Week 5 Workshop

Alice: Your model reduces the most interesting information to something flat and boring.

ing a database.

Riccardo: Also, the system knows so l

s hard to obtain good perfo

Alice: Are you telling me that the m

Vittorio: No, wait, we are going to fix it!

(Foundations of Databases, S. Abiteboul, R. Hull, V. Vianu, Addison-Wesley, 1995)

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Housekeeping

1. **Assignment 1 (SQL)** (due 11.59pm 3 Sep 2021)

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   - Aug 30 (Mon) 2-3 pm
   - Aug 31 (Tue) 2-3 pm
   - Sep 1 (Wed) 8-9 pm

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Update Anomalies

- What could happen to insert, delete and update operations?

ENROLMENT					
Name	StudentID	DoB	CourseNo	Semester	Unit
					6
					12
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Fran	123457	11/09/1987	COMP2400	2009 S2	6

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- **Modification anomalies**: If changing the DoB of Michael, then ...

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- **Insertion anomalies**: If inserting a new c (i.e., cannot insert NULL values into C integrity constraint).
- **Deletion anomalies**: If deleting the enrolled course COMP2400 of Fran, then ... the personal information of Fran, such as DoB, will be lost as well.
- **Modification anomalies**: If changing the DoB of Michael, then ... update every tuple that records the DoB of this student.

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Update Anomalies?

ENROLMENT					
Name	StudentID	DoB	CourseNo	Semester	Unit
Tom	123456	25/01/1988	COMP2400	2010 S2	6
Tom	123456	25/01/1988	COMP8740	2011 S2	12
					6
					12
					6

STUDENT		
Name	StudentID	DoB
Tom	123456	25/01/1988
Michael	123458	21/04/1985
Fran	123457	11/09/1987

St[illegible]	[illegible]	[illegible]ster
1[illegible]	[illegible]	[illegible] S2
123456	COMP8740	2011 S2
123458	COMP2400	2009 S2
123458	COMP8740	2011 S2
123457	COMP2400	2009 S2

COURSE	
CourseNo	Unit
COMP2400	6
COMP8740	12

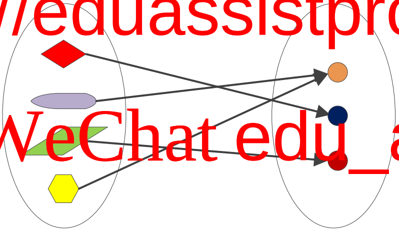
Why Functional Dependencies?

FD

- F ... ional, da
  - **Top down**: start with a relation schema and ... relation schemas in certain normal form (ca
  - **Bottom up**: start with attributes and FDs, a ... schemas (*not popular in practice*).

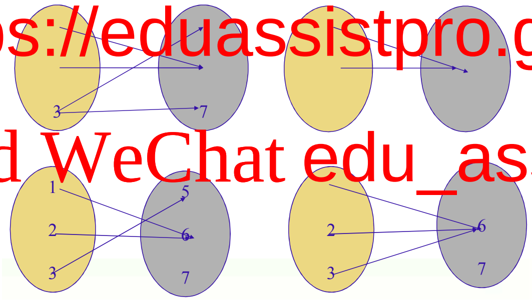
What is "Functional" about Functional Dependencies?

- The notion of functional dependency is very close to the notion of function.
- A (t : X an Y.



What is "Functional" about Functional Dependencies?

- A (total) **function** $f : X \rightarrow Y$ describes a relationship between two sets X and Y such that each element of X is mapped to a unique element of Y.
- **E**



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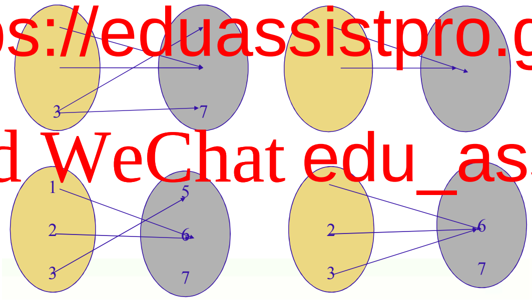
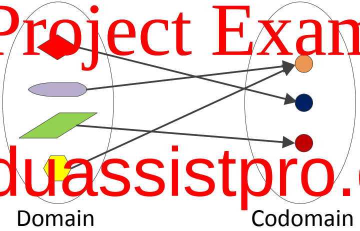
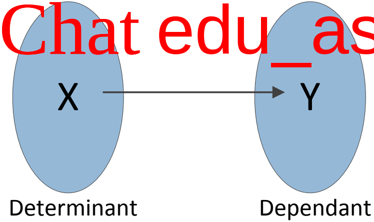


Answer: The ones at the bottom.

