Leader Election

(Par

Outline

- Non-Comparison Based Alg
  - Time Slice
  - Variable Speeds
- Lower Bounds
- Randomized
  - identity selec
  - Leader election

Comparison of Leader Election Algorithms

- Resulting Tradeoffs

Algorithm	Ro	e	# Messages
TimeSlice	$O(u_{\min} n)$	$O(u_{\min} n)$	$O(n)$
VariableSpeed	$O(2^{u_{\min}} n)$	$O(2^{u_{\min}} n)$	$O(n)$
Randomiz			$O(n \log n)$

where u_i denot ... $u_{\min}$ the minimum identifier among the node ide

No ison

Assignment Project Exam Help

https://eduassistpro.github.io/

Bas

Add WeChat edu_assist_pro

Time Slice Algorithm

- Uses the *strong* assumpti
  - the ring size n is know es (non-uniform).
- It assumes unidirectional communication.
- It elects the process with the minimum user ID.

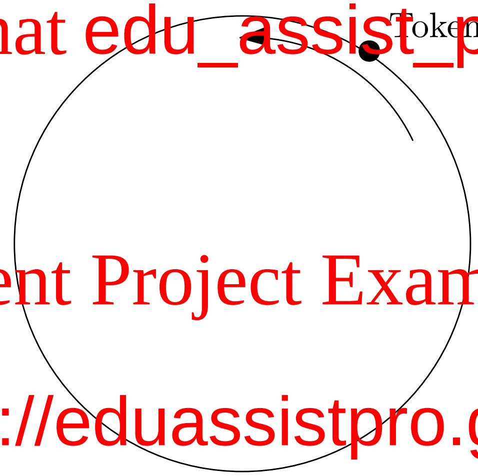
- Assumes process

  - Each process i o the rest of the processors).

- Employs synchrony in a deeper way in that
  - it uses a token to convey information.

Time Slice Algorithm: Searhing for IDs!

- Employs a *circulating toke* D around the ring.

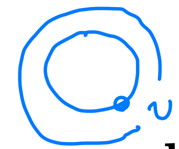


- Let v denote the phase. Starting from d each phase incrementing by 1, it attempts to elect v as leader.
- For $v = 1, 2, \ldots$: in phase v only a token carrying ID v is permitted to circulate.
- Processor IDs are unknown to other processors (and can be large numbers).

Time Slice Algorithm

1. Computation proceeds in phase ... and each phase consists of n consecutive r (n is known).

2. Each phase devoted to the possible circulation, all the way around the ring, of a token carrying a particular value. Nodes check if their ID is equal to the value of the token.



3. In **phase** v, whi $1)n+1, \ldots, vn$, **only a token carrying value** v **is per** ate.

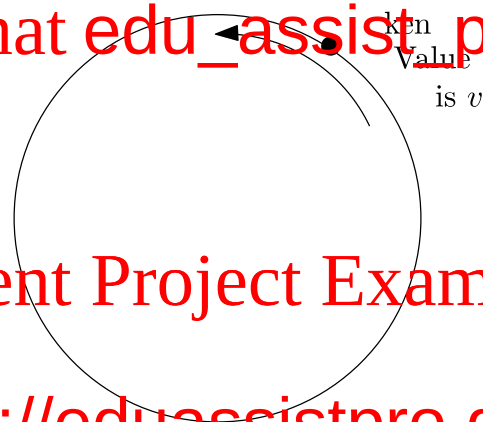
4. **If a process** i **with** ID **equal t** nd round $(v-1)n+1$ is reached then process i elects itself the leader and sends a token carrying its ID around the ring.

5. As this token travels, all the other processes note that they have received it, which prevents them from electing themselves as leader or initiating the sending of a token at any later phase.

i.e., you assume the nodes have MAC addresses: in the Network Card

Example of Time Slice Algorithm

- A token **carrying a certai** irculates around the ring.



1. Phase $v = 1$: Token carrying ID equ
   around the ring; nodes check if their ID i

2. Phase $v = 2$: Token carrying ID equal to 1 circulating around the ring; nodes check if their ID is equal to 1;
3. etc.

- Note that in each phase, it takes n steps for the token to go around the ring.

Correctness of Time Slice Algorithm

- The minimum ID $u_{\min}$ e ll the way around, which causes its originating pr lected.

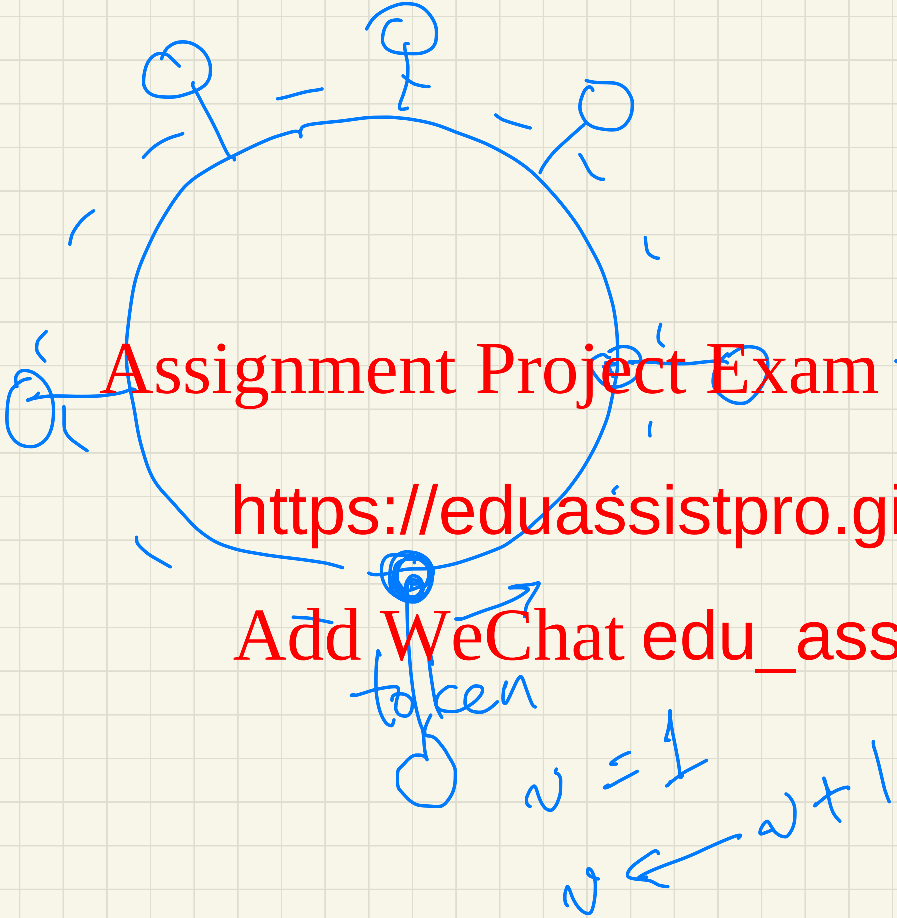
  - No messages are sent before round $(u_{\min} - 1)n + 1$, and
  - no messages are sent after round $u_{\min} n$.

- The total number o n.
  - These are the nimum ID u_{min} claiming to be the leader.

- If we prefer to elect the process with the maximum ID rather than the process with the minimum, we can simply let the minimum send a special message around after it is discovered in order to determine the maximum.

$u_{\min} \cdot n$

- The communication complexity is still $O(n)$.


token
n = 1
n ← n+1

Correctness of Time Slice Algorithm

- The good property of the TimeS hat the total number of messages is n.

