COMP 250

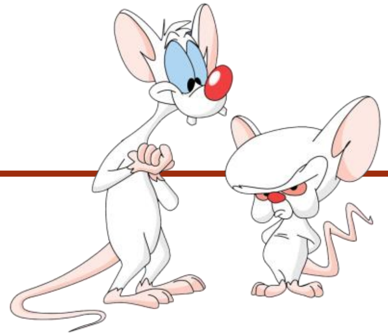
INTRODUC TER SCIENCE

Week 13-3 :

Giulia Alberini, Fall 2020

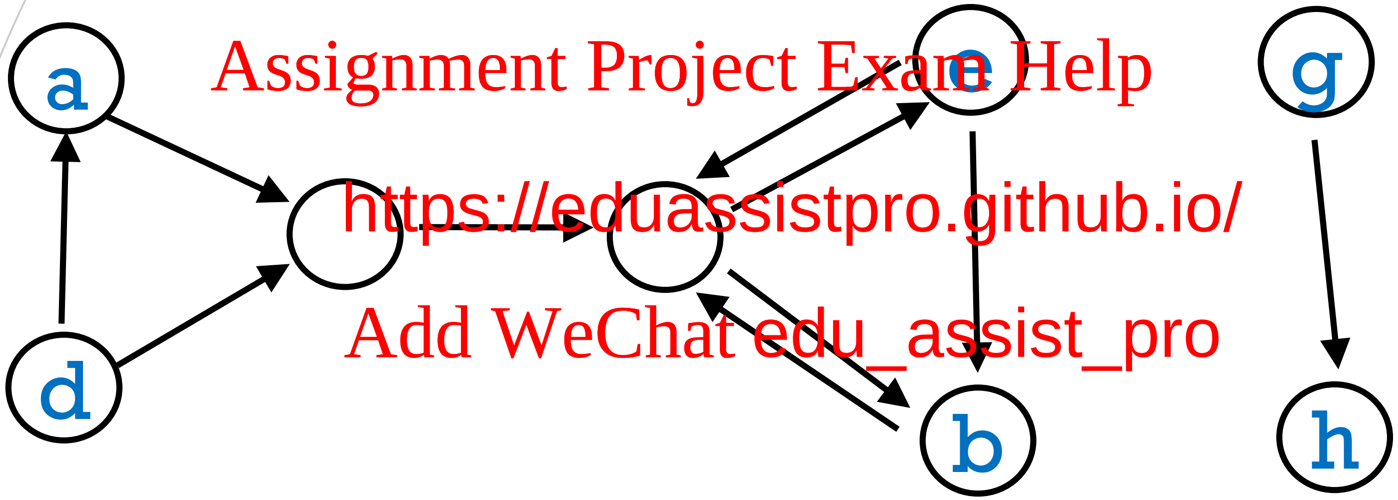
Slides adapted from Michael Langer's

WHAT ARE WE GOING TO DO IN THIS VIDEO?

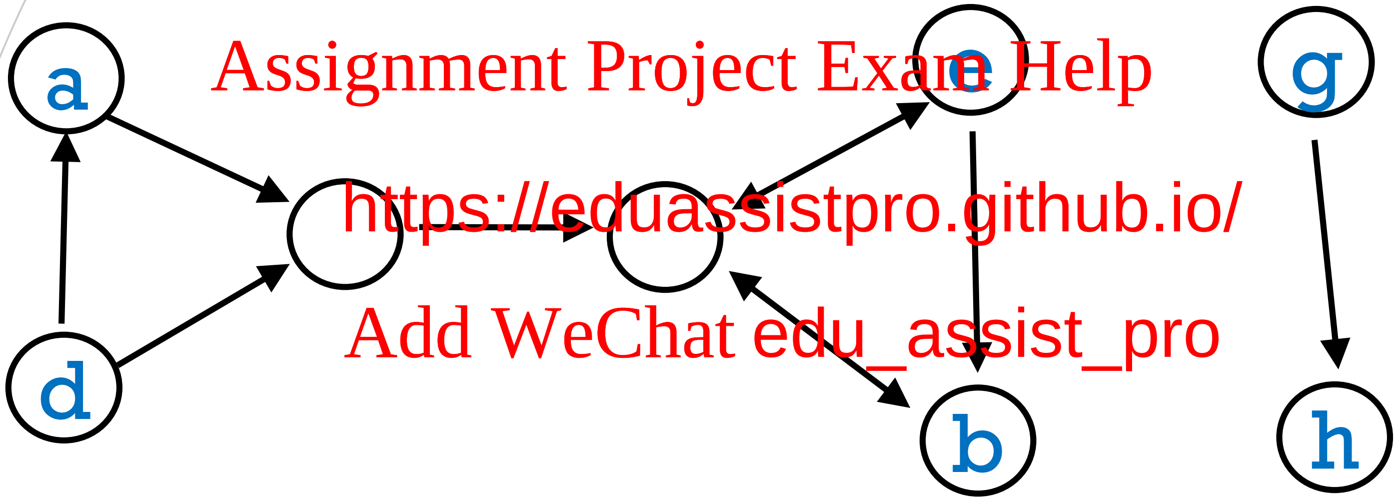


- Graphs
  - Definitions

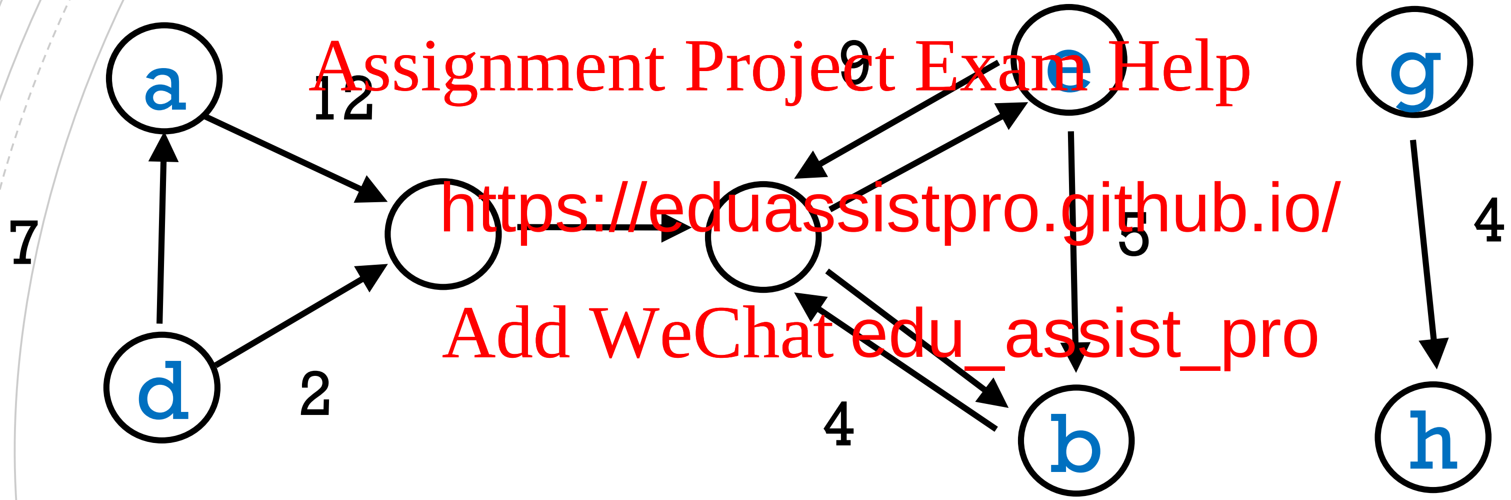
EXAMPLE



SAME EXAMPLE – DIFFERENT NOTATION



WEIGHTED GRAPH



DEFINITION

A *directed graph* is a set of *vertices*

$$V = \{ v_i : i \in \{1, \dots, n\} \}$$

and set of ordered p alled *edges*.

$$E = \{ (v_i , v_j) : i, j \in \{1, \dots, n\} \}$$

In an *undirected* graph, the edges are *unordered* pairs.

