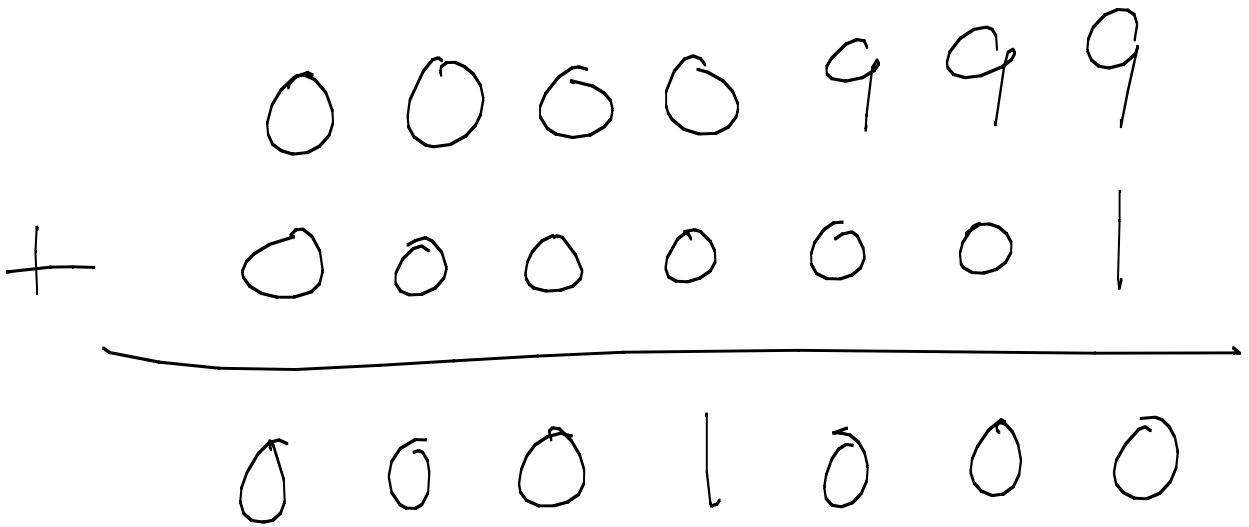
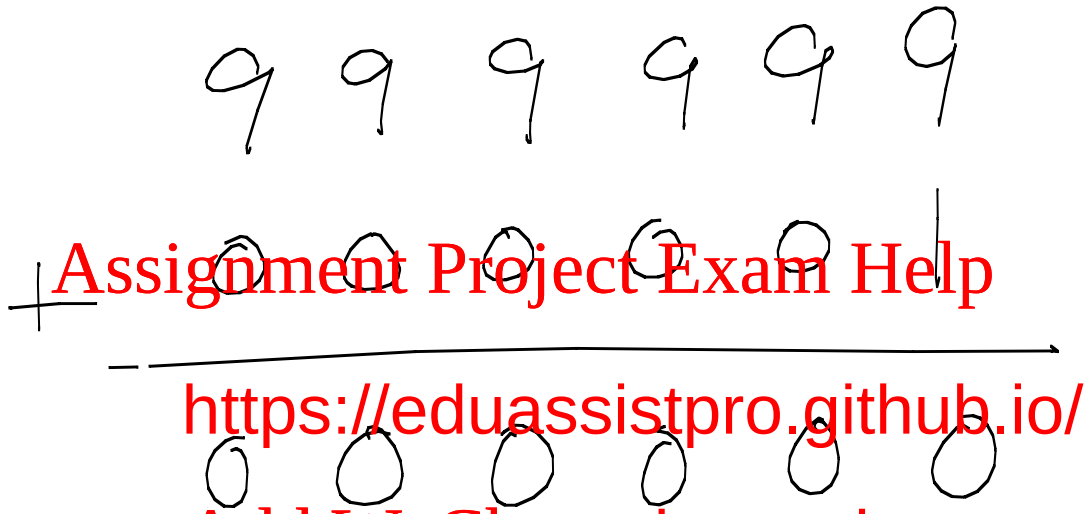
lecture 1