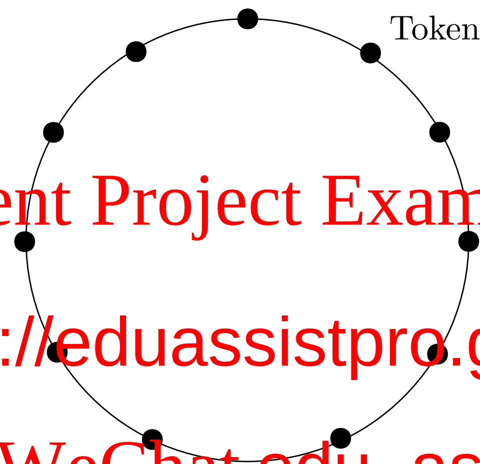
- Unfortunately, the time complexity is about $nu_{\min}$, which is an unbounded number, even in a fixed-size ring.

- This time comple rithm.

- It is only useful in pra h IDs are assigned from among the small pos

Variable Speed Algorithm

- Employs *circulating token* ens as nodes) carrying certain IDs.



- Each process i has the ID u_i (unknown to the rest of the processors).
- Process i initiates a token, which travels around the ring, carrying the ID u_i of the originating process i.
- Different tokens travel at different speeds.

Variable Speed Algorithm

1. Each process i initiates a to els around the ring, carrying the value u_i of th rocess i.

2. Different tokens travel at different speeds.

   - A token with value v travels at the speed of one message transmission per 2^v rounds, that is, each process along its path waits 2 before sending it out. / kes time $n2^v$ to circulate around the ring (if it does) */

3. Each process keeps track of the smallest value it has seen so far and simply discards any token carrying an identifier that is larger than this smallest one.

4. If a token returns to its originator the originator is elected leader.

Correctness of Variable Speed Algorithm

- Algorithm guarantees that
  - by the time the token carryi entifier $u_{\min}$ gets all the way around the ring,
  - the second smallest identifier could only get at most halfway around, the third smallest could only get at most a quarter of the way arou
  - the kth small $/2^{k-1}$ of the way around.
- Therefore, up to the time of election, the token carrying $u_{\min}$ uses more messages than all the others combined.
- Since $u_{\min}$ uses exactly n messages, the total number of messages sent, up to the time of election, is less than $2n$.

Correctness of Variable Speed Algorithm

- By the time $u_{\min}$ gets all th … ing, all nodes know about this value, and so wil … ny other tokens.

- It follows that $2n$ is an upper bound on the total number of messages that are ever sent by the algorithm (including the time after the leade …

- The time complexity, as mentioned abo … $n2^{u_{\min}}$, since each node delays the token carrying ID … $u_{\min}$ … time units.

Assignment Project Exam Help

Le n

https://eduassistpro.github.io/

Lower B

Add WeChat edu_assist_pro

Comparison Based Algorithms

- An algorithm is comparison ba e same on rings that have the same order pa ifiers.

  - in two rings with corresponding processors $p_1, p_2, \ldots, p_n$ and $q_1, q_2, \ldots, q_n$ the actions of the algorithm depend only on the order of the identifiers $ID(p_1), ID(p_2), \ldots, ID(p_n)$ and $ID(q_1), ID($.

- We have seen that t m achieves the following bounds:

  - communication complexity of $O(n \log n)$ messages, and
  - time of $O(n)$.

Radius
Growth

Non-Comparison Based Algorithms

- Non-comparison based algo er hand, use $O(n)$ messages, but (can) have a huge r

- A lower bound of $\Omega(n \log n)$ messages can be shown for
  1. *comparison based* algorithms; lower bound holds even if we assume that communication is bidirectional and the ring size n is kno
  2. *non-comparison* based algorith ed time complexity (i.e., bounded numbe [a]

- In the sequel we discuss only the first bound for comparison based algorithms.[b]

---

[a] This can be proved using Ramsey Theory.

[b] Greg N. Frederickson and Nancy A. Lynch. Electing a leader in a synchronous ring. Journal of the ACM, 34(1):98-115, January 1987.

The Plan: What Are We Going to Do?

- Assume we are given a uniform (known) algorithm $\mathcal{A}$ that solves the above comparis t of the leader election problem.

- We will show that there exists an admissible execution (i.e., an execution that conforms to the model being considered) of $\mathcal{A}$ in which $\Omega(n \log$
  - So not all execu $\log n)$ lower bound condition!

- **Theorem 1** *For any comparison based leader election algorithm on a ring of size n there is an execution of the algorithm in which $\Omega(n \log n)$ messages are being sent.*

Rad Gro.

$O(n \log n)$

How Do You Prove a $\Omega(n \log n)$ Lower Bound?

- The quantity being considere : $M(n) =$ "the number of messages required t n-node ring".

- This means that we must find a constant $c > 0$ independent of n such that

for all n.

we do not really care about $c>0$

c independent of n

- But how do we accomplish this task?
  - Using a recurrence!

A Simple Recurrence

- We will show that

$$M(n \qquad\qquad\qquad\qquad n/4$$

This is what we have to prove

- What does this mean?

- Lets denote our problem $LE(n)$:
  - Leader Electi

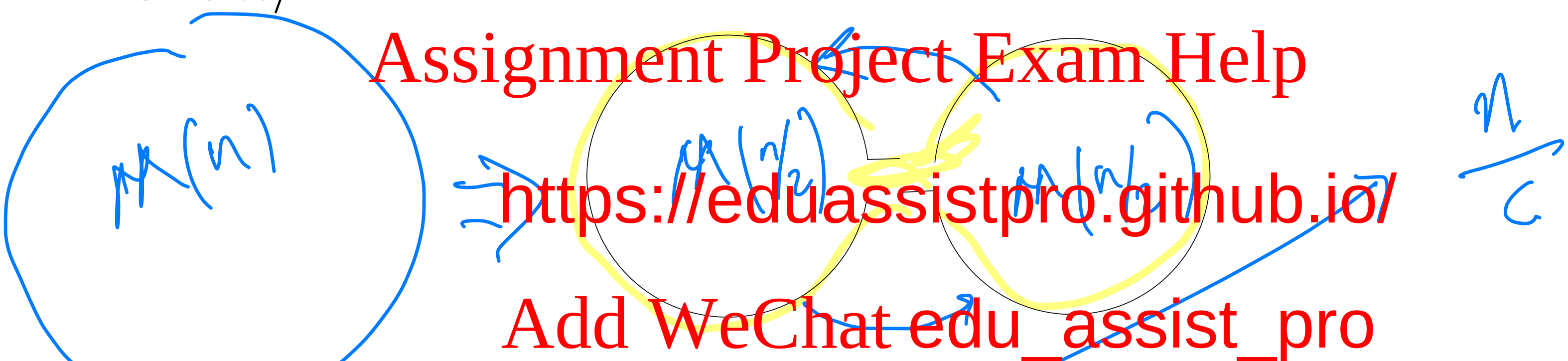
- One way to interpret Inequality (1) is the f

1. Split $LE(n)$ into two subproblems $LE(n/2)$.
2. Show that the number of messages required to solve $LE(n)$ is at least the number of messages required to solve two $LE(n/2)$ problems plus $n/4$.

What Does it Mean to Split?

- From our definition $M(n$ … er of messages required to elect a leader in an … de ring".

