CIS 471/571(Fall 2020): Introduction to Artificial Intelligence

Lecture 16 – Sampling

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Source: http://ai.berkeley.edu/home.html

Bayes' Nets

- ✓ Representation
- ✓ Conditional Independences

- Proba
  - ✓ Enu
    com

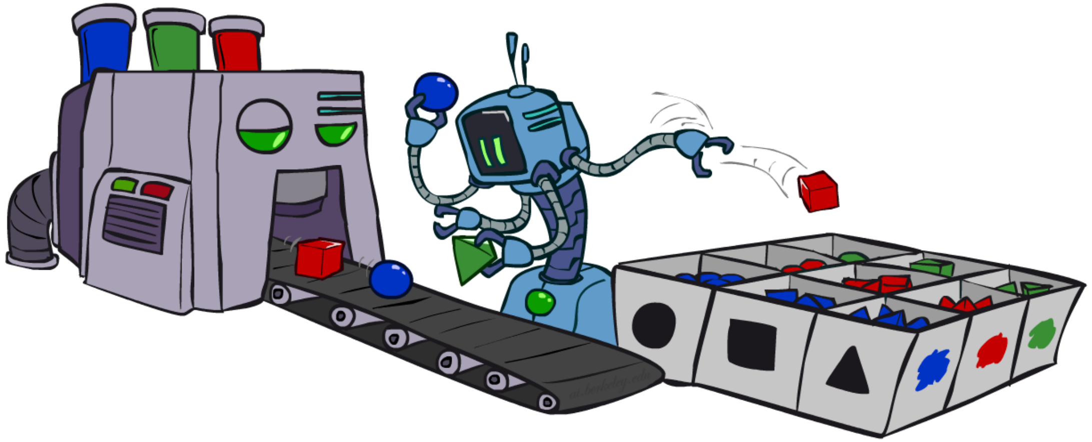
  - ✓ Variable elimination (ase
    exponential complexit r)
  - ✓ Inference is NP-complete
  - Sampling (approximate)
- Learning Bayes' Nets from Data

Approximate Inference: Sampling

Sampling

- Sampling is a lot like repeated simulation
  - Predicting the weather, basketball games, …
- Basic idea
  - Draw N samples from a sampl
  - Compute an approximate posterior probability
  - Show this converges to the true probability P

- Why sample?
  - Learning: get samples from a distribution you don't know
  - Inference: getting a sample is faster than computing the right answer (e.g. with variable elimination)



Sampling

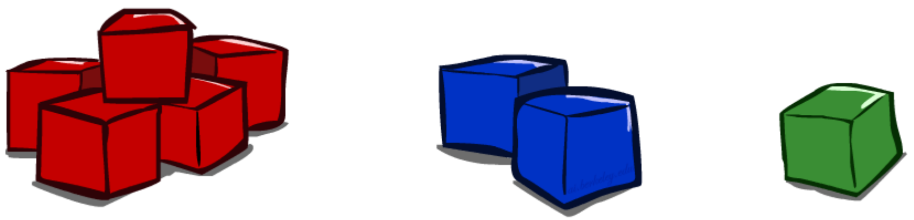
- Sampling from given distribution
  - Step 1: Get sample u from uniform distribution over [0, 1)
    - E.g. random() in python
  - Step 2: Convert this sample u i
outcome for the given distribut
    - Each target outcome is associated with a sub-interval of [0,1)
    - Sub-interval size is equal to probability of the outcome.

- Example

C	P(C)
	.6
	.1
bl	

$$0 \le u < 0.6, \rightarrow C = red$$
$$0.6 \le u < 0.7, \rightarrow C = green$$
$$0.7 \le u < 1, \rightarrow C = blue$$

- If random() returns $u = 0.83$, then our sample is C = blue
- E.g, after sampling 8 times:



Sampling in Bayes' Nets

- Prior Sampling

- Rej

- Likelihood We

- Gibbs Sampling

Prior Sampling

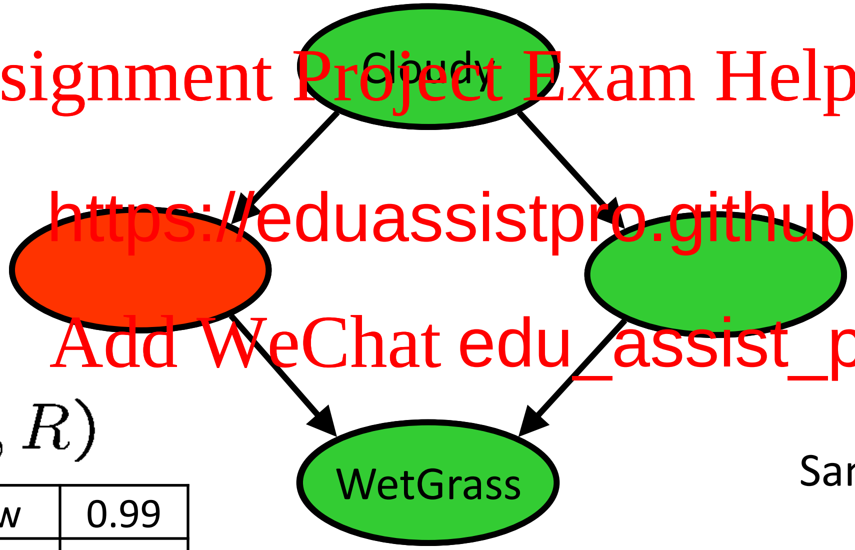
Prior Sampling

$P(C)$

+c	0.5
-c	0.5

$P(S|C)$

+c	+s	0.1
	-s	0.9
-c	+s	0.5
	-s	0.5



$P(R|C)$

+c	+r	0.8
	-r	0.2
-c	+r	0.2
	-r	0.8

$P(W|S, R)$

+s	+r	+w	0.99
		-w	0.01
	-r	+w	0.90
		-w	0.10
-s	+r	+w	0.90
		-w	0.10
	-r	+w	0.01
		-w	0.99

Samples:

+c, -s, +r, +w

-c, +s, -r, +w

…

Prior Sampling

- For i = 1, 2, …, n
  - Sample x_i from $P(X_i \mid Parents(X_i))$
- Return $(x_1, x_2, \dots, x_n)$

Prior Sampling

- This process generates samples with probability:

$$S_{PS}(x_1 \ldots x_n) = \prod_{i=1}^{n} P(x_i|\text{Parents}(X_i)) = P(x_1 \ldots x_n)$$

...i.e. the BN's

- Let the number of samples $N_{PS}(x_1 \ldots x_n)$

- Then

$$\begin{aligned}\lim_{N\to\infty} \widehat{P}(x_1, \ldots, x_n) &= \lim_{N\to\infty} N_{PS}(x_1, \ldots, x_n)/N \\ &= S_{PS}(x_1, \ldots, x_n) \\ &= P(x_1 \ldots x_n)\end{aligned}$$

- I.e., the sampling procedure is consistent

Example

- We'll get a bunch of samples from the BN:

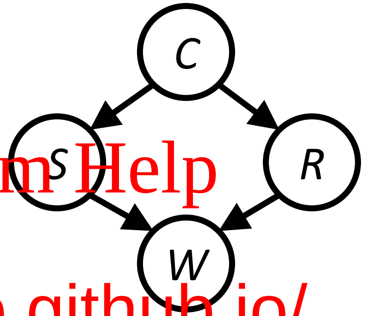
  +c, -s, +r, +w

  +c, +s, +r, +w

  -c, +s, +r, -w

  +c, -s, +r, +w

  -c, -s, -r, +w

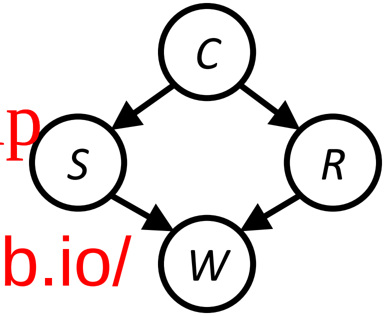


- If we want to know P(W)
  - We have counts <+w:4, -w:1>
  - Normalize to get P(W) = <+w:0.8, -w:0.2>
  - This will get closer to the true distribution with more samples
  - Can estimate anything else, too
  - What about P(C | +w)? P(C | +r, +w)? P(C | -r, -w)?
  - Fast: can use fewer samples if less time (what's the drawback?)