- two's complement
- floating point numbers
- hexadecimal

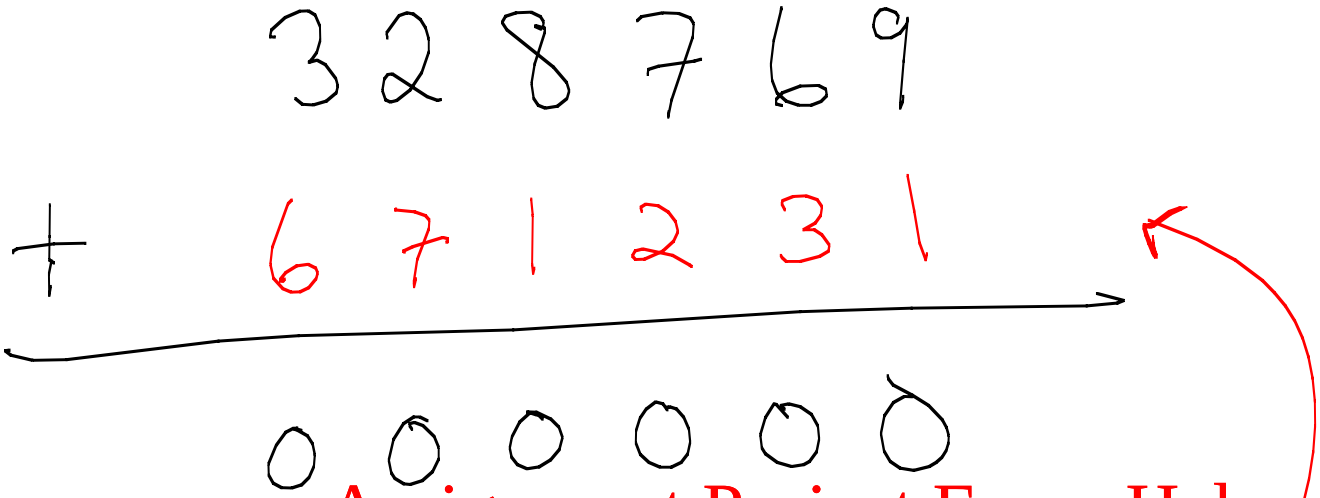
Car odometer (fixed number of digits)





$\Rightarrow \quad 999999 \equiv -1$

$$
\begin{array}{r}
3\,2\,8\,7\,6\,9 \\
+\quad ? \\
\hline
0\,0\,0\,0\,0\,8
\end{array}
$$

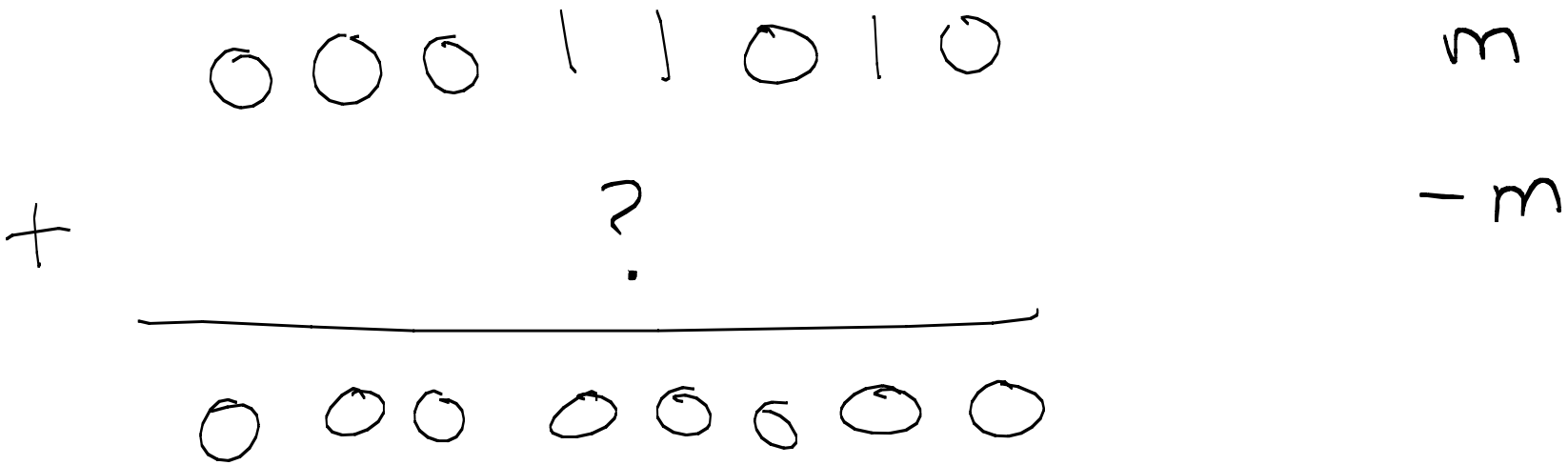


3 2 8 7 6 9

If you know what "modular arithmetic" is (MATH 240), then you recognize this: addition of integers mod 10^6.

Q: How to represent negative numbers in binary ?

