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Conne s

## Outline



- Trees
- Canonical Form
- Distributed Views
- Broadcast
- BFS/DFS
- Flooding
- Convergecast
- Applications

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## Trees and Communication



- Trees are everywhere: sapling cal compounds.
  - There is something about t d economy.





- Trees form a natural communication structure in distributed computing.

## Main Concepts on Trees



- A tree is a connected graph that ha



- Start with the tree of one vertex: w ree we wish by successively adding a new edge and a new vertex.
  - At each stage, the # of vertices exceeds the number of edges by 1, so every tree with $n$ vertices has exactly $n-1$ edges

## Characterization of Trees



- Let $T$ be a graph with $n$ he following statements are equivalent.



1. $T$ is connected and has no cydes.
2. $T$ has $n-1$ edges and has no cycles.
3. $T$ is connected and has $n-1$ edges.
4. $T$ is connecte nects $T$.
5. Any two vertices ctly one path.
6. $T$ contains no cycles, but the additi creates a cycle.

## Spanning Trees



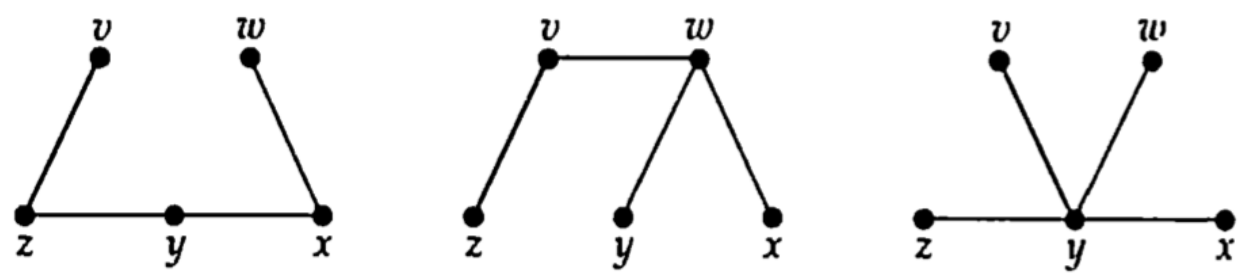
- Let $G$ be a connected graph. T ee in $G$ is a subgraph of $G$ that inclu d is also a tree.



- A graph . . .



- . . . and possible spanning trees




- Spanning trees emerge naturally in communication.

## Forests



- A Forest is a collection of vertex di



- Forests arise naturally in clustering.
- A Spanning Forest is a collection of vertex disjoint spanning trees.

# Ca m

## How Do Nodes Build their Knowledge?



- They learn by exchanging mes





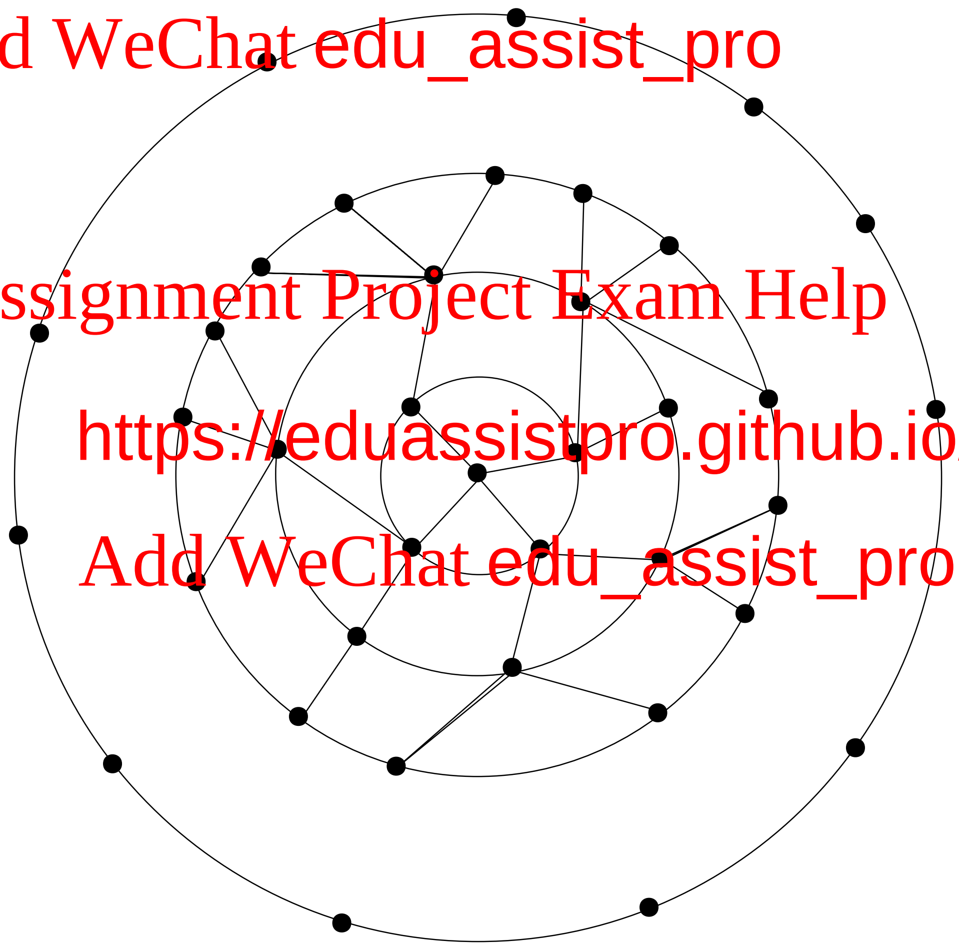
- At the same time, different nodes learn different things!