$$E = \{ \{v_i, v_j\} : i, j \in \{1, \dots, n\} \}$$

EXAMPLES

Vertices	Edges
airports	
web pages	
Java objects	

EXAMPLES

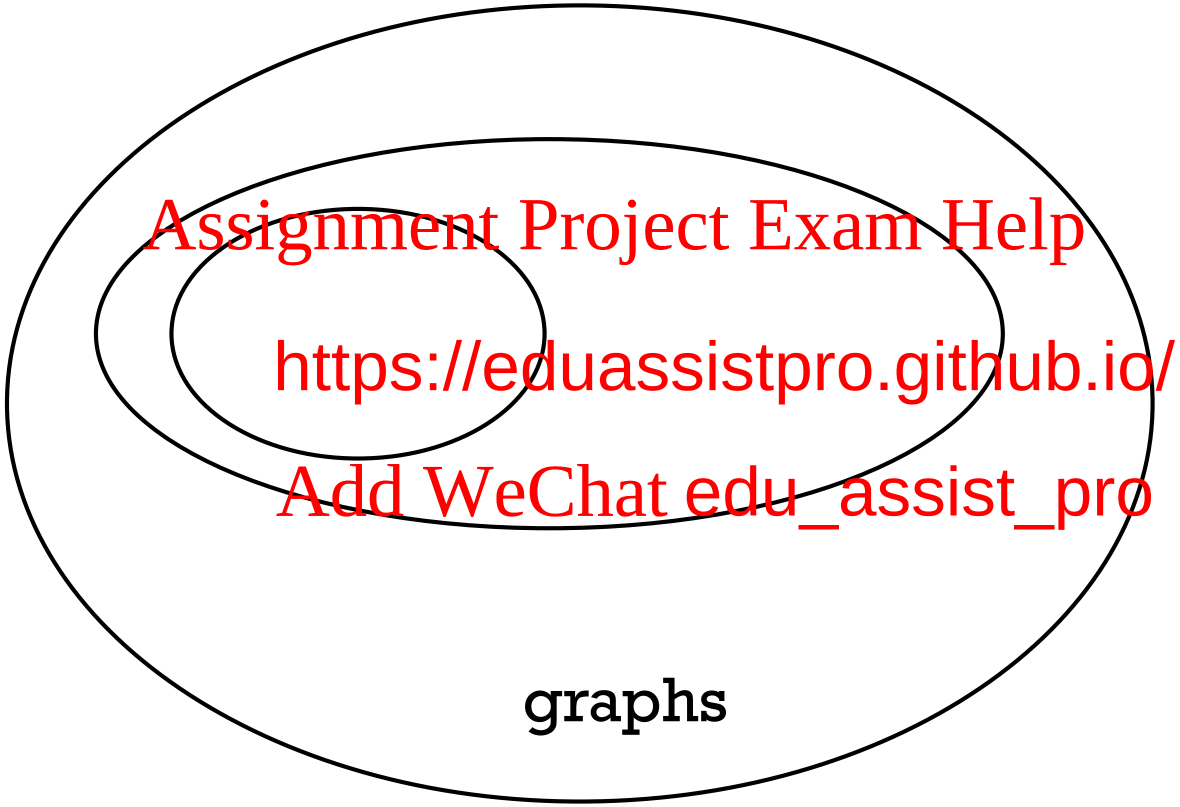
Vertices	Edges
airports	ts
web pages	
Java objects	

EXAMPLES

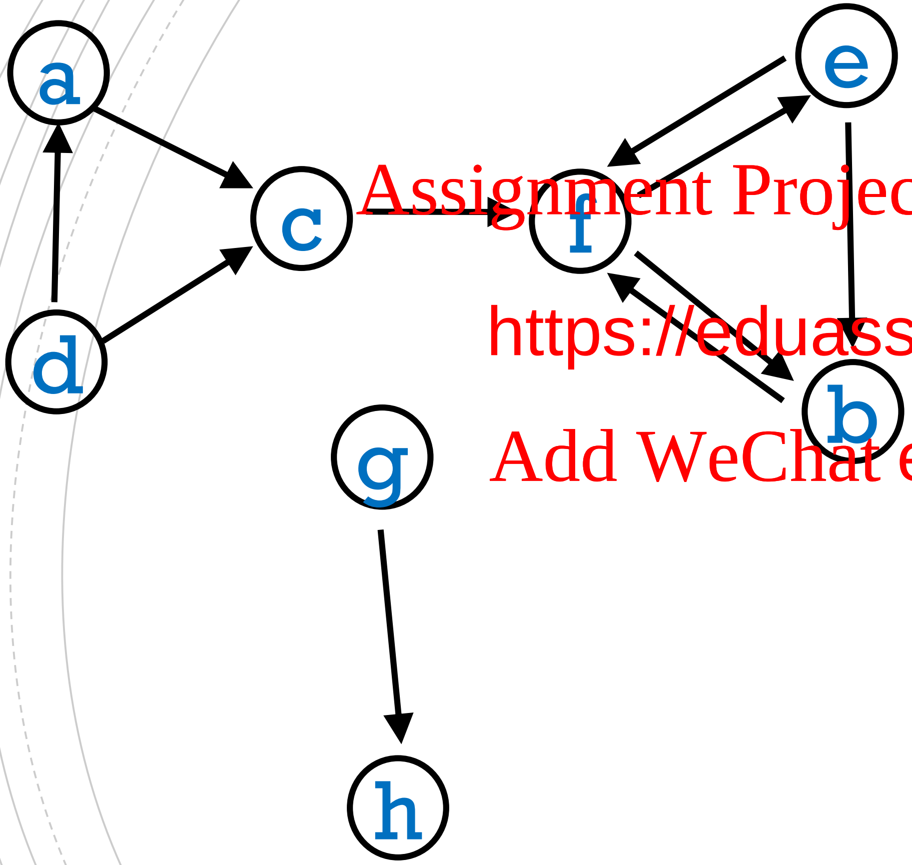
Vertices	Edges
airports	ts
web pages	URLs)
Java objects	

EXAMPLES

Vertices	Edges
airports	ts
web pages	URLs)
Java objects	references

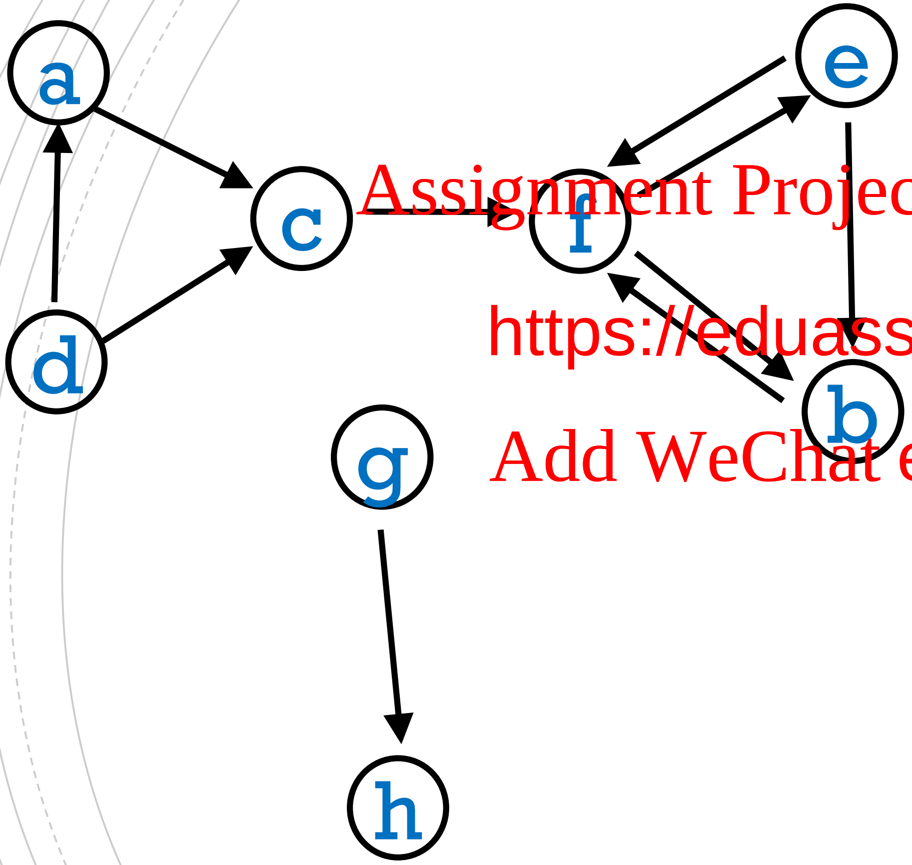

graphs

TERMINOLOGY: "IN DEGREE"