Functions vs Functional Dependencies



Functional dependency



Functions vs Functional Dependencies

$$f(x) = x^2$$

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3	9
4	16
5	25
6	36
...	...

Functions vs Functional Dependencies

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3	9
4	16
5	25
6	36
...	...

$$X \rightarrow f(x)$$

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Formal Definition

- Let R be a relation schema.
  - A **FD** on R is an expression $X \rightarrow Y$ with attribute sets $X, Y \subseteq R$.
  - A relation $r(R)$ **satisfies $X \rightarrow Y$ on R** if, for any two tuples

    f X,

    $t_1[Y] = t_2[$

- A FD is **trivial** if it can *always* be satisfied e
  - $\{A, B\} \rightarrow \{A\}$
  - $\{A, B, C\} \rightarrow \{A, B, C\}$
- **Syntactical convention**: (1) Instead of $\{A, B, C\}$, we may use ABC. (2) $A, B, \ldots$ for individual attributes and $X, Y, \ldots$ for sets of attributes.

Exercise - Functional Dependencies

- A functional dependency specifies a constraint on the relation schema that must hold **at all times**.
- C atisfy th

1	2	3	4	
1	2	2	2	
1	2	3	2	
2	2	2	4	

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3. $E \rightarrow ABCD$

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  (1) **Analyse data requirements**
  Can be provided in the form of discussion wit and/or data requirement specifications

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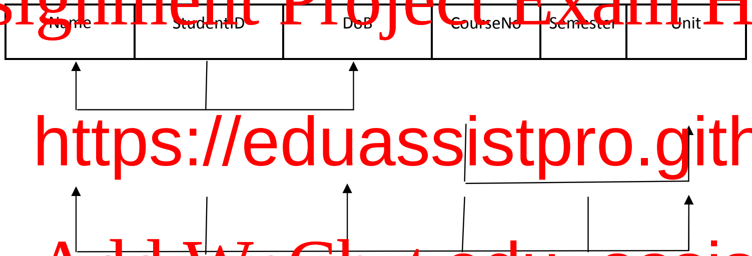
(1) **Analyse data requirements**
Can be provided in the form of discussion wit and/or data requirement specifications.

(2) **Analyse sample data**
Useful when application users are unavailable for consultation and/or the document is incomplete.

(1) Analyse Data Requirements and FD Diagram



- StudentID $\rightarrow$ Name, DoB;
- CourseNo $\rightarrow$ Unit;
- StudentID, CourseNo, Semester $\rightarrow$ Name, DoB, Unit.

(2) Analyse Sample Data

- Can you find some FDs on ENROLMENT based on the sample data?

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  - {StudentID} → {Name, DoB};
  - {StudentID, Name} → {DoB};

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  - ......

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  - {Name} → {StudentID} ×;
  - ......

Limitations:
(1) Sample data needs to be a true representation of **all possible values** in the database.
(2) Do we need all FDs?

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Inference?

Assignment Project Exam Help

- To design a good database, we need to consider **all possible FDs**.

E

If https://eduassistpro.github.i }, we

ca { } → { }

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ca $\{ \quad \} \rightarrow \{ \quad \}$

If each student works on one project and each proje

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  ca { } $\rightarrow$ { }

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- **Can we systematically infer all possible FDs?**

Armstrong's Inference Rules (Slides 16-25 will not to be assessed)

- Th

  - **Augmentation rule**: $\{X \rightarrow$

  - **Transitive rule**: $\{X \rightarrow$

- We use the notation $\Sigma \models X \rightarrow Y$ to denote that $X \rightarrow Y$ is **inferred** from the set Σ of functional dependencies.

