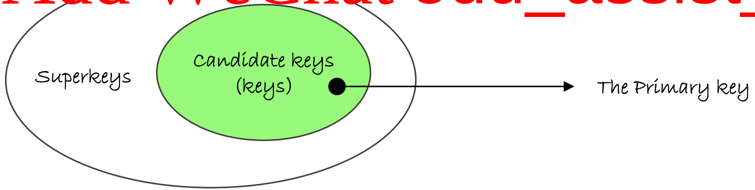
Functional Dependencies – Part 3

A Bunch of Keys

- We will need keys for defining the normal forms later on.
  - A *subset of the attributes* of a relation schema *R* is a **superkey** if it uniquely determines all attributes of *R*.
  - *K* is a

  - ere is not
  - **Candidate keys** are also called **ke** osen from them.



Australian National University

Finding Keys

Assignment Project Exam Help

- Gi https://eduassistpro.github.i

Add WeChat edu_assist_pr

Implied Functional Dependencies

- To design a good database, we need to consider **all possible FDs**.

- If e
  d

$\{\{\text{StudentID}\} \rightarrow \{\text{ProjectNo}\},$ $\models$ r$\}$
$\{\text{ProjectNo}\} \rightarrow \{\text{Supervisor}\}\}$

- We use the notation $\Sigma \models X \rightarrow Y$ to den y the set Σ of FDs.

- We write Σ^* for all possible FDs **implied** by Σ.

Equivalence of Functional Dependencies

- Σ_1 and Σ_2 are **equivalent** if $\Sigma_1^* = \Sigma_2^*$

- **Example:** Let $\Sigma_1 = \{X \rightarrow Y, Y \rightarrow Z\}$ an … $\rightarrow Z\}$.
  We have $\Sigma_1 \neq \Sigma_2$ but $\Sigma_1^* = \Sigma_2^* = \{X \rightarrow Y$ …
  Hence, Σ_1 and Σ_2 are equivalent.

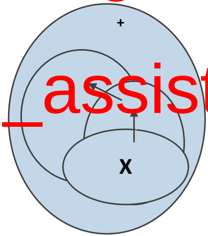
- **Questions:**
  1. Is it possible that $\Sigma_1^* = \Sigma_2^*$ but $\Sigma_1 \neq \Sigma_2$? **Yes**
  2. Is it possible that $\Sigma_1^* \neq \Sigma_2^*$ but $\Sigma_1 = \Sigma_2$? **No**

Implied Functional Dependencies

- Let Σ be a set of FDs. Check whether or not $\Sigma \models X \rightarrow W$ holds? We need to
  1. Compute **the set of all attributes** that are dependent on X, which is
  2.

- Al
  - $X^+ := X$;
  - repeat until no more change on X^+

    - for each $Y \rightarrow Z \in \Sigma$ with $Y \subseteq$
      add all the attributes in Z to X^+, i.e., replace X^+ by $X^+ \cup Z$.



[1] See Algorithm 15.1 on Page 538 in [Elmasri & Navathe, 7th edition] or Algorithm 1 on Page 555 in [Elmasri & Navathe, 6th edition]

Implied Functional Dependencies – Example

- Consider a relation schema $R = \{A, B, C, D, E, F\}$, a set of FDs $\Sigma = \{AC \to B, B \to CD, C \to E, AF \to B\}$ on R.

- D

1.

$$\begin{aligned} (AC)^+ &\supseteq AC \\ &\supseteq ACB \\ &\supseteq ACBD \\ &\supseteq ACBDE \\ &= ACBDE \end{aligned}$$

i
u
u
u

2. Then we check that $ED \subseteq (AC)^+$. Hence $\Sigma \models AC \to ED$.

- **Can you quickly tell whether or not $\Sigma \models AC \to EF$ holds?**

Finding Keys

- **Fact**: A key K of R always defines a FD $K \rightarrow R$
- **Algorithm**[2]:

  In
  o

  - for every subset X of the relation R, compute its closure X^+
  - if $X^+ = R$, then X is a superkey.
  - if no proper subset Y of X with Y^+

- A **prime attribute** is an attribute occurring in a key, and a **non-prime attribute** is an attribute that is not a prime attribute.

[2] It extends Algorithm 15.2(a) in [Elmasri & Navathe, 7th edition, pp. 542], or Algorithm 2(a) or in Algorithm 2(a) in [Elmasri & Navathe, 6th edition pp. 558] to finding all keys of R

Exercise – Finding Keys

- Consider a relation schema $R = \{A, B, C, D\}$ and a set of functional dependencies $\Sigma = \{AB \rightarrow C, AC \rightarrow D\}$.
  1. List all the keys and superkeys of R.
  2. [illegible]
- S[illegible]

  1. [illegible]
     - $(A)^+ = A$, $(B)^+ = B$, $(C)^+ =$ [illegible]
     - $(AB)^+ = ABCD$, $(AC)^+ = AC$[illegible]
     - $(BD)^+ = BD$, $(CD)^+ = CD$

     - $(ABC)^+ = ABCD$, $(ABD)^+ =$ [illegible] $(BCD)^+ = BCD$
  2. Hence, we have
     - AB is the only key of R.
     - AB, ABC, ABD and $ABCD$ are the superkeys of R.
     - A and B are the prime attributes of R.

Exercise – Finding Keys

- Checking all possible combinations of the attributes is too tedious!

  E
  Σ

- **S**
  - If an attribute *never* appears in the dep
    must **be part of each key**

  - If an attribute *never* appears in the dete
    in the dependent of any FD, this attribute must **not be part of each key**.
  - If a proper subset of X is a key, then X must **not be a key**.

Finding Keys - Example

- Consider ENROLMENT and the following FDs:
  - {StudentID} → {Name};
  - StudentID, CourseNo, Semester ConfirmedBy, Office};
  -

					Office
Tom	123456	COMP2400	2		1
Mike	123458	COMP2400	2		3
Mike	123458	COMP2600	2		3

- What are the keys, superkeys and prime attribute
  - {StudentID, CourseNo, Semester} is the only key.
  - Every set that has {StudentID, CourseNo, Semester} as its subset is a superkey.
  - StudentID, CourseNo and Semester are the prime attributes.