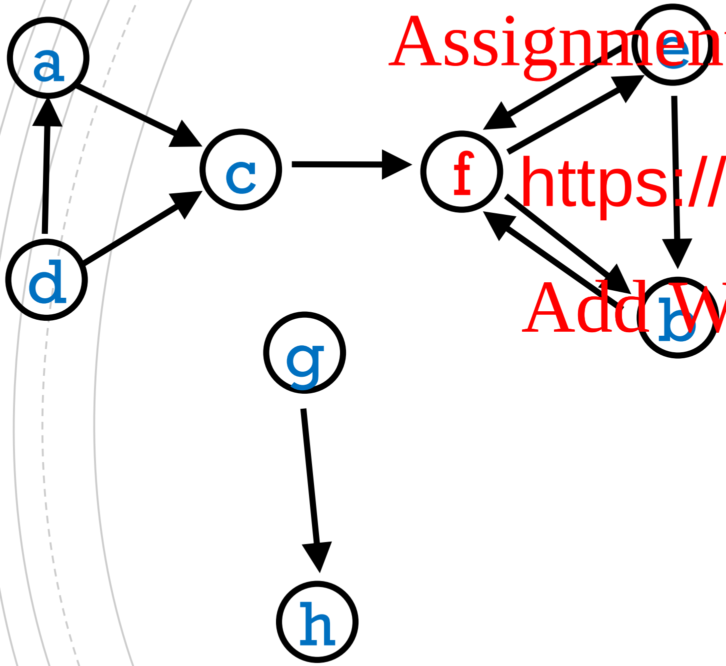
v	in degree
a	1
b	2
c	2
d	0
e	1
f	3
g	0
h	1

TERMINOLOGY: "OUT DEGREE"



v	out degree
a	1
b	1
c	1
d	2
e	2
f	2
g	1
h	0

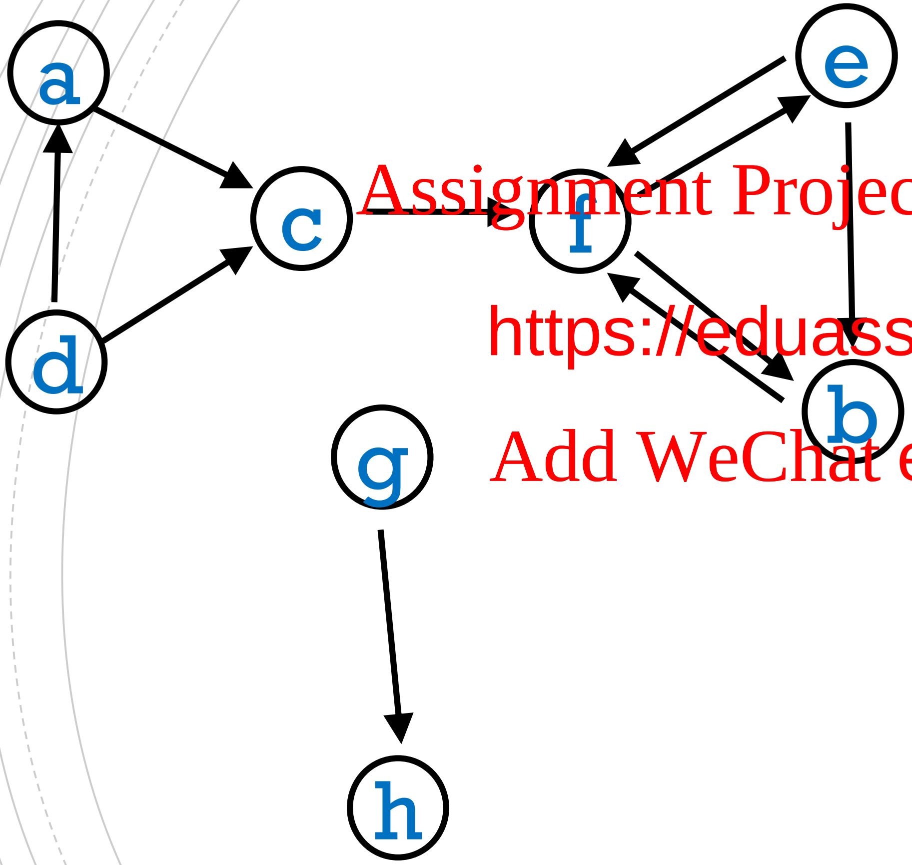
EXAMPLE: WEB PAGES



In degree: How many web pages link to some web ...g. f) ?

...ee: How many web pages does some web page (e.g. f) link to ?

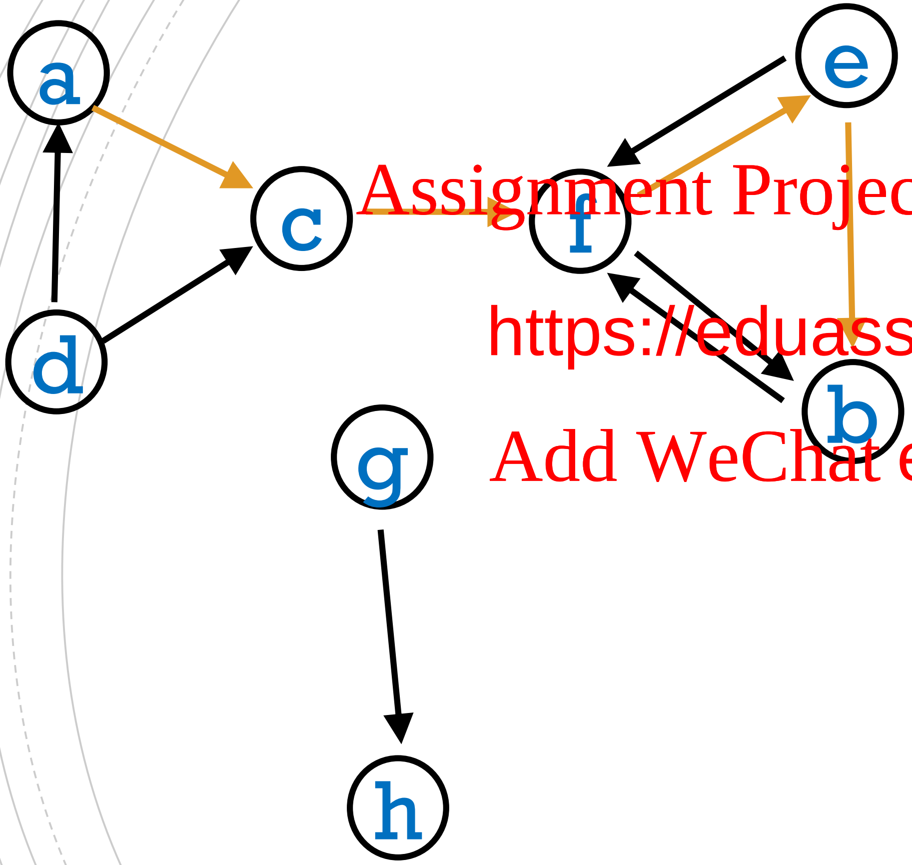
TERMINOLOGY: PATH



A *path* is a sequence of edges such that the end vertex of one edge is the rt vertex of the next edge

no vertex is repeated pt maybe first and last.

TERMINOLOGY: PATH



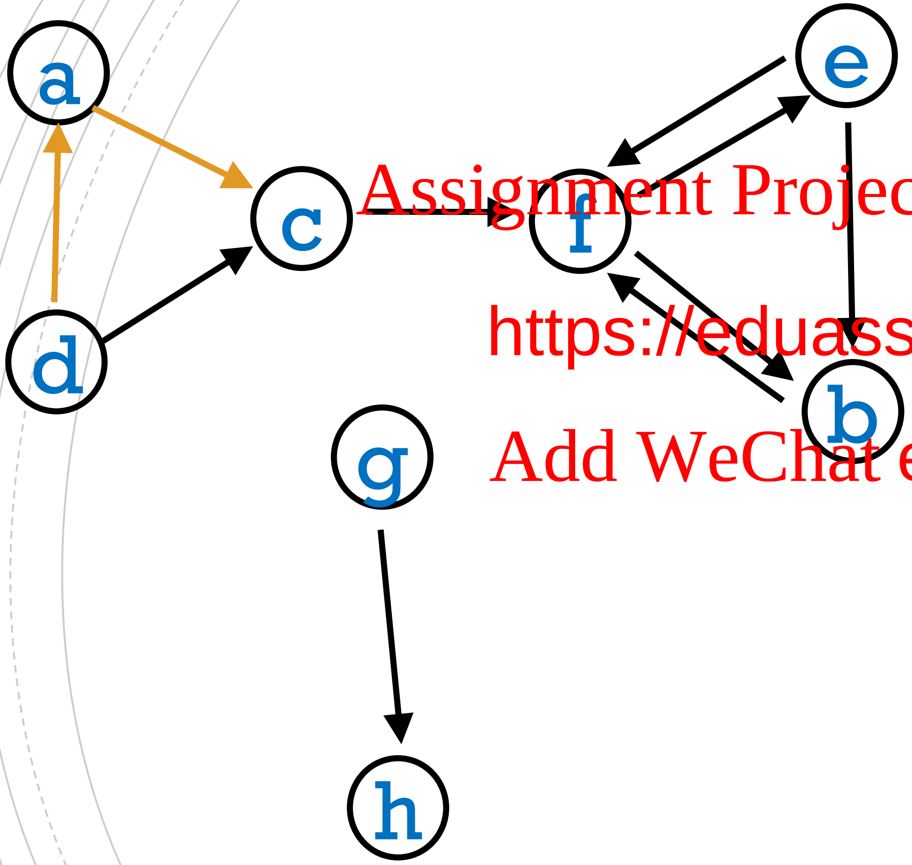
A *path* is a sequence of edges such that the end vertex of one edge is the rt vertex of the next edge

no vertex is repeated pt maybe first and last.