## Information Growth and Knowledge Discovery



- Start from node $u$







- When does the growth stop?

## In a Line Graph



- In a typical synchronous distri m each node executes the following atomic



$$\text{send} \rightarrow \text{receive} \rightarrow \text{process}$$

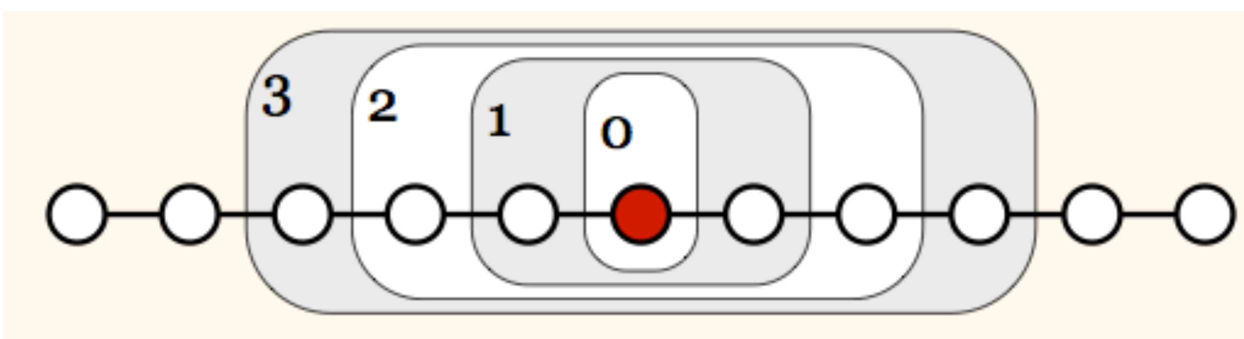
in synchronous rounds.



- Node $v$, by excha



- ...receives information about distance $1, 2, 3, \ldots$ nodes.



## Canonical Form



- Assume that initially, all node [illegible] wn identifier and potentially some additio [illegible]



- Information needs at least $r$ rounds to travel $r$ hops.
- After $r$ rounds, a node $v$ can only learn about other nodes at distance at most



- If message size and l [illegible] ed, it is in fact not hard to see, that



  - in $r$ rounds, a node $v$ can learn exactly all the node labels and inputs up to distance $r$ from $v$.
- This allows us to transform every deterministic $r$-round synchronous algorithm into a simple canonical form.

## Cumulative Messages



- The idea is to “simplify commu mulative messages



- A typical synchronous distributed algorithm at each node consists of a sequence of executions



ss



in synchronous ro



- Often what matters is the source and the de
- Can we first do a sequence of $r$ executions “send $\rightarrow$ receive$''$ followed by a single “process$''$ at the end?
- In other words, can we send “cumulative” messages for $r$ rounds and finally do the processing?

