CMSC5741 Big Data Tech. & Apps.

Lecture 7: Recommender Systems / M[illegible]on

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The Netflix Problem

- Netflix database
  - About half a million users

  - About 18,

- People assign ratings to movies

- A sparse matrix

The Netflix Problem

- Netflix database
  - Over 480,000 users
  - About 18,000 movies
  - Over 100,00 ratings

- People assign ratings to movies
- A sparse matrix
  - Only 1.16% of the full matrix is observed

The Netflix Problem

- Netflix database
  - About half a million users
  - About 18,000 movies
- People assign r
  movies
- A sparse matrix

Challenge:
Complete the “Netflix Matrix”

Many such problems: collaborative filtering, partially filled out surveys …

BellKor Recommender System

- **The winner of the Netflix Challenge!**
- **Multi-scale modeling of the data:**
  Combine top level, “regional”

  modeling of the d

  a refined, local vi

  - **Global:**
    - Overall deviations of users/movies
  - **Factorization:**
    - Addressing “regional” effects
  - **Collaborative filtering:**
    - Extract local patterns

Global effects

Factorization

Collaborative filtering

Modeling Local & Global Effects

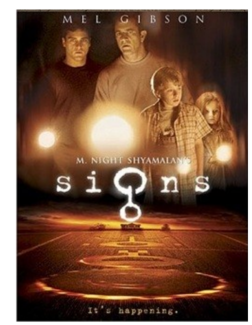
- **Global:**
  - Mean movie rating: **3.7 stars**
  - *The Sixth Sense* is **0.5** stars above avg.
  - Joe rates **0.2** st

    ⇒ Baseline estimation:

    ***Joe* will rate *The Sixth Sense* 4**

- **Local neighborhood (CF/NN):**
  - *Joe* didn't like related movie *Signs*
  - **⇒ Final estimate:**

    ***Joe* will rate *The Sixth Sense* 3.8 stars**



Outline

- Introduction
- LU Decomposition
- Singular Val
- Probabilistic Matrix Fa on
- Non-negative Matrix Factorization
- Recent Development of Matrix Factorization methods in Collaborative Filtering

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High Dimensional Data

High dim. data	Graph data	Infinite data	Machine learning	Apps
Locality Sensitive Hashing		Stream		Recommender Systems
Clustering	Community Detection	Web Advertising	Decision Trees	Association Rules
Dimensionality Reduction	Spam Detection	Queries on Streams	Perceptron, kNN	Duplicate Document Detection

Matrix Completion

- Matrix $X \in R^{N \times M}$

- Observe su entries

- Can we guess the missing entries?

Everyone would agree this looks impossible.

Massive High-dimensional Data

Engineering/scientific applications:
Unknown matrix often has (approx.) low rank.

Images

Videos

High-dimensionality but often low-dimensional structure

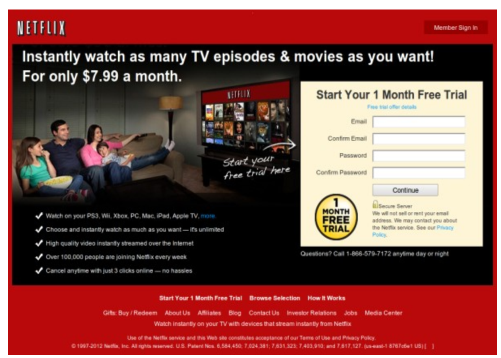
Dear reader, I want you to ask yourself this question: What caused me to become shy? Yes, I'm talking about your present shyness or any you may have suffered in the past. It's quite possible that your story may have a lot in common with that of Joman, the novel's main character. In this book, which unfolds in a spellbinding atmosphere of suspense, the factors that contribute to Joman's becoming a shy child are recounted in detail. You will see that many factors that contributed to his shyness started or existed before he was even born, and this could be your case as well.

What happened after your shyness took root?

You will see how Joman's shyness interfered with his relationships with other people, with his family life, and in matters as diverse as dating, sex, work, and general well-being.

Throughout most of the book, you will enjoy reading how he managed to overcome his shyness.

