COMP 250

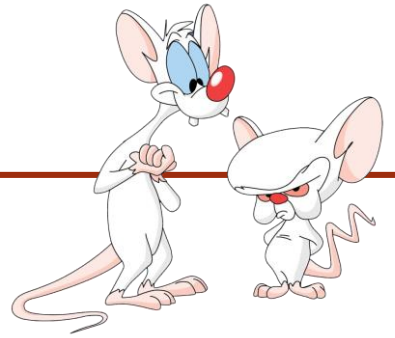
INTRODUCTION TO COMPUTER SCIENCE

Week 12-2 :

Giulia Alberini, Fall 2020

Slides adapted from Michael Langer's

WHAT ARE WE GOING TO DO IN THIS VIDEO?



- Heaps

PRIORITY QUEUE

Assume a set of comparable elements or "keys".

Like a queue, but now [illegible] definition of which element to remove next, namely the [illegible] hest priority.

e.g. hospital emergency room

PRIORITY QUEUE ADT

- add(key)
- removeMin()

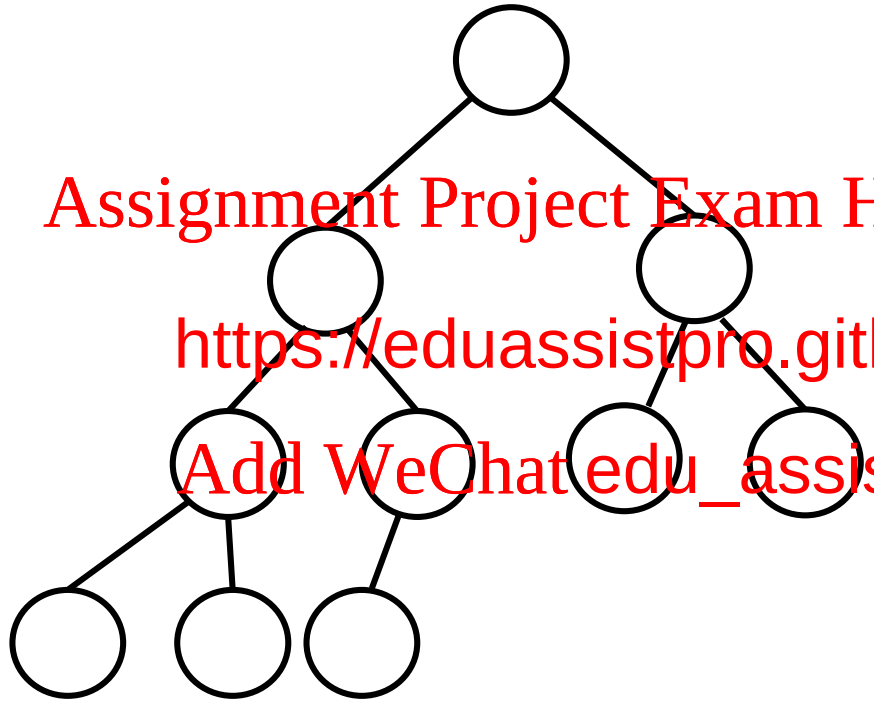
  "highest" priority =
- peek()
- contains(element)
- remove(element)

HOW TO IMPLEMENT A PRIORITY QUEUE ?

- sorted list ?
- binary search tree (la
- balanced binary search tree (COMP
- heap (next 2 lectures)

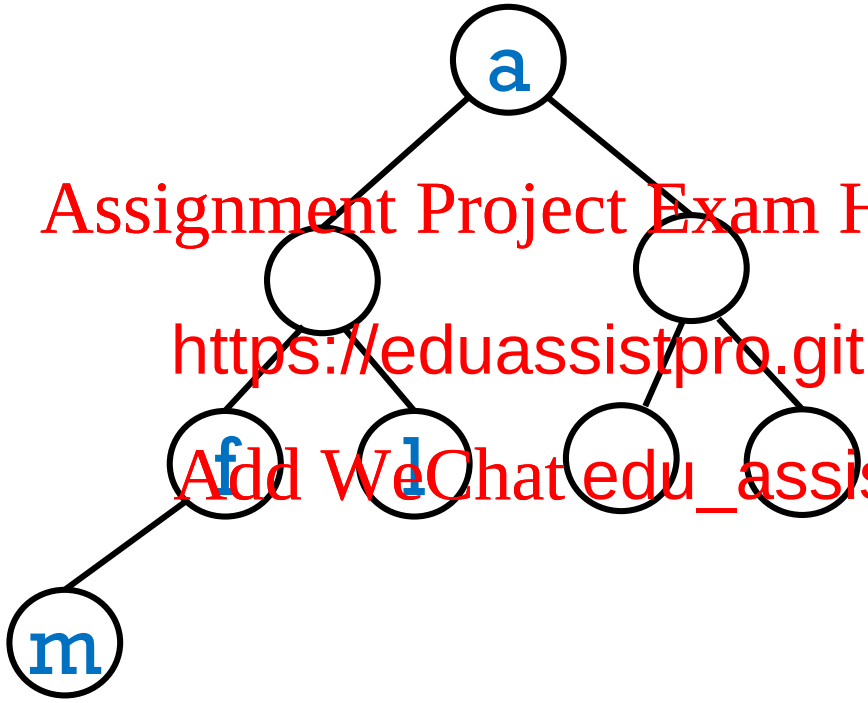
Not the same "heap" you hear about in COMP 206.

COMPLETE BINARY TREE (DEFINITION)



Binary tree of height h such that every level less than h is full, and all nodes at level h are as far to the left as possible

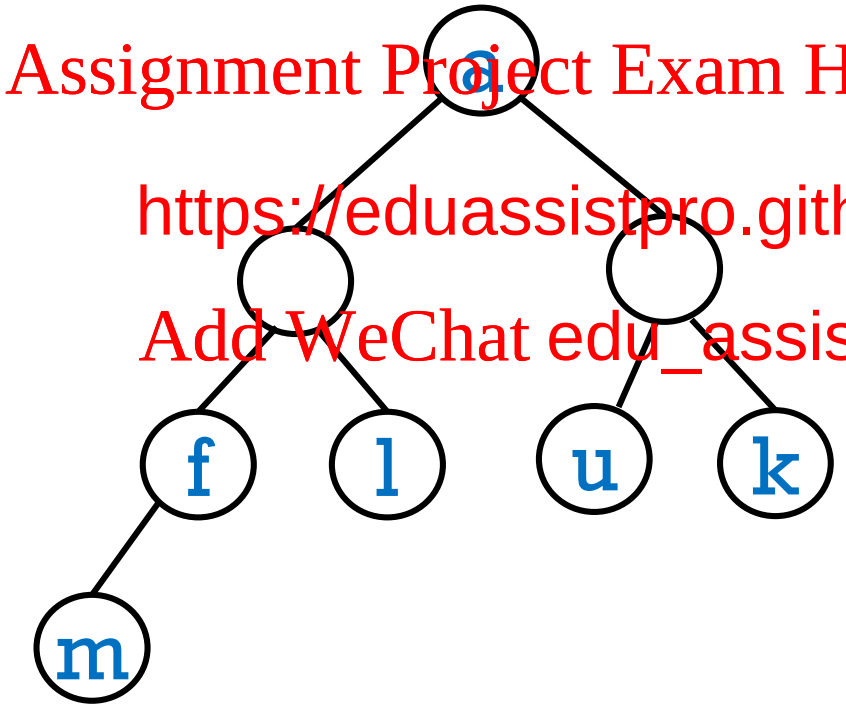
MIN HEAP (DEFINITION)



Complete binary tree with unique comparable elements, such that each node's element (key) is less than its children's element (key).

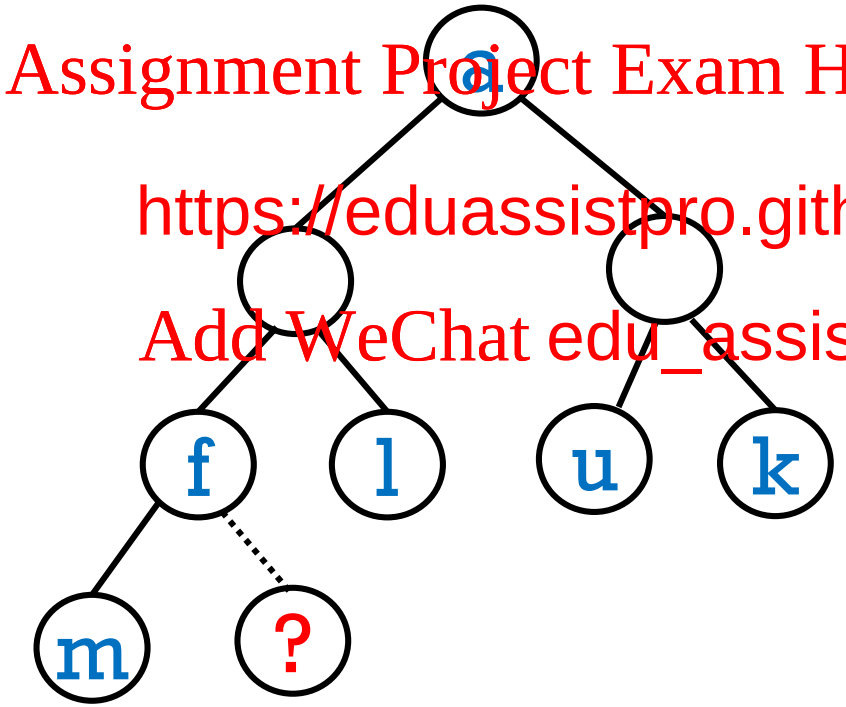
ADD()

For example, `add(c)`



ADD()

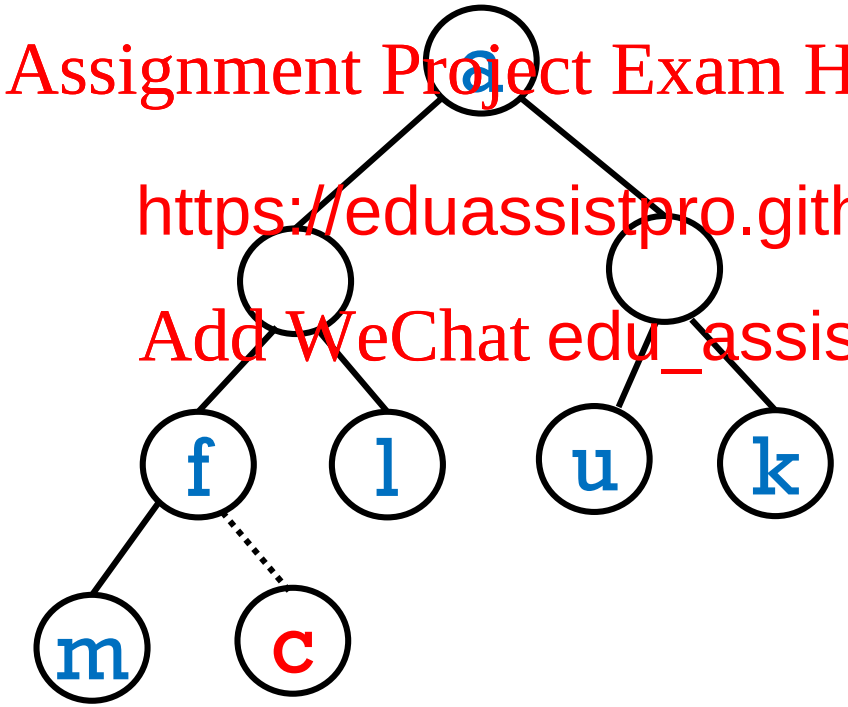
For example, `add(c)`



ADD()

For example, `add(c)`

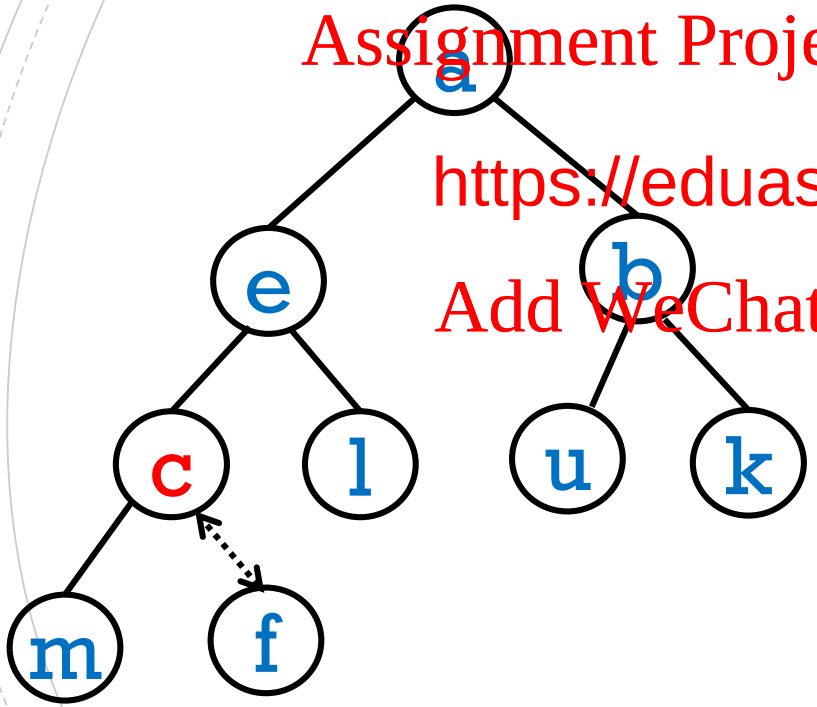
 What can we do?