## Example: Computing the Sum in a Ring



- Consider a ring of $n$ nod rs $ID_i$ and weights $w_i$ at each node, for $i = 1, 2$



- In a typical distributed computation for a node $i$:

  **for** $r$ rounds **do**



  1. $i$ sends pair $(ID_i, w_i)$ to $i+1$, and receives pair $(ID_{i-1}, w_{i-}$



  2. process by adding $w_i + w_{i-1}$



- This can be done in a cumulative manner a $i$ as follows:

  1. $i$ sends $(ID_i, w_i), (ID_{i-1}, w_{i-1}), \ldots, (ID_{i-1}, w_{i-r})$ to $i+1$, receives $(ID_{i-1}, w_{i-1}), (ID_{i-2}, w_{i-2}), \ldots, (ID_{i-r-1}, w_{i-r-1})$ from $i-1$

  2. process by adding $w_{i-1} + w_{i-2} + \cdots + w_{i-r-1}$

## Synchronous Algorithm: Canonical Form



- **Synchronous Algorith** **orm**

  1. In $r$ rounds: send comple  odes at distance at most $r$ /* all the communication first */
  2. Compute output based on complete information about $r$-neighborhood /* do all the computation in the end */



- Example: inform





## Main Claim on Canonical Form



- **Theorem 1** *If message siz … tations are not bounded, every deterministi … r-round algorithm can be transformed into an algorithm having the canonical form (i.e., it is possible to first communicate for r rounds and then do all the computations in the end).*



- Notice the import … ges of arbitrary size:
  - this size will depend on the number of rounds, and
  - it can be exponetial in $r$



- To handle "large size messages" you need "large memory"

## Main Argument (1/2)



- Consider an $r$-round algor ant to show that $\mathcal{A}$ can be brought to a canonical form.



- First, let the nodes communicate for $r$ rounds.
- Assume that in every round, every node sends its complete state to all of its neighbors.



- By induction, afte he initial state of all other nodes at distance at most





- Hence, after $r$ rounds, a node $v$ ed initial knowledge of all the nodes in its $r$-neighborhood.
- We want to show that this suffices to simulate locally (at node $v$) enough of Algorithm $\mathcal{A}$ to compute all the messages that $v$ receives in the $r$ communication rounds of a regular execution of Algorithm $\mathcal{A}$.

## Main Argument (2/2)



- We prove the following statem n on $i$.

  **Claim.** For all nodes at dista  $r - i + 1$ from $v$, node $v$ can compute all messages of the first $i$ rounds of a regular execution of $\mathcal{A}$.



- $i = 1$: $v$ knows the initial state of all nodes in the $r$-neighborhood ound.



- Induction Step: from $i$ to $i + 1$.



  – By the induction hypothesis, e the messages of the first $i$ rounds of all nodes in its $(r - i + 1)$-neighborhood.

  – It can therefore compute all messages that are received by nodes in the $(r - i)$-neighborhood in the first $i$ rounds.

  – This is exactly what is needed to compute the messages of round $i + 1$ of nodes in the $(r - i)$-neighborhood.

## Issues



- It is straightforward to general l form to randomized algorithms:



  - Every node first computes all the random bits it needs throughout the algorithm.



- The random bits are then part of the initial state of a node.

# Dis  ws

## Views: Undirected Networks



- Each node has port labels and can mulating its knowledge.



- The view of depth $k$ of a node is a tree containing information on all the walks of length $k$ leaving that node.
- Views contain all the information that nodes could obtain by exchanging mess

## $r$-View (or $r$-Hop View or $r$-Neighborhood)



- Collection of initial states of all n $r$-neighborhood of a node $v$, is called $r$-hop v rhood) of $v$.



  – For a given graph $G$, it is denoted by

$$V_r^G(v) \text{ or } N_r^G(v).$$



  – We usually om ar from the context) and d





$$V_r(v) \text{ or}$$

- A view can be enriched as needed by including information:
  – on node states,
  – node topology $r$ hops away from the source $v$,
  – etc

**Example: $r$-Hop Views for $r = 0, 1, 2, 3$**

## Issues



- Assume that initially, every no ee, its label (identifier) and potentially so put.



- The $r$-hop view of a node $v$ then includes
  - the complete topology of the $r$-neighborhood,
  - possibly edge $r$ in the subgraph, and
  - the labels and additional inputs of all n $r$-neighborhood.

## The View as a Function



- **Theorem 2** *A determinis* *lgorithm $A$ is a function that maps every possible*  *et of possible outputs.*

- By Theorem 1, we know that we can transform Algorithm $A$ to the canonical form.



- After $r$ commu  $v$ knows exactly its $r$-hop view.



- This information suffices to compute th $v$.