Examples

- acfeb

TERMINOLOGY: PATH



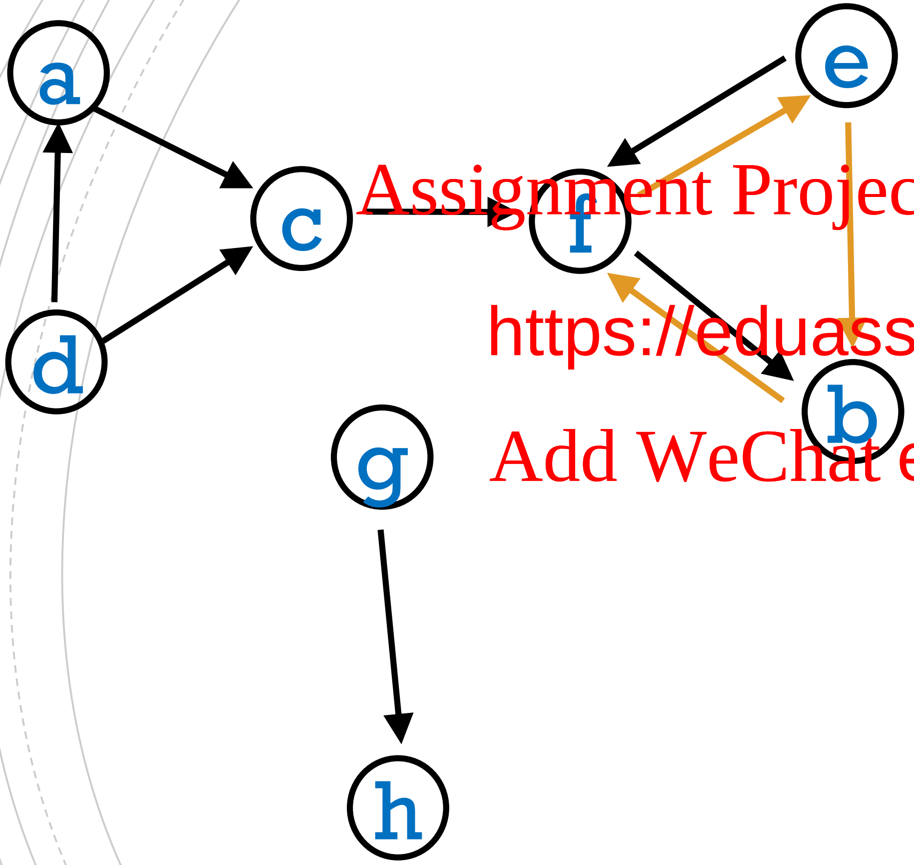
A *path* is a sequence of edges such that the end vertex of one edge is the start vertex of the next edge

no vertex is repeated except maybe first and last.

Examples

- acfeb
- dac

TERMINOLOGY: PATH



A *path* is a sequence of edges such that the end vertex of one edge is the start vertex of the next edge

no vertex is repeated except maybe first and last.

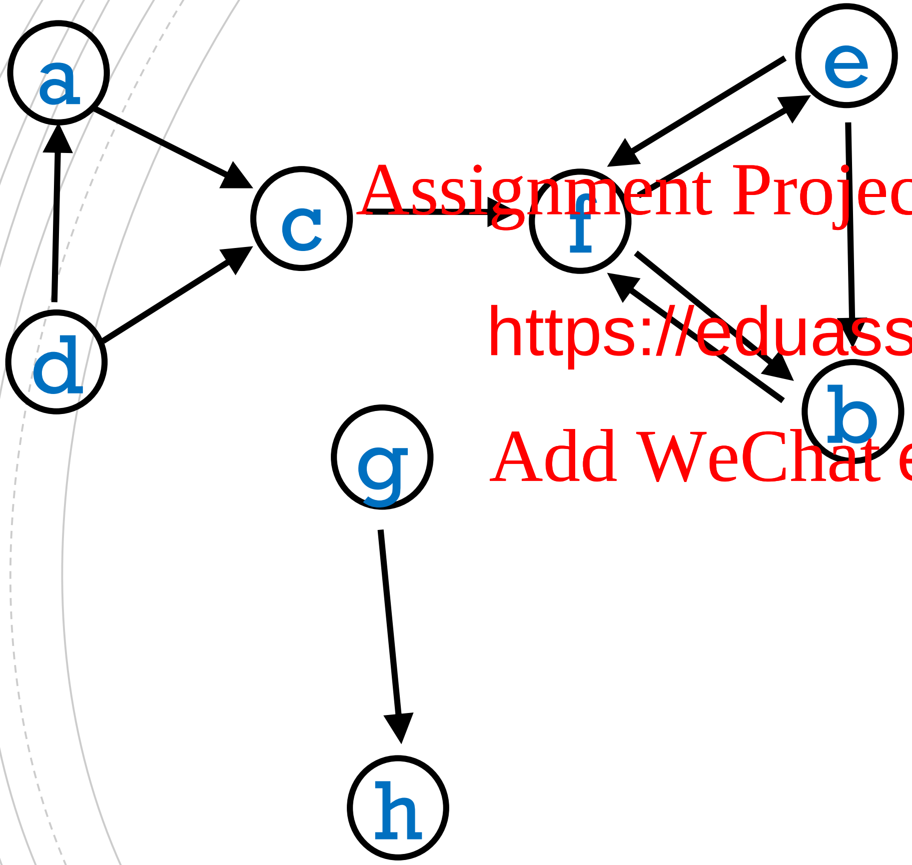
Examples

- acfeb
- dac
- febf
- .....

GRAPH ALGORITHMS IN COMP 251 (DIJKSTRA'S ALGORITHM)

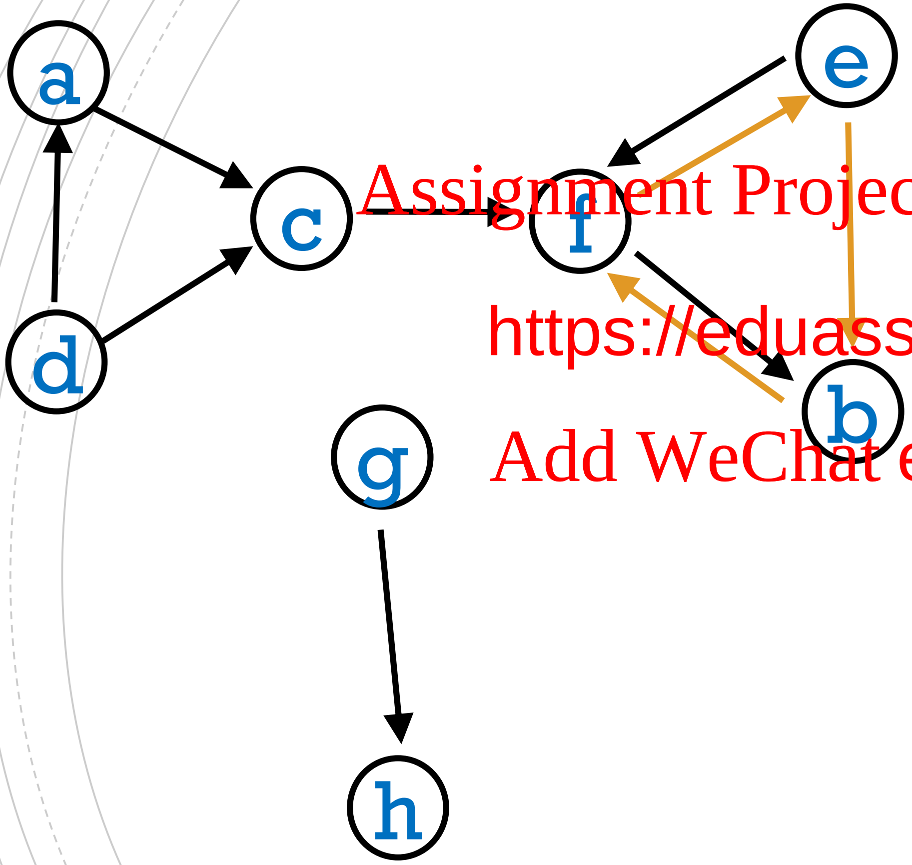
Given a graph, what is the shortest (weighted) path between two vertices?