Problem : adding at the next available slot typically destroys the heap property.

ADD()

For example, `add(c)`

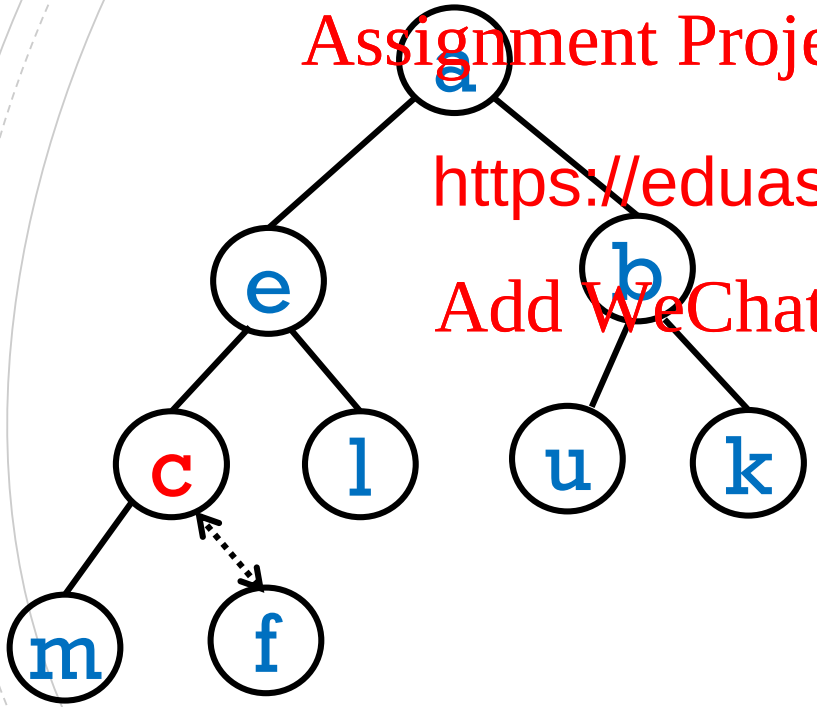


What can we do?

- swap c with its parent f.
- is create a problem with c's former sibling, who is now c's child?

ADD()

For example, `add(c)`



What can we do?

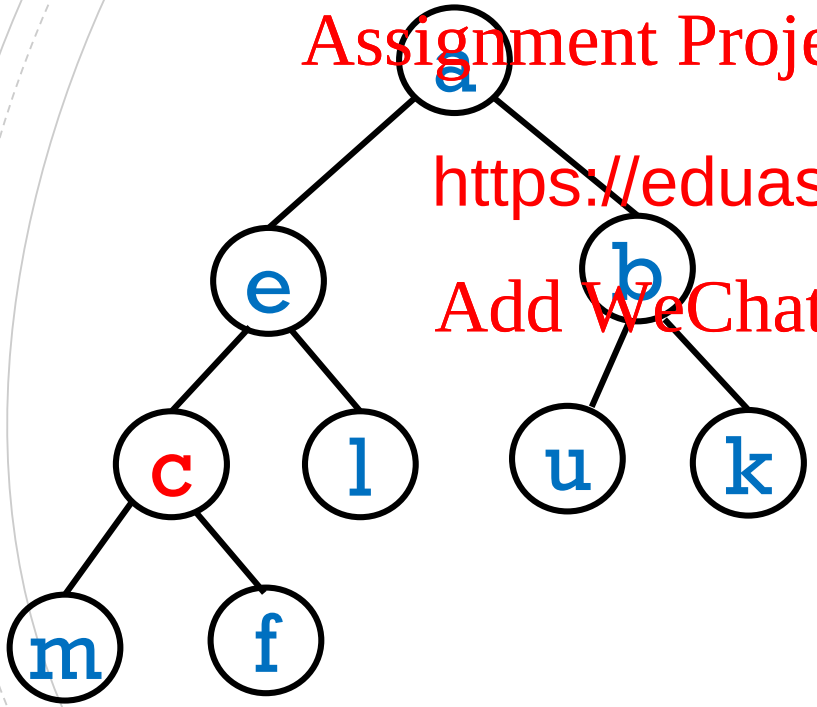
swap c with its parent f.

is create a problem with c's former sibling, who is now c's child?

A: No. Why?

ADD()

For example, `add(c)`

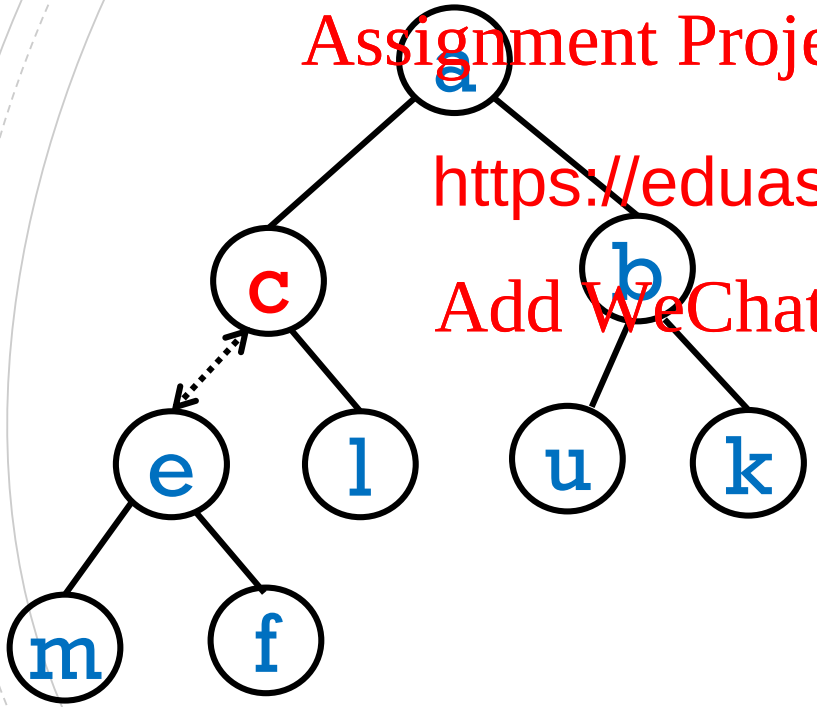


Q: Are we done?

: Unnecessarily. What about c's

ADD()

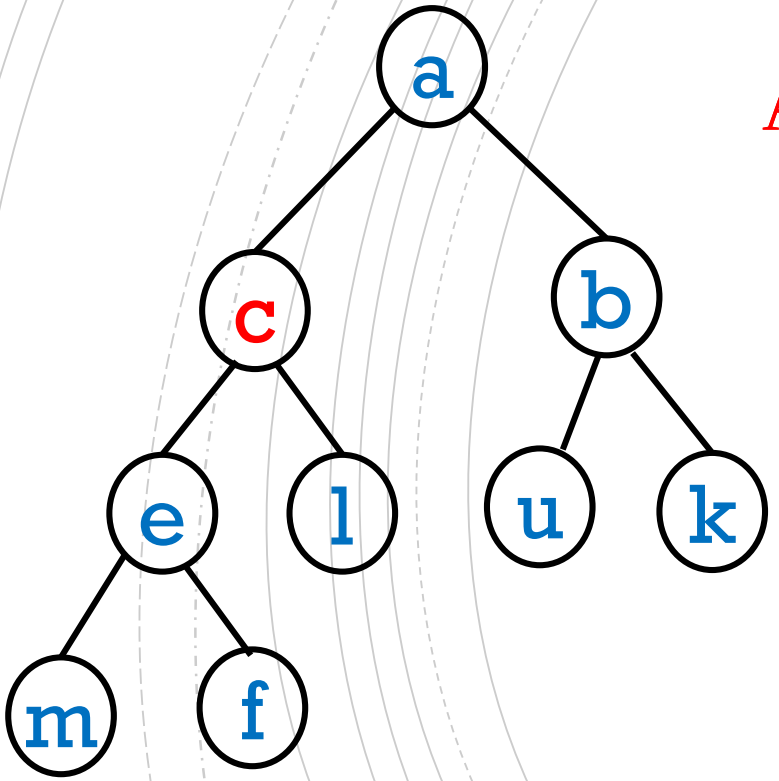
For example, `add(c)`



We swap c with its (new) parent e.

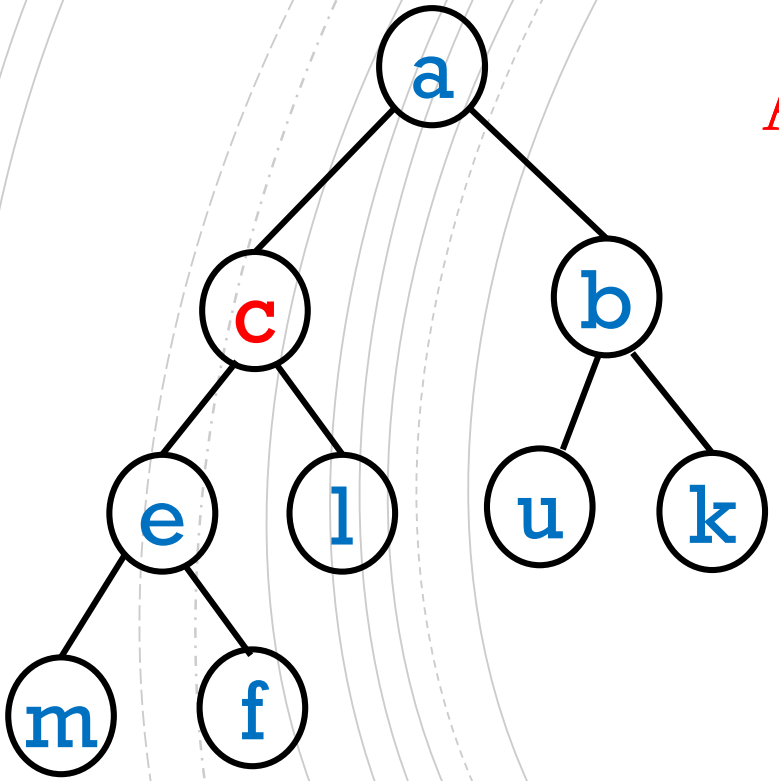
e are done because c is greater
arent a

ADD() - IMPLEMENTATION



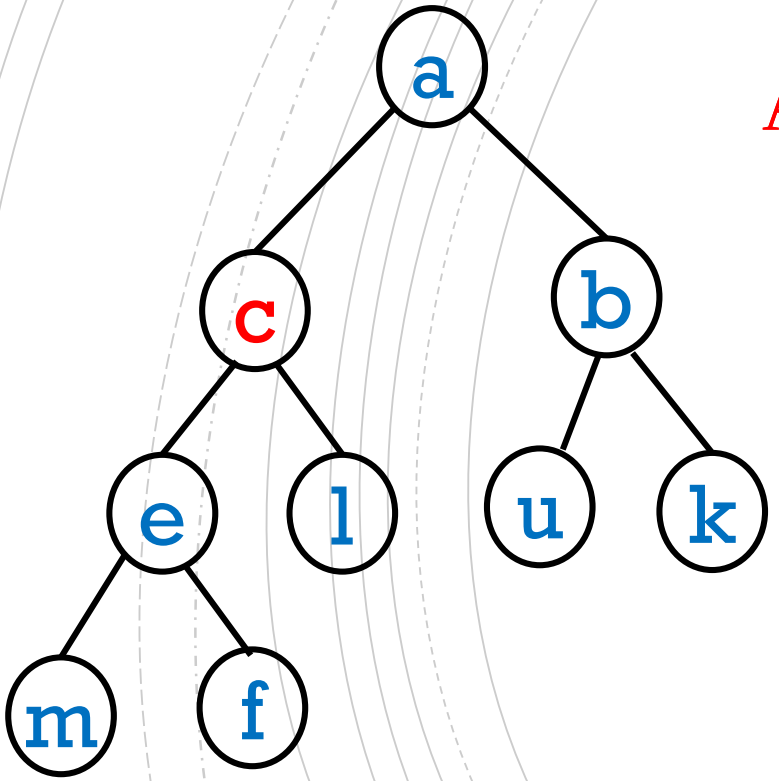

```
add(key) {
    cur = new node at next available leaf position
    cur.key = key

}
```

ADD() - IMPLEMENTATION




```
add(key){
    cur = new node at next available leaf position
    cur.key = key
                                        empty tree
}
```

ADD() - IMPLEMENTATION




```
add(key){
    cur = new node at next available leaf position
    cur.key = key
                                    empty tree
    else {
        while (c                & cur.key<cur.parent.key) {
            swapKeys(cur, cur.parent)
            cur = cur.parent
        }
    }
}
```

HOW TO BUILD A HEAP?