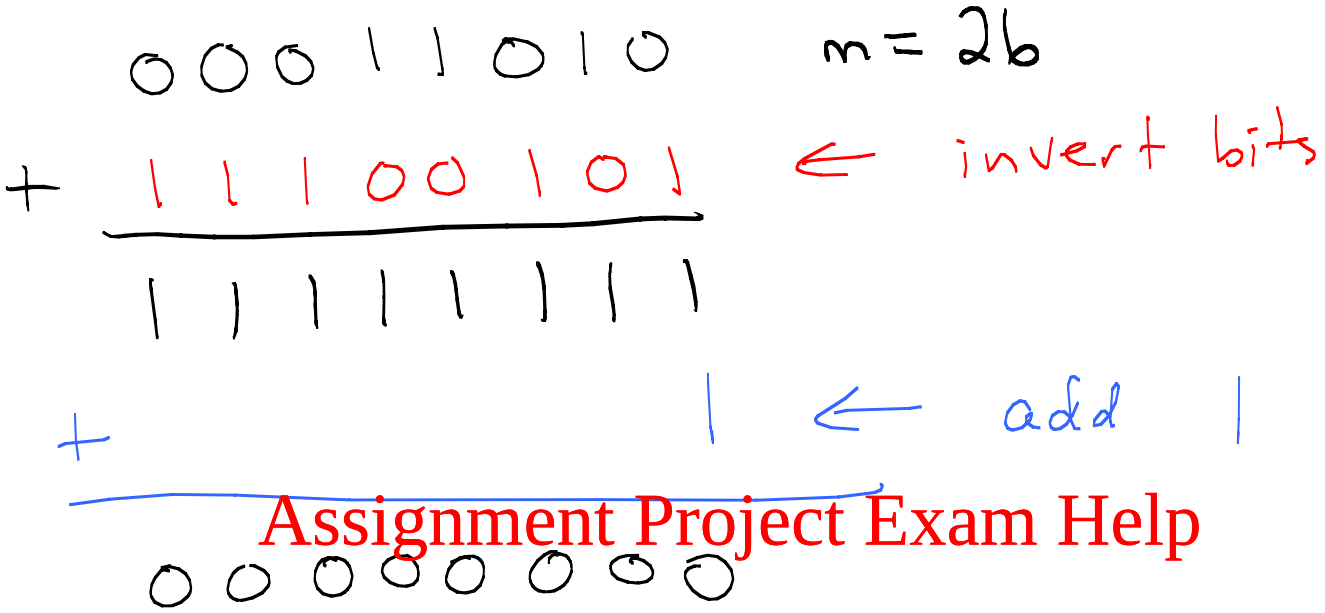
A: Given an 8 bit binary number m, define -m so that m + (-m) = 0.