TERMINOLOGY: CYCLE



A *cycle* is a path such that the last vertex is the same as the first vertex.

TERMINOLOGY: CYCLE

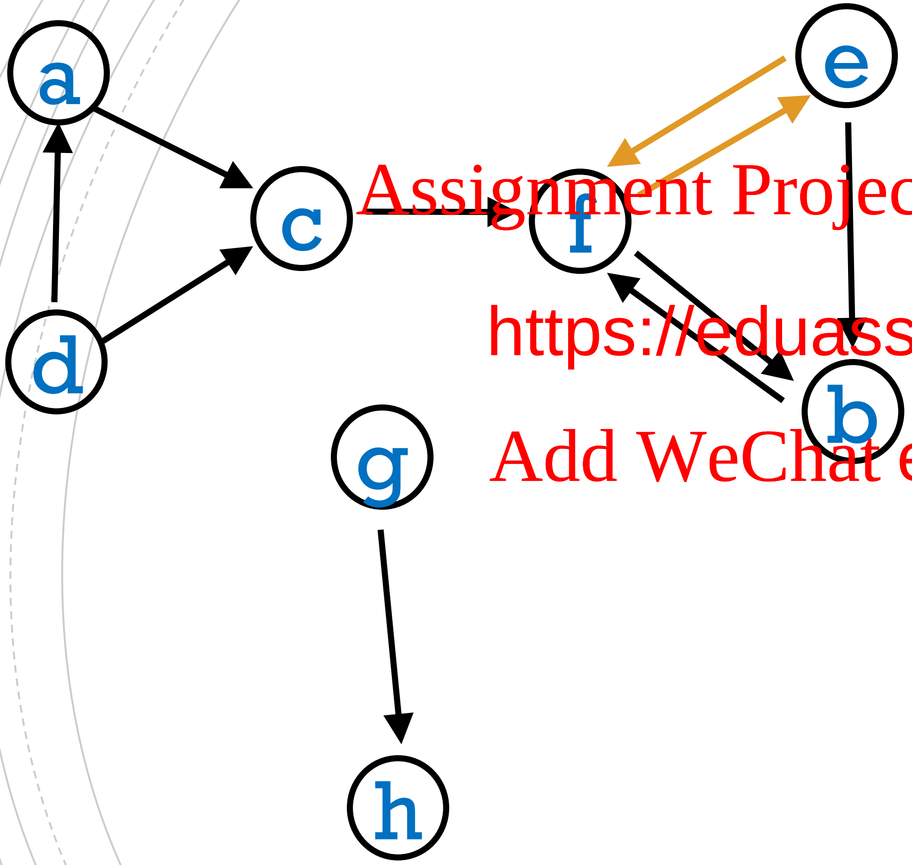


A *cycle* is a path such that the last vertex is the same as the first
rtex.

mples

- febf

TERMINOLOGY: CYCLE

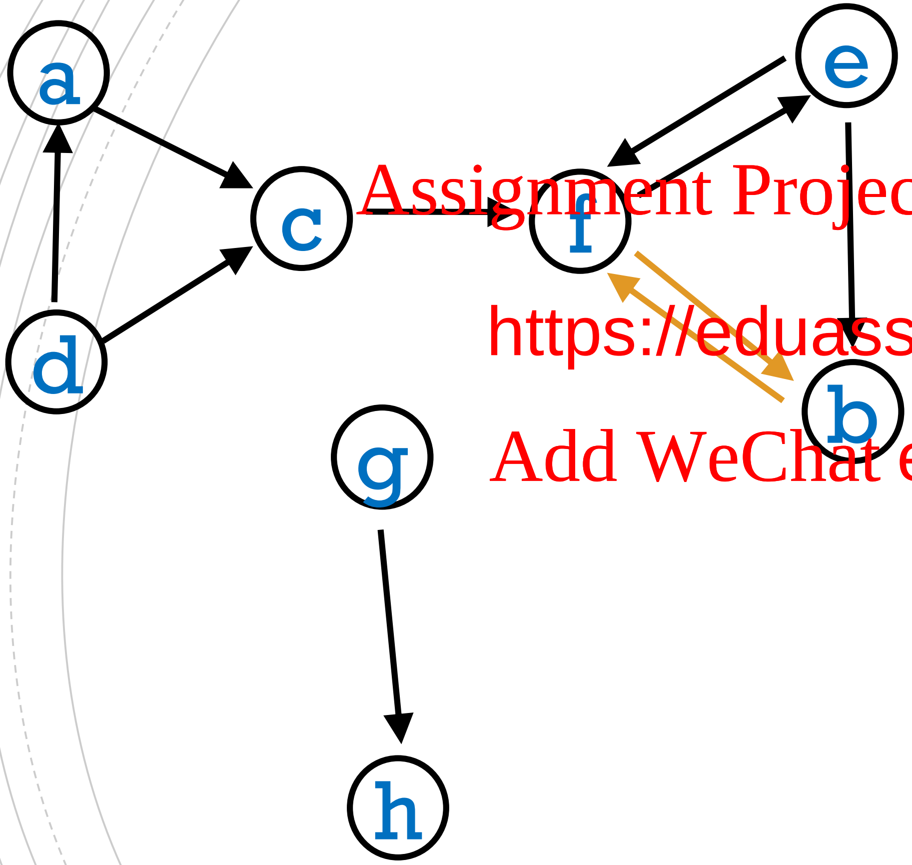


A *cycle* is a path such that the last vertex is the same as the first rtex.

mples

- febf
- efe

TERMINOLOGY: CYCLE

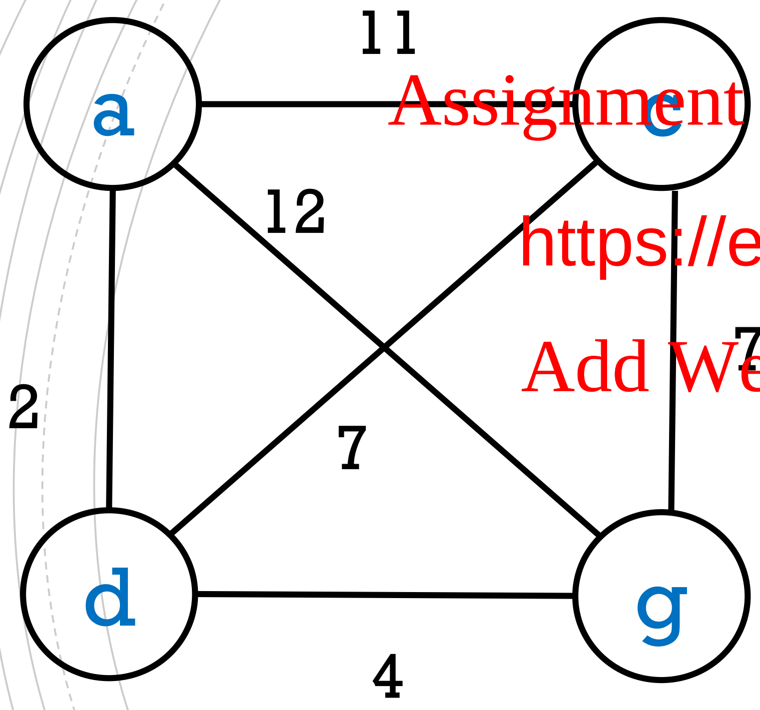


A *cycle* is a path such that the last vertex is the same as the first
rtex.

mples

- febf
- efe
- fbf
- …

"TRAVELLING SALESMAN" COMP 360 - (HAMILTONIAN CIRCUIT)



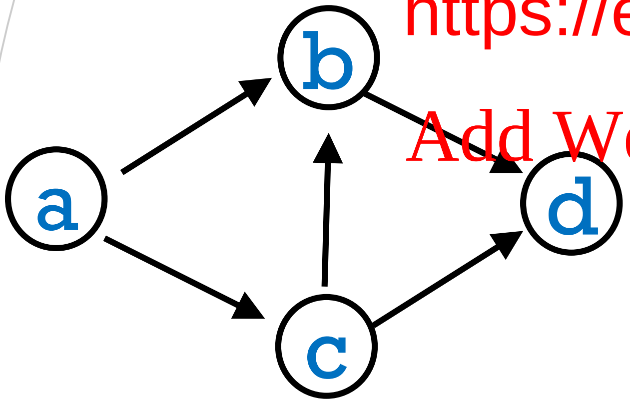
Find the shortest cycle that all vertices once.

any potential cycles are there in a graph of n vertices ?