Rejection Sampling

Rejection Sampling

- Let's say we want P(C)
  - No point keeping all samples around
  - Just tally counts of C as we go

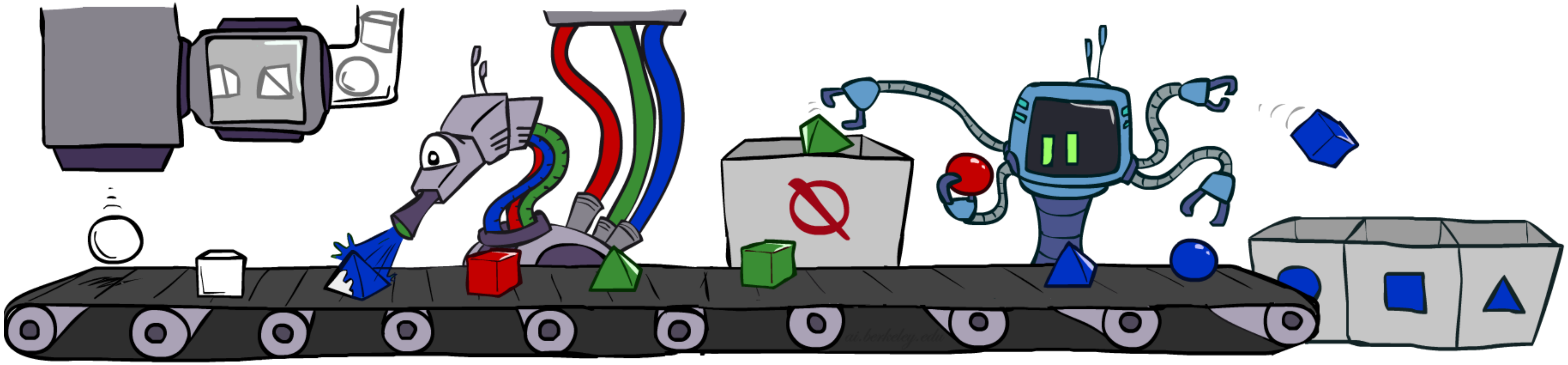


- Let's say we want
  - Same thing: tally C outcomes, but (reject) samples which don't have
  - This is called rejection sampling
  - It is also consistent for conditional probabilities (i.e., correct in the limit)

+c, -s, +r, +w
+c, +s, +r, +w
-c, +s, +r, -w
+c, -s, +r, +w
-c, -s, -r, +w

Rejection Sampling

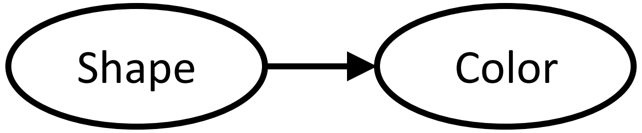
- Input: evidence instantiation
- For i = 1, 2, …, n
  - Sample x_i from $P(X_i \mid \text{Parents}(X_i))$
  - If x_i
    - R ated in this cycle
- Return $(x_1, x_2, \ldots, x_n)$



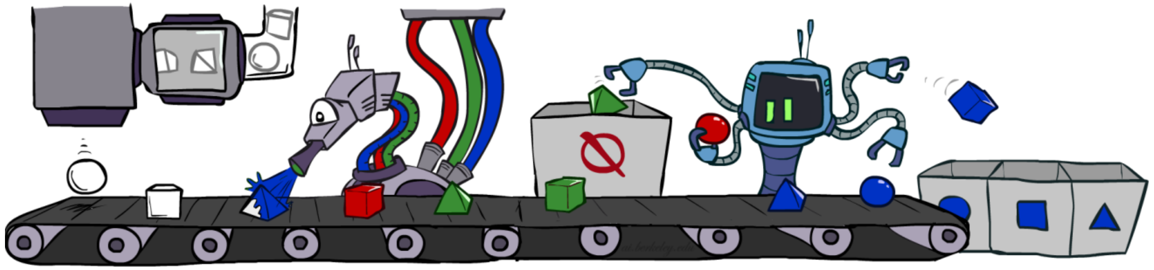
Likelihood Weighting

Likelihood Weighting

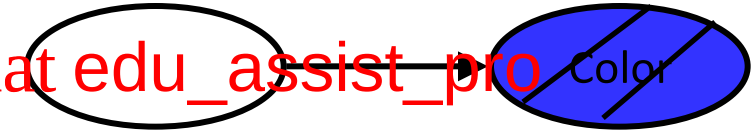
- Problem with rejection sampling:
  - If evidence is unlikely, rejects lots of samples
  - Evidence not exploited as you sample
  - Consider P(Shape | blue)