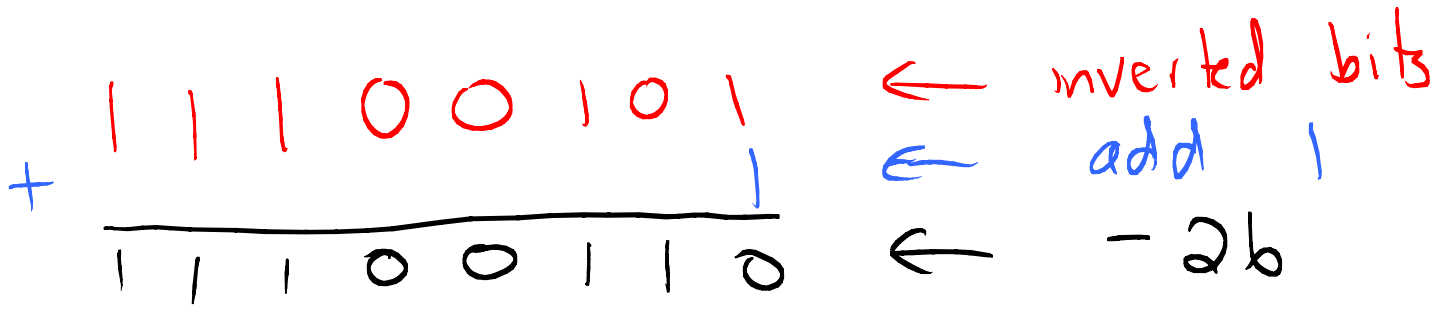


Two's complement representation of integers

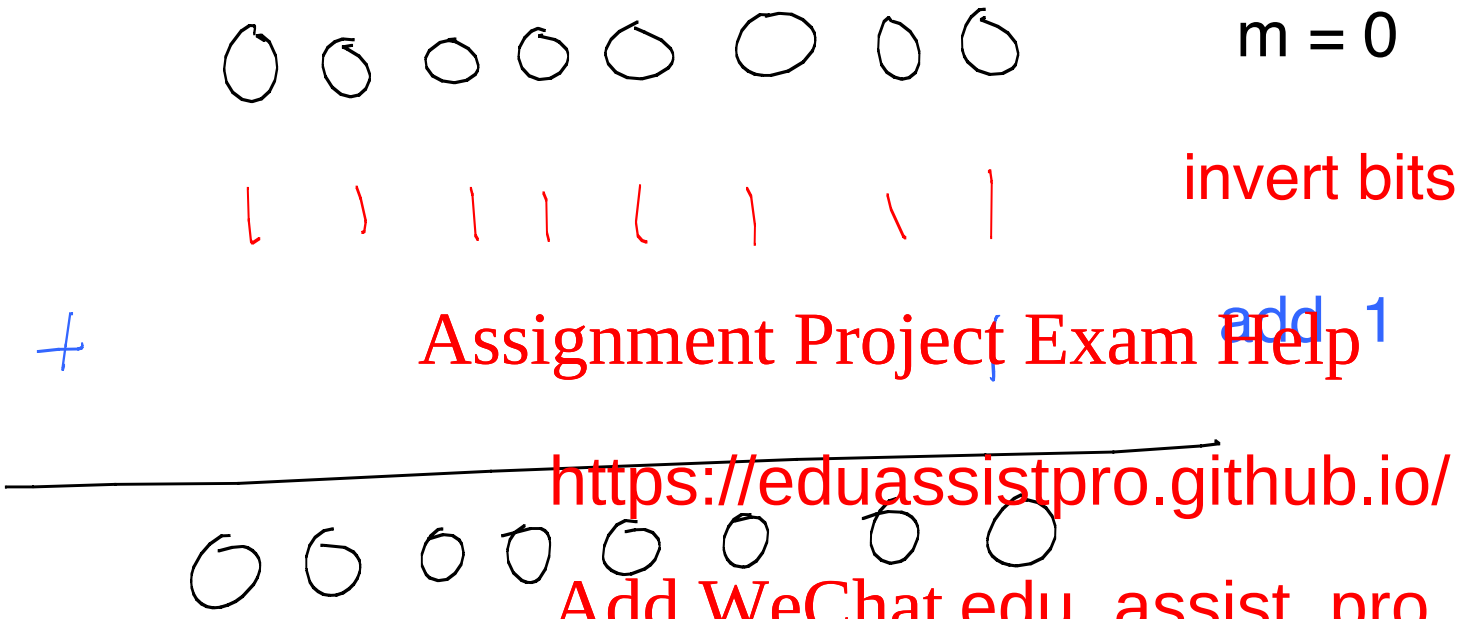
Example: How to represent -26 ?

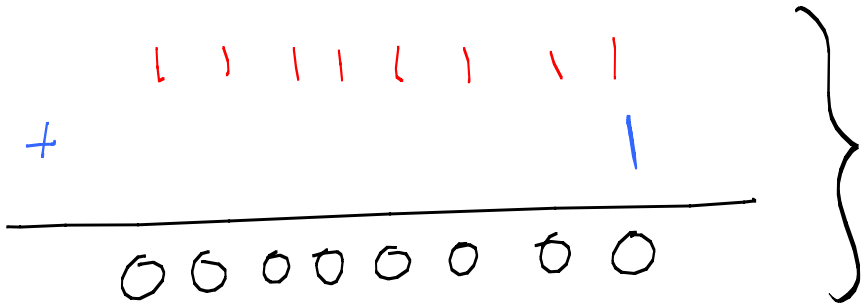
Use a trick!





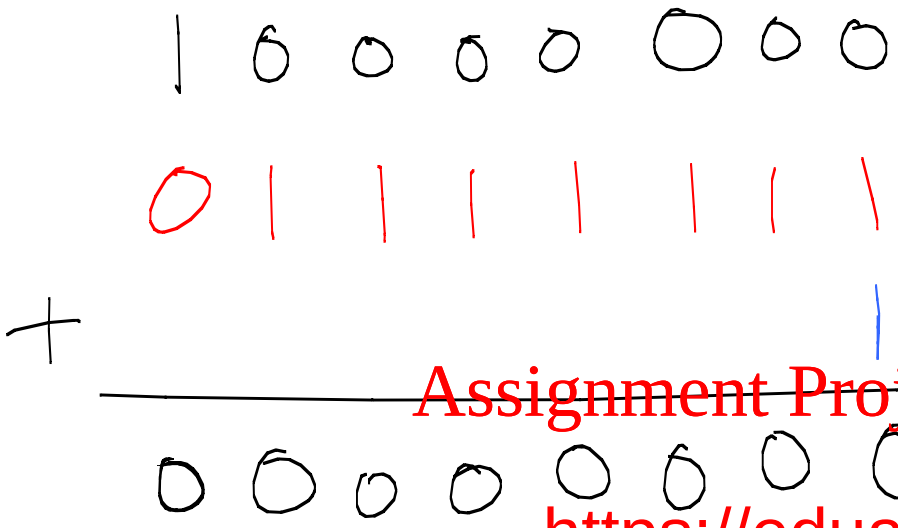
Another example: What is -0 ?