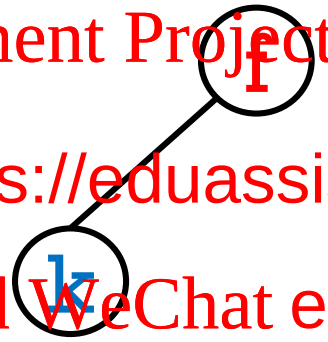
`add( k )`

HOW TO BUILD A HEAP?

```
add( k )
add( f )
```

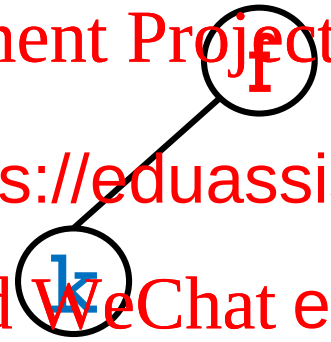
HOW TO BUILD A HEAP?

```
add( k )
add( f )
```

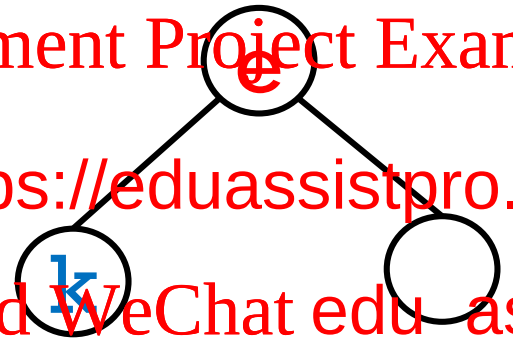
HOW TO BUILD A HEAP?

```
add( k )
add( f )
add( e )
```

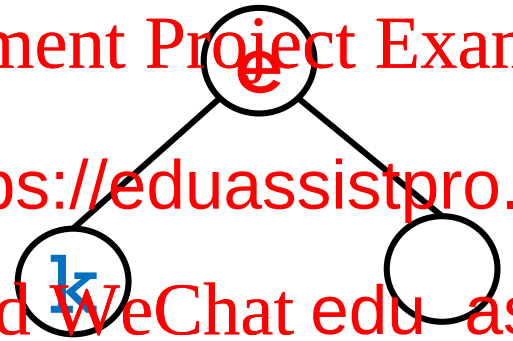
HOW TO BUILD A HEAP?

```
add( k )
add( f )
add( e )
```

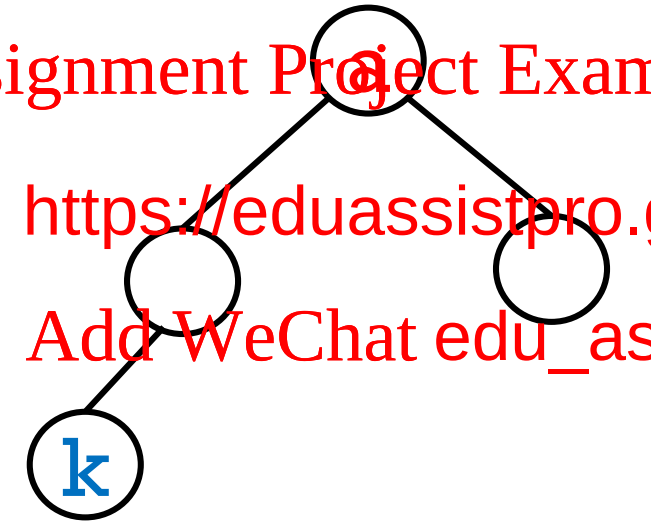
HOW TO BUILD A HEAP?

```
add( k )
add( f )
add( e )
add( a )
```

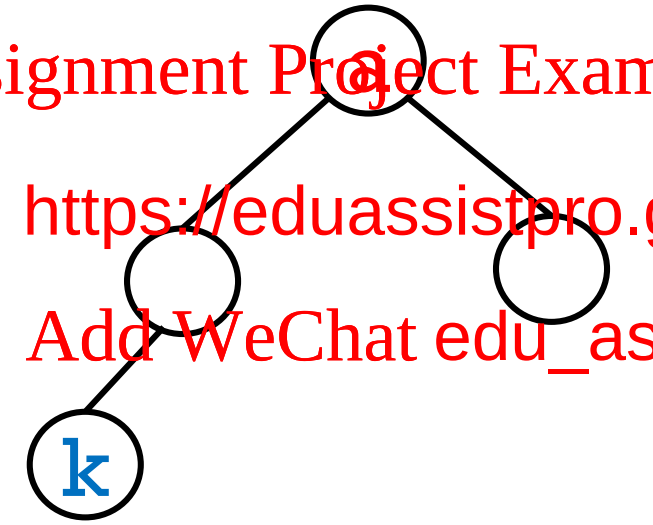
HOW TO BUILD A HEAP?

```
add( k )
add( f )
add( e )
add( a )
```

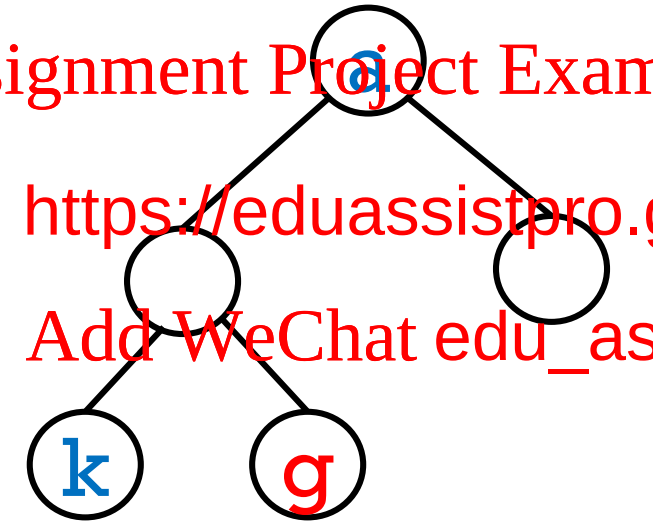
HOW TO BUILD A HEAP?

```
add( k )
add( f )
add( e )
add( a )
add( g )
```

HOW TO BUILD A HEAP?

```
add( k )
add( f )
add( e )
add( a )
add( g )
```

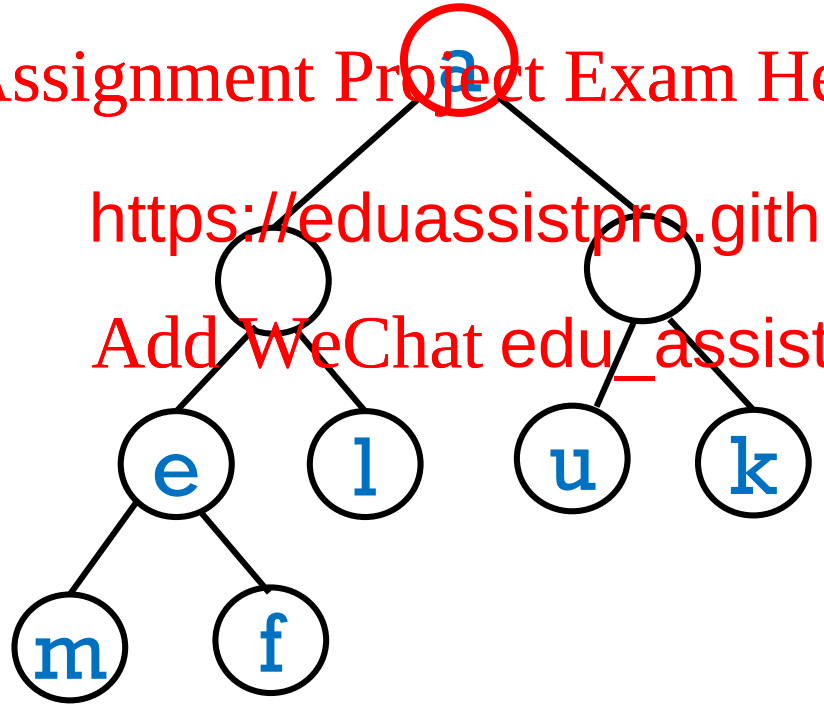



This method of building a heap is slow.

We will see a faster method next video.

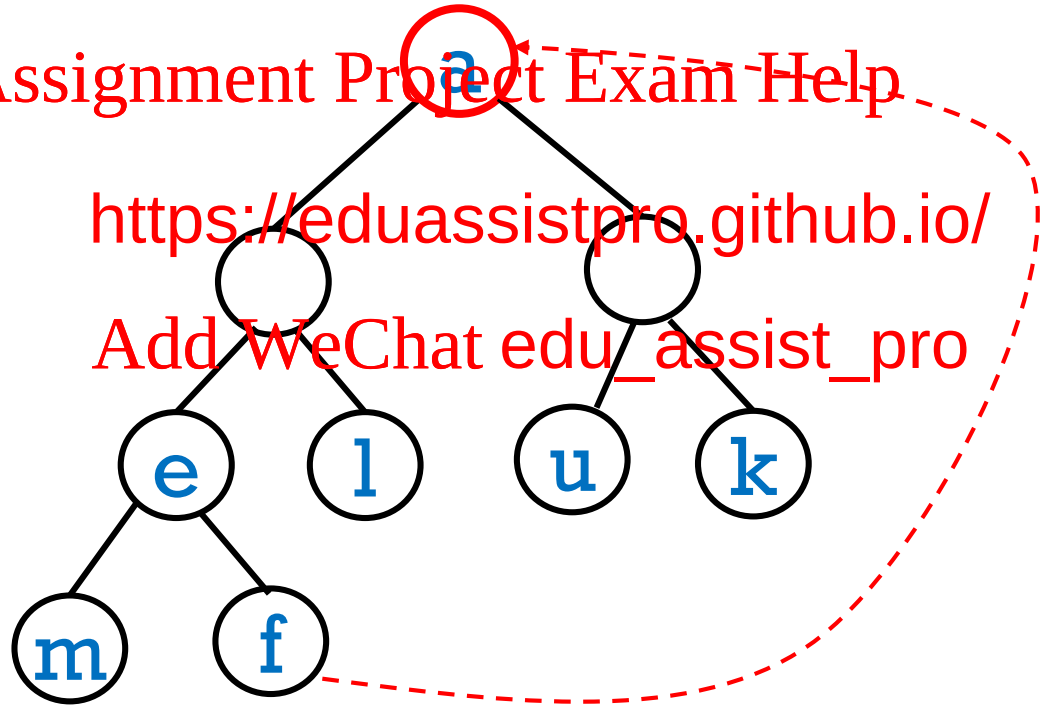
REMOVEMIN()

returns root element



REMOVEMIN()

returns root element



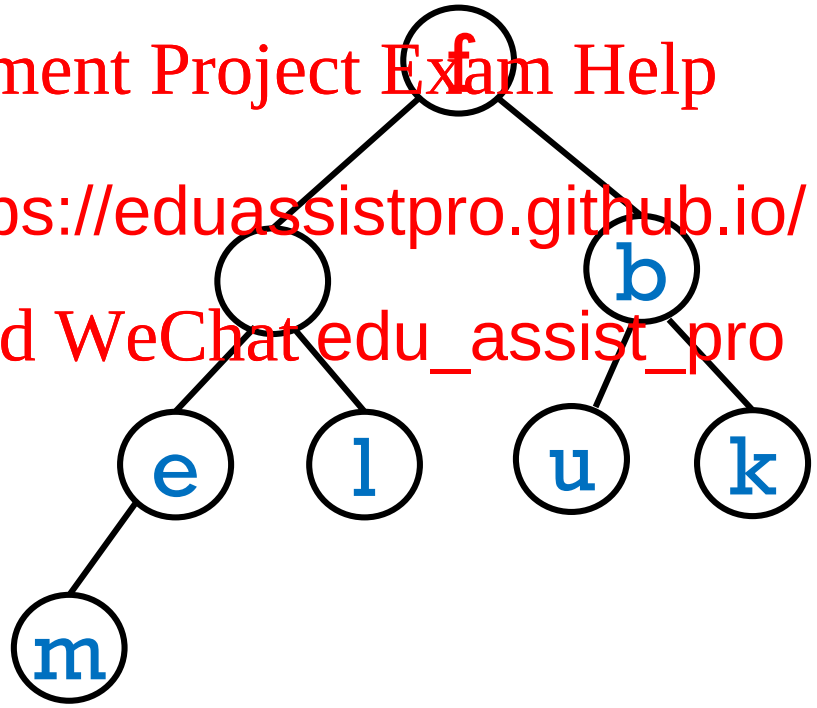
REMOVEMIN()

a

Claim: if the root has two children, then the new root *will* be greater than at least one of its children.

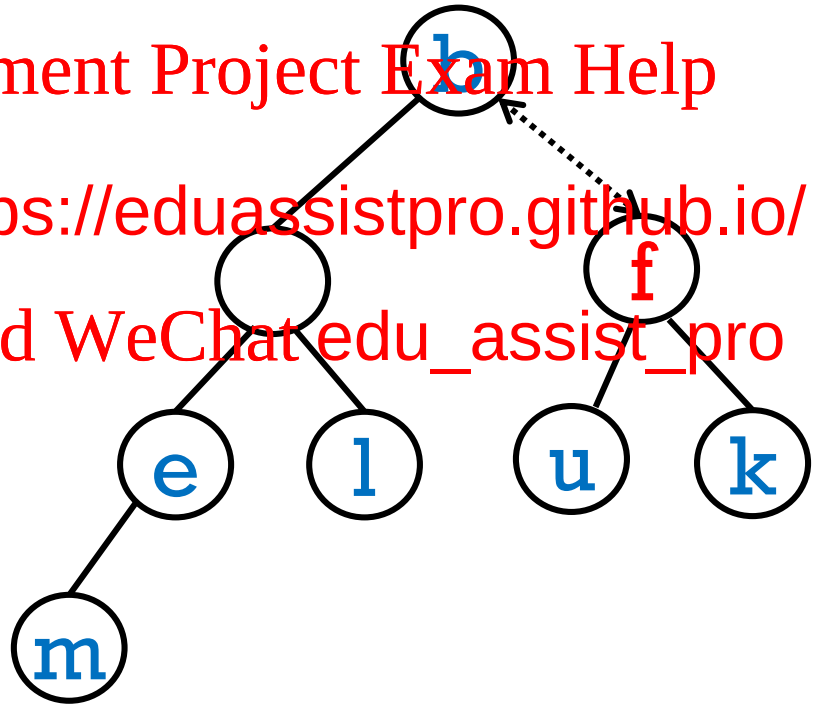
Why?