Get ready to live through a diversified range of emotions in eleven chapters. The story will grab hold of you in the first few pages and carry you all the way to the end. And there's really nothing to be gained by going directly to the very last page to see how things turn out because the plot presents new elements in each chapter. Although instructive, and even pedagogical in certain aspects, the book tells the saga of the main character and his family.



Text

Web data

Matrix Recovery Algorithm

Observation:
Try to recover a lowest complexity (rank) matrix that agrees with the observation

Recovery by mini ming no noise)

$$\text{subject to} \quad \hat{X}_{ij} = X_{ij} \quad (i,j) \in \mathcal{Q}_{obs}$$

- NP hard: not feasible for N > 10!
- Resort to other approaches
  - Select a low rank K, and approximate X by a rank K matrix X'

Low Rank Factorization

- Assume X can be recovered by a rank K matrix X'
- Then X' can be factorized into the product of

$U \in R^{K \times N}, V \in R^{K \times M}$

- Let $\mathcal{E}$ be the loss function

Recovery by rank K matrix

$$\text{minimize} \quad \sum_{i,j \in \mathcal{Q}_{obs}} \mathcal{E}(\hat{X}_{ij} - X_{ij})$$

$$\text{subject to} \quad \hat{X} = U^T V$$

Overview of Matrix Factorization Methods

- Some methods are traditional mathematical way of factorizing a matrix.
  - SVD, LU, Eigen Decomposition
- Some methods are used to factorize partially observed matrix.
  - PMF, SVD++, MMMF
- Some methods have multiple applications.
  - NMF in image processing
  - NMF in collaborative filtering

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LU Decomposition

LU Decomposition

The LU Decomposition factors a matrix as the product of a lower triangular matrix (L) and an upper triangular matrix (U).

$$A = LU$$

Lower triangular matrix:
Every entry above the main diagonal are zero.

Upper triangular matrix:
Every entry below the main diagonal are zero.

LU Decomposition

- LU Decomposition is useful when
  - Solving a system of linear equations
  - Inverting a
  - Computing the deter a matrix
- LU Decomposition can be computed using a method similar to Gaussian Elimination

LU Decomposition

- Computing LU Decomposition of a matrix A
  - Using Gaussian elimination to compute U
  - Apply inv he corresponding entry to I

$$\underset{A}{\begin{bmatrix} 1 & 2 & 3 \\ 2 & -4 & 6 \\ 3 & -9 & -3 \end{bmatrix}} \xrightarrow[R3 - 3R1]{R2 - 2R1} \begin{bmatrix} 1 & 2 & 3 \\ 0 & -8 & 0 \\ 0 & -15 & -12 \end{bmatrix} \xrightarrow{R2 * (-1/8)} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 0 & -15 & -12 \end{bmatrix} \xrightarrow{R3 + 15R2} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & -12 \end{bmatrix} \xrightarrow{R3 * (-1/12)} \underset{U}{\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}$$

LU Decomposition

- Computing LU Decomposition of a matrix A
  - Using Gaussian elimination to compute U
  - Apply inverse operation on the corresponding entry to I to

    - Any row operations that ding a multiple of one row to another, for + kRj, put the value –k in the ith-row, jth-column of the identity matrix.

    - Any row operations that involves getting a leading one on the main diagonal, for example, kRi, put the value 1/k in the position of the identity matrix where the leading one occurs.

LU Decomposition

- Computing LU Decomposition of a matrix A
  - Using Gaussian elimination to compute U
  - Apply inverse operation on the corresponding entry to I t

I

L

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{\text{R2 + 2R1, R3 + 3R1}} \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix} \xrightarrow{\text{R2 * (-8)}} \xrightarrow{\text{R3 - 15R2}} \xrightarrow{\text{R3 * (-12)}} \begin{bmatrix} 1 & 0 & 0 \\ 2 & -8 & 0 \\ 3 & -15 & -12 \end{bmatrix}$$

R2 + 2R1
R3 + 3R1

R2 * (-8)

R3 - 15R2

R3 * (-12)