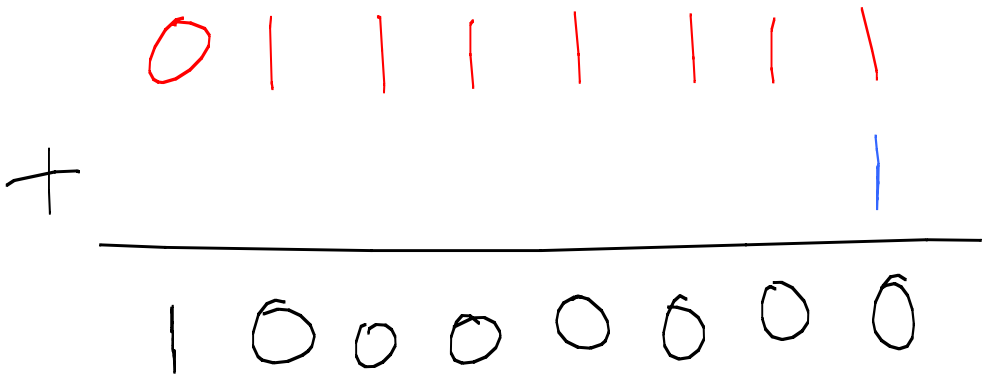




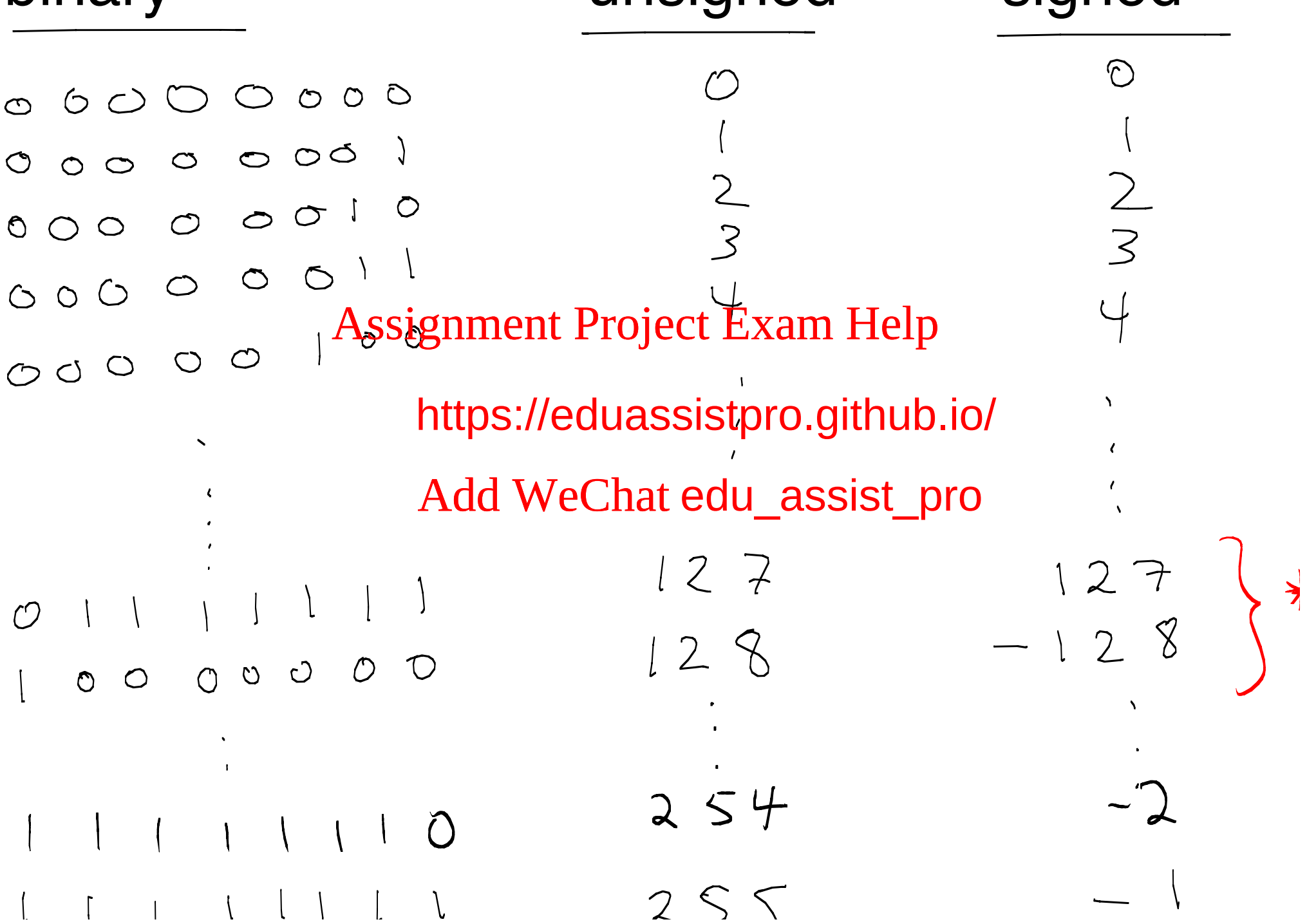
We have verified that -0 = 0.