How to solve this problem?



REMOVEMIN()

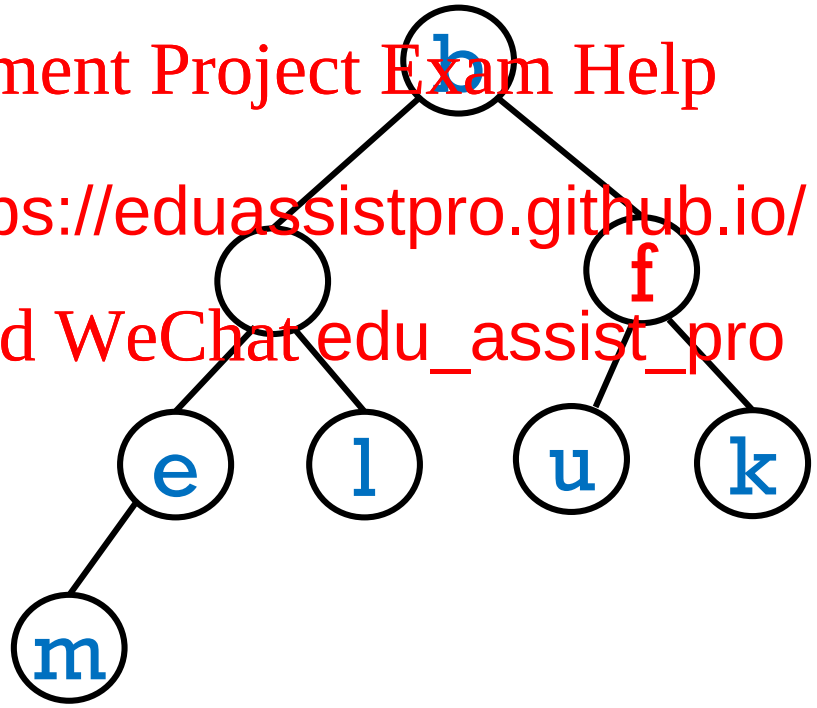
Swap keys with the smaller child!



REMOVEMIN()

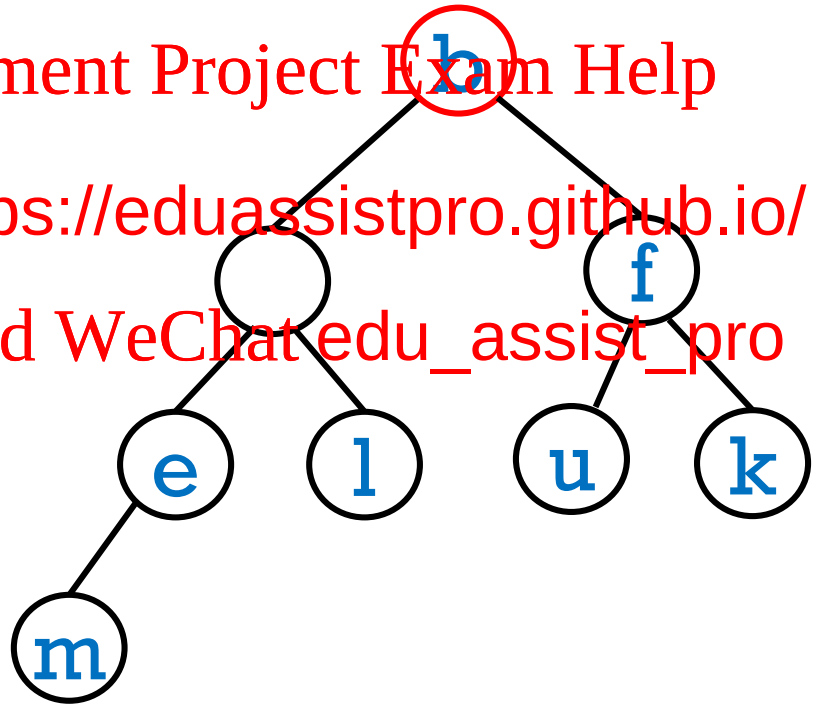
Swap keys with the smaller child!

Keep swapping with keys with the smaller child until it's necessary.



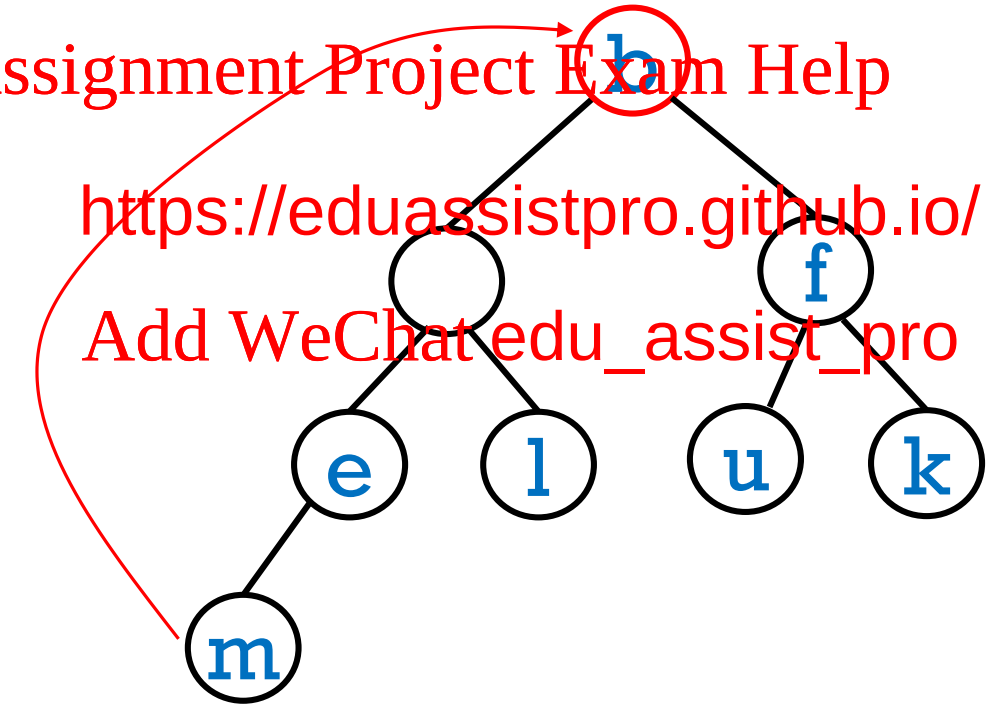
REMOVEMIN()

Let's removeMin() again!



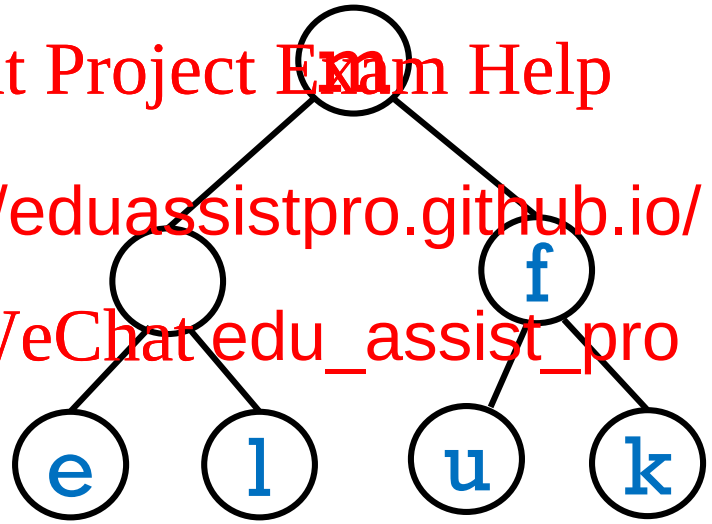
REMOVEMIN()

Let's removeMin() again!



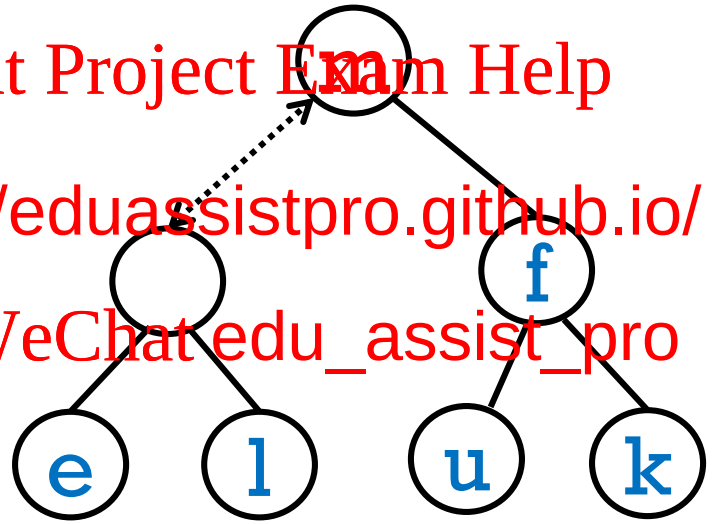
REMOVEMIN()

Let's removeMin() again!



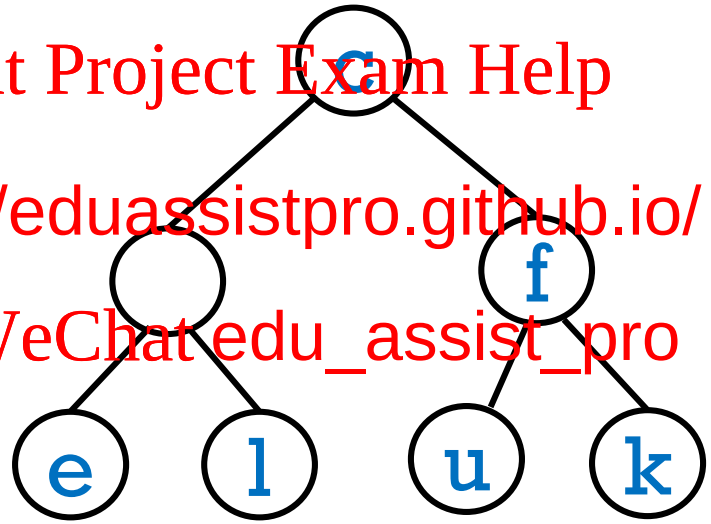
REMOVEMIN()

Now swap with smaller child, if necessary, to preserve heap property.



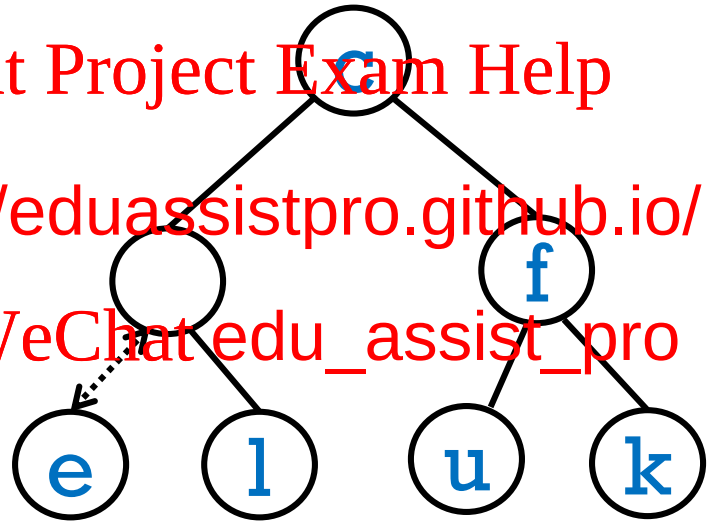
REMOVEMIN()

Now swap with smaller child, if necessary, to preserve heap property.



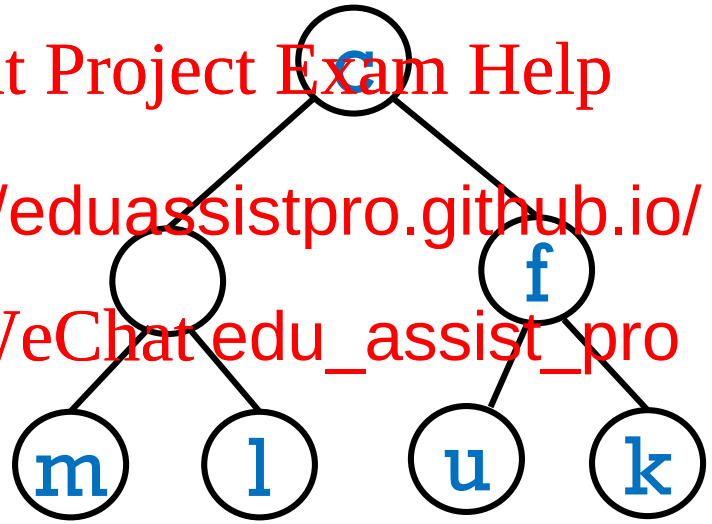
REMOVEMIN()

Keep swapping with smaller child, if necessary.