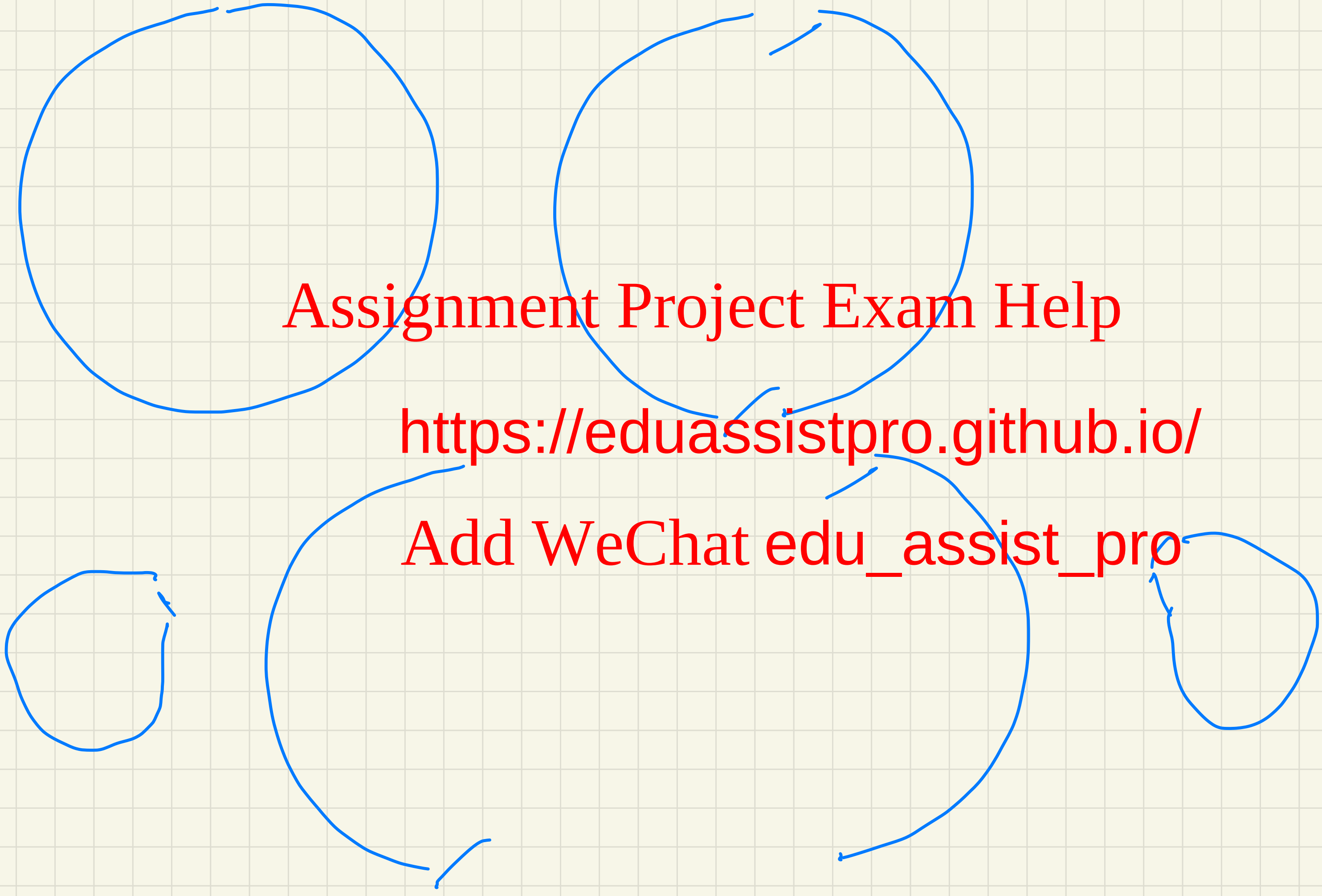
- So we need to split the ring of size n into two subrings each of size $n/2$:



  – The term $2M(n/2)$ in Inequality (1) should come from this two subrings in this splitting!

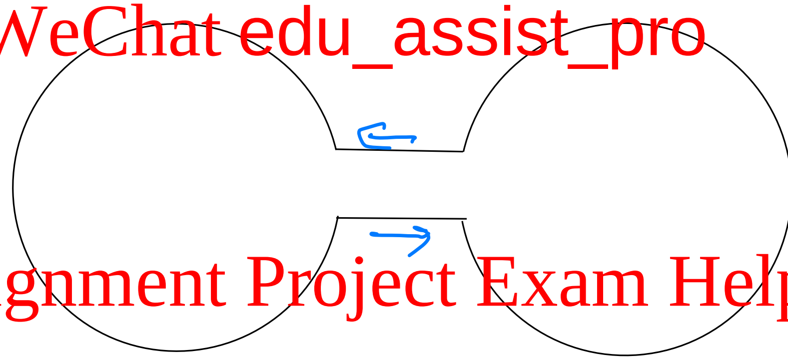
- How about the term $n/4$ in Inequality (1)?
  – The term $n/4$ in Inequality (1) should come from messages which are in transit from one subring to the other and needed to elect a leaser! (Discussed later.)

~~log(n/c)~~



Glueing!

- How do you glue two subrings int



- You must find two r s of size $n/2$ which send at least $M(n/2)$ subring and also leave at least one edge unused!
  - You need this so that you can do the glueing!

- We will call such schedules which leave an edge of the ring unused *open*
  - *open* because they leave an edge unused.

Inequality (1) Implies $M(n) \geq \frac{n}{4}(\log n + 1)$

$c = \frac{1}{4}$

- For simplicity, assume n

- Assume that $M(n) \geq 2$ as been proved[a]
- By induction: we will prove
  - Base case:

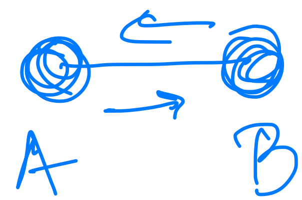
$$2$$

$\frac{2}{4} \cdot 2 = 1$

  - Inductive Ca

    Assuming the inductive assumpti

$$M(n/2) \geq \frac{n}{8}(\log \frac{n}{2} + 1)$$



we'll prove

$$M(n) \geq \frac{n}{4}(\log n + 1)$$

[a] We have not proved this yet! This is our goal in this lecture!

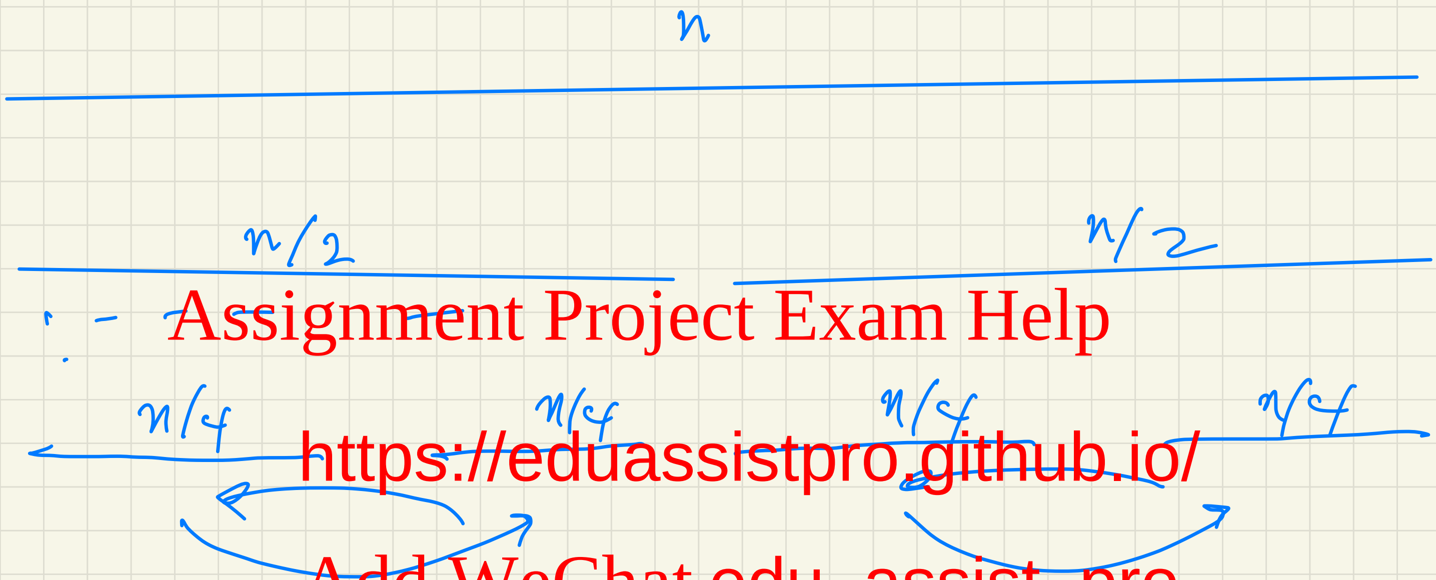
$$S(n) \leq 2\,S\left(\frac{n}{2}\right) + \frac{n}{4}$$

$$S(n) = \# \text{ of steps}$$

You don't know what the explicit formula for $S(n)$ is

Example Radius Growth satisfies this inequality

what is $S(n)$?



