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Outline

- Independent Set (IS)

- Distributed IS
  1. Distributed Slow MIS
  2. Distributed Fast MIS

Ind ts

Inependent Sets

- **IS:** Given an undirected Gra[illegible] $V, E)$ an independent set is a subset of nodes $U \subseteq$ [illegible]o nodes in U are adjacent.

- **MIS:** An independent set is maximal if no node can be added without violating independence.

- **MaxIS:** An ind[illegible]lity is called maximum.

Independent Set (IS)

- An IS is a set of nodes of the graph suc em are not adjacent.

- We also have maximal and maximum independent sets.

- Every MIS (Maximal Independent Set) is a dominating set.
- In general, the size of every MIS can be larger than the size of an optimal minimum dominating set by a factor of $\Omega(n)$.[a]

[a] We won't prove this here.

Coloring and Independent Sets

- **Example 1** Graph has two endent sets (MIS), but only one is a maximum indep IS).

- **Example 2**

Coloring and Independent Sets

- There is a relation between inde d node coloring:
  - each color class is an indepen

  - however, it is not necessarily a MIS.

From Coloring to Independent Sets

- Starting with a coloring, one ca rithm:

1. We first choose all nodes of the fir
2. Then, for each additional color we add "in parallel" (without conflict) as many nodes as possible.

From Coloring to Independent Sets: Analysis

- **Theorem 1** *Given a colori* [illegible] *t needs C colors and runs in time T, we can c* [illegible] *ime $C + T$.*

- Time complexity:
  - the T in the time complexity comes from the coloring algorithm, and
  - the C in the ti [illegible] ber of colors.

Related Topic: Set Cover (SC)

- Given a set of elements { called the universe) and a collection S of m sets whe e universe, the set cover problem is to identify the smallest sub-collection of S whose union equals the universe.

  1. For example, consider the universe $U = \{1, 2, 3, 4, 5\}$ and the collection 4},{3,4},{4,5}}. Clearly the uni an cover all of the elements with the following, smalle $\{\{1, 2, 3\}, \{4, 5\}\}$.

  2. A company needs to buy a certain amount of varied supplies and there are suppliers that offer various deals for different combinations of materials (Supplier A: 2 tons of steel + 500 tiles for $\$x$; Supplier B: 1 ton of steel + 2000 tiles for $\$y$; etc.). You could use set covering to find the best way to get all the materials while minimizing cost.

Issues

- Computing a maximum inde xIS) is a notoriously difficult proble

  - Equivalent to maximum clique on the complementary graph.
  - Both problems are NP-hard, in fact not approximable within $n^{1/2-\epsilon}$.

- MIS and MaxIS ca

  - On a star graph MIS is $\Theta(n)$ s axIS.

Examples: MIS

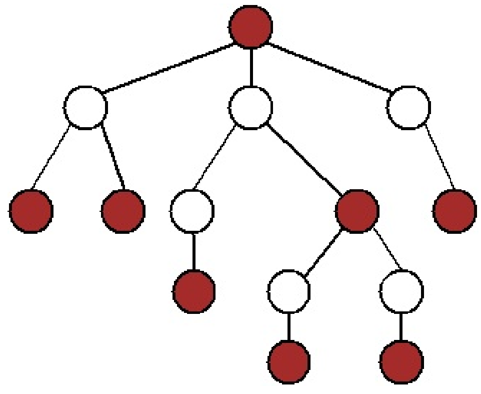
- Example 1

- Example 2

Examples: MIS

- Example 3

- Example 4



Computing MIS

- Computing a MIS sequentiall

1. Scan the nodes in arbitrary ord
2. If a node u does not violate independence,
   - add u to the MIS.
3. If u violates independence,
   - discard u

Algorithm: Lexicographic $MIS(G)$

- Previous algorithm sometim ws. Consider a graph $G = (V, E)$ in which th icographically ordered.

1: $I = \emptyset, V' = V$

2: **while** $V' \neq \emptyset$ **do**

3: Choose

4: $I \leftarrow I \cup$

5: $V' \leftarrow V' \setminus (\{v\} \cup N(v))$

6: Return I;

- With this simple greedy algorithm, we can find a MIS in $O(|V| + |E|)$ time.
- The main question is how to compute a MIS in a distributed manner.

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Distributed Slow MIS

- Main idea is to give priority to nod .

- $\boxed{\textbf{Slow MIS}}$

- Requires Node IDs

- Every node v executes the following code:

1. **if** all neighbo ave decided not to join the MIS t
2. v decides to join the MIS
3. **end if**

Complexity

- **Theorem 2** *Algorithm Sl* … *mplexity of* $O(n)$ *and a message complexity of*

- Slow MIS is not better than the sequential algorithm in the worst case, because there might be one single point of activity at any time.

Issues

- Using Theorems 1 and 2 we get a di ministic MIS algorithm for cycles (also for tre degree graphs) with time complexity $O(\log^* n)$ (will cover this later in class).

  - First do the colouring in $O(\log^* n)$ rounds.
  - Choose all nodes of the first color.
  - For each addit ut conflict) as ma

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- With a lower bound argument one can sho deterministic MIS algorithm for rings is asymptotically optimal.
  - Because in the ring MIS is "essentially" the same as coloring.
- There have been attempts to extend the 6-Color Algorithm to more general graphs, however, so far without much success.

Is There a Faster Algorithm?

- Given that "Slow MIS" is not bet ential algorithm in the worst case

  - Is there a faster MIS?

- In the sequel we give a probabilistic algorithm with $O(\log n)$ expected termination time.

Goal: Find a parallel MIS algorithm

- Consider algorithms of the for

1. $I = \emptyset, V' = V$
2. While G' is not the empty graph
   (a) Choose a random set of vertices $S \subseteq V$ by selecting each vertex v independently with probability $\frac{1}{d_v}$, where d_v is the degree o

   (b) For every edge points are in S, then remove the vertex of lower de S (break ties). Denote the set after this step as S'.

   (c) Remove S' and $Neighbor(S')$ and all adjacent edges from G'.
   (d) $I \leftarrow I \cup S'$

Distributed Fast MIS[a]

- Algorithm operates in synchr rouped in phases.

- A single phase is as follows:

1. Each node v marks itself with probability $\frac{1}{2d(v)}$, where $d(v)$ is the current degree of v.

2(a) If no higher degree neighbor of v is also marked, node v joins the MI e */

(b) If a higher degre v unmarks itself again.

/* If neighbors have same degree, ties broken by ID */

3. Delete all nodes that joined the MIS and their neighbors /* as they cannot join the MIS anymore. */

[a] A more general form of this algorithm assigns real numbers (in the range $[0, 1]$) as weights at the nodes. An alternative version is to label the vertices with a random permutation.

Issues

- Correctness in the sense that th uces an independent set is relatively si

  – Steps 1 and 2 make sure that if a node v joins the MIS, then v's neighbors do not join the MIS at the same time.

  – Step 3 makes sure that v's neighbors will never join the MIS.

- The algorithm ev e node with the highest degree will mark itself at s 1.

- The only remaining question is how fast t terminates.

  – This is not easy to figure out!

Exercises[a]

1. Show that any maximal matchin [illegible] ation of a maximum matching.

2. Let $G = (V, E)$ be the graph for which we want to construct the matching. Define the auxiliary graph G' as follows:

   - for every edge in G there is a node in G';
   - two nodes in [illegible] r respective edges in G are adjacent.

   Show that a (maximal) independent set i [illegible] ′ is a (maximal) matching in G, and vice versa.

[a] Do not submit!