LU Decomposition

- Computing LU Decomposition of a matrix A
  - Using Gaussian elimination to compute U
  - Apply inve e corresponding entry to I t

$$\overset{A}{\begin{bmatrix} 1 & 2 & 3 \\ 2 & -4 & 6 \\ 3 & -9 & 3 \end{bmatrix}} = \overset{L}{\begin{bmatrix} 1 & 0 & 0 \\ 2 & -8 & 0 \\ 3 & -15 & -12 \end{bmatrix}} \overset{U}{\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}$$

In-class Practice 1

- Go to practice

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Singular Value Decomposition

Singular Value Decomposition
The Singular Value Decomposition (SVD) of an NxM matrix A is a factorization of the form:

- V^* is the conjugate trans V
- $U \in \mathbb{R}^{N \times N}$ is orthonormal matrix, i.e., $UU^* = I$
- $\Sigma \in \mathbb{R}^{N \times M}$ is rectangular diagonal matrix with positive entries
- $V^* \in \mathbb{R}^{M \times M}$ is orthonormal matrix, i.e., $VV^* = I$

SVD v.s. Eigen Decomposition

Singular Value Decomposition

The Singular Value Decomposition (SVD) of an NxM matrix A is a factorization of the form:

- Diagonal entries of Σ are ca lar values of A.
- Columns of U and V are called left singular vectors and right singular vectors of A, respectively
- The singular values $\Sigma_{ii}s$ are arranged in descending order in Σ

SVD v.s. Eigen Decomposition

Singular Value Decomposition

The Singular Value Decomposition (SVD) of an NxM matrix A is a factorization of the form:

- The left singular vectors of A are eige A*, because

- The left singular vectors of A are eigenvectors of A*A, because

$$A^*A = (U\Sigma V^*)^*(U\Sigma V^*) = V\Sigma^T\Sigma V$$

- The singular values of A are the square roots of eigenvalues of both AA* and A*A.

SVD Example

- We give an example of a simple SVD decomposition

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 2 \\ 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 \end{bmatrix} = \qquad \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ \sqrt{0.2} & 0 & 0 & 0 & \sqrt{0.8} \\ 0 & 0 & 0 & 1 & 0 \\ -\sqrt{0.8} & 0 & 0 & 0 & \sqrt{0.2} \end{bmatrix}$$

$A \qquad U \qquad \Sigma \qquad V^*$

SVD as Low Rank Approximation

Low Rank Approximation

$$\operatorname{argmin}_{\tilde{A}} \quad \|A - \tilde{A}\|_{Fro}$$

$$\text{s.t.} \quad \operatorname{Rank}(\tilde{A}) = r$$

SVD gives the opti

Solution (Eckart-Young Theorem)

Let $A = U\Sigma V^*$ be the SVD for A, and $\tilde{\Sigma}$ is the same as Σ by keeping the largest *r* singular values. Then,

$$\tilde{A} = U\tilde{\Sigma}V^*$$

Is the solution to the above problem.

SVD as Low Rank Approximation

Solution (Eckart-Young Theorem)

Let $A = U\Sigma V^*$ be the SVD for A, and $\tilde{\Sigma}$ is the same as Σ by keeping the largest r singular values. Then,

Is the solution to the above probl

- It works when A is fully observed.
- What if A is only partially observed?

Low Rank Approximation for Partially Observed Matrix

Low Rank Approximation for Partially Observed Matrix

- I_{ij} is the indicator that equals 1 if A_{ij} is observed and 0 otherwise
- We consider only the observed entries.
- A natural probabilistic extension of the above formulation is Probabilistic Matrix Factorization

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Probabilistic Matrix Factorization

- A popular c ring (CF) method

- Follow the low rank matrix factorization framework

Collaborative Filtering

Collaborative Filtering

The goal of collaborative filtering (CF) is to infer user preferences for items given a large but incomplete collection of preferences for many users.

- For example:

  - Suppose you infer f users who like "Star Wars" also like "Lord of th

  - Then if a user watched and liked "Star uld recommend him/her "Lord of the Rings" but not "Dune".