$$\boxed{S(n) \leq \underbrace{2\,S(n/2)} + \frac{n}{4}}$$

$$S(n) \le 2\,S(n/2) + n/4$$

$$\le 2\left(2\,S(n/4) + n/8\right) + n/4$$

$$= 2^2\,S(n/2^2) + 2 \cdot \frac{n}{4}$$

$$= 2^2\left(2 \cdot S(n/2^3) + n/2^4\right) + 2 \cdot \frac{n}{4}$$

$$= 2^3 \cdot S(n/2^3) + 3\,\frac{n}{4}$$

$$\vdots$$

$$= n \cdot S(1) + \log n \cdot \frac{n}{4}$$

$$S(1) = 1$$

$$S(2) \le 2 \cdot S(1) + \frac{1}{4}$$

$$S(4) \le 2 \cdot S(2) + \frac{4}{4}$$

$$S(8) \le 2 \cdot S(4) + \frac{8}{4}$$

$$\vdots \qquad \vdots$$

Base Case

- Here we must show that

$$M(2) \quad \frac{-}{4} \quad 1$$

- Somehow this seems like a simple statement about rings of just 2 nodes.

  – Lets postpon i

Inductive Step

- Assumie the inductive assum

$$M(n \quad [illegible] \quad \frac{[illegible]}{8} \quad [illegible] \quad \frac{n}{2} + 1)$$

- Therefore

$$
\begin{aligned}
M(n) &\geq 2 \quad [illegible] \\
&\geq 2 \quad [illegible] \quad \text{[illegible]uctive Assumption)} \\
&= \frac{n}{4}\log n + \frac{n}{4} \\
&= \frac{n}{4}(\log n + 1)
\end{aligned}
$$

log n −1

- Lets move now to the details! We are missing the proofs of
  - the base case, and
  - Inequality (1).

Basic Concepts/Assumptions

- We prove the lower bound for a sp e leader election problem, where the ele t be the processor with the maximum identifier in the ring;
  - in addition, all the processors ~~must know the identifier of~~ the elected leader.[a]

$$\boxed{M(n) \geq 2M\left(\frac{n}{2}\right) + \frac{n}{4}}$$

- We only accept uni th the maximum identifier can be the leader.

- Additionally, every node that is not the le he identity of the leader.
- Ring is asynchronous: nodes may wake up at arbitrary times (but at the latest when receiving the first message).

[a] Unless this is done, no leader has been elected.

Rules of the Game

- Recall execution model

- Nodes wake up at the latest when r essage

- Algorithms must be uniform

- We assume we have an algorithm and show it cannot complete faster than $O($

- Algorithm needs t re scheduled

  – And when nodes wake up

  – Otherwise it is not a solution

- But communication links must be FIFO

Schedules

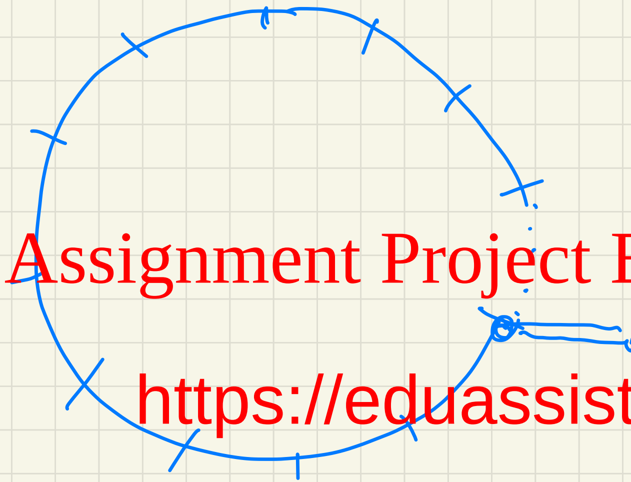
- An execution of a distributed al ents, sorted by time.

  - An event is a record (time, node, type, message) where type is "send" or "receive".

- A schedule σ of for a particular ring is open if there exists an edge e of the ring s s delivered over the edge e in eit

  - **Edge is open** if no message will e edge has been received so far.
  - **Schedule is open** if there is an open edge in the ring.

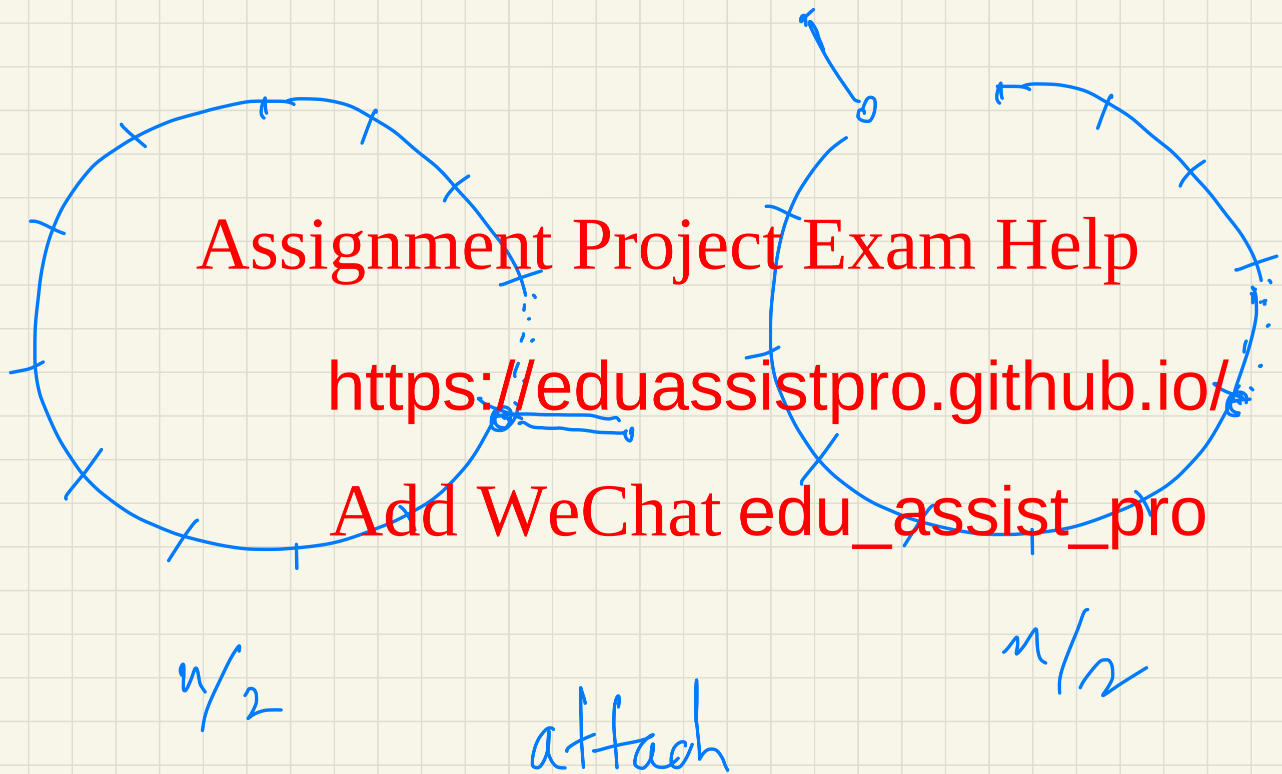
We need to "cut" the ring into two subrings



A schedule in which edges are "left open".