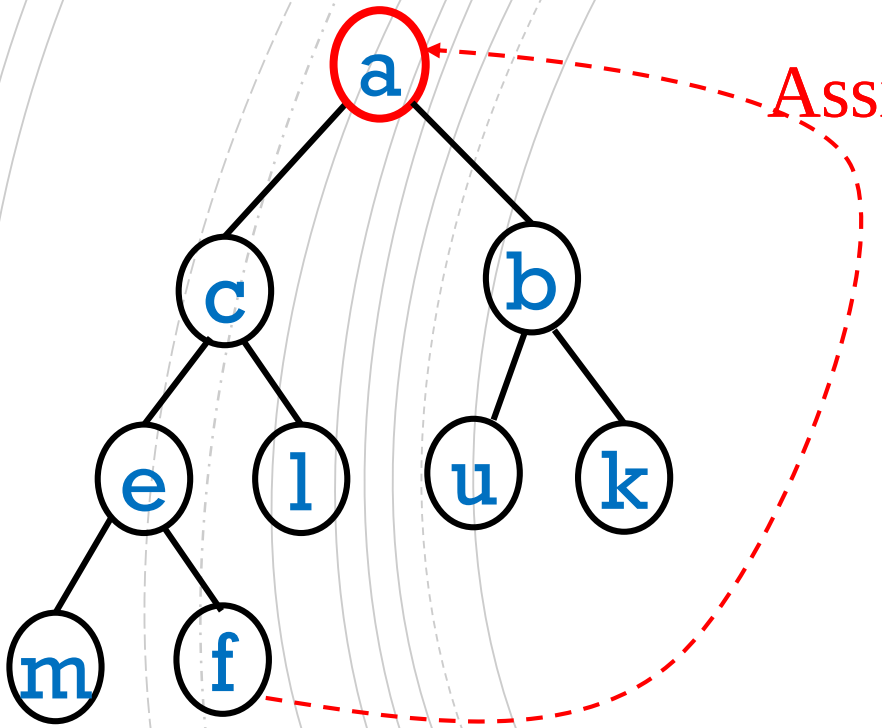


REMOVEMIN()

Keep swapping with smaller child, if necessary.

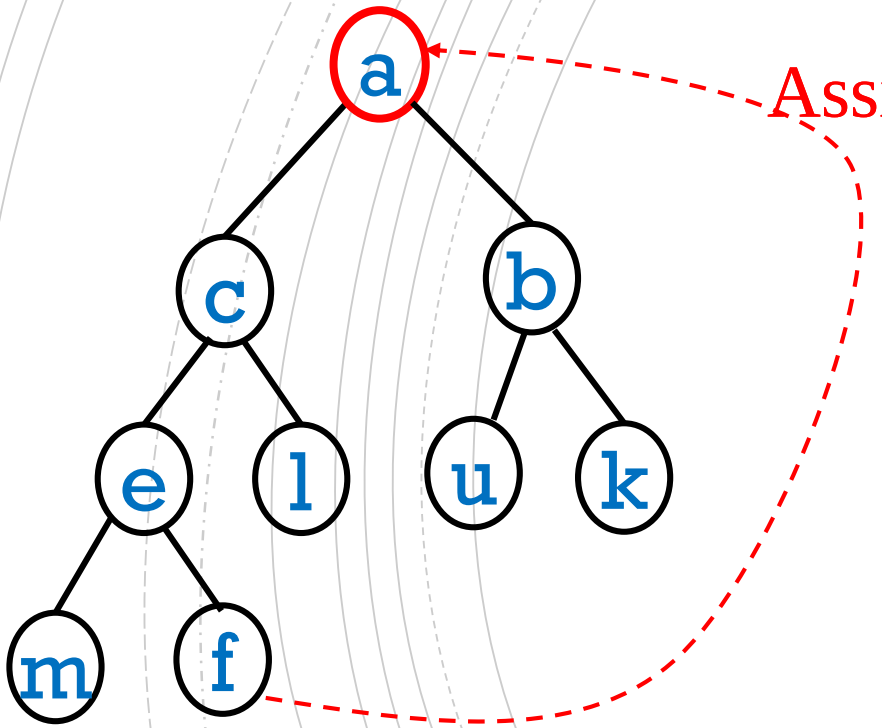


REMOVEMIN() - IMPLEMENTATION



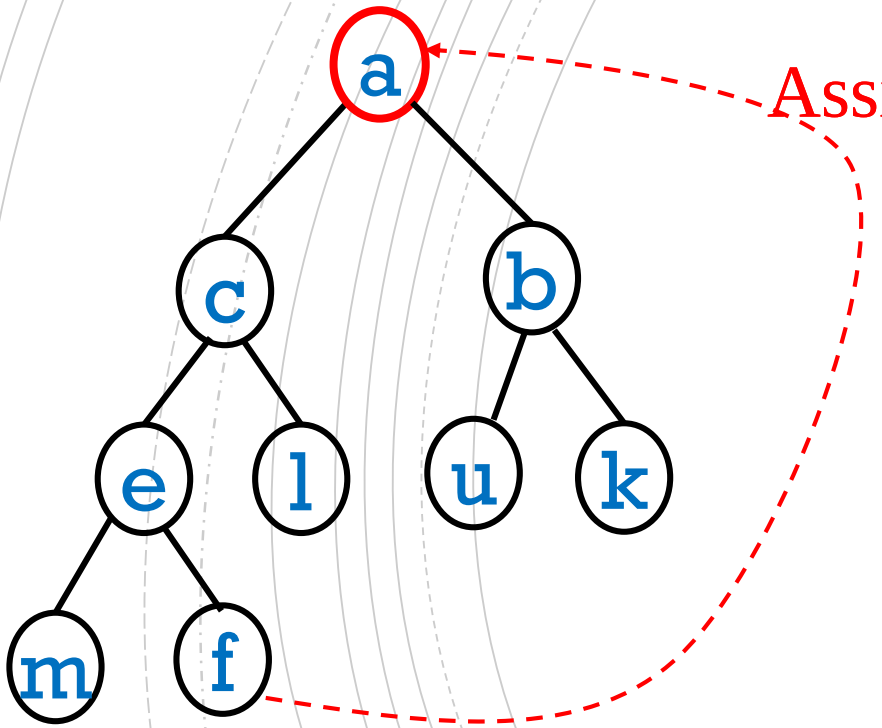

```
removeMin(){
    temp = root.key
    remove the last leaf node and
                        o the root
    return temp
}
```

REMOVEMIN() - IMPLEMENTATION




```
removeMin(){
    temp = root.key
    remove the last leaf node and
                          o the root

    while((cu                l && cur.key > cur.left.key)
    || (cur.ri                & cur.key > cur.right.key)) {




    }
    return temp
}
```

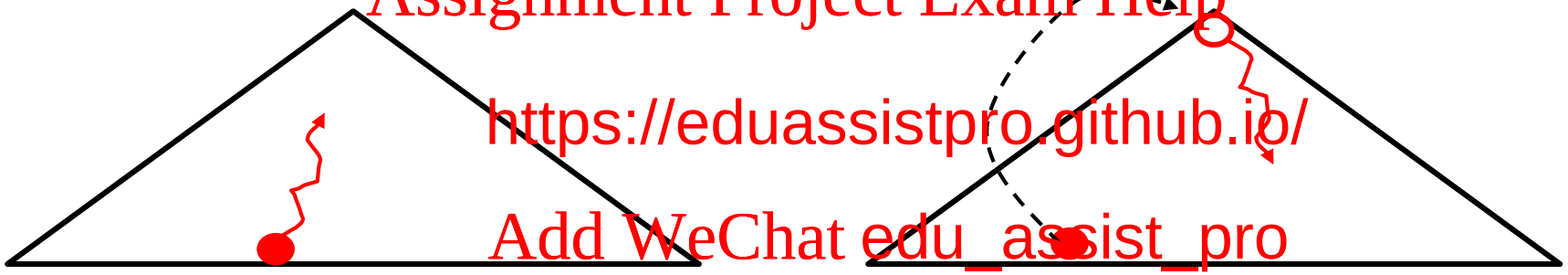
REMOVEMIN() - IMPLEMENTATION




```
removeMin(){
    temp = root.key
    remove the last leaf node and
                          o the root

    while((cu                 l && cur.key > cur.left.key)
    || (cur.ri                & cur.key > cur.right.key)) {
        minChild = child with smaller key
        swapKeys(cur, minChild)
        cur = minChild
    }
    return temp
}
```

add()

removeMin()



“upHeap”

“downHeap”

REMOVE()

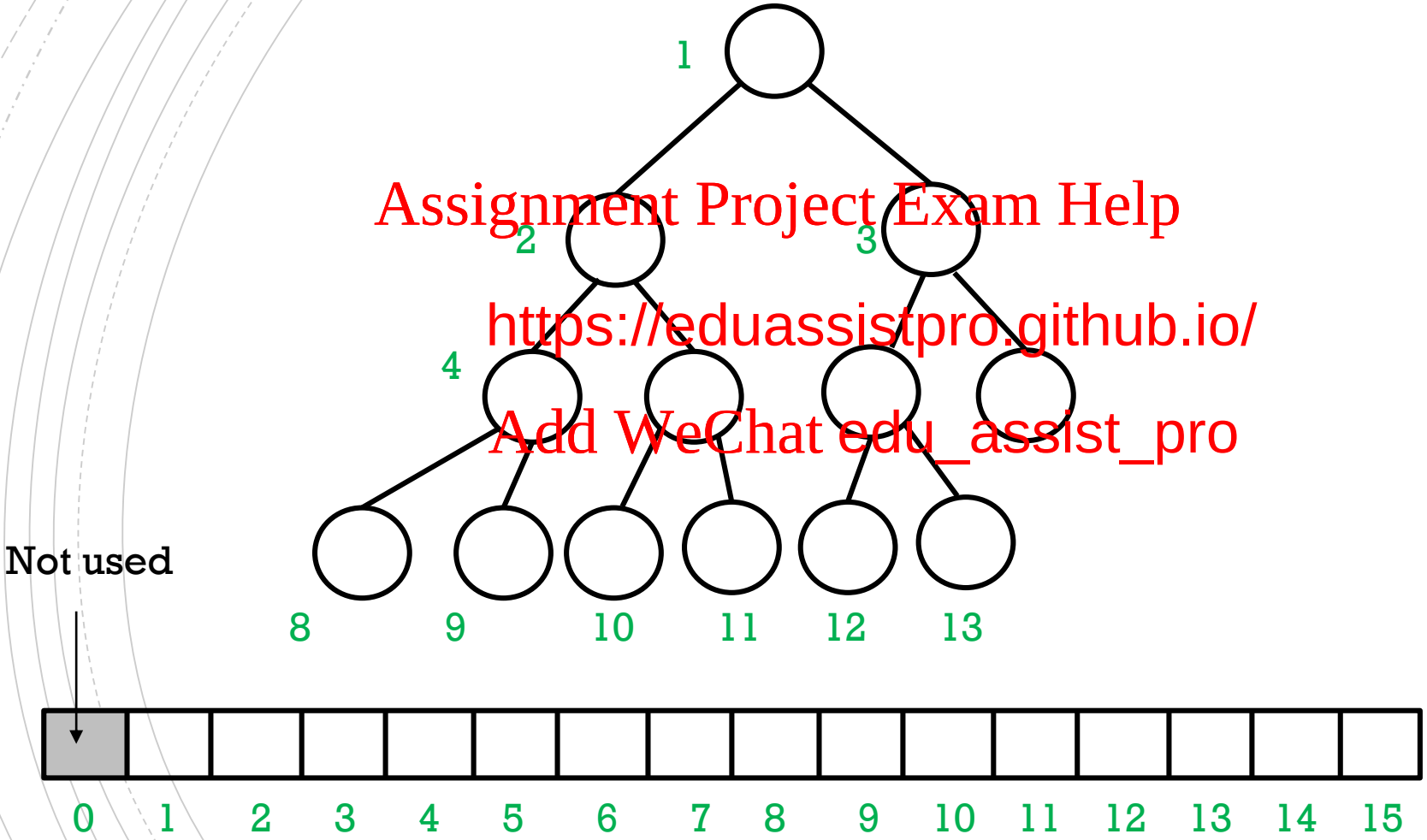
Q: What about remove(key) ?