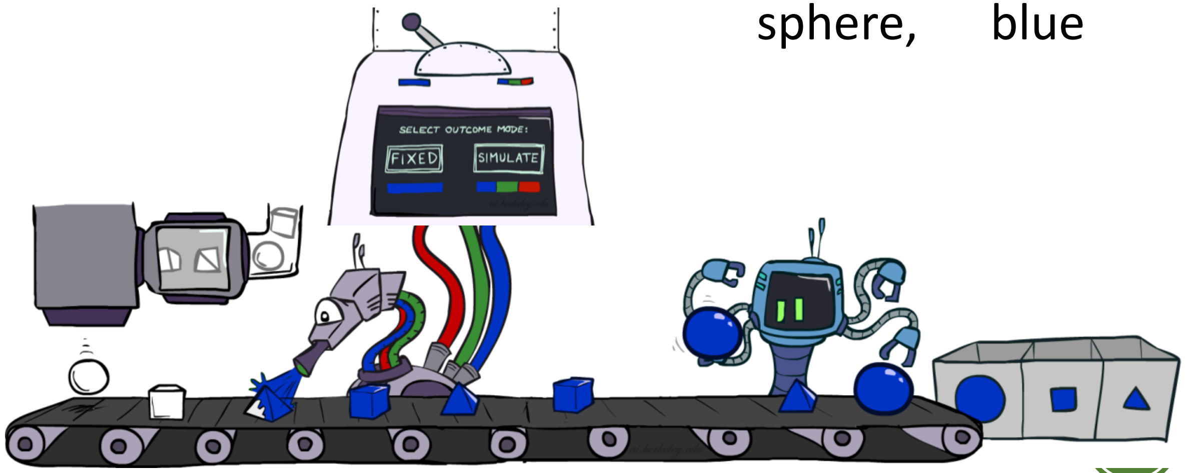
~~pyram~~
~~pyram~~
sphere, blue
~~cube, red~~
~~sphere, green~~



- Idea: fix evidence variables and sample the rest
  - Problem: sample distribution not consistent!
  - Solution: weight by probability of evidence
    ents



pyramid, blue
pyramid, blue
sphere, blue
cube, blue
sphere, blue



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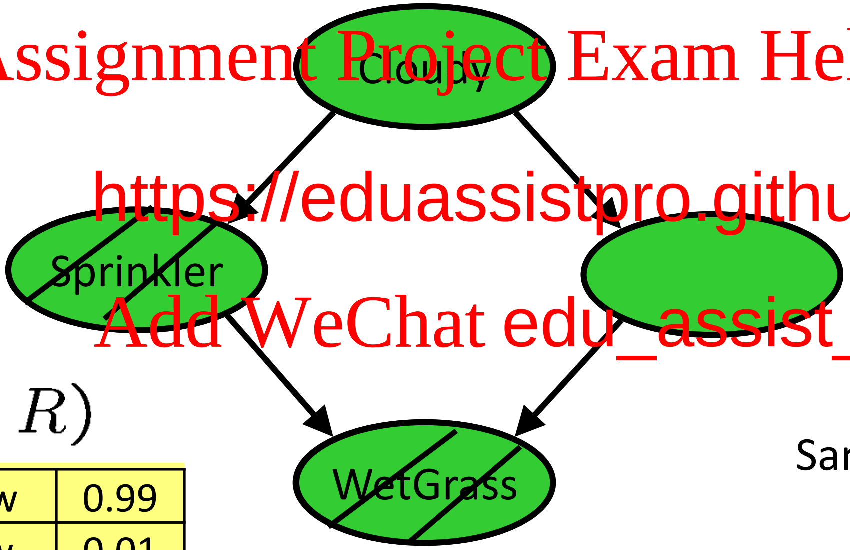

$P(W|S,R)$

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		-w	0.10
-s	+r	+w	0.90
		-w	0.10
	-r	+w	0.01
		-w	0.99

Samples:

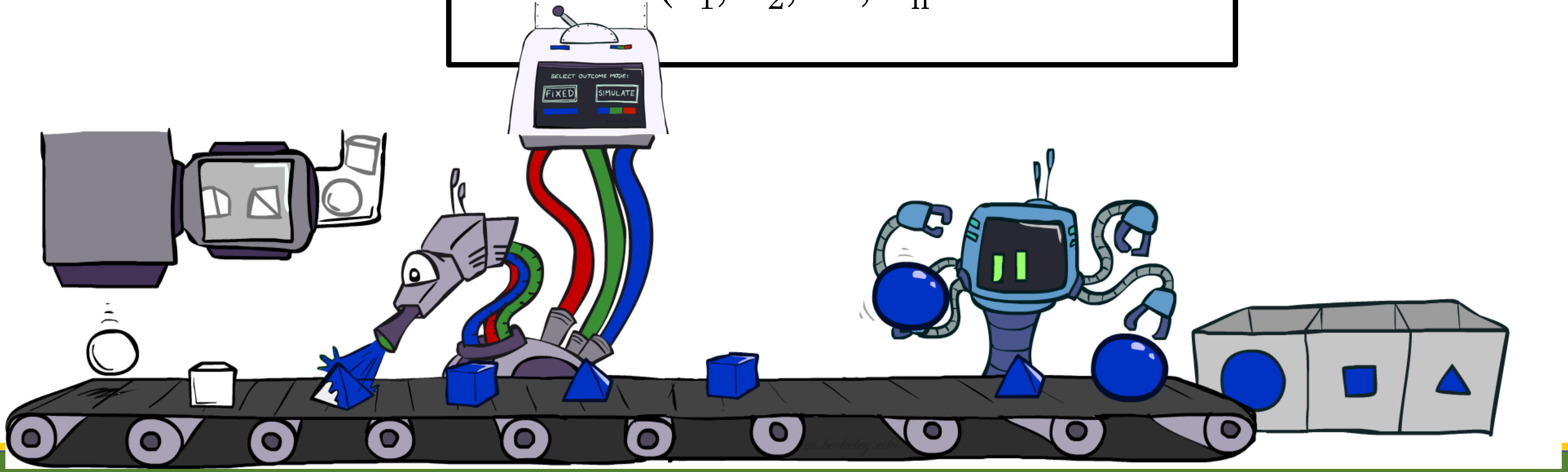
+c, +s, +r, +w

...

$$w = 1.0 \times 0.1 \times 0.99$$

Likelihood Weighting

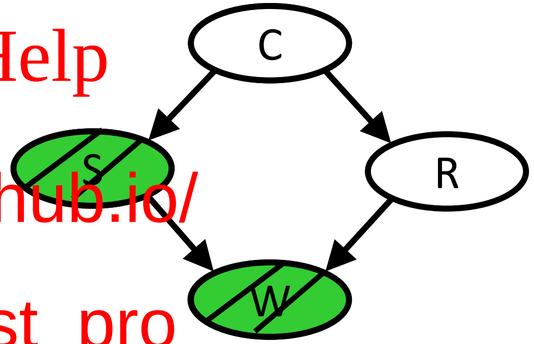
- Input: evidence instantiation
- w = 1.0
- for i = 1, 2, …, n
  - if X_i is an evidence variable
  -
    - Sample x_i from P(
- return (x_1, x_2, …, x_n