- Preferences can be explicit or implicit:
  - Explicit preferences
    - Ratings assigned to items
    - Facebook "Like", Google "Plus“
  - Implicit preferences
    - Catalog browse history
    - Items rented or bought by users

Content Based Filtering vs. Collaborative Filtering

Content Based Filtering

- Analyze the content of the item
- Match the item with users preferences
- Item features are hard to extract
  - Music, Movies
- Can recommend new items

Collaborative Filtering

- User preferences are d from ratings
- eat ferred ngs
- Cannot recommend new items
- Very effective with sufficient data

CF as Matrix Completion

- CF can be viewed as a matrix completion problem

Items

- Task: given a user/item matrix with only a small subset of entries present, fill in (some of) the missing entries.
- PMF approach: low rank matrix factorization.

Collaborative Filtering and Matrix Factorization

- Collaborative filtering can be formulated as a matrix factorization problem.
- Many matrix factorization methods can be used to solve collaborative filtering problem.
- The above is only a partial list.

Notations

- Suppose we hav integer rating values from 1 to D.
- Let *ij*th entry of X, X_{ij}, be the rating of user i for item j .
- $U \in R^{K\times N}$ is latent user feature matrix, U_i denote the latent feature vector for user i .
- $V \in R^{K\times M}$ is latent item feature matrix, V_j denote the latent feature vector for item j .

Matrix Factorization: the Non-probabilistic View

- To predict the rating given by user i to item j,

$$V_{kj}$$

- Intuition

  - The item feature vector can be he input.

  - The user feature vector can be viewed as the weight vector.
  - The predicted rating is the output.
  - Unlike in linear regression, where inputs are given and weights are learned, we learn both the weights and the input by minimizing squared error.
  - The model is symmetric in items and users.

Probabilistic Matrix Factorization

- PMF is a simple probabilistic linear model with Gaussian observation noise.
- Given the feature vectors for the user and the item, the distribution of the :

P

- The user and item feature vectors adopt zero-mean spherical Gaussian priors:

$$P(U|\sigma_U^2) = \prod_{i=1}^{N} \mathcal{N}(U_i|0, \sigma_U^2 I) \qquad P(V|\sigma_V^2) = \prod_{j=1}^{M} \mathcal{N}(V_j|0, \sigma_V^2 I)$$

Probabilistic Matrix Factorization

- Maximum A Posterior (MAP): Maximize the log-posterior over user and item features with fixed hyper-parameters.
- MAP is equivalent to minimizing the following objective function:

PMF objective function

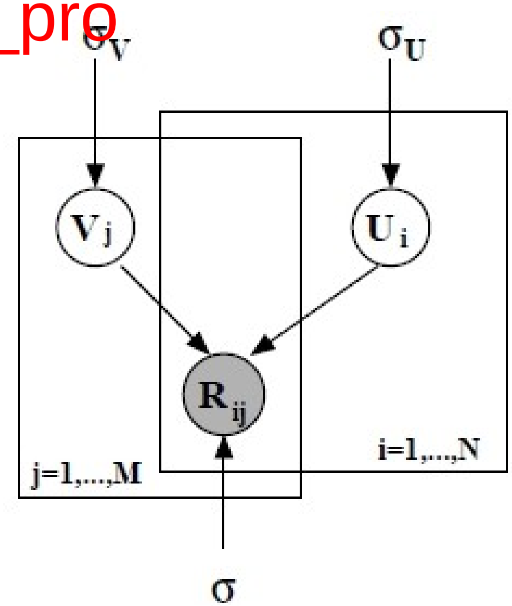
$$\mathcal{E} = \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{M} I_{ij}(R_{ij} - U_i^T V_j)^2 + \frac{\lambda_U}{2}\sum_{i=1}^{N}\|U_i\|_{Fro}^2 + \frac{\lambda_V}{2}\sum_{j=1}^{M}\|V_j\|_{Fro}^2$$

Probabilistic Matrix Factorization

PMF objective function

$$\mathcal{E} = \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{M} I_{ij}(R_{ij} - U_i^T V_j)^2 + \frac{\lambda_U}{2}\sum_{i=1}^{N}\|U_i\|_{Fro}^2 + \frac{\lambda_V}{2}\sum_{j=1}^{M}\|V_j\|_{Fro}^2$$

- of whether user i rated item j
- First term is the sum-of-squared-error.
- Second and third term are quadratic regularization term to avoid over-fitting problem.