REMOVE()

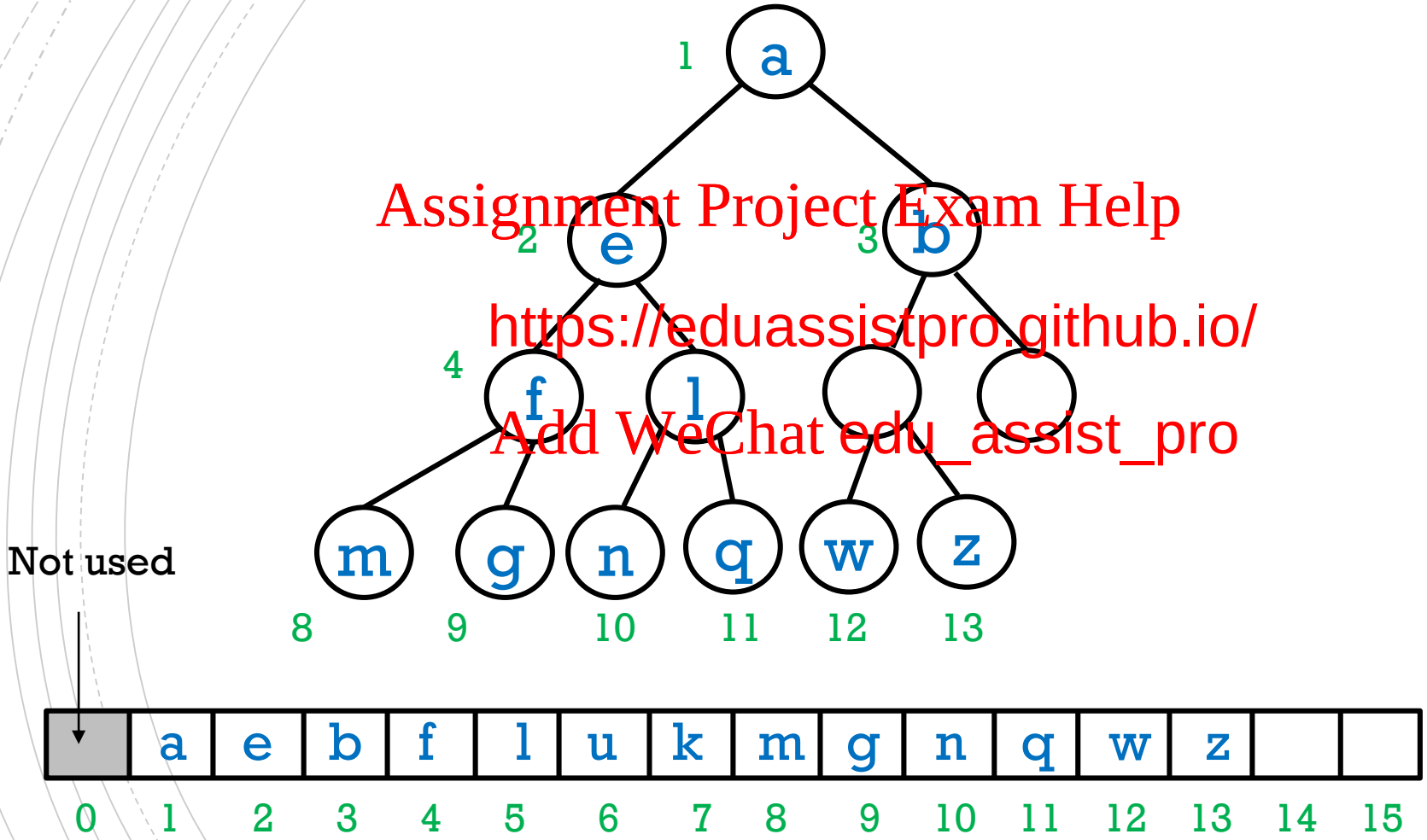
Q: What about remove(key) ?

A: Worst case $O(\ )$

HEAP (ARRAY IMPLEMENTATION)

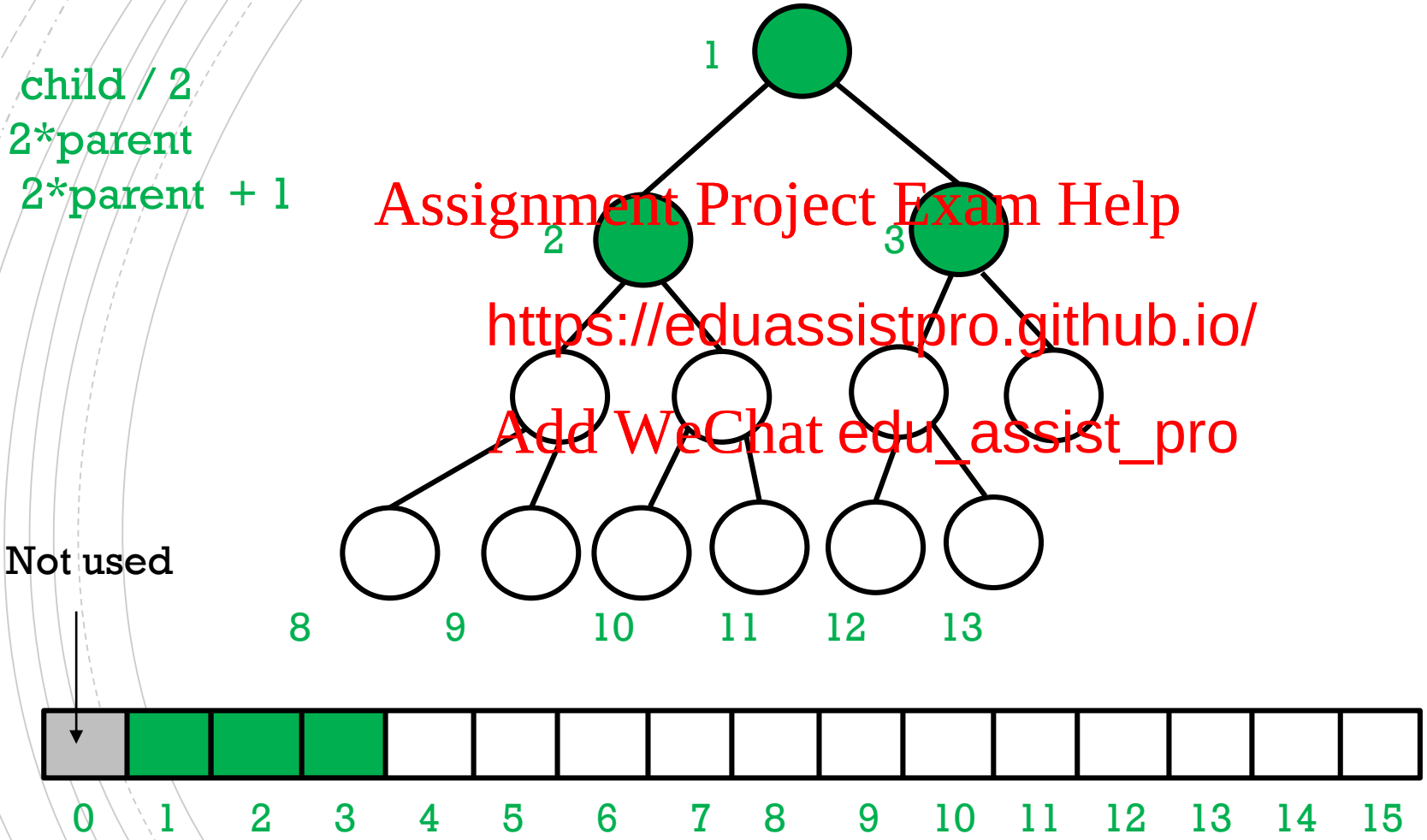


HEAP (ARRAY IMPLEMENTATION)



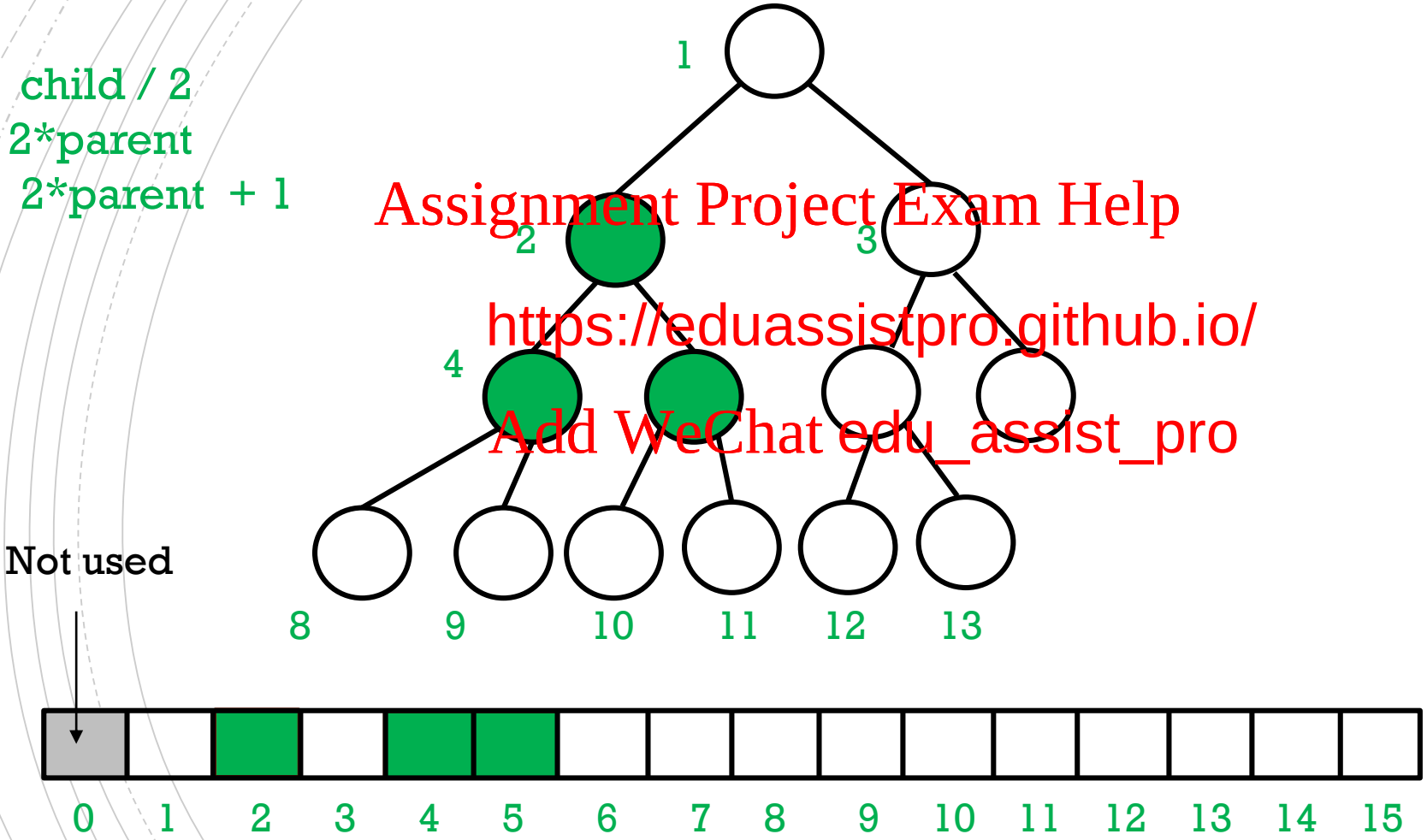
HEAP INDEX RELATIONS

```
parent  =  child / 2
left    =  2*parent
right   =  2*parent + 1
```

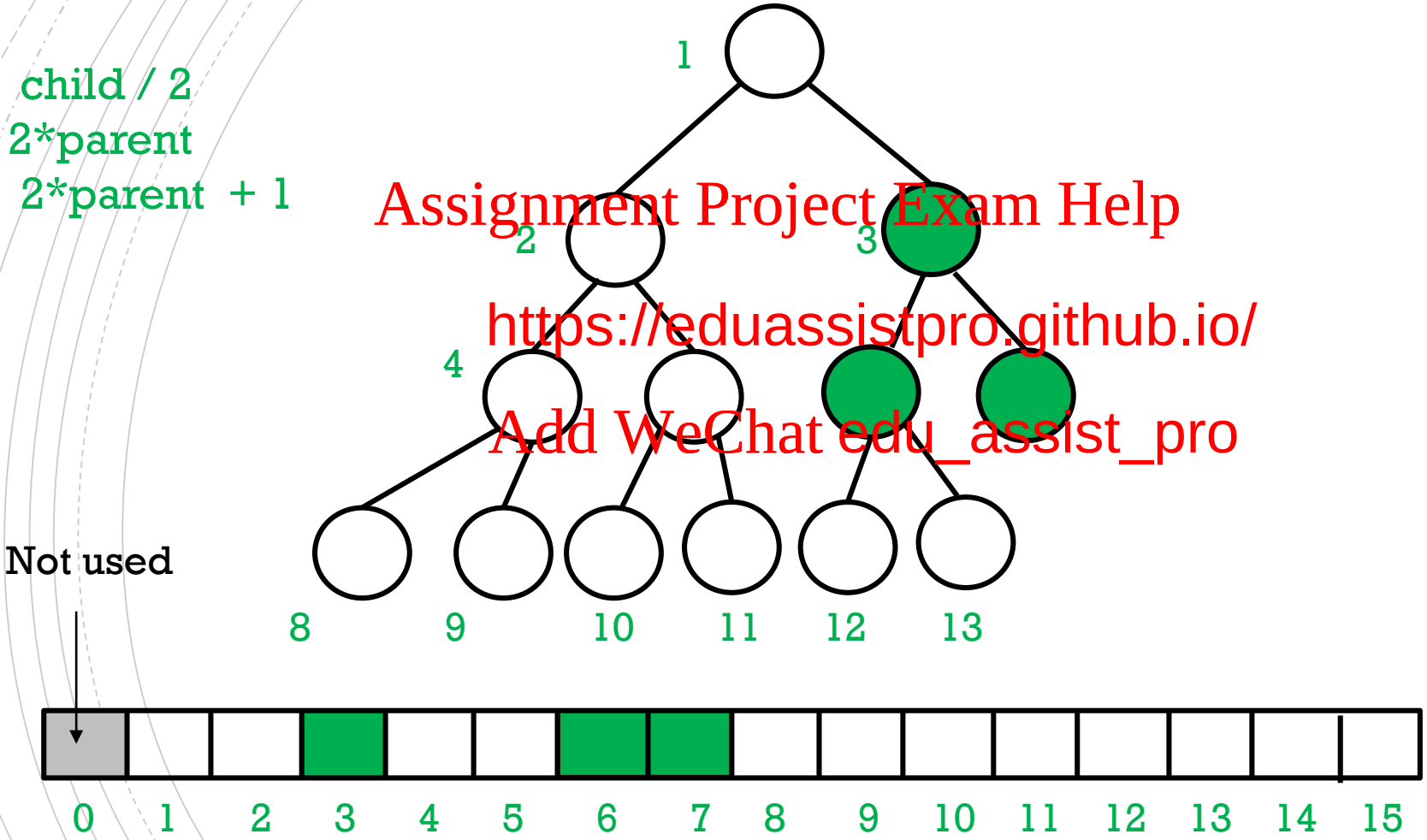
HEAP INDEX RELATIONS

```
parent = child / 2
left   = 2*parent
right  = 2*parent + 1
```