What about m = 128 ? What is -128 ?

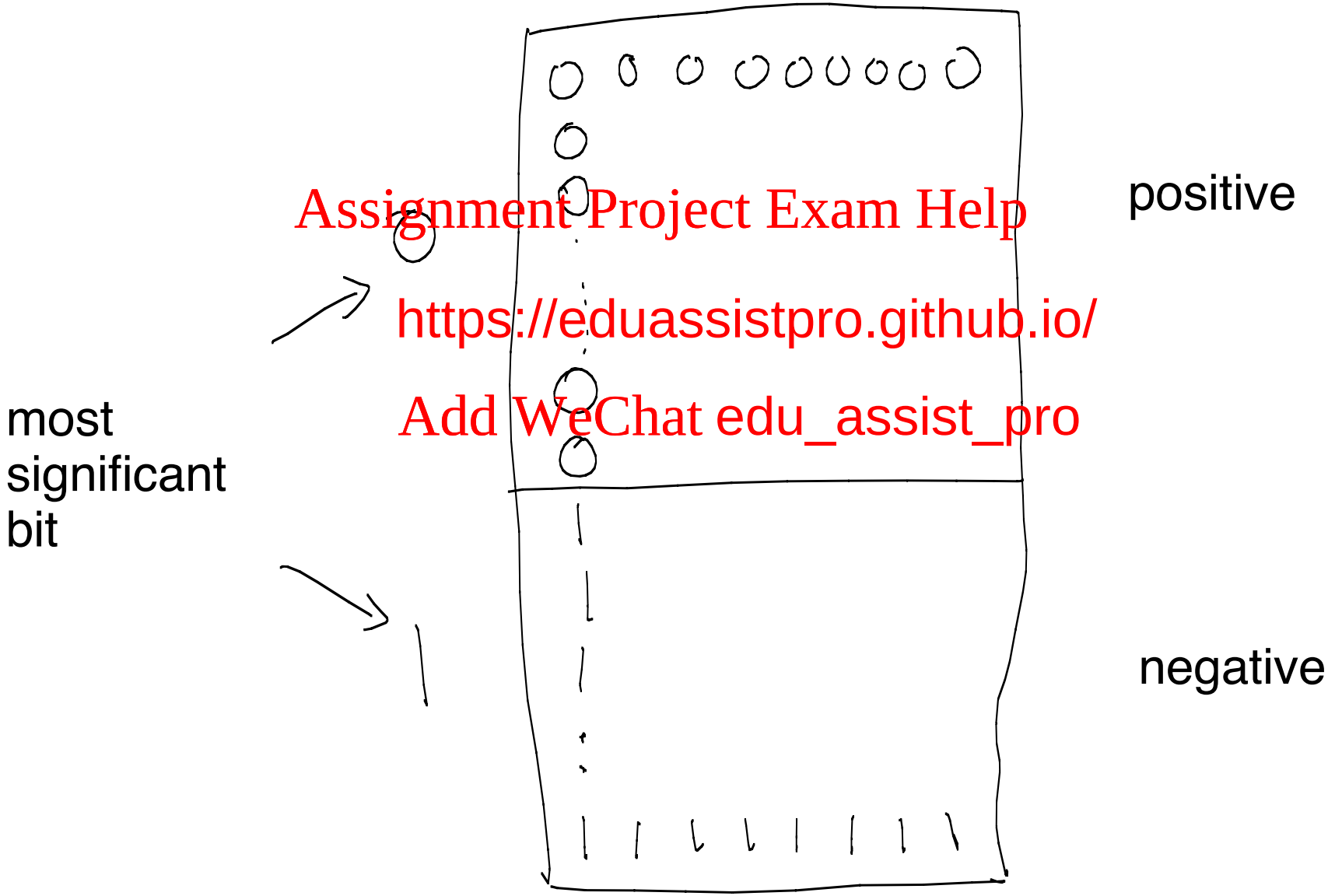




Thus, 128 is equivalent to -128.



signed integers



8 bit integers (unsigned vs. signed)

n bits defines 2^n integers

unsigned

$0, 1, \ldots, 2^n - 1$

signed

$-2^{n-1}, \ldots, 0, \ldots, 2^{n-1} - 1$

Take n = 32.