Rule 1 – Reflexive Rule

- $XY \rightarrow Y$.

ENROLMENT					
	______		______	______	Unit
					6
					12
					6
					12
Fran	123457	11/09/1987			

- **Example**: {StudentID, CourseNo, Semester} where
  - X={StudentID};
  - Y={CourseNo, Semester}.

Rule 2 – Augmentation Rule

- $\{X \to Y\} \models XZ \to YZ$

ENROLMENT					
					Unit
					6
					12
					6
					12
Fran	123457	11/09/1987			

- **Example**:
  $\{\{CourseNo\} \to \{Unit\}\} \models \{CourseNo,$ ... er$\}$,
  where
  - X={CourseNo};
  - Y={Unit};
  - Z={Semester}.

Rule 3 – Transitive Rule

- $\{X \rightarrow Y, Y \rightarrow Z\} \models X \rightarrow Z$.

ENROLMENT					
					Unit
					6
					12
					6
					12
Fran	123457	11/09/1987			

- **Example**: $\{\text{StudentID, CourseNo}\} \rightarrow$ … $\rightarrow$ $\{\text{Unit}\} \models \{\text{StudentID, CourseNo}\} \rightarrow \{$
  - $X=\{\text{StudentID, CourseNo}\}$;
  - $Y=\{\text{CourseNo}\}$;
  - $Z=\{\text{Unit}\}$.

Other Derived Rules

- From Armstrong's axioms (i.e. reflexive, augmentation, transitive rules), we can derive the following rules:

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    - Y=Name;
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  - , then we

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    - Y=Name;
    - Z=DoB.

  - **Decomposition rule:** If $X \rightarrow YZ$, th
  - **Example:** If StudentID $\rightarrow$ Name, Do $\rightarrow$ Name and StudentID $\rightarrow$ DoB, where
    - X=StudentID;
    - Y=Name;
    - Z=DoB.

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Example on Armstrong's Inference Rules

- If e
  do

{ProjectNo}→{Supervisor}} ⊨ StudentID Supervisor}

Example on Armstrong's Inference Rules

- If e
  do

$\{$ProjectNo$\}\rightarrow\{$Supervisor$\}\}$ $\models$ StudentID Supervisor$\}$

- This can be proven by using the Transitive rule:

$$\{X \rightarrow Y, Y \rightarrow Z\} \models X \rightarrow Z$$

Example on Armstrong's Inference Rules

- Can we use the following rules to infer FDs, i.e., are they correct?
  (1) $\{X \rightarrow Y\} \models XZ \rightarrow YZ$

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  ([illegible]

X	Y	Z
a	b	c
a	c	d

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(

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(3) $\{X \rightarrow Y\} \models Y \rightarrow X$

Example on Armstrong's Inference Rules

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(

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(3) $\{X \rightarrow Y\} \models Y \rightarrow X$

No. See the counter-example below:

X	Y
0	2
1	2

Armstrong's Inference Rules

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- **Two questions:**

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[1] William Ward Armstrong: Dependency Structures of Data Base Relationships, page 580-583. IFIP Congress, 1974.

Armstrong's Inference Rules

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Armstrong's Inference Rules

- **Two questions:**
  -
  -
    ⇝ **completeness** (you can prove anyt

- **Theorem** (W. W. Armstrong, 1974[1])
  - The Armstrong's inference rules are both **sound** and **complete**.

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Implied Functional Dependencies

- We write Σ^* for all possible FDs **implied** by Σ.

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- Σ^* can be computed using the Armstrong's inference rules.

- **Why can we compute Σ^* using the Arms**

  Because the Armstrong's inference rules are both **sound** and **complete**.
- Nonetheless, computing Σ^* using the Armstrong's inference rules is **not efficient**.

Implied Functional Dependencies

- Computing Σ^* using the Armstrong's inference rules is **not efficient**.