Open means unused.

We need to ensure that an execution will take place that will transmit "enough" messages and still leave an edge used.


n/2
n/2
attach

Scheduler

- We assume no two events happ me time.

- During the proof we can "play go ich message in transmission arrives next in the execution.

- If more than one message is in transit, the scheduler can choose which one arrives fi

- If two messages are t d edge, then it is sometimes required that the mes ed first will also be received first.

  – We respect the FIFO conditions for links.

Open Schedules

- Schedule: Execution chosen b

- Open schedule:
  - Schedule with an open edge / communication link

- Open edge:
  - Edge along which no message has yet been scheduled

Main Idea: Ring of n nodes

- We want to count how many me [illegible] end to elect a leader and at the same time main [illegible] dule, i.e. there is an edge for which no message has been received so far.

  - We prove it by induction on n
  - Will assume n is a power of 2.

- The proof is by indu[illegible]

  - The base case is for $n = 2^1$;
  - The inductive step is for $n =$ [illegible] re [illegible] 1.

$$\underbrace{M(n)}_{\checkmark\checkmark} \geq \underbrace{M\left(\frac{n}{2}\right)}_{\checkmark} + \underbrace{M\left(\frac{n}{2}\right)}_{\checkmark} + \frac{n}{4}$$

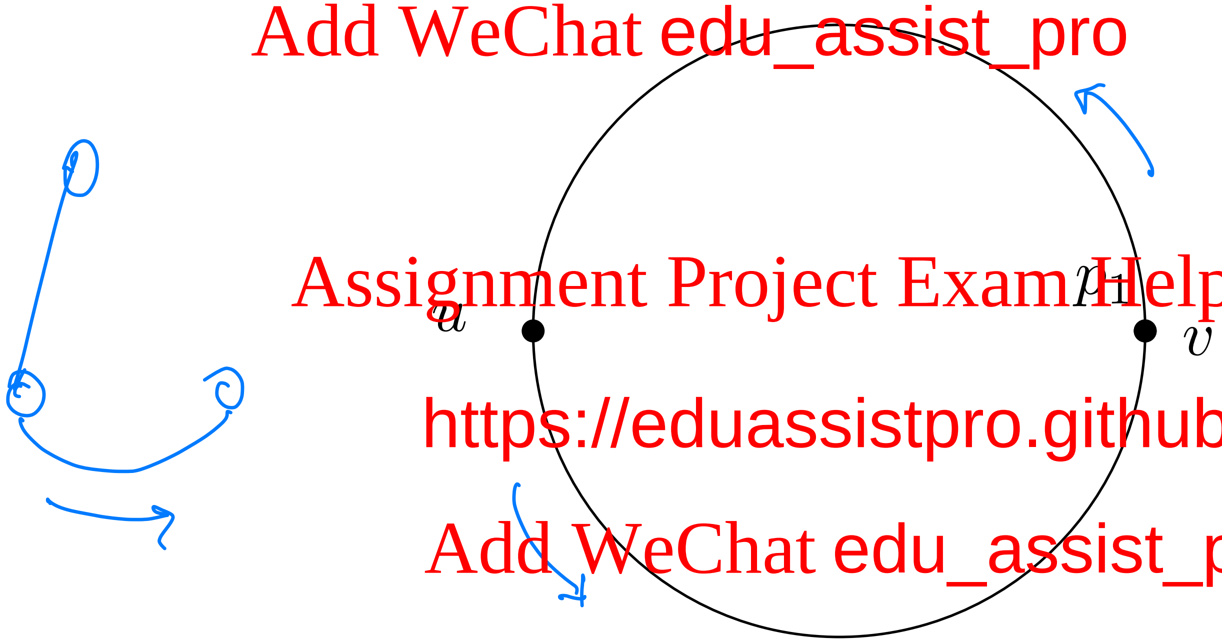
Base Case: Starting the recursion in a 2-node Ring

- **Lemma 1** *Given a ring [illegible] es, we can construct an open schedule in which at least o[illegible] eived.*

  - *The nodes cannot distinguish this schedule from one on a larger ring with all other nodes being where the open edge is.*

2-node Ring

- Two processors p_0, p_1 ha u and v s.t. $u > v$.



- Processor p_1 must learn the identity of node v, thus receive at least one message.

- We stop the execution of the algorithm as soon as the first message is received.

Induction Step

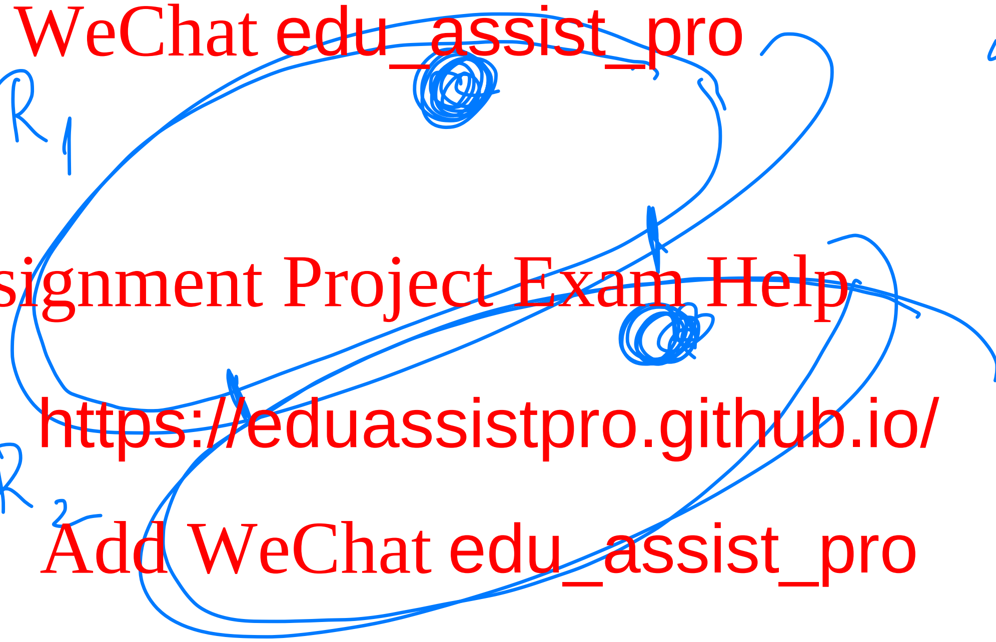
- Assume two rings of size hedules

Induction Step

- Assume two rings of size hedules

leader in R_1

Def. of LE problem is critical



leader in R_2

We can construct an open schedule on a ring of size n

- If $M(n/2)$ is number of messages can construct schedule with $2M(n/2)$ without scheduling either of the two open edges
- Remember: We decide when edges are scheduled and when nodes wake up