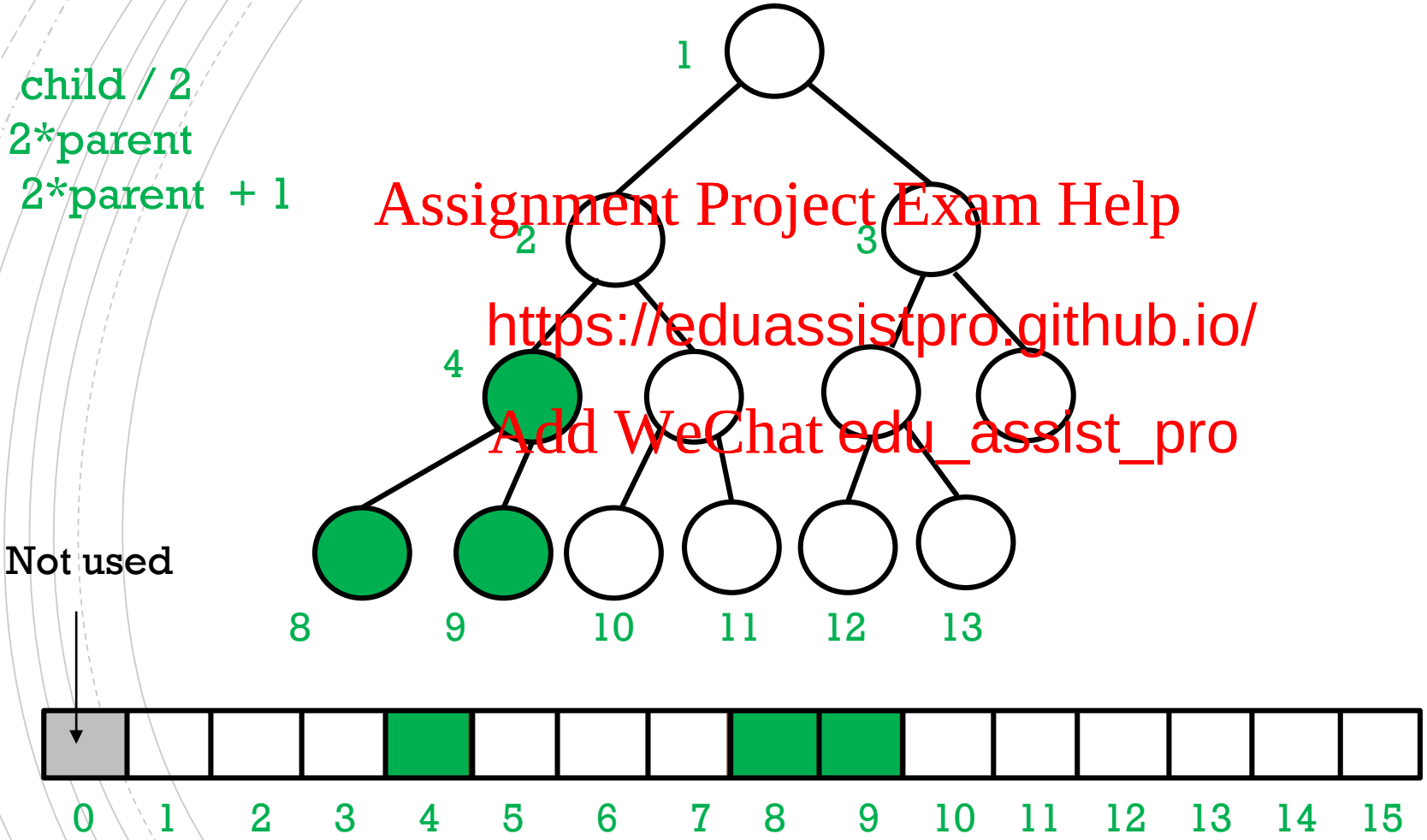
HEAP INDEX RELATIONS

```
parent = child / 2
left   = 2*parent
right  = 2*parent + 1
```

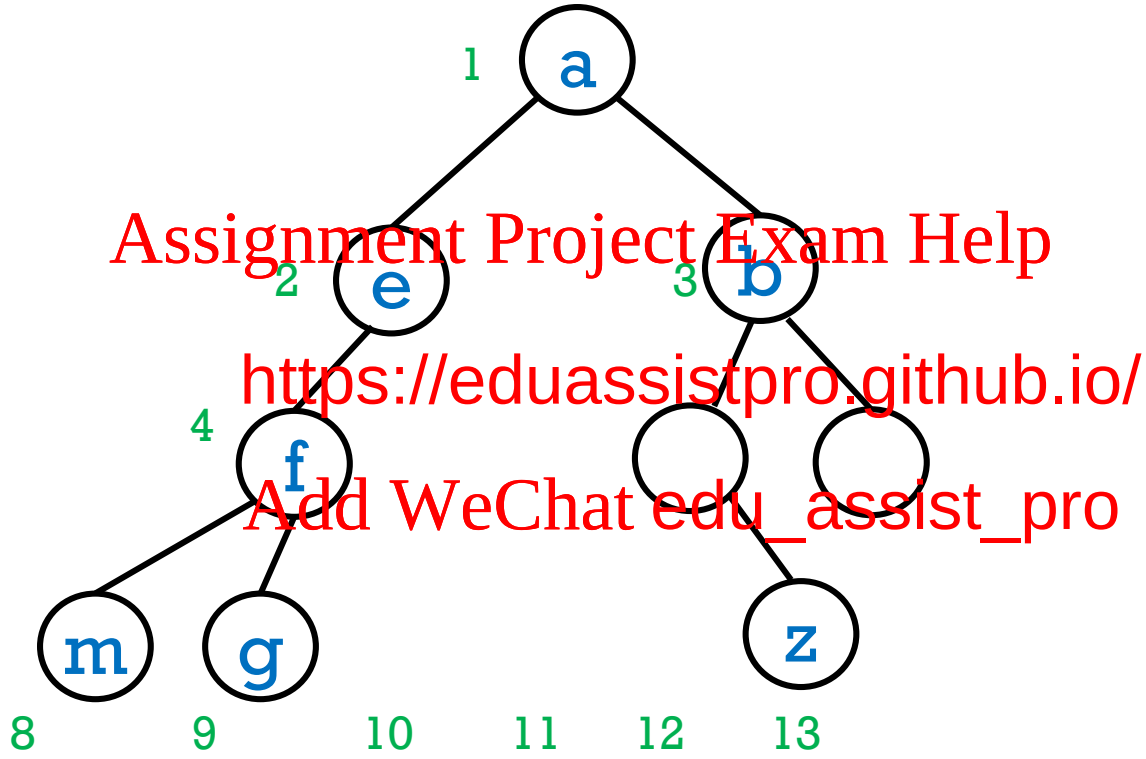



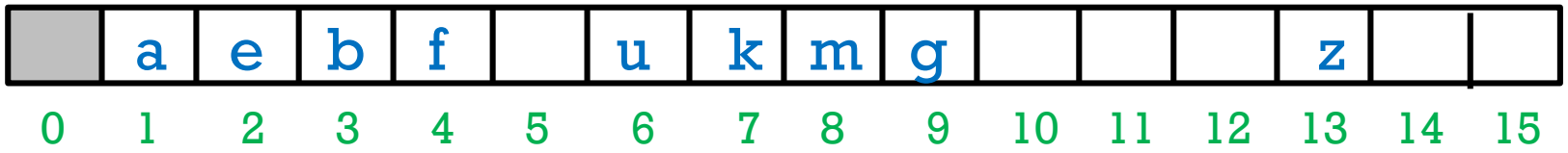
HEAP INDEX RELATIONS

```
parent = child / 2
left   = 2*parent
right  = 2*parent + 1
```

ASIDE: an array data structure can be used for *any* binary tree. But this is uncommon and often inefficient.