In-class Practice 2

- Go to practice

Probabilistic Matrix Factorization

PMF objective function

$$\mathcal{E} = \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{M} I_{ij}(R_{ij} - U_i^T V_j)^2 + \frac{\lambda_U}{2}\sum_{i=1}^{N}\|U_i\|_{Fro}^2 + \frac{\lambda_V}{2}\sum_{j=1}^{M}\|V_j\|_{Fro}^2$$

- If all ratings were observed, t e reduces to the SVD objective in the limit of prior variances going to infinity.
- PMF can be viewed as a probabilistic extension of SVD.

Probabilistic Matrix Factorization

PMF objective function

$$\mathcal{E} = \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{M} I_{ij}(R_{ij} - U_i^T V_j)^2 + \frac{\lambda_U}{2}\sum_{i=1}^{N}\|U_i\|_{Fro}^2 + \frac{\lambda_V}{2}\sum_{j=1}^{M}\|V_j\|_{Fro}^2$$

A trick to improve g values)

- Map ratings to [0,1] by $(R_{ij} - 1)/(D - 1)$
- Pass $U_i^T V_j$ through logistic function

$$g(x) = \frac{1}{1 + \exp(-x)}$$

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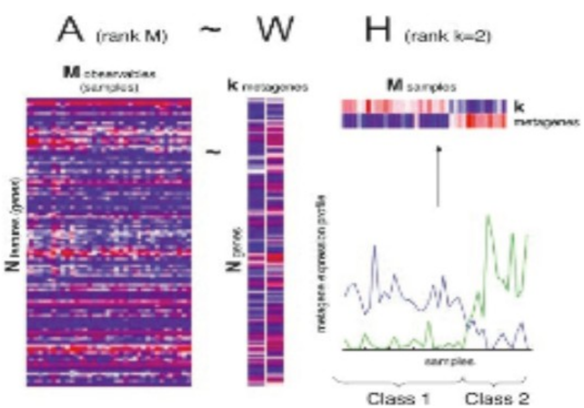
- **Non-negative Matrix Factorization**
- Recent Development of Matrix Factorization methods in Collaborative Filtering

Non-negative Matrix Factorization

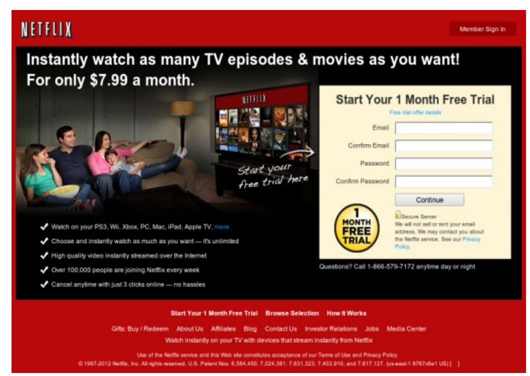
NMF is a popular method that is widely used in:

Images Mining

t Mining



Metagenes Study



Collaborative Filtering

Non-negative Matrix Factorization

- NMF fits in the low rank matrix factorization framework with additional non-negativity constraints.
- NMF can only factorize a Non-negative matrix $A \in \mathbb{R}^{N \times M}$ into basis matr ght matrix $H \in \mathbb{R}^{K \times M}$

$$\text{s.t.} \quad W, H \geq 0$$

Interpretation with NMF

- Columns of W are the underlying basis vectors, i.e., each of the M columns of A can be built from K columns of W.
- Columns of H give the weights associated with each basis vector.

- W,H >= 0 commands additive parts-based representation.

NMF in Image Mining

Additive parts-based representation

NMF in Image Mining

- In image processing, we often assume Poisson Noise

NMF Poisson Noise

- Objective function can be changed t , the non-negative constraint is more important than the form of the objective function.