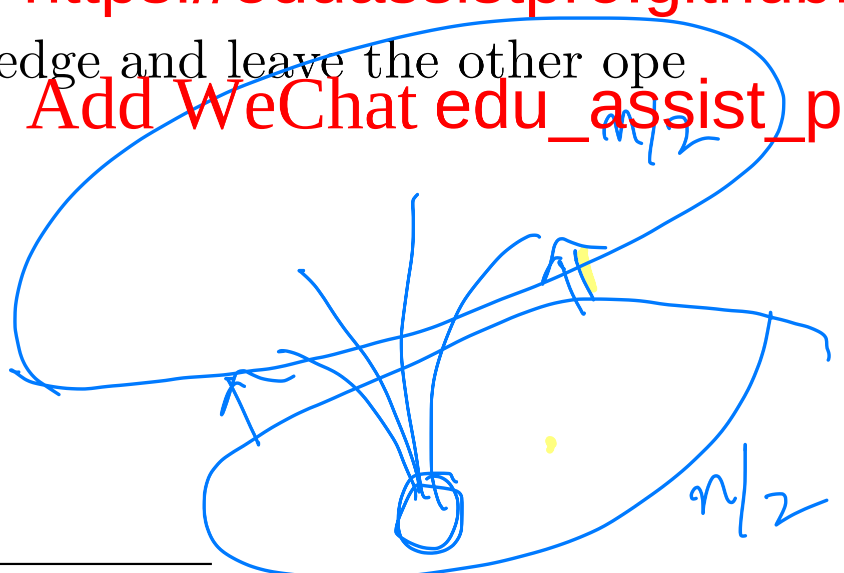
Induction Step

- Each node in, say R_1, mu e node in R_2

- At least $n/2$ messages must R_1 to R_2[a]
- But some messages use e_1 and others use e_2 is not good enough as an argument!

- Closing one of the edges will cause at least $n/4$ messages to be passed (not necess

- Schedule this edge and leave the other ope



$\frac{n}{2}$

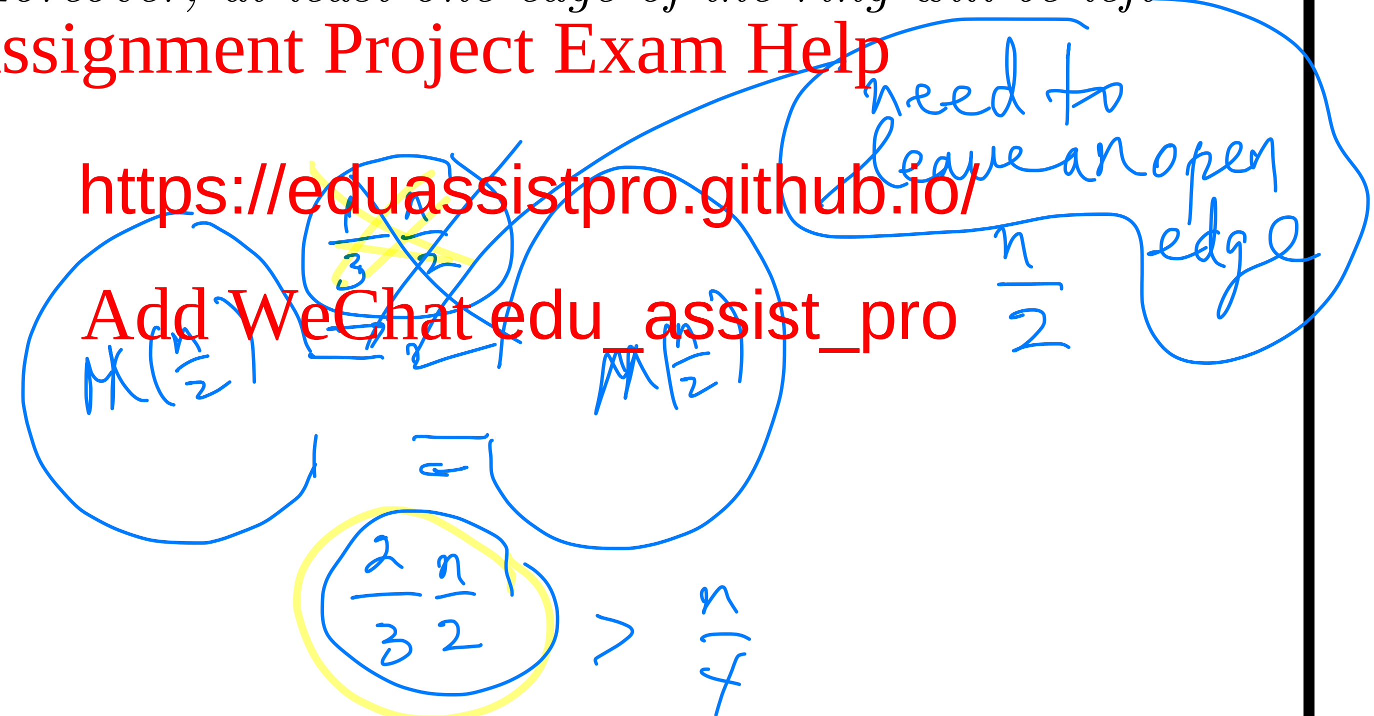
At least n/4 messages must pass through e_1 or e_2

[a] This is crucial to the leader election process!

Gluing Together

- We can show that

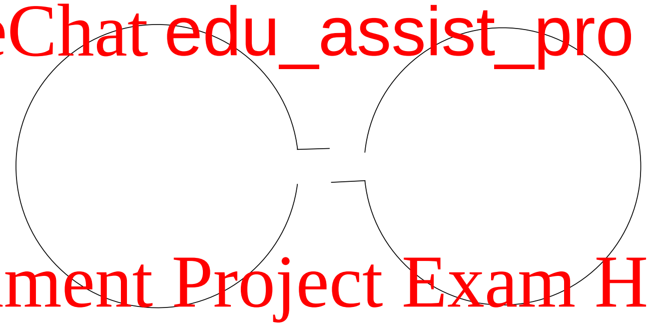
  Lemma 2 *Gluing togethe[illegible] $_2$, at least $2M(n/2) + n/4$ messages must be exchanged to solve leader election on the ring of size n. Moreover, at least one edge of the ring will be left open.*[a]



[a]This is crucial to the validity of the induction step.

Recursive Step: Stitching

- Take two size $n/2$ subring $_2$ with open schedules.



  – Take the open e ese edges to glue th n.

- Electing a leader in the resulting "glued" r messages that

  1. stay within each subring, plus
  2. move from one ring to the other.

Stitching

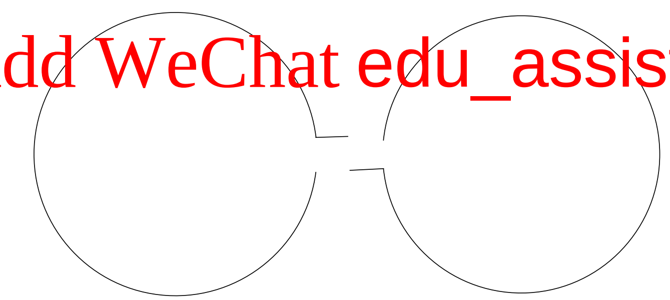
- Glue two rings of size $n/$

n-node Ring: Idea of Inductive Hypothesis

- **Lemma 3** *By glueing toget … e $n/2$ for which we have open schedules, we can c … en schedule on a ring of size n.*

  - *If $M(n/2)$ denotes the number of messages already received in each of these schedules, at least $n/4$ additional messages have to be excha … .*

- Divide the ring into … $_2$ of size $n/2$.



- These subrings cannot be distinguished from rings with $n/2$ nodes if no messages are received from "outsiders".
- Can ensure this by not scheduling such messages until we want.

n-node Ring: Idea of Inductive Hypothesis

- Executing both given open sch[illegible] R_1 and R_2 "in parallel" is possible because we [illegible] he scheduling of the messages, but also when nodes wake up.

- This ensures that $2M(n/2)$ messages are sent before the nodes in R_1 and R_2 learn anything of each other!

Without loss of generality, R_1 contains the maximum identifier.

- Each node in R_2 must learn the identity of the max identifier, thus at least $n/2$ additional messages must be received.

