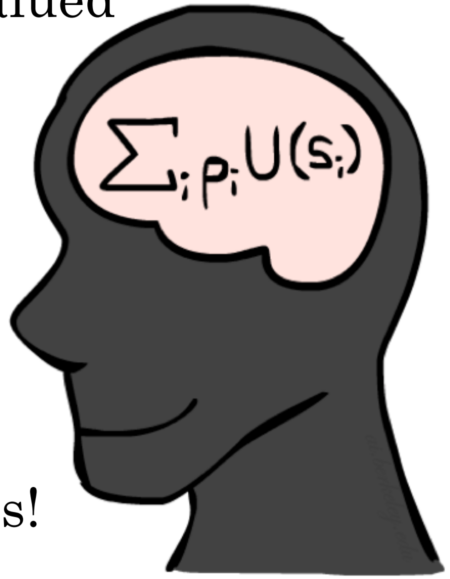
Rational Preferences

The Axioms of Rationality

Theorem: Rational preferences imply behavior describable as maximization of expected utility

MEU Principle

- Theorem [Ramsey, 1931; von Neumann & Morgenstern, 1944]
  - Given any preferences satisfying these constraints, there exists a real-valued function U such that:



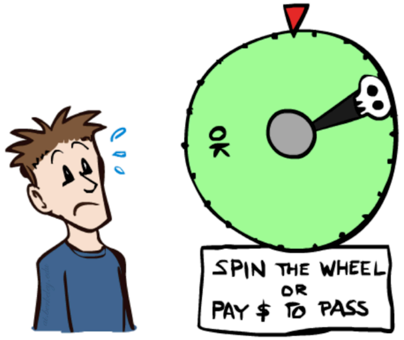
  - I.e. values assigned by U preserve preferences of both prizes and lotteries!

- Maximum expected utility (MEU) principle:
  - Choose the action that maximizes expected utility

Human Utilities

Human Utilities

- Utilities map states to real numbers. Which numbers?
- Standard approach to assessment (elicitation) of human utilities:
  - Compare a prize A to a standard lottery L_p between
    - “best possible prize” u_+ wit
    - “worst possible catastrophe
  - Adjust lottery probability p
  - Resulting p is a utility in [0,1]



Human Utilities: Example

- A person is given the choice between 2 scenarios:
  - Guaranteed scenario: the person receives $50
  - Uncertain scenario: a coin is flipped to decide the person receive $100 or not.

- Which choice would that pers ?

Risk Aversion

- Risk averse: would accept the guaranteed payment of (less than) $50 rather than take the gamble

- Risk neutral: indiff[illegible]e bet and the guaranteed $50 payment

- Risk seeking: would accept the bet even when the guaranteed payment is more than $50

Prospect Theory: Utility Function

- Risk aversion: convexity
  - Risk averse regarding gain
  - Risk seeking regarding loss

- Loss aversion
  - Losses are felt more strong than gains

- Endowment effect
  - We values things we own more highly
  - Reference point: differentiate gains and loss

Source: https://www.economicshelp.org/blog/glossary/prospect-theory/