NMF Gaussian Noise

$$\begin{aligned} \min \quad & \|A - WH\|_{Fro}^2 \\ \text{s.t.} \quad & W, H \geq 0 \end{aligned}$$

Inference of NMF

NMF Gaussian Noise

$$\min \|A - WH\|_{Fro}^2$$

- Convex in W or H, but not bo
- Global min generally not achievable.
- Many number of unknowns: N×K for W and M×K for H (or H^T)

Inference of NMF

NMF Gaussian Noise

$$\min \quad \|A - WH\|_{Fro}^2$$
$$\text{s.t.} \quad W, H \geq 0$$

- Alternating g get a local minima

W

Properties of NMF

- Basis vectors W_i are not orthogonal
- W_k, $H_k \geq 0$ Have immediate interpretation
  - Example: large w_{ij}'s implies basis vector W_i is mostly about terms j
  - Example: h_{i1} den pointing in the "direction" of topic vector W_1

$$Ae_1 = WH_{*1} = [W_1]\, H_{11} + [W_2]\, H_{21} + \cdots + [W_K]\, H_{K1}$$

- NMF is algorithm-dependent: W, H not unique

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Recent Development of MF methods in Collaborative Filtering

- The basic form of matrix factorization has been extended to improve prediction accuracy
  - SVD++ [Yehuda Koren
  - RLFM [Agarwal 2009]
  - Etc.

SVD++

- SVD++ is a matrix factorization model which makes use of implicit feedback.

- In general, i can refer to any kinds of users' his ormation that can help indicate user rences.

$$
\begin{aligned}
\hat{r}_{ui} = \quad & \mu + b_u + b_i \\
& + q_i^T \left(p_u + |N(u)|^{-\frac{1}{2}} \sum_{j \in N(u)} y_j \right) \\
& + |R^k(i;u)|^{-\frac{1}{2}} \sum_{j \in R^k(i;u)} (r_{uj} - b_{uj}) w_{ij} + |N^k(i;u)|^{-\frac{1}{2}} \sum_{j \in N^k(i;u)} c_{ij}
\end{aligned}
$$

1^{st} Tier

- The first term is the basis rate; it takes in account a global mean and the bias of both user a

2^{nd} Tier

- The second term is similar to the original SVD but takes in account the implicit feedback pr of rated items N(u)

3rd Tier

- The third and fourth terms are the neighborhood terms. The former is the weighted bi te and the actual rate, and the la e local effect of the implicit feedba

$$\begin{aligned}\hat{r}_{ui} = \quad & \mu + b_u + b_i \\ & + q_i^T \left(p_u + |N(u)|^{-\frac{1}{2}} \sum_{j \in N(u)} y_j \right) \\ & + |R^k(i;u)|^{-\frac{1}{2}} \sum_{j \in R^k(i;u)} (r_{uj} - b_{uj}) w_{ij} + |N^k(i;u)|^{-\frac{1}{2}} \sum_{j \in N^k(i;u)} c_{ij}\end{aligned}$$

RLFM

- Regression-based Latent Factor Model makes use of the side information that is available in der systems
  - User demographic inf
  - Properties of items (e.g. director, leading actor of a movie, genre of a movie)

One-slide Takeaway

- Matrix Factorization is the key to recommender systems
- LU-decomposition
  - Decompose a matrix into a lower triangular matrix and an upper triangular matrix

- SVD decompositi

  - Decompose a matrix into , where rmal matrices and is a diagonal matrix, whose values are lar values

- Probabilistic Matrix Factorization
  - Factorize a partially observed matrix into the product of two low-rank matrices, usually used in recommender systems
- Non-negative Matrix Factorization
  - Factorize a matrix into the produce of two non-negative matrices, can be used to learn the "parts"

References

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Netflix algorithm: Prize Tribute Recommendation Algorithm in Python, Danny Tarlow

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In-class Practice 1

LU decomposition

Perform LU decomposition of the following matrix A:

$$\begin{pmatrix} 2 & -5 \\ 0 & 2 \end{pmatrix}$$

In-class Practice 2

PMF objective function

Write out the partial derivative of the above objective function with respect to U_i and V_j.

We will explain how to solve the equation using the partial derivatives.