DIRECTED *ACYCLIC* GRAPH

no cycles

capture
encies.



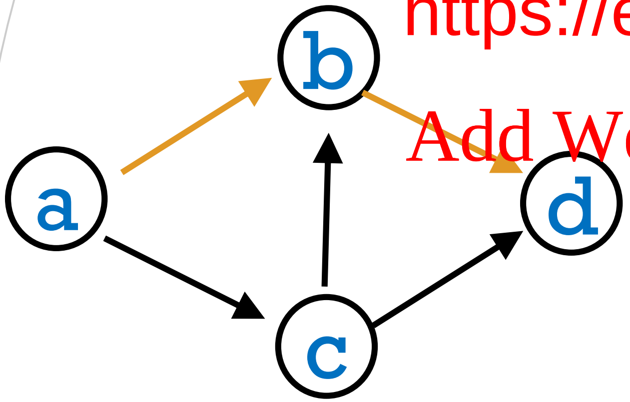
There are three paths from a to d.

DIRECTED *ACYCLIC* GRAPH

no cycles

capture

encies.



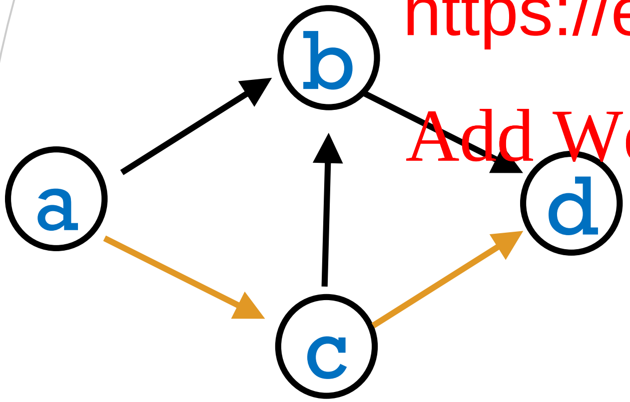
There are three paths from a to d.

DIRECTED *ACYCLIC* GRAPH

no cycles

capture

encies.



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DIRECTED *ACYCLIC* GRAPH

no cycles

capture

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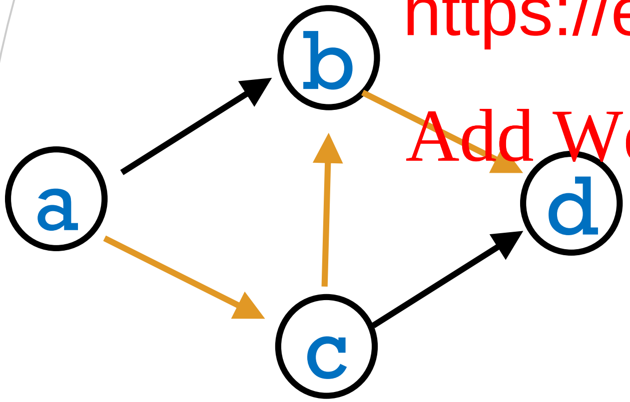

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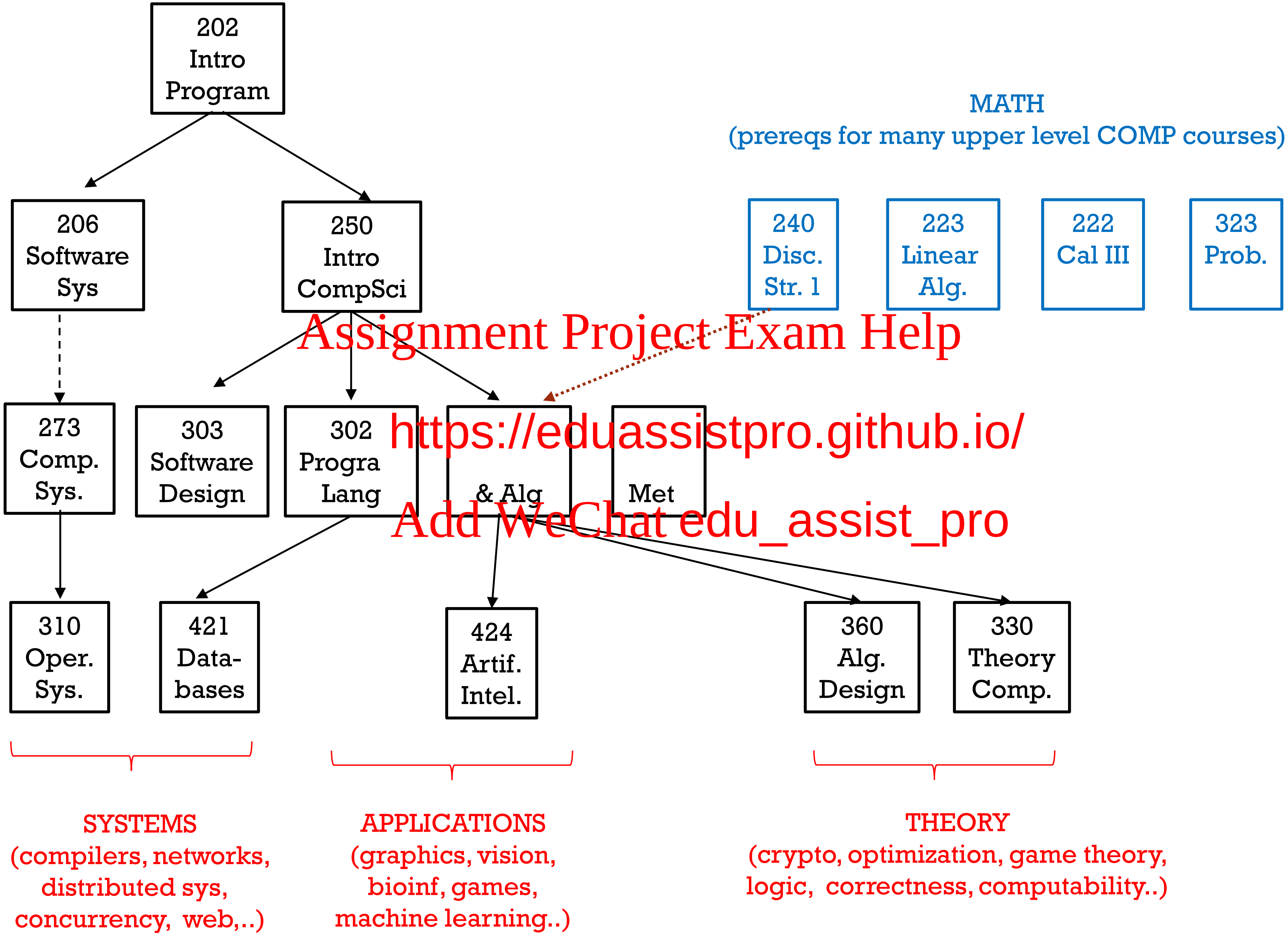

202
Intro
Program
206
Software
Sys
250
Intro
CompSci
MATH
(prereqs for many upper level COMP courses)
240
Disc.
Str. 1
223
Linear
Alg.
222
Cal III
323
Prob.
273
Comp.
Sys.
303
Software
Design
302
Progra
Lang
& Alg
Met
310
Oper.
Sys.
421
Data-
bases
424
Artif.
Intel.
360
Alg.
Design
330
Theory
Comp.
SYSTEMS
(compilers, networks,
distributed sys,
concurrency, web,..)
APPLICATIONS
(graphics, vision,
bioinf, games,
machine learning..)
THEORY
(crypto, optimization, game theory,
logic, correctness, computability..)