Close One and Open the Other

- The only problem is that we cann o subrings with both edges since the new ri pen.

- Thus, only messages over one of the edges can be received.
- We look into the future: we check what happens when we close only one of these connecting edges.

Count Messages

- Since we know that $n/2$ n ormed in R_2, there must be at least $n/2$ mess eceived by R_2.

- Closing both edges must inform $n/2$ nodes, thus for one of the two edges there must be a node in distance $n/4$ which will be informed upon creating that edge.

- This results in $n/4$ additional messages. Thus, we pick this edge and leave the other one open which yields the claim.

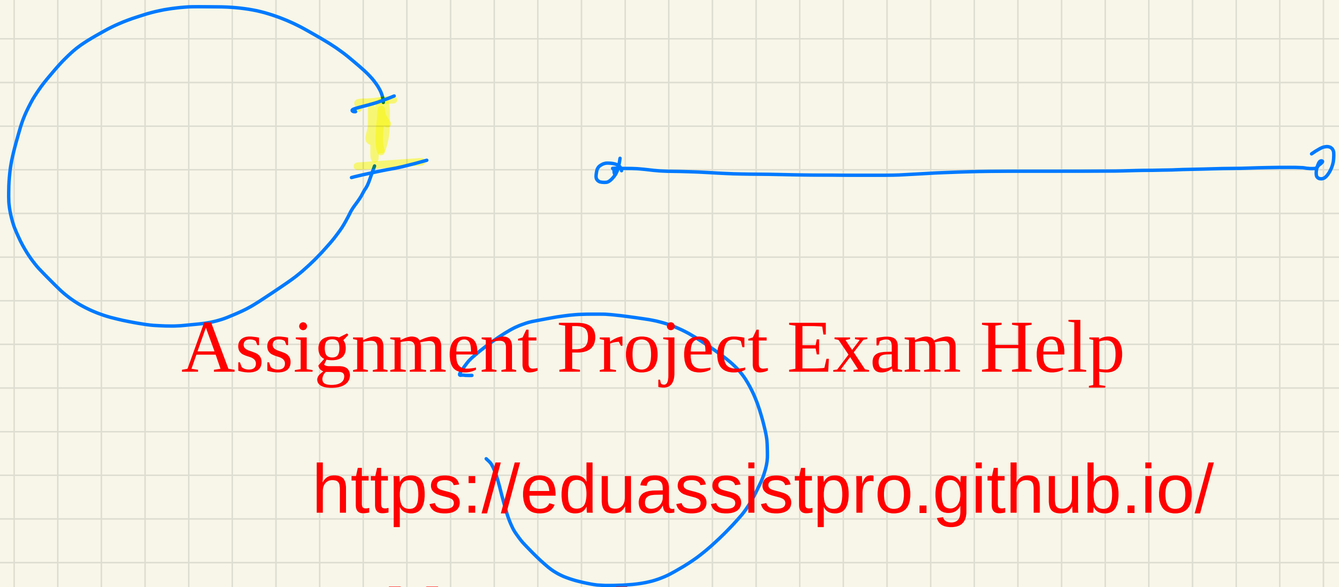
We know that $n/2$ messages must pass through either via e_1 or via e_2

why isn't the overhead $n/2$

In the "ring glued" from R_1, R_2

~~wee~~ we need to leave an edge open!

$$M(n) \geq M(n/2) + M(n/2) + \frac{n}{4}$$



$\Omega(n \log n)$ **messages**

- So we have proved.

- **Theorem 2** *Any compar[illegible]r election algorithm on a ring of size n needs at least $\Omega(n \log n)$ messages.*

Leader El d

Determ. lead. El.
Ref. LE with random labels

A Simple Way to Break Symmetry

- Assume that each node is equip tor of random bits.

- Each node i can flip a fair coin X_i, for $i = 0, 1, \ldots, n-1$.
  - Warning: we are not using i as an identity!

  - X_i is a "fair coi
    $\Pr[X_i = 0] = \Pr$

- The coins are independent of each other.

- Observe that for $i \neq j$,

$$\Pr[X_i = X_j] = \frac{1}{2}$$

$\underbrace{X_i = X_j = 0}_{1/4}$ or $\underbrace{X_i = X_j = 1}_{1/4}$

Randomized Identity Selection (1/2)

- For simplicity assume the ring i l.

1. Each node "flips a coin" and cho or 1; the selection of each node is independent of the others.
2. **For** $c \log n$ rounds each node sends and receives bits from its neighbour.[a] /* We must specify how the hidden constant c in $c \log n$ i

   $c > 4$

3. Each node uses as identity the number w representation is the sequence of its it has collected, in the order received.

Assignment Project Exam Help

https://eduassistpro.github.io/

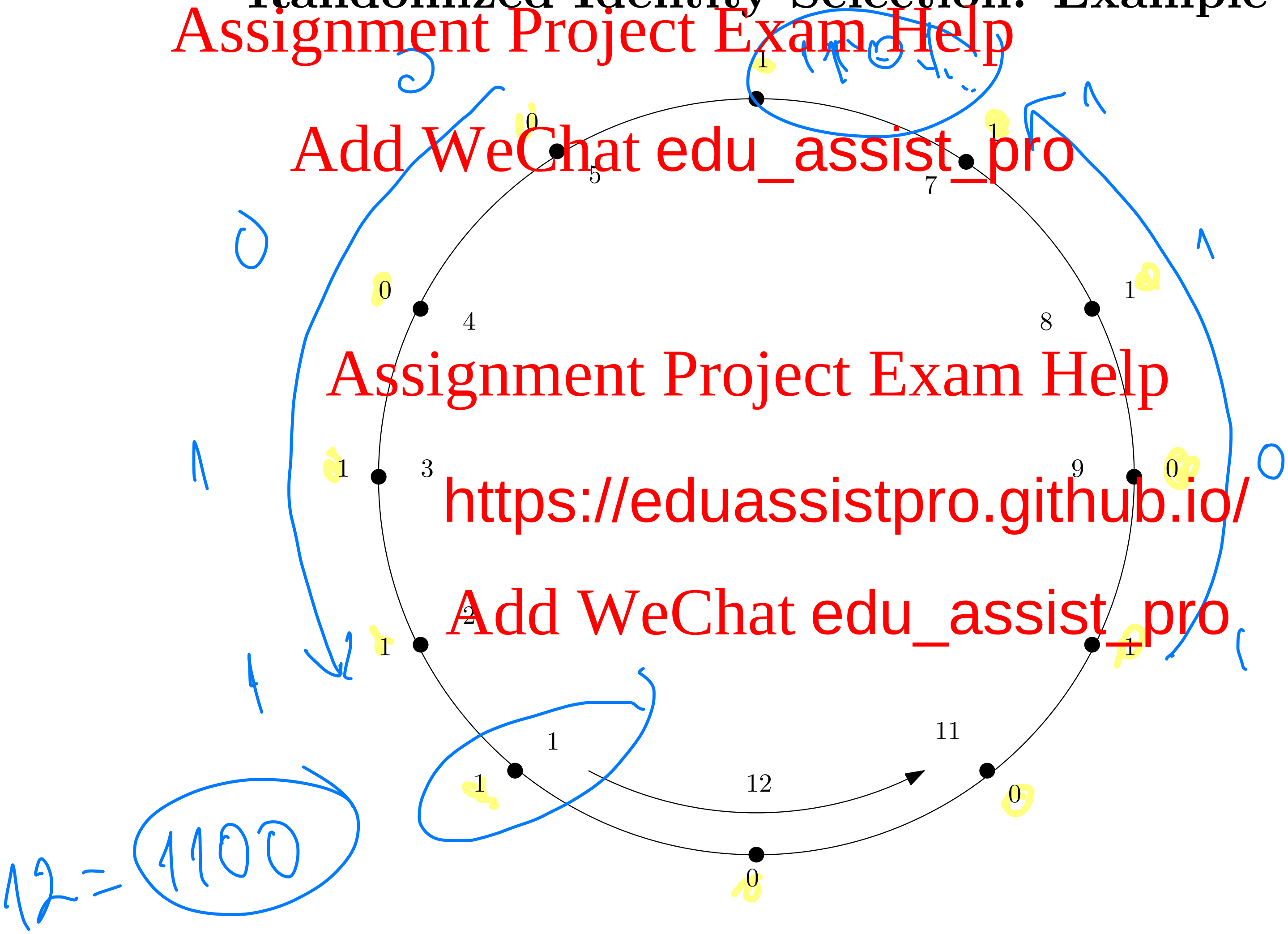
Add WeChat edu_assist_pro

- **NB:** The input collection phase is $c \log n$ rounds.

