Com S 311, Final Exam

Problem	Max Points	Score
1 (Basics)	20	
2 (Hash and Heaps)	20	
3 (Recurrence)	10	
4 (Graph)	15	
6 (Dyna		
(Extra Credit-1: Graphs)	20	
(Extra Credit-2: NP)	20	
Total	100 + 40 (EC)	

- If you write *Do not grade* and nothing else, you will receive 15% cre
- For all problems that involve writing an algorithm, please write clear pseudocode, not Java or C.
- In general you may use the f
  exam (that is, unless m
  for them): sorting, brea hm,
  Dijkstra's algorithm, topological sort. You can assume that hash (1)
  However, if you modify any of these methods, then you must write a co ed method. Merely stating the modification does not suffice.

- Level of points for solutions related to design and analysis algorithm

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if designed algorithm is correct and efficient as expected then
   if proof of correctness is unambiguous and justifiable then
      if run-time analysis shows derivation steps and is correct then
           points = 100%
      else
           points = 75%
   else if run-time analysis shows derivation steps and is correct then
           points = 75%
         else
           points = 50%
else if designed algorithm is correct and brute force then
   points = 30%
else if designed algorithm is incorrect then
   points = 0--20% (at the discretion of grader)
else if answer is "DO NOT GRADE" (YOU NEED TO EXPLICITLY WRITE DO NOT GRADE)
   points = 15%
else
   points = 0%
```

1. Short Answer questions. Do not write explanations. There is *Do not grade* option for this question.

   (a) True or False: $2^{3n} \in O(2^{2n})$.

   (b) True or False: $2^{3^n} \in O(2^{2^n})$.

   (c) True or False: Every divide and conquer algorithm runs in $O(n \log n)$ time.

   (d) True or False: If G has a topological ordering, then G must be a DAG.

   (e) True or False: Every undirected graph with more than n edges has a cycle.

   (f) True of False: The runtime for solving any decision problem in NP is not polynomial with respect to the size of the problem..

   (g) What is the solution to the recurrence $T(n) = T(n/2)$, $T(1)$ $O(1)$.

   (h) What is the runtime for rem

   (i) Assuming that the comp to output the smallest element of

   (j) What is the runtime of Prim's Algorithm?

2. *Hashing and Heap*

   (a) Let A_1, A_2 be two arrays each consisting of n integers and T be an integer. Give an algorithm using hashing that checks whether there exist integers at $x + y$ equals T. State the run-time of your algorithm.

   (b) Given an array A of n integers and an integer k, give an kth smallest element. Express runtime as a function of k and n.

3. Solve the recurrence rela

4. You are given a directed gr e graph has a vertex v with the foll

   - Number of vertices from which there is a path to k_1.
   - Number of vertices that have a path from v (bt $= 1$.
   - There is no vertex $u \neq v$ such that there is a path from th from v to u.

   Write the verification algorithm.

5. You are given n jobs numbered $1, 2, \cdots, n$ to complete and each job i comes with a difficulty d_i. Each job takes exactly one week to complete irrespective of its difficulty. If you complete job i during week $j(\leq n)$, then you earn a profit of $d_i(n - j)$. You plan to complete all the jobs in n weeks, since the goal is to maximize the profit. Consider greedy algorithm that completes that jobs in the based on the *decreasing order* of difficulty. Use exchange argument to prove that the greedy algorithm produces optimal solution. You may assume that all d_i's are distinct.

6. Let M be a matrix of integers with n rows (rows numbered 1 to n) and m columns (columns numbered 1 to m). Let $M[i, j]$ denote the entry in ith row and jth column. A *horizontal cut* in M is a sequence $[c_1, c_2, \cdots, c_m]$ such that

   - For every i, $1 \leq c_i \leq n$
   - For $1 \leq i \leq m - 1$, $c_{i+1} \in \{c_i - 1, c_i, c_i + 1\}$

   Given a horizontal cut $[c_1, c_2, \cdots, c_m]$, its cost is $M[c_1, 1] + M[c_2, 2] + \cdots + M[c_m, m]$. A horizontal cut is a *max-cost cut* if its cost is at least the cost of any other horizontal cut.

   Give a dynamic programming algorithm, that takes a matrix M as input, and outputs the *cost of the* max-cost cut. The following needs to be presented as part of your answer.

(a) Recurrence relation (recursive characterization or recursive definition) describing the value of max-cost cut.

(b) Iterative algorithm for computing the value of max-cost cut.

(c) Runtime of the iterative algorithm.

* **Extra Credit-1.** Let G be an undirected graph where every edges has exactly the same weight and let s be a vertex of the graph. Write an algorithm that takes s as input and outputs for every vertex v, the number of shortest path from s to v.

* **Extra Credit-2.** Let $L = \{S_1, S_2, \cdots, S_n\}$ be a collection of sets. A set H is a *blanket* for L if $\forall i \in [1, n] : H \cap S_i \neq \emptyset$. Show that the following decision problem is in NP:

Input: $L = \{S_1, S_2, \cdots$
Decision: Does L have a b