GRAPH ADT

- addVertex( ), addEdge( )
- containsVertex( ), containsEdge( )
- getVertex( ), g
- removeVertex( ), removeEdg
- numVertices( ), numEdges( )
- …

How to implement a Graph class?

A graph is a generalization of a tree, so …

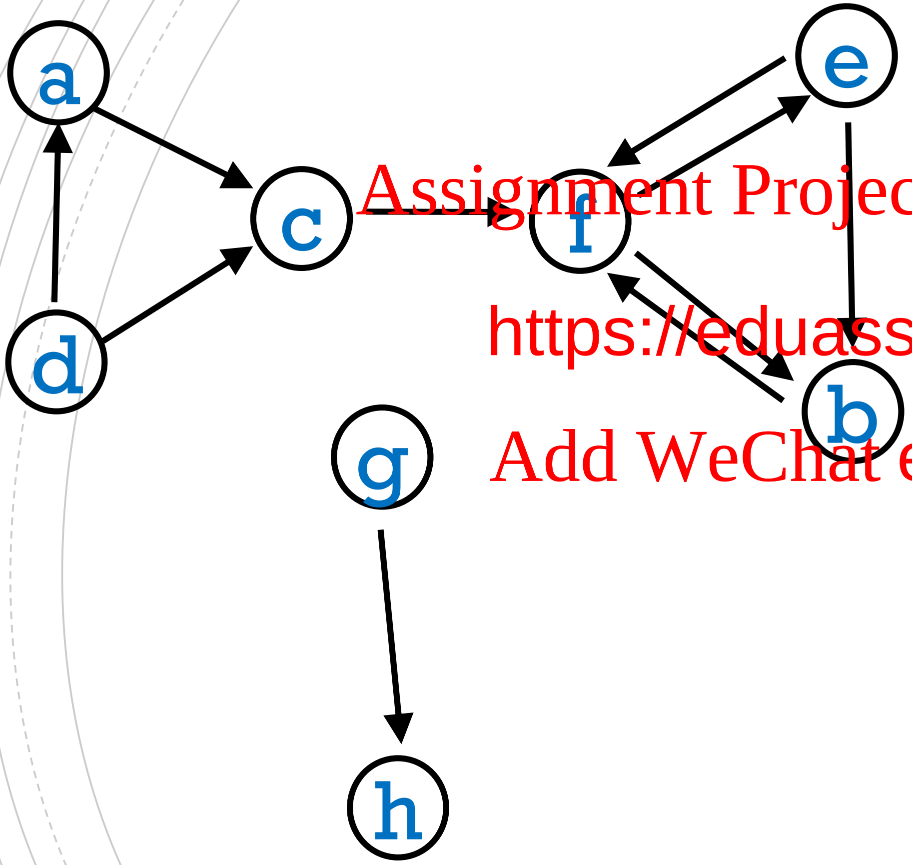
RECALL: HOW TO IMPLEMENT A ROOTED TREE IN JAVA ?

```
class Tree<T>{
   TreeNode<T> root;
      :

   class TreeNode<T>{
      T element;
      ArrayList<TreeNode<T>> children;
      TreeNode<T> parent; // optional
   }
}
```

// alternatively….

```
class Tree<T>{
   TreeNode<T> root;
      :

   class TreeNode<T>{
      T element;
      TreeNode<T> firstChild;
      TreeNode<T> nextSibling;
   }
}
```

ADJACENCY LIST
(GENERALIZATION OF CHILDREN FOR GRAPHS)



v	v.adjList
a	c
b	f
c	f
d	a, c
e	b, f
f	b, e
g	h
h	

Here each adjacency list is sorted, but that is not always possible (or necessary).

HOW TO IMPLEMENT A GRAPH CLASS IN JAVA?

A very basic Graph class:

```
class Graph<T> {
   class Vertex< [illegible] e called it GNode
      ArrayList<Vertex> adjL
      T element;
   }
}
```

HOW TO IMPLEMENT A GRAPH CLASS IN JAVA?

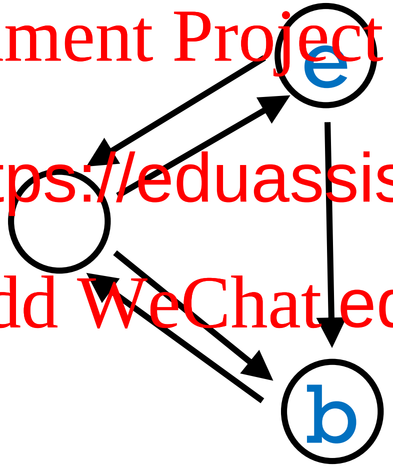
```
class Graph<T> {
    class Vertex<T> {
        ArrayList<Edge> adjList;
        T element;
        boolean visi
    }

    class Edge {
        Vertex endVertex;
        double weight;
               :
    }
}
```


Note that, unlike a rooted tree, there is no notion of a root vertex in a graph.

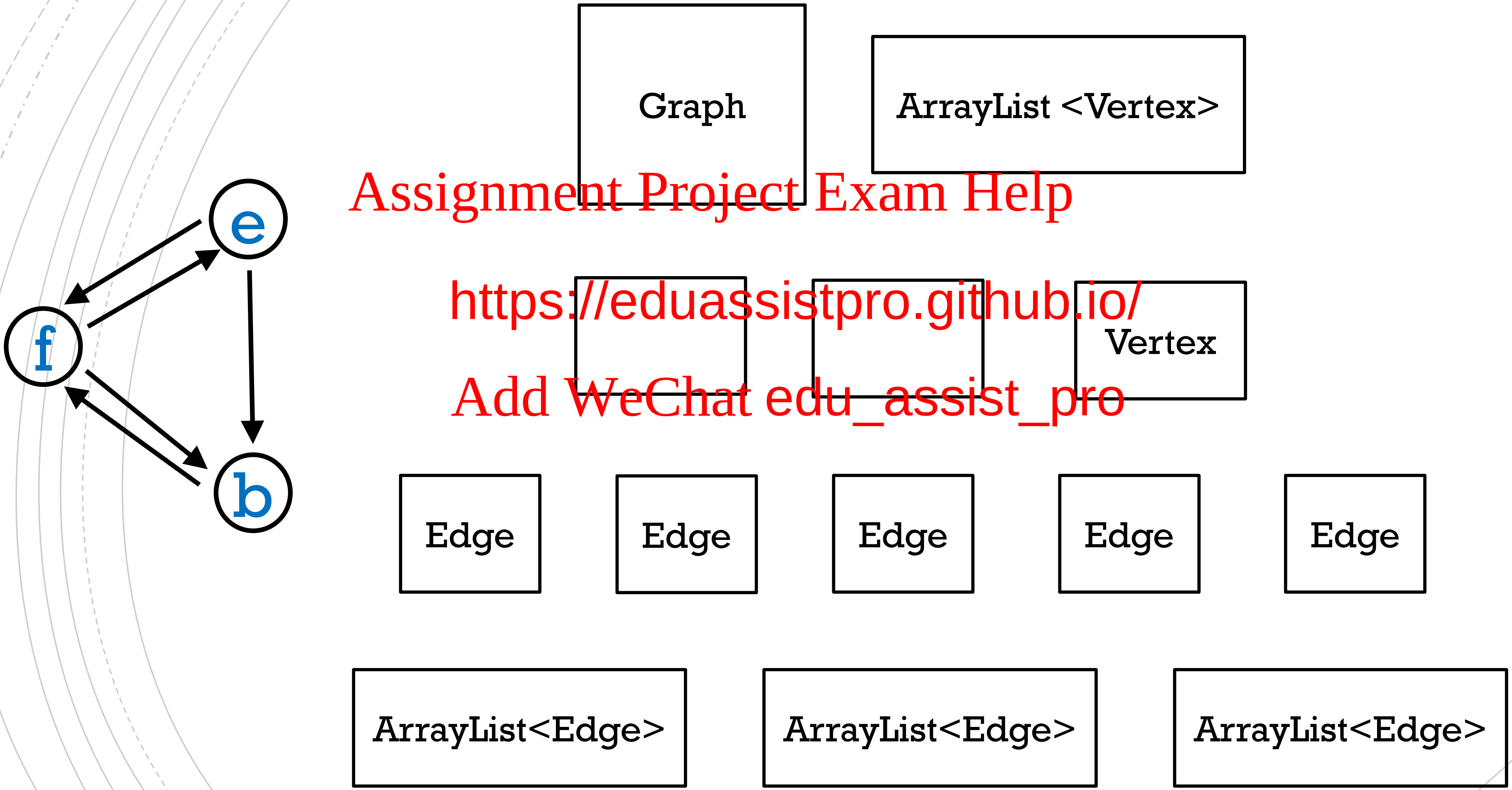
HOW TO REFERENCE VERTICES?

```
class  Graph<T> {
      ArrayLis                        exList;
          :
      class Vertex<T> { .
      class  Edge<T>  { ... }
}
```