## Issues

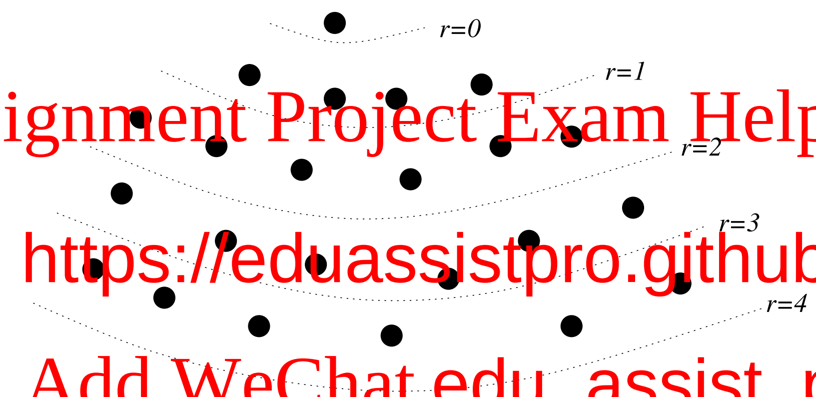


- Two nodes with equal $r$-
  - have to compute the same o  $r$-round algorithm.
- For coloring algorithms, the only input of a node $v$ is its label.
  - The $r$-hop vi
    $r$-neighborh

## Power of Views[a]



- For a graph of $n$ nodes, N ed that if two nodes have the same view of depth  $n$ they have the same views for all depths.






- Taking the diameter $\delta$ of a graph into account, can improve $n-1$ to

$$O(\delta + \delta \log(n/\delta))$$

for bidirectional graphs with port numberings

[a]We won't discuss details for these claims.

## Views: Directed Networks



- A view can be computed by a node o g a distributed deterministic al









- In directed networks we have "in" and "out" views at a node.

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## Broadcasting



- Broadcasting refers to a metho a message to all recipients "at once" in a networ



  - It is initiated by a single processor, the source.
  - The source sends a message to all other nodes in the system.
- In a typical network it may not be possible to send a message "at once" since there might be multiple hops from the source to the rest of the nodes.

## Graph Concepts in Broadcasting



- The distance $d(u, v)$ betw and $v$ in an undirected graph $G$ is the number of hops gth path between $u$ and $v$.



- The radius
  - of a node $u$ is the maximum distance between $u$ and any other node in th



$$R(u) = \mathrm{ma}$$



  - of a graph is the minimum radius of any node in the graph.

$$R = \min_u R(u)$$

- The radius and diameter of a graph are called graph eccentricities.

**Examples: Graph Eccentricities**



- Distance



- Radius
- Diameter

## Examples: Graph Eccentricities



- Radius, Diameter



- There is a close relationship between the radius $R$ and the diameter $D$ of a graph
  - $R \leq D \leq 2R$.

## Examples: Graph Eccentricities



- What are the Radius and Diame

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## BFS Spanning Trees



- Traversal of a graph is performe ts vertices in some predefined order.



- **Breadth-First-Search Tree.** A breadth-first-search tree $T$ of a graph $G$ is a spanning tree of $G$ such that for every node of $G$, the tree path is a minimum hop path to the root.



- Of course a root must be specified!

## BFS Algorithm[a]



- **BFS Algorithm:** Input [illegible] $(V, E)$

  Proceed by layers;

  1. mark the root $r$;
  2. mark all neighbor vertices that are one hop away from $r$;
  3. mark new vertices that are one hop away from these neighbors (th[illegible] $r$);
  4. and so on.







- It uses a FIFO queue
- It checks whether a vertex has been discovered before enqueueing the vertex rather than delaying this check until the vertex is dequeued from the queue

[a] Invented in 1945 by Konrad Zuse

## BFS Algorithm



- How do you construct a BFS tree f h?

## BFS (Distance Computation (1/2))



- It starts by placing the source no at distance $d(s) = 0$; the distance of all other nodes starts $(i) = \infty$.



- At the $k$th step (starting at $k = 0$), all nodes $i$ at distance $d(i) = k$ are examined, and any neighbors $j$ with $d(j) = \infty$ (i.e., not yet discovered) have their distance $d(j)$ set to $k + 1$.



- The process halts w ighbors; $d(j)$ is then the length of the shortest path from o $j$, or $d(j) = \infty$ if there is no such path.

## BFS (Distance Computation (2/2))



- BFS is the simplest way to search



- It is suited only for unweighted g ge weights.
- **Example 1:**



- **Example 2:** In a social network, you el one and your friends of friends are at level two in a BFS starting at your node.