  Example: Consider a relation schema $R = \{A, B, C, D, E\}$ and a set of FDs
  Σ o
  sh

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Implied Functional Dependencies

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  Example: Consider a relation schema $R = \{A, B, C, D, E\}$ and a set of FDs Σ … o

  sh…

- **How can we derive the proof more efficiently?**

Implied Functional Dependencies

- Let Σ be a set of FDs. Check whether or not $\Sigma \models X \rightarrow W$ holds?

[2] See Algorithm 15.1 on Page 538 in [Elmasri & Navathe, 7th edition] or Algorithm 1 on Page 555 in [Elmasri & Navathe, 6th edition]

Implied Functional Dependencies

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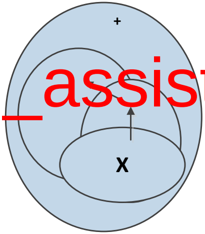

  - $X^+ := X$;
  - repeat until no more change on X^+

    - for each $Y \rightarrow Z \in \Sigma$ with $Y \subseteq$ add all the attributes in Z to X^+, i.e., replace X^+ by $X^+ \cup Z$.

[2] See Algorithm 15.1 on Page 538 in [Elmasri & Navathe, 7th edition] or Algorithm 1 on Page 555 in [Elmasri & Navathe, 6th edition]

Implied Functional Dependencies – Example

- Consider a relation scheme $R = \{A, B, C, D, E, F\}$, a set of FDs $\Sigma = \{AC \rightarrow B, B \rightarrow CD, C \rightarrow E, AF \rightarrow B\}$ on R.
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$\supseteq ACB$ u

Implied Functional Dependencies – Example

- Consider a relation scheme $R = \{A, B, C, D, E, F\}$, a set of FDs $\Sigma = \{AC \rightarrow B, B \rightarrow CD, C \rightarrow E, AF \rightarrow B\}$ on R.
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$= ACBDE$

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2. Then we check that $DE \subseteq (AC)^+$. Hence $\Sigma \models AC \to DE$.

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- **Can you quickly tell whether or not $\Sigma \models AC \rightarrow EF$ holds?**

Implied Functional Dependencies – Example

- Consider a relation schema $R = \{A, B, C, D, E, F\}$, a set of FDs $\Sigma = \{AC \rightarrow B, B \rightarrow CD, C \rightarrow E, AE \rightarrow B\}$ on R.
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$$
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- **Can you quickly tell whether or not $\Sigma \models AC \rightarrow EF$ holds?**

  $\Sigma \models AC \rightarrow EF$ does not hold because $EF \not\subseteq (AC)^+$

Exercise – Implied Functional Dependencies

- Consider a relation schema $R = \{A, B, C, D, E\}$ and a set of functional dependencies $\Sigma = \{A \rightarrow C, B \rightarrow C, CD \rightarrow E\}$ on R.

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  2. $\Sigma \models BD \rightarrow AC$ holds

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- We build the closure for the set of attributes and chec

  1. $(AD)^+ = (ACD)^+ = (ACDE)^+ = A$
     $\Sigma \models AD \rightarrow CE$.
  2. $(BD)^+ = (BCD)^+ = (BCDE)^+ = BCDE$ and $AC \not\subseteq (BD)^+$, hence
     $\Sigma \not\models BD \rightarrow AC$.

Equivalence of Functional Dependencies

- Σ_1 and Σ_2 are **equivalent** if $\Sigma_1^* = \Sigma_2^*$.

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- If $\Sigma_1 \models \Sigma_2$ and $\Sigma_2 \models \Sigma_1$, are Σ_1 and Σ_2 equivalent? Yes.
- **Questions:** Can we find the **minimal** one among equivalent sets of FDs?

Minimal Cover – The Hard Part!

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Minimal Cover – The Hard Part!

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4. Remove a FD from Σ_m if it is redundant.

Minimal Cover - Examples

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     If $\Sigma \models \mathbf{A} \rightarrow \mathbf{D}$, then $\Sigma \models \Sigma_1 \quad \Sigma_1 \models \Sigma \quad \Sigma \quad = \Sigma_1^*$.