The largest signed integer is 2^31 - 1.

2 ^ 10 = 1024 ~ 10 ^ 3 = one thousand.

2 ^ 20 ~ 10 ^ million

2 ^ 30 ~ 10 ^ 9 = one billion

2 ^ 31 ~ 2,000,000,000 = two billion

Java Example

```
int  j = 4000000000;   // 4 billion  >  2^31
```

This gives a compiler error. "The literal of type int is out of range."


```
int< j2≙32000000000
System.out.println( 2 * j )

//  This prints out  -294967296.
//   To understand why these particular digits are printed,  you
//   would need to convert 4000000000 to binary, which I don't
//    recommend.)
```

lecture 1

- two's complement
- floating point numbers
- hexadecimal

Floating Point

"decimal point"

$$26.375 = 2 \times 10^{1} + 6 \times 10^{0} + 3 \times 10^{-1} + 7 \times 10^{-2} + 5 \times 10^{-3}$$

"binary point"

$$(11010.011)_2 = 2^4 + 2^3 + 2^1 + 2^{-2} + 2^{-3}$$

$$= 16 + 8 + 2 + 0.25 + 0.125$$

$$= 26.375$$

Convert from binary to decimal

We must use both positive and negative powers of 2.

n	2^n
-1	.5
-2	.25
-3	.125
-4	.0625
-5	.03125
-6	.015625
-7	.0078125
⋮	etc.

Sum up the contributing 1 bits as on previous slide.

How to convert from decimal to binary ?

$$26.375 = (\underline{\quad ? \quad} . \underline{\quad ? \quad})_2$$

To find the bits for the positive powers of 2, use the algorithm from lecture ("repeated division") .

m	b
26	0
13	1
6	0
3	1
1	1
0	

$\Rightarrow$ 26 $= (11010)_2$

What about negative powers of 2 ?

In general, note that multiplying by 2 shifts bits to the left (or shifts binary point to the right)

Example:

$(11010.011)_2 \times 2$

$= (110100.11)_2$

Similarly....dividing by 2 and not ignoring remainder shifts bits to the right (or shifts binary point to the left)

$$(1101.0011)_2 / 2$$

$$= (1101.0001\ 1)_2$$

For the negative powers of 2, use "repeated multiplication"

$$.375$$

$$= 375 \times 2^{1} \times 2^{-1}$$

$$= 75 \times 2^{-2}$$

$$= 15 \times 2^{-3}$$

$$= 3.0 \times 2^{-3}$$

convert decimal to binary

$$= (11)_2 \times 2^{-3}$$

$$= (.011)_2$$

A more subtle example:

$$19.243 = (\ ?\ )_2$$

First, find the bits for the positive powers of 2 using "repeated division" (last lecture).

m	b_i
19	
9	1
4	1
2	0
1	0
0	1

$\therefore 19 = (10011)_2$

Then find the bits for the negative powers of 2 using repeated multiplication.

$.243$

$= .243 \times 2^{1} \times 2^{-1}$

$= (.486) \times 2^{-1}$

$= ?$

Then find the bits for the negative powers of 2 using repeated multiplication.

$$.243$$

$$= (0)_2 \; .486 \times 2^{-1}$$

$$= (00)_2 \; .972 \times 2^{-2}$$

$$= (001)_2 \; .944 \times 2^{-3}$$

$$= (0011)_2 \; .888 \times 2^{-4}$$

$$\text{Thus } (.243)_{10} = (.0011)_2 + \sum_{i=-5}^{-\infty} b_i 2^i$$

Note the summation is over bits bi from -5, -6, ..., -infinity.

$19.243 = (10011.0011\_\_)_2$

We cannot get an exact representation using a finite number of bits for this example.

Can we say anyt out what happens ?

$(0.05)_{10}$

$= (0)_2.1 \times 2^{-1}$

$= (00)_2.2 \times 2^{-2}$

$= (000)_2.4 \times 2^{-3}$

$= (0000)_2.8 \times 2^{-4}$

$= (00001).6 \times 2^{-5}$

$= (000011)_2.2 \times 2^{-6}$

$= .00\ 0011\ 0011\ 0011$ etc.

This will repeat over and over again.

When we convert a floating point decimal number with a finite number of digits into binary, we get:

- a finite number of non-zero bits to left of binary point

- an infinitely repeating sequence of bits to the right of the binary point

Why ?

