



Relational Algebra (Part 2)

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Summary of Relational Operators

Operator	Notation	Meaning
Selection	$\sigma_\varphi(R)$	choose rows
Projection	$\pi_{A_1, \dots, A_n}(R)$	choose columns
Union Intersection Difference	$R_1 \cup R_2$ $R_1 \bowtie_\varphi R_2$ $R_1 \bowtie R_2$	https://eduassistpro.github.io/functions Add WeChat edu_assist_pro
Cartesian product Join Natural-join	$R_1 \times R_2$ $R_1 \bowtie_\varphi R_2$ $R_1 \bowtie R_2$	combine tables
Renaming	$\rho_{R'}(A_1, \dots, A_n)(R)$ $\rho_{R'}(R)$ $\rho_{(A_1, \dots, A_n)}(R)$	rename relation and attributes



A Complete Set of Relational Operators

- The following six operators constitute **a complete set**:

- **selection** σ ;
- **projection** π ; <https://eduassistpro.github.io/>
- **renaming** ρ ; [Add WeChat edu_assist_pro](#)
- **union** \cup ;
- **difference** $-$;
- **Cartesian product** \times .



A Complete Set of Relational Operators

- Six operators (i.e., **selection** σ , **projection** π , **renaming** ρ , **union** \cup , **difference** - and **Cartesian product** \times) constitute **a complete set**.
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in are not necessary and can <https://eduassistpro.github.io/>
- **join:** $R_1 \bowtie_{\varphi} R_2 = \sigma_{\varphi}(R_1 \times R_2)$ Add WeChat edu_assist_pro
- **intersection:** $R_1 \cap R_2 = R_1 - (R_1 - R_2)$
- Hence, **intersection** and **join** do not increase the expressive power of RA.
- Nonetheless it is important to include **intersection** and **join** because they are convenient to use and commonly applied in database applications.



Relational Algebra Queries

- The output of each RA operation is a relation, which can be used again as the input for another RA operation.
- RA operations ~~can be nested to arbitrary depth~~ for expressing complex queries, as in arithmetic.
 - Parentheses affect evaluation:
from highest to lowest: $\{\sigma, \pi, \rho\}$, $\{\times, \bowtie\}$
 - Operators with the same precedence from left to right.
 - Use brackets if you are not sure.
- A **query** in RA is a sequence of RA operations and each RA operation takes one or two relations as its input and produces one relation as its output.
- Different from SQL, RA considers **relations as sets** (not **multisets** as in SQL). Hence, relations produced by an RA operation **have no duplicate tuples**.



Hints for Writing RA Queries

- ① Firstly, identify which relations need to be involved, while ignoring the rest.
- ② Then break the answer down by considering intermediate relations, i.e., queries may be expressed as **a sequence of assignment statements**.

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Example: $R := \pi_{HT}$ $\text{ore}, GScore, GTeam$ (Soccer))

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- Use good names
- Keep track of attributes you have at each step

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- ③ When combining relations, check attribute names and make sure that:
 - attributes that should match are to match.
 - attributes that shouldn't match are not to match.
- ④ When using set operations, make sure that two relations of an operation have the same type (i.e., **type compatibility**).



RA Queries – Exercises (Self Join)

- Given the following relation schema:

STUDENT={StudentID, Name, DoB}

- Query 1:** Find ~~Assignment Project Exam Help~~ students who have the same birthday. Show their names.

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STUDENT		
StudentID	Name	DoB
457	Lisa	18-Oct-1993
458	Mike	16-May-1990
459	Peter	18-Oct-1993



RA Queries – Exercises (Self Join)

- Given the following relation schema:

$\text{STUDENT} = \{\text{StudentID}, \text{Name}, \text{DoB}\}$

- Query 1:** Find ~~passing~~ students who have the same birthday. Show their names.