---

[a] $c > 0$ is a constant that will be determined later.

Randomized Identity Selection: Example



- Each node collects the k bits it receives and converts the result to decimal.

Randomized Identity Selection: Example for 4 Steps

Node 1	1100	12
Node 2	1001	9
Node 3	0011	3
Node 4	0111	7
	1110	14
	1101	13
Node 7	1010	10
Node 8	0100	4
Node 9	1001	9
Node 10	0011	6
Node 11	0111	7
Node 12	1110	14

Correctness of Randomized Identity Selection

- **Theorem 3** *For $c \geq 3$, w.* … *obability $\geq 1 - 1/n$)*
  Algorithm Randomized ID Se… *s that the identities selected are pairwise distinct. Moreover, the algorithm*

  1. *uses a total of n random bits,*
  2. *terminates in $c \log n$ rounds,*
  3. *the total number of* … $n \log n$.

Correctness of Randomized Identity Selection

- Consider the i-th node. Lets $k = c \log n$.

- After $c \log n$ rounds node ived the following sequence of bits

$$X_i, X_{(i+1) \bmod n}, X_{(i+2) \bmod n}, \cdots, X_{(i+k) \bmod n}$$

and form its identit

$$ID_i := X_i X_{(i+1) \bmod n} X_{(} \cdots X_{(i+k) \bmod n}$$

$X_j X_{j+1} \cdot -$

- We now ask the question.

  How likely is it that two different nodes $i \neq j$ of the ring will obtain the same identity?

Correctness of Randomized Identity Selection

- Assume $i \neq j$.

- Observe that

$$ID_i = ID_j \text{ iff } X_i X_{(i+1) \bmod n} \cdots X_{(i+k) \bmod n}$$

$$= X_j X_{(j+1) \bmod n} \cdots X_{(j+k) \bmod n}$$

$$\leq l \leq k.$$

- Therefore

$$\begin{aligned} \Pr[ID_i = ID_j] &= \Pr[\forall l \leq k (X_{i+l} = X_{j+l})] \\ &= \prod_{l=1}^{k} \Pr[X_{i+l} = X_{j+l}] \\ &= 2^{-k} = \frac{1}{n^c} \text{ (since } k = c \log n) \end{aligned}$$

1/2

$(1/2)^k$

Correctness of Randomized Identity Selection

- However since there are at most $\binom{n}{2}$ pairs $i \neq j$ we get

$$\Pr[\exists i \neq j (ID_i = ID_j)] \leq \sum_{i \neq j} \underbrace{\Pr[ID_i = ID_j]}$$

Boole's Rule

Union Rule

$$\leq \frac{\binom{n}{2}}{n^c}$$

- Hence,

$$\underbrace{\Pr[\forall i \neq j (ID_i \neq ID_j)]} \geq \underbrace{1 - \frac{1}{n^{c-2}}}.$$

$c = 4$

$1 - \frac{1}{100^2}$

Randomized Leader Election (2/2)

- A leader election algorithm in a ri n has to run for n rounds so as to ensure that every f the leader.

- **Algorithm Randomized Leader Election**

1. Each node chooses a random bit 0 or 1 independently of each other.

2. **For** n roun om its neighbour.

3. Each node computes its identity as the se n bits it receives /* in the order received */

4. A node becomes a leader if its identity is the largest among everybody else's identities.

Randomized Identity Selection: Example

- Lets run the algorithm for 4 steps

Node 1	1100	12
Node 2	1001	9
Node 3	0011	3
		7
		14
Node 6	1101	13
Node 7	1010	10
Node 8	0100	4
Node 9	1001	9
...	...	...

- Which one of the 12 nodes receives the largest identifier?

Correctness of Randomized Leader Election

- **Theorem 4** *With probab* $\geq 1 - n^2/2^n$ *Algorithm Randomized Leader Electio* *ique leader is elected. The algorithm uses a total of n random bits, terminates in n rounds and the total number of bits transmitted is n^2.*[a]

- The algorithm terminates in n rounds, because Step 2 of the algorithm runs fo identity selection).

- In Step 4, how does each node compute the i other node without additional commu

[a] Notice that the IDs constructed by this algorithm can be as large as 2^n.

Correctness of Randomized Leader Election

- For the sake of simplicity, assu w is mod$n$

- If

$$ID_i = X_i X_{i+1} \cdots X_{i+n-1}$$

is i's identity as computed by the algorithm then i can compute ID_{i+} k positions.

- Indeed

$$\begin{aligned}
ID_i &= X_i X_{i+1} X_{i+} \quad {}_{n-2} X_{i+n-1} \\
ID_{i+1} &= X_{i+1} X_{i+2} X_{i+3} \cdots X_{i+n-2} X_{i+n-1} X_i \\
ID_{i+2} &= X_{i+2} X_{i+3} \cdots X_{i+n-2} X_{i+n-1} X_i X_{i+1} \\
\vdots &= \qquad\qquad \vdots
\end{aligned}$$

- No additional communication round is needed.

Correctness of Randomized Identity Selection

- Finally, the claim on the probab

- Just repeating the previous arg $k = n$ we see that

$$
\begin{aligned}
\Pr[ID_i = ID_j] &= \Pr[\forall l \leq k (X_{i+l} = X_{j+l})] \\
&= \prod^{k} \Pr[X_{i+l} = X_{j+l}]
\end{aligned}
$$

- Therefore since there are at most $\neq j$ we get

$$
\begin{aligned}
\Pr[\exists i \neq j (ID_i = ID_j)] &\leq \sum_{i \neq j} \Pr[ID_i = ID_j] \\
&\leq \binom{n}{2} / 2^n \\
&\leq n^2 / 2^n .
\end{aligned}
$$

Sources

- H. Attiya, J. Welch, Distribut , ohn Wiley and Sons, 2E, 2004.

- R. Wattenhofer, Lecture Notes on Principles of Distributed Computing, ETH Spring 2012.

- N. Lynch, Distrib mann, 1996.

Exercises[a]

1. Can paiwise distinct identifiers b … es have independent random genera…

2. Why are we allowed to interpret an event E that is valid "with probability at least $\geq 1 - n^2/2^n$" as "it is valid with high probability"?

3. Justify the validity of t … $^{-1} \approx 1 - x$, for $|x|$ sufficiently small … he function e^u, where u is a real numbe…

4. Consider the following variant of randomized ID selection: Each node selects k random bits $b_1, b_2, \ldots, b_k$ and makes the sequence $b_1 b_2 \cdots b_k$ its identifier. Show that by choosing k appropriately with high probability the identifiers chosen by the nodes are pairwise distinct. **NB.** This algorithm differs

[a] Do not submit!

from the one discussed in class in that it does not require any message exchanges.

5. How do “TimeSlice” and “Varia [illegible] r from the traditional “C;lock-wise” and “RadiusGrowth” algorithms?