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       If $\Sigma \not\models \mathbf{A \rightarrow D}$, then $\Sigma^* \neq \Sigma_1^*$.
     - $\Sigma \not\models \mathbf{A \rightarrow D}$ because $D \not\subseteq (A)^+$.

     No. $AB \rightarrow D$ cannot be replaced by $A \rightarrow D$.

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     - $B \rightarrow A$ is redundant because $\{D \rightarrow A, B \rightarrow D\} \models B \rightarrow A$;

Therefore, the minimal cover of Σ is $\{D \rightarrow A, B \rightarrow D\}$.

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The minimal cover of a set of functional dependencies Σ always

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- The algorithm in the previous slide can find one, but not all minimal covers of a set of functional dependencies Σ.

Finding Keys

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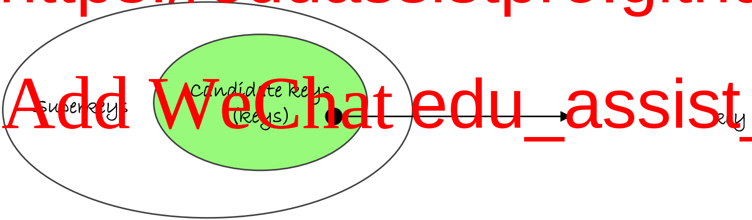

Finding Keys

- **Fact**: A key K of R always defines a FD $K \to R$.

[3] It extends Algorithm 15.2(a) in [Elmasri & Navathe, 7th edition, pp. 542], or Algorithm 2(a) or in Algorithm 2(a) in [Elmasri & Navathe, 6th edition pp. 558] to finding all keys of R

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- A **prime attribute** is an attribute occurring in a key, and a **non-prime attribute** is an attribute that is not a prime attribute.

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Exercises - Keys and Minimal Cover

- Consider RENTAL = {CustID, CustName, PropertyNo, DateStart, Owner} and the following set Σ of FDs:
  -
  -
  -
  - $\{$CustID, StartDate$\} \rightarrow \{$PropertyNo
  - $\{$Owner$\} \rightarrow \{$PropertyNo$\}$

- **Questions:**
  1. What are the keys of RENTAL?
  2. What is a minimal cover of Σ?

Exercises - Keys and Minimal Cover

- Consider RENTAL={CustID, CustName, PropertyNo, DateStart, Owner} and its FDs in the abbreviated form as
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  -

- W ENTAL

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  - $(O)^+ = OP$
  - $(CO)^+ = CPNDO$, $(DO)^+ = CPNDO$ ...
  - Thus, {CustID, Owner} and {Owner, DateStart} are the keys.

Exercises - Keys and Minimal Cover

- Consider RENTAL={CustID, CustName, PropertyNo, DateStart, Owner} and its FDs in the abbreviated form as
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Therefore, Σ is a minimal cover itself.

Accommodation Database

- Consider the following:
  - HOTEL(hotelNo, hotelName, city) with PK {hotelNo}
  - ROOM(roomNo, hotelNo, type, price) with PK {roomNo, hotelNo}
  - stNo}
  -

Accommodation Database

- Consider the following:
  - HOTEL(hotelNo, hotelName, city) with PK {hotelNo}
  - ROOM(roomNo, hotelNo, type, price) with PK roomNo, hotelNo}
  - stNo}
  -

- We have some requirements on BOOKING

R1 A booking can be made for one day only.

R2 A guest can make several bookings in a hotel f

R3 A guest cannot make two or more bookings in t same day.

R4 A guest can make two or more bookings in different hotels for the same day.

R5 A room in any hotel can only be booked by one guest on the same date, i.e., no *double-booking*.

How to Identify FDs?

- Consider the following
  - HOTEL(hotelNo, hotelName, city) with PK {hotelNo}
  - lNo}
  - stNo}
  -

- Which functional dependency does the followin

R1 A booking can be made for one day only.

How to Identify FDs?

- Consider the following
  - HOTEL(hotelNo, hotelName, city) with PK {hotelNo}
  - lNo}
  - stNo}

- Which functional dependency does the followin

  R1 A booking can be made for one day only.