## What is BFS Tree Used for?



- Finding all nodes within one con nent
  
  - BFS by itself is not enough: so sing is needed!
- Finding the shortest path between two nodes $u$ and $v$ (with path length measured by number of edges)
  
  - $u$ and $v$ cou  respectively.



- Testing a graph for bipartiteness
  - Construct a BFS tree from a vertex $v$ and look at all other vertices at odd or even distance from $v$.
- Doing efficient broadcast
  - from any any node.

## DFS Spanning Trees



- For a rooted spanning tree $G$,



let us denote by

- $S(u)$ all the nodes in the subtree of $u$, and
- $P(u)$ denote all the vertices that exist in a path between $u$ and the root.

## DFS Spanning Trees



- **DFS Algorithm:** Input $(V, E)$



1. Start from a vertex $r$;
2. visit all possible vertices as far as you can reach;
3. when all vertices are visited, return to the current parent node.

## DFS (Depth-First Search) (1/2)



- DFS visits the same nodes as BFS … rder.



- If it sees an unvisited node … mining node $i$, it fully discovers all unvisited nodes reachable from $j$ and then backtracks to node $i$ to consider the remainder of the nodes adjacent to $i$.



- It is best described recursively.

## DFS (Depth-First Search) (2/2)



- All nodes start out unvisited.



- DFS(i):

1. mark $i$ as visited
2. for all nodes $j$ adjacent to $i$ do:
3. if node $j$ is not visited $DFS(j)$



- Example

## What is DFS Tree Used for?



- Finds all of the vertices reachab rtex $r$ in a graph 
  - unlike BFS it does not need to search the whole graph.
- Topological sorting.

  - this is because of the way it traverses a directed graph.
- Finding the brid 
  - these are edges whose removal disco  h.
- Finding connected components.
  - like BFS.
- Finding strongly connected components.
  - these are maximal “strongly connected components” of a directed graph.

## Lower Bounds



- The message complexity of bro $n$ node graph is at least $n-1$.

  - This is because every node must receive the message.
  - Which graphs require $n-1$ message complexity?

- The source's radi ity.
  - This is becau .

- You can use a pre-computed spanning tr t with tight message complexity.

- If the spanning tree is a BFS spanning tree (for a given source), then the time complexity is tight as well.

## More on BFS and DFS

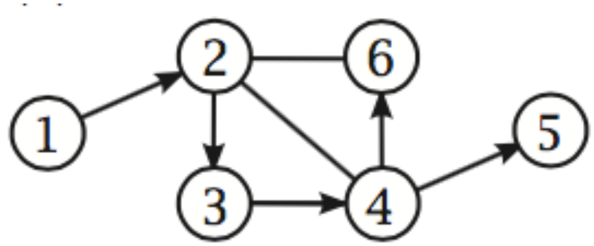


- Both BFS and DFS describe a tre is the parent of $j$ if the unvisited node $j$ is discov ing node $i$.



- The DFS tree has a rich set of mathematical properties.
  - For example, if "($i$" is printed at the start of $DFS(i)$ and "$i$)" when it finishes (after traversing all its neighbors $j$), then the result i d and matching par
  - The parentheses of two nodes ither nested one within the other, or they are disjoint.






(1 (2 (3 (4 (5 5) (6 6) 4) 3) 2) 1)

## Impact of Knowledge: Clean Graphs



- If the graph is stored in adjacency l S and DFS take an amount of time that i he graph: $O(|V| + |E|)$, where $|V|$ and $|E|$ are the number of nodes and edges, respectively.



- Knowledge can affect the message complexity!



- Call a graph (netw now the topology of the graph.



- If the nodes do not know the topology of the g clean network) then the number of edges is a lower bound for the broadcast message complexity.
  - If you do not try every edge, you might miss a whole part of the graph behind it.