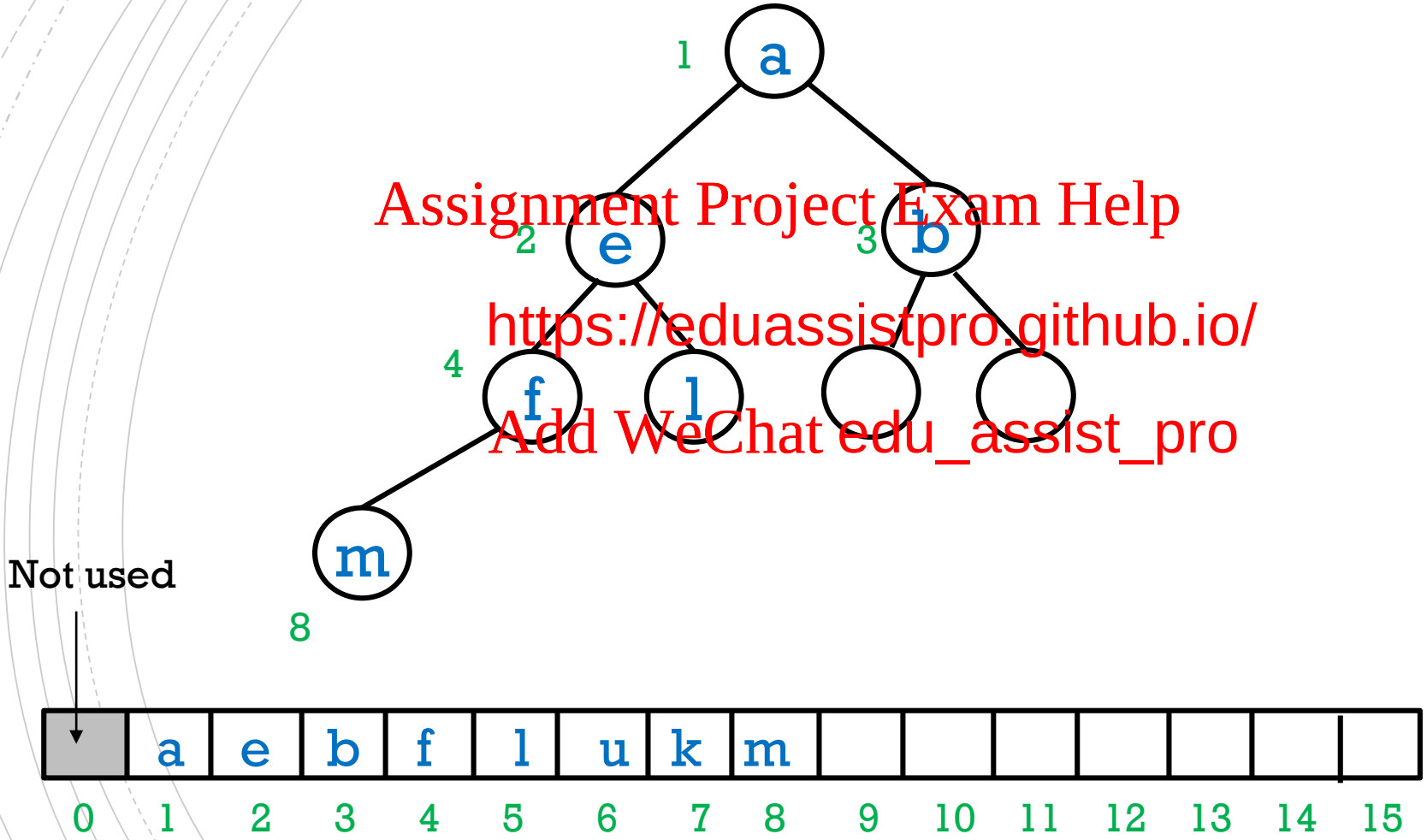




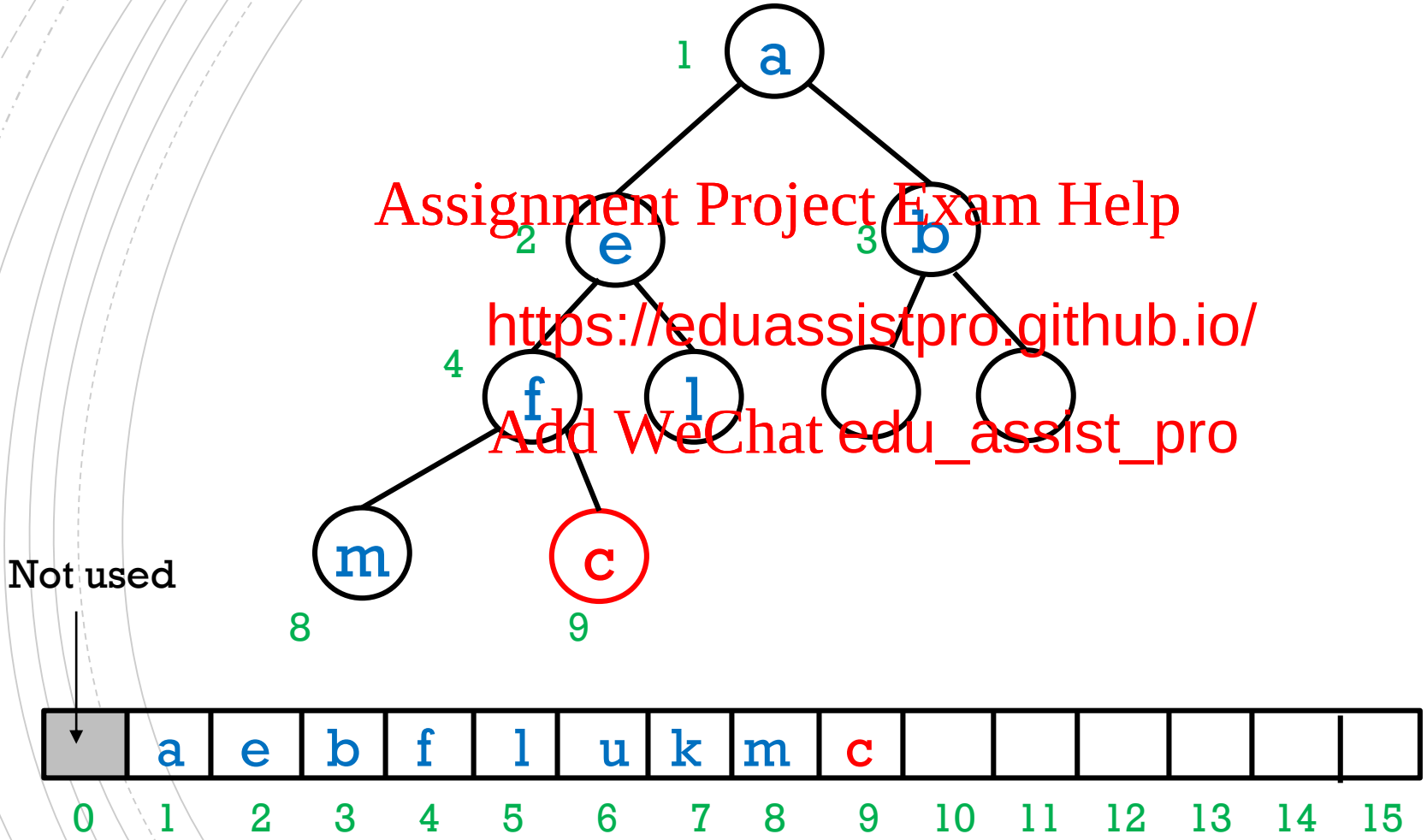
ADD() - IMPLEMENTATION

```
add(key){
    size = size + 1  // number of elements in heap
                     Assignment Project Exam Help
    // assuming                              nother element
    heap[ size      https://eduassistpro.github.io/

    i = size        Add WeChat edu_assist_pro

    //  the following is sometimes called "upHeap"
    while ( i > 1  &&  heap[i] < heap[ i/2 ]){
        swapElements( i, i/2 )
        i = i/2
    }
}
```

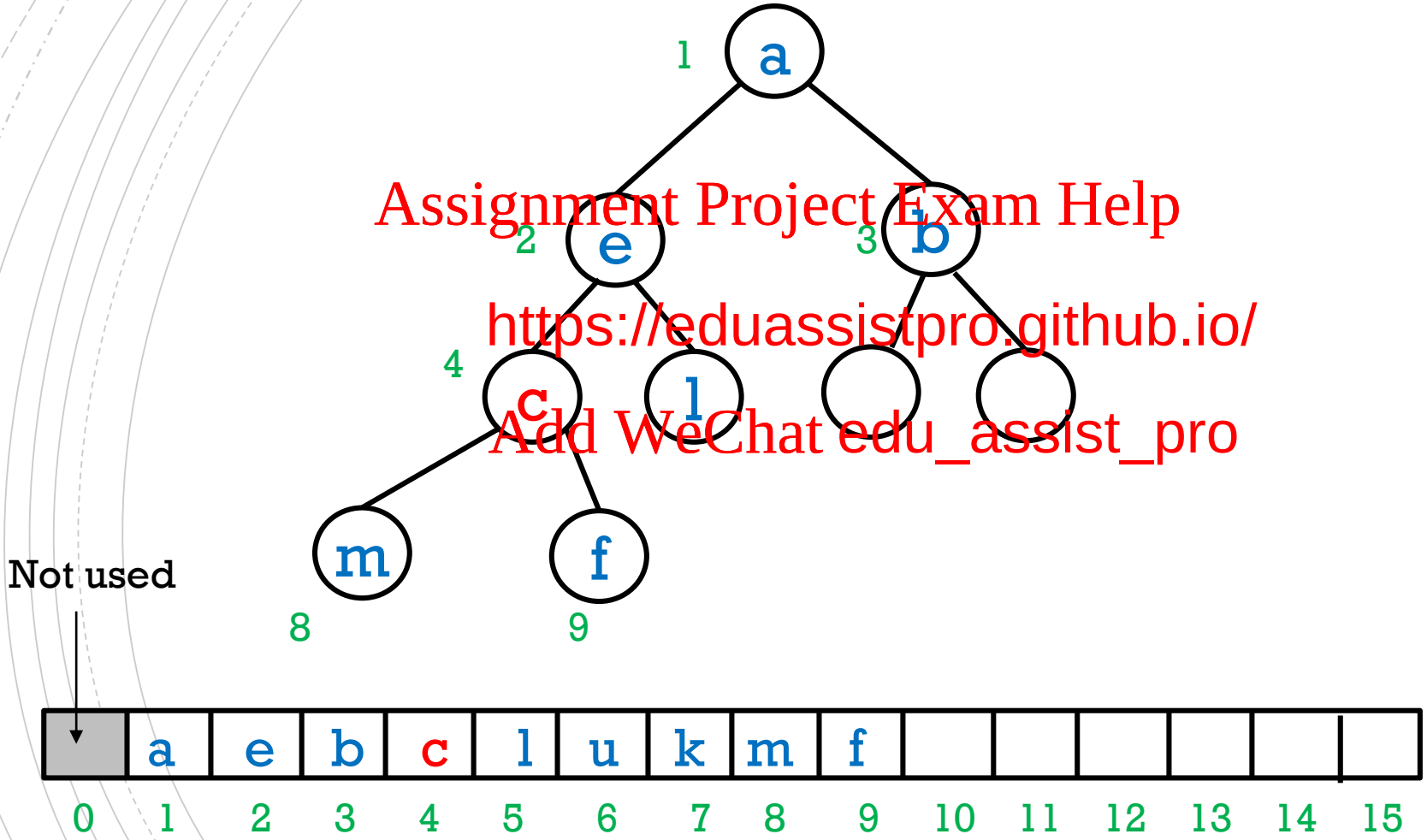
E.G. add(c)



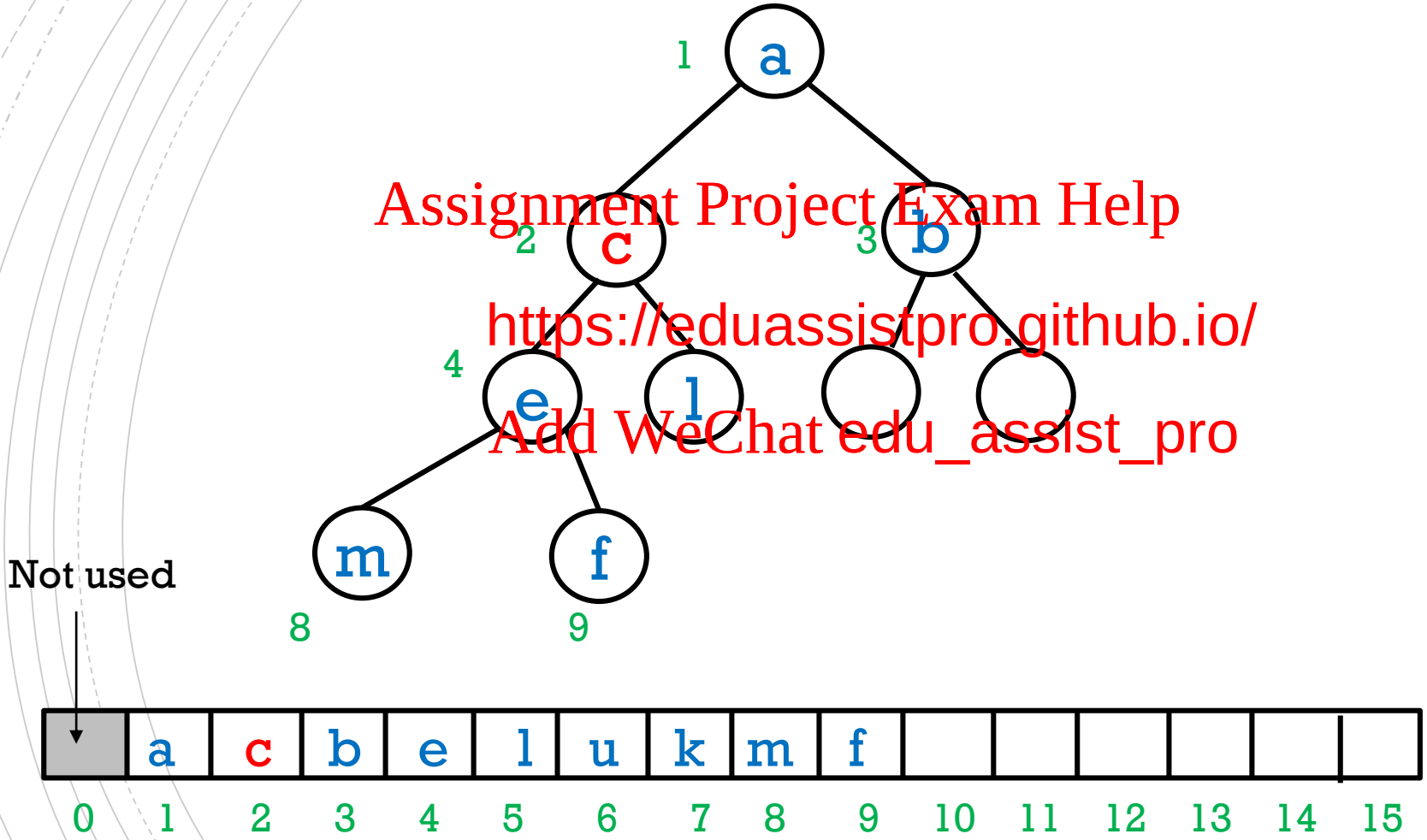
E.G. add (c)



E.G. add (c)

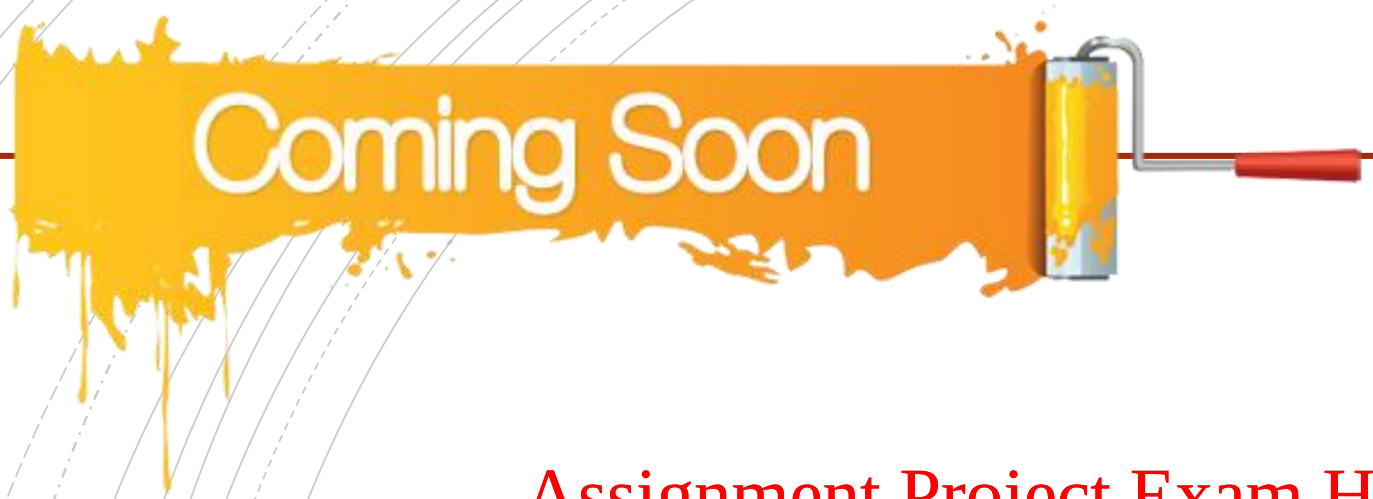


E.G. add(c)



NEXT VIDEO

- write removeMin() using array indices
- discuss best and wor
- faster algorithm for building a heap



In the next

- More on