  $\hookrightarrow$ {guestNo, hotelNo, roomNo} $\rightarrow$ {

How to Identify FDs?

- Consider the following
  - HOTEL(hotelNo, hotelName, city) with PK {hotelNo}
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  -

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  R1 A booking can be made for one day only.

  ↪ {guestNo, hotelNo, roomNo} → {

Australian National University

How to Identify FDs?

- Consider the following
  - HOTEL(hotelNo, hotelName, city) with PK {hotelNo}
  - lNo}
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  -

- Which functional dependency does the followin

R1 A booking can be made for one day only.

$\hookrightarrow$ {guestNo, hotelNo, roomNo} $\rightarrow$ {

guestNo	hotelNo	roomNo	Date
001	H1	R101	28/08/2020
001	H1	R101	29/08/2020

How to Identify FDs?

- Consider the following:
  -
  - lNo}
  - stNo}
  - BOOKING(guestNo, hotelNo, date, roomNo) with PK ?

- Which functional dependency does the followin

R2 A guest can make several bookings in a hotel f

Australian National University

How to Identify FDs?

- Consider the following:
  -
  - lNo}
  - stNo}
  - BOOKING(guestNo, hotelNo, date, roomNo) with PK ?

- Which functional dependency does the followin

R2 A guest can make several bookings in a hotel f

None

Australian National University

How to Identify FDs?

- Consider the following:
  - HOTEL(hotelNo, hotelName, city) with PK {hotelNo}
  - ROOM(roomNo, hotelNo, type, price) with PK roomNo, hotelNo}
  - stNo}
  -

- Which functional dependency does the followin

R3 A guest cannot make two or more bookings in t same day.

How to Identify FDs?

- Consider the following:
  - HOTEL(hotelNo, hotelName, city) with PK {hotelNo}
  - ROOM(roomNo, hotelNo, type, price) with PK roomNo, hotelNo}
  - stNo}
  -

- Which functional dependency does the followin

  R3 A guest cannot make two or more bookings in t same day.

  $\hookrightarrow$ {guestNo, hotelNo, date} $\rightarrow$ {ro }

How to Identify FDs?

- Consider the following:
  - HOTEL(hotelNo, hotelName, city) with PK {hotelNo}
  - ROOM(roomNo, hotelNo, type, price) with PK roomNo, hotelNo}
  - stNo}
  -

- Which functional dependency does the followin

R3 A guest cannot make two or more bookings in t same day.

$\hookrightarrow$ {guestNo, hotelNo, date} $\rightarrow$ {ro }

guestNo	hotelNo	roomNo	Date
001	H1	R101	29/08/2020
001	H1	R102 ×	29/08/2020

How to Identify FDs?

- Consider the following:
  -
  - lNo}
  - stNo}
  - OOKING

- Which functional dependency does the followin

R4 A guest can make two or more bookings in diff same day.

Australian National University

How to Identify FDs?

- Consider the following:
  -
  - lNo}
  - stNo}
  - OOKING

- Which functional dependency does the followin

R4 A guest can make two or more bookings in diff same day.

None

Australian National University

How to Identify FDs?

- Consider the following:
  - HOTEL(hotelNo, hotelName, city) with PK {hotelNo}
  - ROOM(roomNo, hotelNo, type, price) with PK {roomNo, hotelNo}
  - stNo}
  -

- Which functional dependency does the followin

R5 A room in any hotel can only be booked by one g
date, i.e., no *double-booking*.

$\hookrightarrow$ {hotelNo, date, roomNo} $\rightarrow$ {gu }

Australian National University

How to Identify FDs?

- Consider the following:
  - HOTEL(hotelNo, hotelName, city) with PK {hotelNo}
  - ROOM(roomNo, hotelNo, type, price) with PK roomNo, hotelNo}
  - stNo}
  -

- Which functional dependency does the followin

  R5 A room in any hotel can only be booked by one g
  date, i.e., no *double-booking*.

  $\hookrightarrow$ {hotelNo, date, roomNo} $\rightarrow$ {gu }

guestNo	hotelNo	roomNo	Date
001	H1	R101	29/08/2020
002 ×	H1	R101	29/08/2020

How to Find Candidate Keys?