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## Outline



- Flooding



- FloodMaxID
- OptFloodMax

## Flooding



- Used by nodes to identify thems



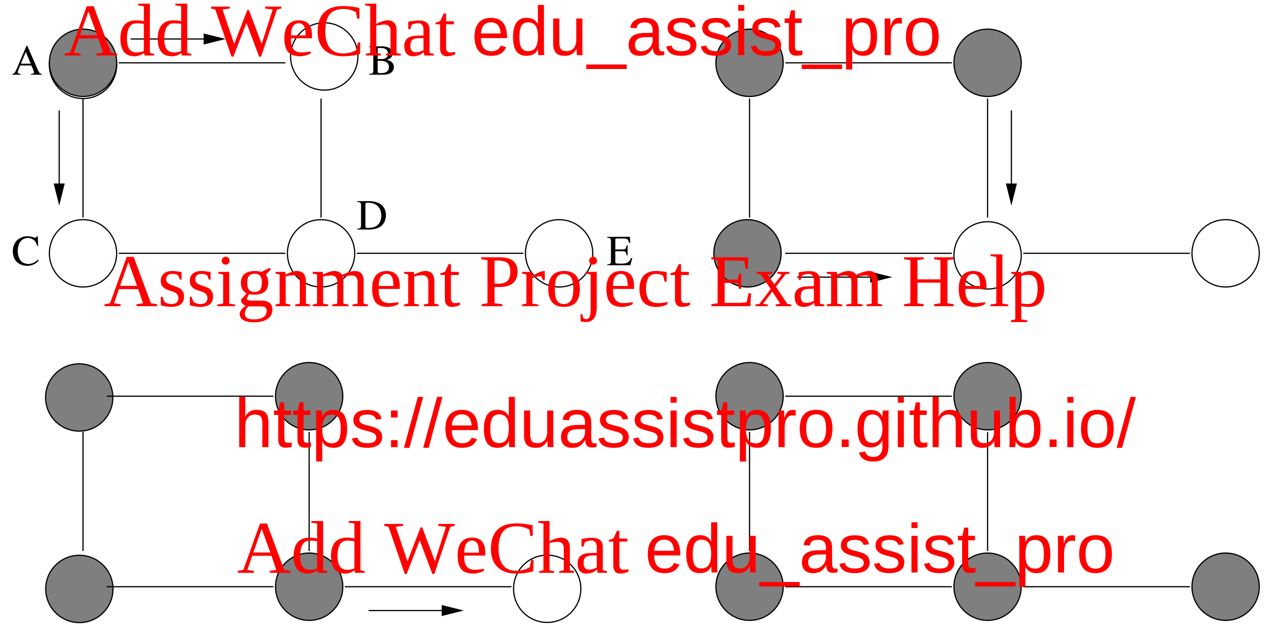
- $\boxed{\textbf{Flooding Algorithm}}$

  1. The source (root) sends the message to all neighbors.
  2. Each other node $v$ upon receiving the message the first time forwards the m
  3. Upon later receiv ges), a node can discard the message.

## Flooding Example



- Let $A$ be the initiating node:






- Note that node $D$ receives two messages.

## Flooding and Trees



- If node $v$ receives the messa u, then node $v$ calls node $u$ parent.



  - Parent relation defines a spanning tree $T$ (nodes receiving more than one message keep message only from one initiator).



  - If flooding algo ystem, then $T$ is a BF oot).



- Let $R(s)$ be the radius of the source twork.



  - In asynchronous systems the flooding algorithm terminates after $R(s)$ time units.
  - However, the spanning tree constructed may not be a BFS spanning tree.

## FloodMaxID



- We give a simple algorithm that ers and non-leaders to identify thems



- The algorithm
  - requires that the processes know the diameter of the network;
  - floods the max .
    - so we call it the FloodMaxID algori
- The algorithm makes leader election po l network.

## Flood MaxID



- **FloodMaxID Algorith**



  1. Every process maintains a rec um ID it has seen so far (initially its own).
  2. At each round, each process propagates this maximum on all of its outgoing edges.



  3. After $D$ (di lue seen is the process's o er; otherwise, it is a non-leader.





- FloodMax elects the process with the maximum ID.

## Analysis of FloodMax



- Define $i_{\max}$ to be the index of the process with the maximum ID, and $u_{\max}$ to be that user ID.



- **Theorem 3** *In the FloodMax algorithm, process $i_{\max}$ outputs leader and each other process outputs non-leader, within diameter rounds.*



- **Main Claim**



  - $status_{i_{\max}} = leader$ and
  - $status_j = non - leader$, for every $j \neq i_{\max}$.



- The key to the proof of this Claim is the fact that
  - after $r$ rounds, the maximum ID has reached every process that is within distance $r$ of $i_{\max}$, as measured along directed paths in $G$.

## FloodMax



- The FloodMax algorithm doe tly to the asynchronous setting, becau nds in the asynchronous model.



- However, it is possible to simulate the rounds asynchronously.
  - We simply require each process that sends a round $r$ message to tag t $r$.
  - The recipient waits to receive round ssages from all its neighbors before performing its rou ransition.