Likelihood Weighting

- Sampling distribution if z sampled and e fixed evidence

$$S_{WS}(\mathbf{z},\mathbf{e}) = \prod_{i=1}^{l} P(z_i|\text{Parents}(Z_i))$$



- Now, samples hav

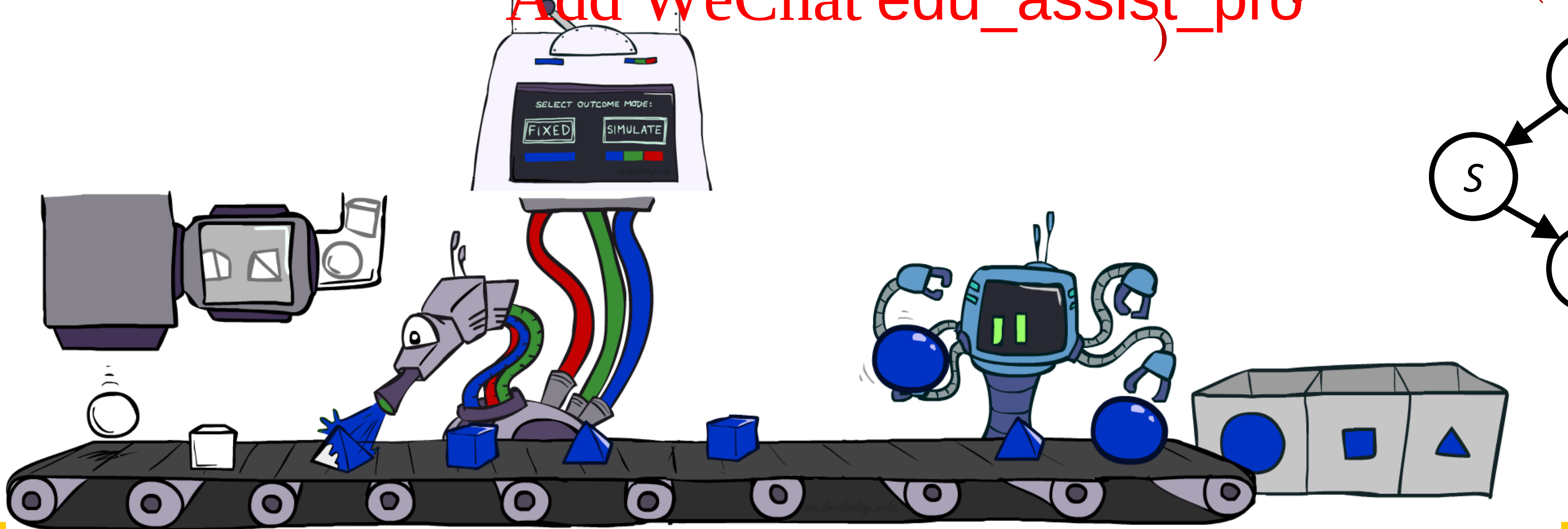
- Together, weighted sampling distribution is consistent

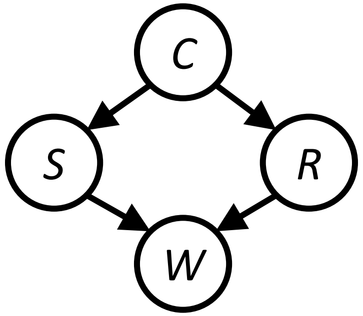
$$S_{\text{WS}}(z,e)\cdot w(z,e) = \prod_{i=1}^{l} P(z_i|\text{Parents}(z_i)) \prod_{i=1}^{m} P(e_i|\text{Parents}(e_i))$$

$$= P(\mathbf{z},\mathbf{e})$$

Likelihood Weighting

- Likelihood weighting is good
  - We have taken evidence into account as we generate the sample
  - E.g. here, W's value will get picked based on the evidence values of S, R
  - More of our samples will reflect the world suggested by the evid
- Likelihood weighting doesn't solve all our problems
  - Evidence influences the choice of downstream variables, but not upstream ones (C isn't more likely to get a value ching the evidence)
  - uld like to consider evidence when e every variable (leads to Gibbs)





Gibbs Sampling

Gibbs Sampling

- *Procedure:* keep track of a full instantiation $x_1, x_2, \ldots, x_n$. Start with an arbitrary instantiation consistent with the evidence. Sample one variable at a time, conditioned on all the rest, but keep evidence fixed. Keep repeating this for a long time.

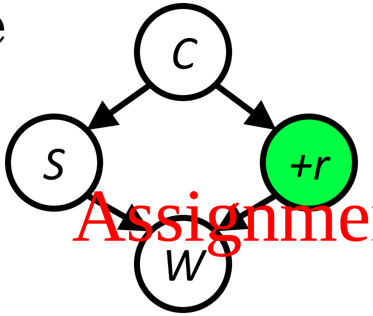
- *Property:* in the limit many times the resulting samples co tion (i.e. conditioned on evidence).

- *Rationale*: both upstream and downstream variables condition on evidence.