[Note: sometimes the infinite number of repeating bits are all 0's, as in the case of 0.375 a few slides back.]

Recall previous example...

$$.243$$

$$= (0)_2 .486 \times 2^{-1}$$

$$= (00)_2 .972 \times 2^{-2}$$

$$= (001)_2 .944 \times 2^{-3}$$

$$= (0011)_2 .888 \times 2^{-4}$$

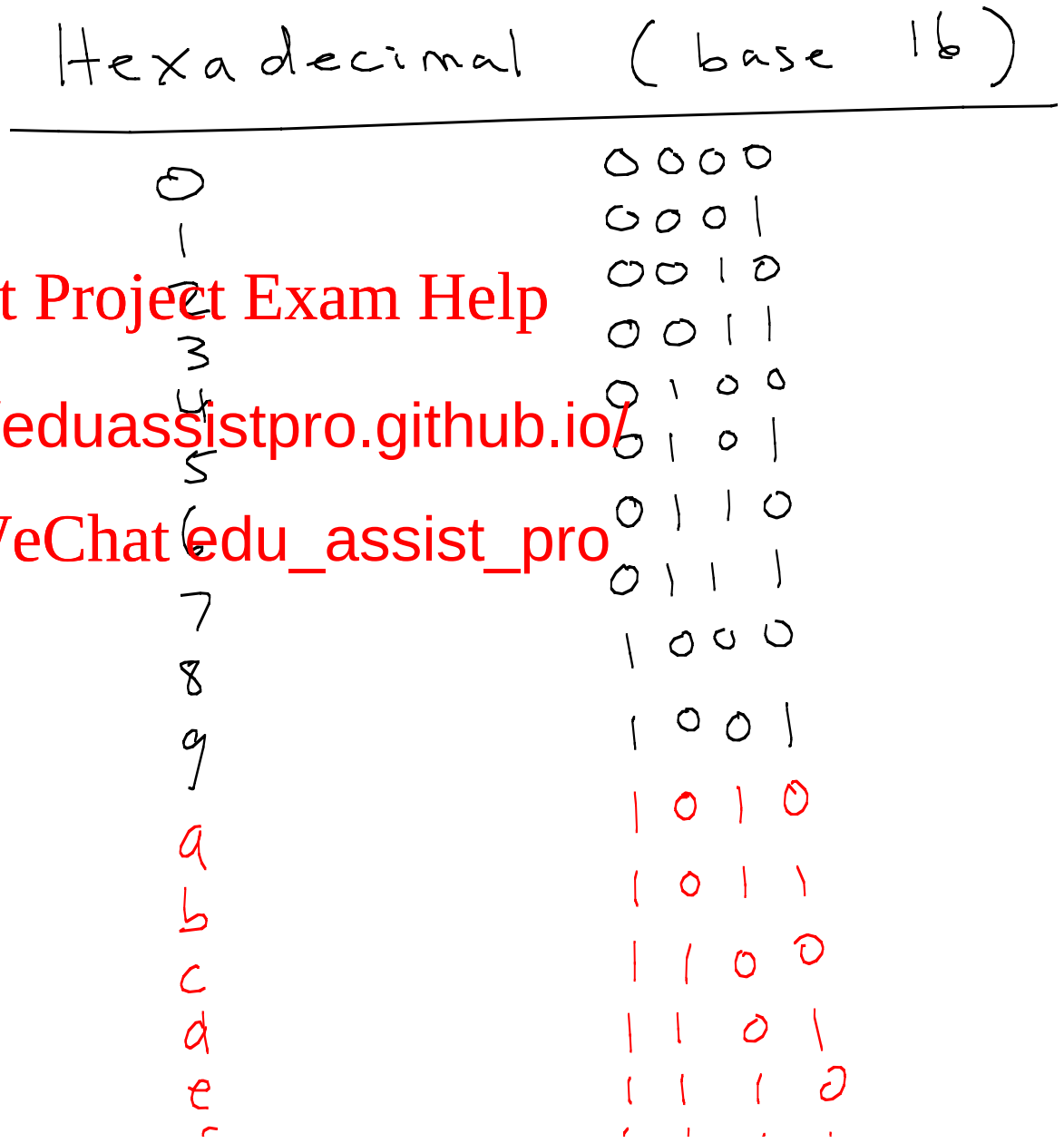
$$= \text{etc}$$

Eventually, the three digits to the right of the decimal point will enter a cycle that repeats forever. This will produce a bit string that repeats forever.

Hexadecimal

Writing down long strings of bits is awk and error prone.

Hexadecimal simplifies the representation.



Examples of hexadecimal

1) 0010 1111 1010 0011

2 f a 3

We write 0x

2) 101100

We write 0x2c (10 1100), not 0xb0 (1011 00)