- By simulating diameter rounds, the algorithm can terminate correctly.

## OptFloodMax Algorithm



- There is a simple improvemen ecrease the communication complex  s, although it does not decrease the order of magnitude in the worst case.
- Namely, processes can send their current max user ID values only when they first learn about them, not at every round.

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## Convergecast



- Convergecast is reversed bro
  - Instead of a root sending a me odes, all other nodes send information to a root.



- Convergecast is useful for input collection.

## Echo Algorithm



**Echo Algorithm**



- **Requirement:** This algo at the leaves.

1. A leaf sends a message to its parent.
2. If an inner node has received a message from each child, it sends a message to the parent.

**Complexity Issues: Broadcast and Convergecast (1/2)**



- The echo algorithm is paired wi [illegible] orithm, which is used to let the leaves kno [illegible] d start the echo process; this is known as flooding/echo.



- One can use convergecast for termination detection.
  - If a root wants to know whether all nodes in the system have finished [illegible] o;
    - the message in the echo algorithm t [illegible] is subtree has finished the task."



- Message complexity of the echo algorithm is $n - 1$,
  - but together with flooding it is $O(m)$, where $m = |E|$ is the number of edges in the graph.

## Complexity Issues: Broadcast and Convergecast (2/2)



- The time complexity of the echo rmined by the depth of the spanning tree (i. oot within the tree) generated by the flooding algorithm.



- The flooding/echo algorithm can do much more than collecting acknowledgements from subtrees.



  – For instance, f nodes in the system, or the maximum ID (for l , or the sum of all values stored in the syste



- By combining results one can compute even fancier aggregations, e.g., with the number of nodes and the sum one can compute the average. With the average one can compute the standard deviation. And so on . . .

## Application to Leader Election



- Asynchronous broadcast an an be used to solve the leader election problem in ar



  - without any distinguished source node and
  - without the processes having any knowledge of the number of nodes or the diameter of the network.



- The processes nee

## Basic Leader Election Algorithm



- Every node can initiate
  - first a broadcast, and
  - next a convergecast

  in order to discover the maximum user ID in the network.



- The node that finds t
  elects itself as leade



- This algorithm uses $O(n|E|)$ mes

# (Dire FS



# and Applic

## Construction of BFS



- How do we perform breadth-fir n a network based on an arbitrary strongly c ted graph having a distinguished source node?



- We consider how to establish a breadth-first spanning tree for the (di)-graph.



- Motivation for c e desire to have a convenient structure to use as a ba communication.



- The BFS tree minimizes the maximum communication time from the process at the distinguished node to all other processes in the network
  - To do this run BFS from each node of the graph and compare values obtained ateach node.

## Construction of BFS



- We suppose that the network is s ted and that there is a distinguished source n $i_0$.

- The algorithm is supposed to output the structure of a breadth-first directed spanning tree of the network graph, rooted at $i_0$.

- The output shou h process other than $i_0$ should have ent that gets set to indicate the node that is its parent i

- As usual, processes only communicate over directed edges.
- Processes are assumed to have user IDs but to have no knowledge of the size or diameter of the network.

## SynchBFS



- The basic idea for this algorithm i e standard sequential breadth orithm.



- **SynchBFS Algorithm**

  1. At any point during execution, there is some set of processes that is *marked*: initially just $i_0$.
  2. Process $i_0$ s o all of its outgoing neighbors.
  3. At any round, if an unmarked process rec message, it marks itself and chooses one of the processes from which the search has arrived as its parent.
  4. At the first round after a process gets marked, it sends a search message to all of its outgoing neighbors.

## Analysis of SynchBFS



- We can prove the invariant tha
  - after $r$ rounds, every pro  $d$ from $i_0$ in the graph, $1 \leq d \leq r$, has its parent pointer defined; moreover, each such pointer points to a node at distance $d-1$ from $i_0$.

  This invariant can, as usual, be proved by induction on the number of rounds.





- The time complexity is at most diameter r



- The number of messages is just
  - a search message is transmitted exactly once on each directed edge.

## Applications of BFS



- Message Broadcast



- Global computation
- Electing a leader
- Computing the diameter

## Message Broadcast: Piggybacking



- The SynchBFS algorithm ca ted to implement message broadca



- If a process has a message $M$ that it wants to communicate to all of the processes in the network,
  - it can simply initiate an execution of SynchBFS with itself as the root, pigg n the search message it sends in roun



- Other processes continue to piggybac ll their search messages as well.
  - Since the tree eventually spans all the nodes, message $M$ is eventually delivered to all the processes.