- Consider the following
  - HOTEL(hotelNo, hotelName, city) with PK {hotelNo}
  - lNo}
  - stNo}
  -

- FDs on BOOKING
  - {guestNo, hotelNo, date} → {room
  - {hotelNo, date, roomNo} → {guest

How to Find Candidate Keys?

- Consider the following
  - HOTEL(hotelNo, hotelName, city) with PK {hotelNo}
  - lNo}
  - stNo}
  -

- FDs on BOOKING
  - {guestNo, hotelNo, date} → {room
  - {hotelNo, date, roomNo} → {guest

- Candidate keys on BOOKING
  - {guestNo, hotelNo, date}
  - {hotelNo, date, roomNo}

How to Identify FDs?

- Consider BOOKING(guestNo, hotelNo, date, roomNo) and the following changes:

~~... tel for the same day.~~

~~R4~~ ~~A guest can make two or more boo ... e same day.~~

R5 ~~A room in any hotel can only be booked by one g~~ ... date, i.e., no *double-booking*.

R6 A guest is not allowed to make more than one booking for the same day even in the different hotels.

How to Identify FDs?

- Consider the following:
  -
  - lNo}
  - stNo}
  -

- Which functional dependency does the followin

R6 A guest is not allowed to make more than one b day even in the different hotels.

How to Identify FDs?

- Consider the following:
  -
  - lNo}
  - stNo}
  -

- Which functional dependency does the followin

R6 A guest is not allowed to make more than one b day even in the different hotels.

$\hookrightarrow$ {guestNo, date} $\rightarrow$ {hotelNo, roomNo}

How to Find Candidate Keys?

- Consider the following
  - HOTEL(hotelNo, hotelName, city) with PK {hotelNo}
  - lNo}
  - stNo}
  -

- FDs on BOOKING
  - {hotelNo, date, roomNo} → {guest
  - {guestNo, date} → {hotelNo, roomN

How to Find Candidate Keys?

- Consider the following
  - HOTEL(hotelNo, hotelName, city) with PK {hotelNo}
  - lNo}
  - stNo}
  -

- FDs on BOOKING
  - {hotelNo, date, roomNo} → {guest
  - {guestNo, date} → {hotelNo, roomN

- Candidate keys on BOOKING
  - {hotelNo, date, roomNo}
  - {guestNo, date}

(credit cookie) Kurt Gödel and Incompleteness Theorem

Kurt Gödel (1906-1978)

Armstrong's Inference Rules

- **Two questions:**
  -
  -
    ⇝ **completeness (you can prove anyt**

- **Theorem** (W. W. Armstrong)
  - The Armstrong's inference rules are both **sound** and **complete**.

Hilbert's program (1920s)

- **Formulation of mathematics**: formalize all true mathematical statements
- **Completeness**: all true mathematical statements can be proved
- **Consistency**: no contradiction can be obtained in the formalism
- **D**

Hilbert's program (1920s)

- **Formulation of mathematics**: formalize all true mathematical statements
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- **Consistency**: no contradiction can be obtained in the formalism
- **D**

David Hilbert (1862-1943)

We must know. We will know.

Kurt Gödel and Incompleteness Theorem

Kurt Göde
(1906-197

- **Theorem** (Kurt Gödel, 1931)
  For any computable axiomatic system that is powerful enough to describe the arithmetic of the natural numbers, **there will always be at least one true but unprovable statement**.

Kurt Gödel and Gödel Prize

Kurt Gödel
(1906-1978)

Jo

- Kurt Gödel's achievement in modern logic is sing
indeed it is more than a monument, it is a landmark which will remain visible far in space and time. — **John von Neumann**

Kurt Gödel and Gödel Prize

Kurt Gödel
(1906-1978)

Jo

- Kurt Gödel's achievement in modern logic is sing indeed it is more than a monument, it is a landmark which will remain visible far in space and time. — **John von Neumann**
- The **Gödel prize** became an annual prize for outstanding papers in the area of theoretical computer science since 1993.