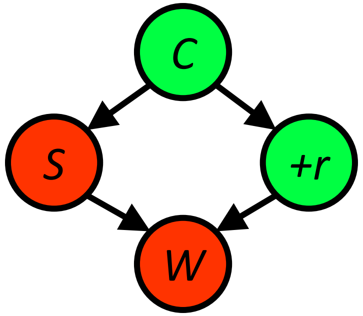
- In contrast: likelihood weighting only conditions on upstream evidence, and hence weights obtained in likelihood weighting can sometimes be very small. Sum of weights over all samples is indicative of how many "effective" samples were obtained, so we want high weight.

Gibbs Sampling Example: P(S | +r)

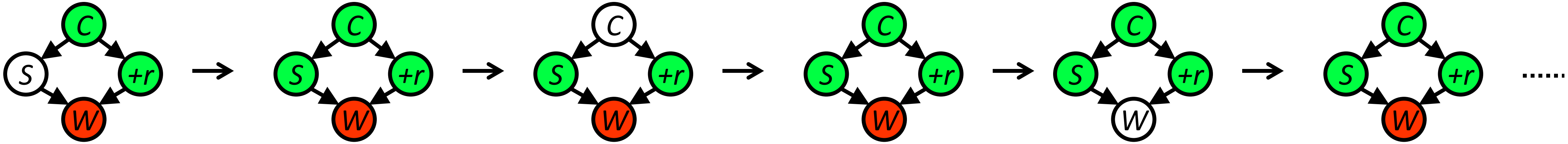
- Step 1: Fix evidence
  - R = +r



- Step 2: Initialize other variables
  - Randomly



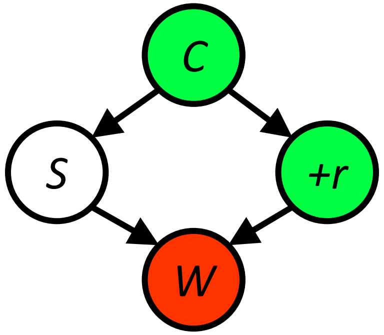
- Steps 3: Repeat
  - Choose a non-evidence variable X
  - Resample X from P(X | all other variables)



Sample from $P(S|+c,-w,+r)$ Sample from $P(C|+s,-w,+r)$ Sample from $P(W|+s,+c,+r)$

Efficient Resampling of One Variable

- Sample from P(S | +c, +r, -w)

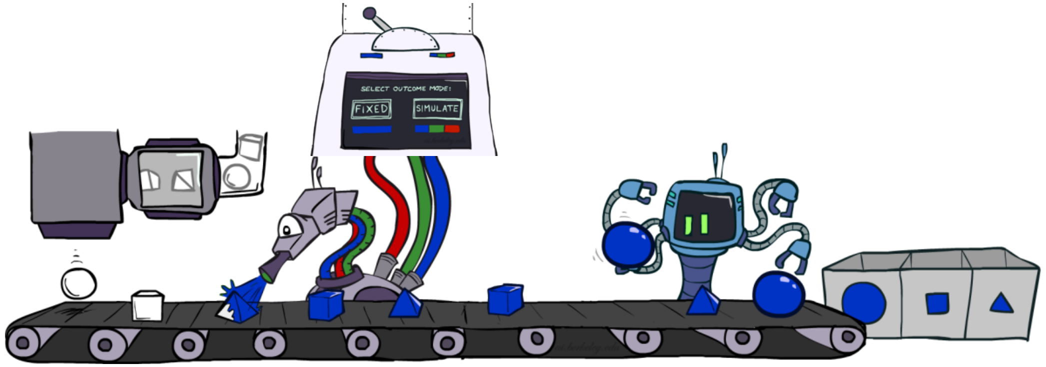


- Many things cancel out – only CPTs with S remain!
- More generally: only CPTs that have resampled variable need to be considered, and joined together

Bayes' Net Sampling Summary

- Prior Sampling P(Q)
- Rejection Sampling P(Q | e)

- Likelihood Weighting P(Q | e)
- ling P(Q | e)



Further Reading on Gibbs Sampling*

- Gibbs sampling produces sample from the query distribution P(Q | e) in limit of re-sampling infinitely often

- Gibbs sampling is a e general methods called Markov chain C) methods

  - Metropolis-Hastings is one of the m s MCMC methods (in fact, Gibbs sampling is a special case of Metropolis-Hastings)

- You may read about Monte Carlo methods – they're just sampling