## Global Computation



- This means



  - Collection of information f ut the network or,
  - more generally, the computation of a function based on distributed inputs.



- For example,



  - consider the pr nonnegative integer input value an e sum of all the inputs in the network.



  - Using a BFS tree, this can be done easily (and efficiently) as follows.

## Global Computation



- Starting from the leaves, “fan in nvergecast procedure, as follows:



  1. Each leaf sends its value to its parent;
  2. each parent waits until it gets the values from all its children, adds them to its own input value, and then sends the sum to its ow





- The sum calculated by the root of the BFS tr answer.

## Electing a Leader



- Using SynchBFS, an algorith d to elect a leader in  a network with IDs, eve ses have no knowledge of $n$ or diameter.

1. Namely, all the processes can initiate breadth-first searches in parallel. 
2. Each process ted and the  global compu etermine the maximum ID of any process in the n 
3. The process with the maximum ID then declares itself to be the leader, and all others announce that they are not the leader.

- If the graph is undirected, the time is $O(diameter)$ and the number of messages is $O(n|E|)$.

## Computing the Diameter



- The diameter of the network ca … aving all processes initiate breadth-fi  rallel.
  1. Each process $i$ uses the tree thereby constructed to determine $\max -dist$, defined to be the maximum distance from $i$ to any other process in the network.
  
  2. Each process … ee for a global computatio  $-dist$ values.



- If the graph is undirected, the time is $O(diam)$ and the number of messages is $O(n|E|)$, where $diam$ is the diameter of the graph.
- The diameter thus computed could be used, for example, in the leader-election algorithm FloodMax.

## Exercises[a]



1. Explain why every tree is a bipartit



2. Let $T$ be a graph with $n$ he following statements are equivalent.
   (a) $T$ is connected and has no cydes.
   (b) $T$ has $n-1$ edges and has no cycles.
   (c) $T$ is connecte
   (d) $T$ is connecte nects $T$.
   (e) Any two vertices of $T$ are con y one path.
   (f) $T$ contains no cycles, but the additi creates a cycle.



3. Give an algorithm to compute the diameter and radius of a tree.

4. Determine the size of a message which propagates for $r$ hops in

[a] Not to hand in!

"Synchronous Algorithm C  ". More specifically, consider a complete binary roo ht $n$. Label the ports at an interior node as $L, R$ (for the Left and Right siblings at a node), and $P$ for its parent. Do the same in an analogous manner for the root and the leaves. For each $r \leq n$ and each node  ode.

5. A connected graph is H  includes every vertex exactly once (suc  Hamiltonian). A connected graph is se n if there is a path (but not a cycle) that includes every vertex exactly once (such a path is called semi-Hamiltonian). Determine which of the following graphs are semi-Hamiltonian, and write down a corresponding semi-Hamiltonian path where possible:

6. A forest is a graph (not necessarily connected), each of whose components is a tre



   (a) Let G be a forest wit $k$ components. How many edges does G have?



   (b) Construct a forest with 12 vertices and 9 e

   (c) Is it true that every forest with k components has at least $2k$ vertices of degree 1?

7. A spanning forest in a graph G (not necessarily connected) is obtained by constructing a spanning tree for each component of G.

(a) Find a spanning forest for the following graph.

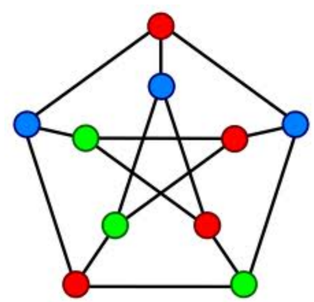


(b) Let $G$ be a graph, and let $F$ be a subgraph of $G$. If $F$ is a forest which includes all vertices of $G$, is $F$ necessarily a spanning fore



8. Find three spanning trees in the Petersen gr below):





9. Prove that trees and forests are bipartite graphs.

10. Prove that, in a bipartite graph, every cycle has an even number of edges. Conversely, prove that, if every cycle of a graph has an even number of edg is bipartite.
**Hint:** Consider a connecte . hoose a vertex $v$ in $G$ and consider those vertices whose minimum distance from $v$ is even and those whose minimum distance from $v$ is odd. To which vertices are the “odd” vertices adjacent? To which vertices are the “ev