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$$\pi_{R_1.\text{Name}, R_2.\text{Name}}(\sigma_{R_1.\text{StudentID} < R_2.\text{StudentID}} =_R R_2.\text{DoB})$$

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```
SELECT R1.name, R2.name
```

```
FROM Student AS R1, Student AS R2
```

```
WHERE R1.DoB = R2.DoB AND R1.StudentID < R2.StudentID;
```

- Why do we need $\sigma_{R_1.\text{StudentID} < R_2.\text{StudentID}}$ in the above query?
- Why do we need to use renaming in the above query?



RA Queries – Exercises (Self Join)

- Given the following relation schema:

$\text{STUDENT} = \{\text{StudentID}, \text{Name}, \text{DoB}\}$

- Query 1:** Find pair of students with same birthday. Show their names.

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Two different solutions:

(1). $\pi_{R_1.\text{Name}, R_2.\text{Name}}(\sigma_{R_1.\text{StudentID} < R_2.\text{StudentID}}(\sigma_{R_1.\text{DoB} = R_2.\text{DoB}}(\rho_{R_1}(\text{STUDENT}) \times \rho_{R_2}(\text{STUDENT}))))$

(2). $\pi_{\text{Name}, \text{Name}'}(\sigma_{\text{StudentID} < \text{StudentID}'}(\text{STUDENT} \bowtie \rho_{S(\text{StudentID}', \text{Name}', \text{DoB})}(\text{STUDENT})))$



RA Queries – Exercises (Self Join)

- **Query 1:** Find **pairs of** students who have the same birthday. Show their names.

(1). $\pi_{R_1.Name, R_2.Name}(\sigma_{R_1.StudentID < R_2.StudentID}(\sigma_{R_1.DoB = R_2.DoB}(\rho_{R_1}(\text{STUDE}$

(2). $\pi_{Name, Name} \text{ https://eduassistpro.github.io/}$
 $\text{STUDENT} \bowtie \rho_S(StudentID', Name')$
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- If evaluating our queries over the following relation, what will be the result?

STUDENT		
StudentID	Name	DoB
457	Lisa	18-Oct-1993
458	Mike	16-May-1990
459	Peter	18-Oct-1993



RA Queries – Exercises (Self Join)

- **Query 1 (solution 1):** $\pi_{R_1.Name, R_2.Name}(\sigma_{R_1.StudentID < R_2.StudentID}(\sigma_{R_1.DoB = R_2.DoB}(\rho_{R_1}(\text{STUDENT}) \times \rho_{R_2}(\text{STUDENT}))))$.

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STUDEN		
StudentID	Name	DoB
	993	
		990
993		

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$R_1.StudentID$	$R_1.Name$	$R_1.DoB$	R_2	Name	$R_2.DoB$
457	Lisa	18-Oct-1993	457	Lisa	18-Oct-1993
457	Lisa	18-Oct-1993	458	Mike	16-May-1990
457	Lisa	18-Oct-1993	458	Peter	18-Oct-1993
458	Mike	16-May-1990	457	Lisa	18-Oct-1993
458	Mike	16-May-1990	458	Mike	16-May-1990
458	Mike	16-May-1990	458	Peter	18-Oct-1993
458	Peter	18-Oct-1993	457	Lisa	18-Oct-1993
458	Peter	18-Oct-1993	458	Mike	16-May-1990
458	Peter	18-Oct-1993	458	Peter	18-Oct-1993



RA Queries – Exercises (Self Join)

- **Query 1 (solution 1):** $\pi_{R_1.Name, R_2.Name}(\sigma_{R_1.StudentID < R_2.StudentID}(\sigma_{R_1.DoB = R_2.DoB}(\rho_{R_1}(\text{STUDENT}) \times \rho_{R_2}(\text{STUDENT}))))$.

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STUDENT		
StudentID	Name	DoB
	993	
		990
	993	

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$R' = \sigma_{R_1.DoB = R_2.DoB}(\rho_{R_1}(\text{STUDENT}) \times \rho_{R_2}(\text{STUDENT}))$

$R_1.StudentID$	$R_1.Name$	$R_1.DoB$	R_2	$R_2.Name$	$R_2.DoB$
457	Lisa	18-Oct-1993	457	Lisa	18-Oct-1993
457	Lisa	18-Oct-1993	459	Peter	18-Oct-1993
458	Mike	16-May-1990	458	Mike	16-May-1990
459	Peter	18-Oct-1993	457	Lisa	18-Oct-1993
459	Peter	18-Oct-1993	459	Peter	18-Oct-1993

$\pi_{R_1.Name, R_2.Name}(\sigma_{R_1.StudentID < R_2.StudentID}(R'))$

$R_1.Name$	$R_2.Name$
Lisa	Peter



RA Queries – Exercises (Self Join)