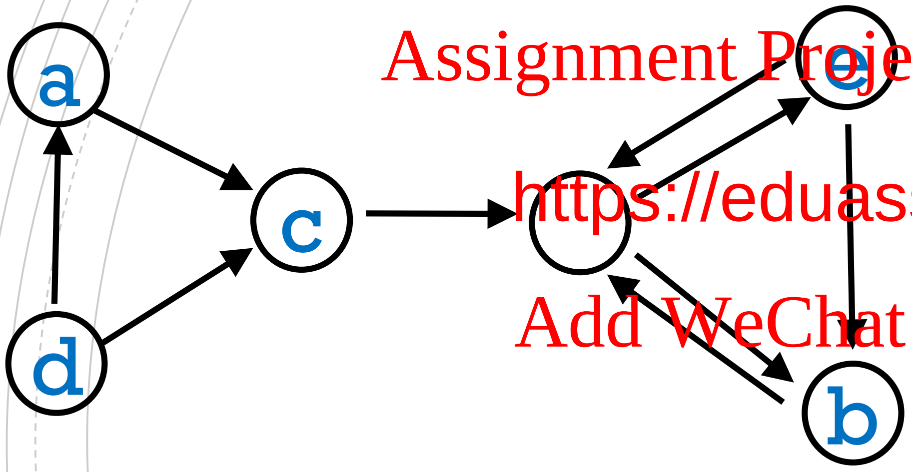
HOW MANY OBJECTS ?



HOW MANY OBJECTS ?



ADJACENCY MATRIX

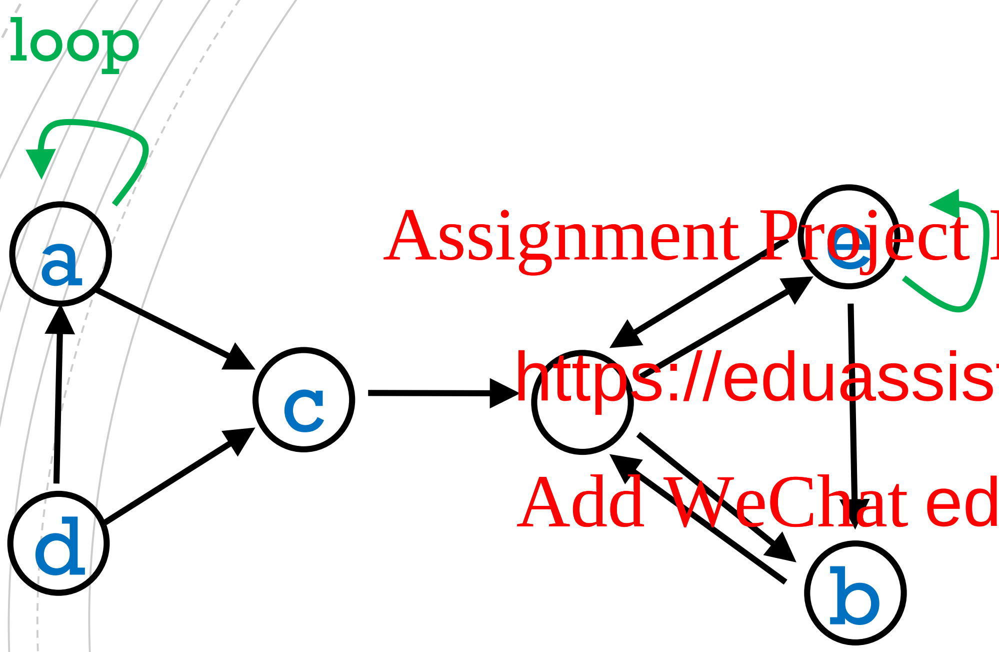


	a	b	c	d	e	f
a	0	0	1	0	0	0
	0	0	0	0	0	1
	0	0	0	0	0	1
	1	0	1	0	0	0
e	0	1	0	0	0	1
f	0	1	0	0	1	0

Assume we have a mapping from vertex names to 0, 1, ... , n-1.

```
boolean[][] adjMatrix = new boolean[6][6]
```

ADJACENCY MATRIX



	a	b	c	d	e	f
a	1	0	1	0	0	0
	0	0	0	0	0	1
	0	0	0	0	0	1
	1	0	1	0	0	0
e	0	1	0	0	1	1
f	0	1	0	0	1	0

Assume we have a mapping from vertex names to 0, 1, …. , n-1.

```
boolean[][] adjMatrix = new Boolean[6][6]
```

"DEFINITIONS"

Consider a graph with n vertices.

We say that the graph i edges is close to n^2.

We say that the graph is *sparse* if the number of edges is close to n.

(These are not formal definitions.)

EXERCISE

Would you use an *adjacency list* or *adjacency matrix* for each of the following?

- The graph is sparse e.g. 10,000 vertices and 20,000 edges and we want to use as little space as possible.
-
-
-
-

EXERCISE

Would you use an *adjacency list* or *adjacency matrix* for each of the following?

- The graph is sparse e.g. 10,000 vertices and 20,000 edges and we want to use as little space as possible.
- The graph is dense e.g. 1 [illegible] ,000 edges, and we want to use as little space as possible.

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- The graph is dense e.g. 1 [illegible] ,000 edges, and we want to use as little space as possible.

- Answer the query areAdjacent() as quick [illegible] le, no matter how much space you use.
-
-

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Would you use an *adjacency list* or *adjacency matrix* for each of the following?

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- The graph is dense e.g. 1 ...00 edges, and we want to use as little space as possible.

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- Perform operation insertVertex(v).
-

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Would you use an *adjacency list* or *adjacency matrix* for each of the following?

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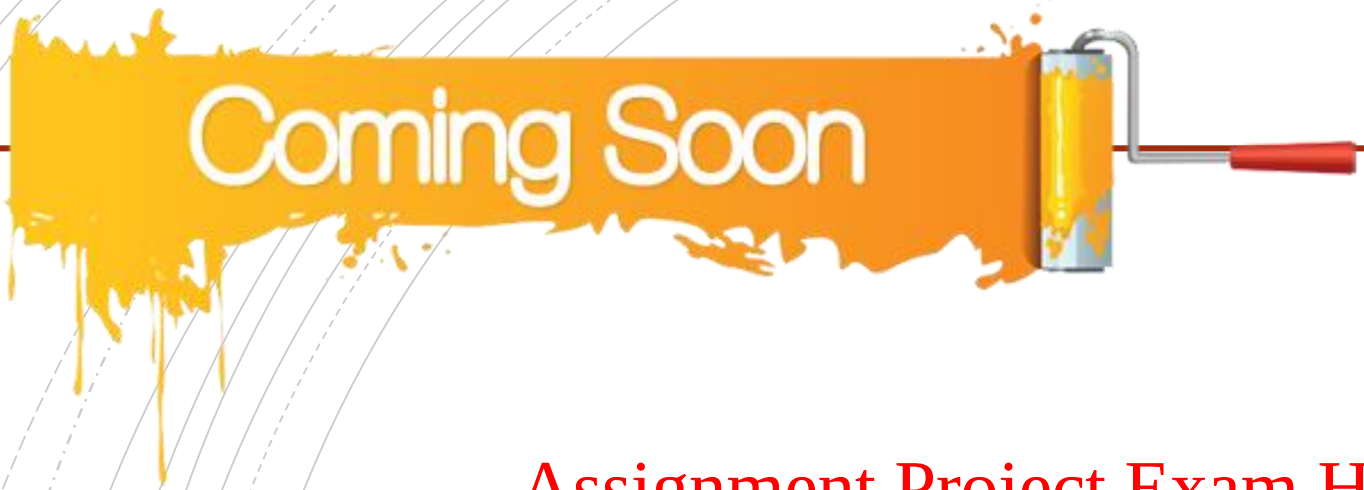
- The graph is dense e.g. 1 ,000 edges, and we want to use as little space as possible.
- Answer the query areAdjacent() as quick le, no matter how much space you use.
- Perform operation insertVertex(v).
- Perform operation removeVertex(v).

COMING UP!

- Recursive graph traversal
  - d
- Non-recursive gra al
  - depth first
  - breadth first



In the next

- More on