Histogram Equalization

Histogram equalization is a technique for adjusting image intensities to enhance contrast.

Let f be a given image represented as a m_r by m_c matrix of integer pixel intensities ranging from 0 to $L-1$. L is the number of possible intensity values, often 256. Let p denote the normalized histogram of f with a bin for each possible intensity. So

$$p_n = \frac{\text{number of pixels with intensity } n}{\text{total number of pixels}} \qquad n = 0, 1, ..., L-1.$$

The histogram equalized image g will be defined by

$$g_{i,j} = \text{floor}((L-1)\sum_{n=0}^{f_{i,j}} p_n), \tag{1}$$

where floor() rounds down to the nearest integer. This is equivalent to transforming the pixel intensities, k, of f by the function

The motivation for this transformation comes from thinking of the intensities of f and g as continuous random variables X, Y on $[0, L-1]$ with

$$Y = T(X) = (L-1) \int_0^X p_X(x)dx, \tag{2}$$

where p_X is the probability density function of f. T is the cumulative distributive function of X multiplied by $(L-1)$. Assume for simplicity that T is differentiable and invertible. It can then be shown that Y defined by $T(X)$ is uniformly distributed on $[0, L-1]$, namely that $p_Y(y) = \frac{1}{L-1}$.

$$\int_0^y p_Y(z)dz = \text{ probability that } 0 \le Y \le y$$

$$= \text{ probability that } 0 \le X \le T^{-1}(y)$$

$$= \int_0^{T^{-1}(y)} p_X(w)dw$$

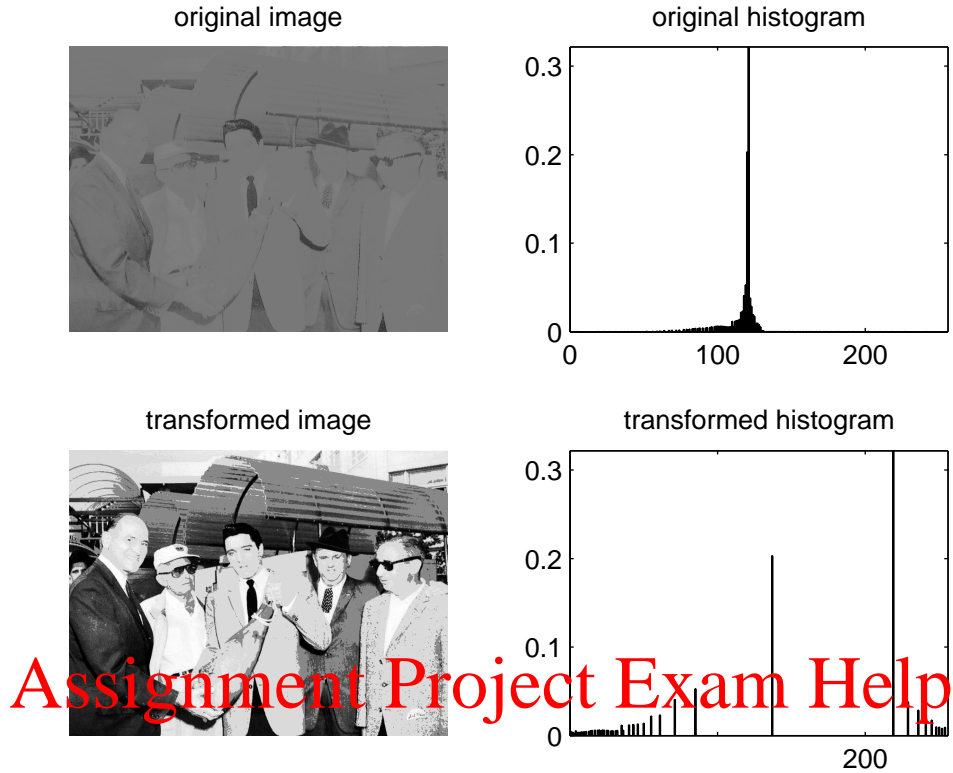$$\frac{d}{dy}\left(\int_0^y p_Y(z)dz\right) = p_Y(y) = p_X(T^{-1}(y))\frac{d}{dy}(T^{-1}(y)).$$

Figure 1: Histogram equalization applied to lo   age

Note that $\frac{d}{dy}T(T^{-1}(y)) = \frac{d}{dy}y = 1$, so

$$\frac{dT}{dx}\big|_{x=T^{-1}(y)}\frac{d}{dy}(T^{-1}(y)) = (L-1)p_X(T^{-1}(y))\frac{d}{dy}(T^{-1}(y)) = 1,$$

which means $p_Y(y) = \frac{1}{L-1}$.

Our discrete histogram is an approximation of $p_X(x)$ and the transformation in Equation 1 approximates the one in Equation 2. While the discrete version won't result in exactly flat histograms, it will flatten them and in doing so enhance the contrast in the image. The result of applying Equation 1 to the elvis_low_contrast.bmp test image is shown in Figure 1.

MATLAB:

To test the accompanying code, hist_eq.m, type

```
g = hist_eq('elvis_low_contrast.bmp');
```

Histogram equalization is also built into MATLAB. Type

```
help histeq
```

to see how it works.

Question:

What happens if Equation 1 is applied twice?

Reference: R. C. GONZALEZ AND R. E. WOODS, *Digital Image Processing*, Third Edition, 2008.

Assignment Project Exam Help

https://eduassistpro.github.io/

Add WeChat edu_assist_pro