- **Query 1 (solution 2):** $\pi_{Name, Name'} (\sigma_{StudentID < StudentID'} (\text{STUDENT} \bowtie \rho_{S(StudentID', Name', DoB)}(\text{STUDENT})))$.

Assignment Project Exam Help		
STUDENT	STUDENT	STUDENT
StudentID	Name	DoB
	993	
		1990
	993	

<https://eduassistpro.github.io/>

R' = STUDENT $\bowtie^1 \rho_S(\text{StudentID}, \text{Name})$ STUDENT				
StudentID	Name	D	ID'	Name'
457	Lisa	18-Oct-1993	459	Peter
459	Peter	18-Oct-1993	457	Lisa
459	Peter	18-Oct-1993	459	Peter
457	Lisa	18-Oct-1993	457	Lisa
458	Mike	16-May-1990	458	Mike

$\pi_{Name, Name'} (\sigma_{StudentID < StudentID'} (R'))$	
Name	Name'
Lisa	Peter



RA Queries – Exercises (Difference 1)

- Given the following relation schemas:

STUDENT={StudentID, Name, DoB}

ENROL={StudentID, CourseNo, Semester, EnrolDate}

- Query 2:** Which students have ~~never~~ enrolled in any course? Show their IDs and names.

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StudentID	Name	DoB
456	Io	991
457	Lisa	18-Oct-1993
458	Mike	16-May-1990

ENROL			
StudentID	CourseNo	Semester	EnrolDate
456	COMP2400	2010 S2	02-Jul-2010
458	COMP2400	2010 S2	23-Jun-2010
458	COMP2600	2010 S2	05-Aug-2010



RA Queries – Exercises (Difference 1)

- Given the following relation schemas:

STUDENT={StudentID, Name, DoB}

ENROL={StudentID, CourseNo, Semester, EnrolDate}

- Query 2:** Which students have ~~never~~ enrolled in any course? Show their IDs and names.

<https://eduassistpro.github.io/>

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Hints:

- (1) All the students
- (2) Students who have enrolled in at least one course

Answer: Students in **the result (1) but not in the result (2)**.



RA Queries – Exercises (Difference 1)

- Given the following relation schemas:

$\text{STUDENT} = \{\text{StudentID}, \text{Name}, \text{DoB}\}$

$\text{ENROL} = \{\text{StudentID}, \text{CourseNo}, \text{Semester}, \text{EnrolDate}\}$

- Query 2:** Which students have ~~never~~ enrolled in any course? Show their IDs and names.

(1) All the students <https://eduassistpro.github.io/>

Add $R_1 := \pi_{\text{StudentID}}(\text{ENROL})$
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(2) Students who have enrolled in at least one course

$R_2 := \pi_{\text{StudentID}}(\text{ENROL})$

Answer: Students in the result (1) but not in the result (2)

$\pi_{\text{StudentID}, \text{Name}}((R_1 - R_2) \bowtie \text{STUDENT})$



RA Queries – Exercises (Difference 1)

- **Query 2:** Which students have **never** enrolled in any course? Show their IDs and names.
- If evaluating our query over the following relations, what will be the result?
 - $R_1 := \pi_{StudentID}(\text{STUDENT})$
 - $R_2 := \pi_{StudentID}$
 - $\pi_{StudentID, Name}((\text{https://eduassistpro.github.io/}$

STU		
StudentID	Name	EnrolDate
456	Tom	02-Jan-1991
457	Lisa	18-Oct-1993
458	Mike	16-May-1990

ENROL			
StudentID	CourseNo	Semester	EnrolDate
456	COMP2400	2010 S2	02-Jul-2010
458	COMP2400	2010 S2	23-Jun-2010
458	COMP2600	2010 S2	05-Aug-2010



RA Queries – Exercises (Difference 1)

- **Query 2:** Which students have **never** enrolled in any course? Show their IDs and names.
- If evaluating our query over the following relations, what will be the result?
 - $R_1 := \pi_{StudentID}(\text{STUDENT})$
 - $R_2 := \pi_{StudentID}$
 - $\pi_{StudentID, Name}((\text{https://eduassistpro.github.io/Assignment Project Exam Help}))$

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R_1
StudentID
456
457
458

R_2
456
458

$\pi_{StudentID, Name}((R_1 - R_2) \bowtie \text{STUDENT})$	
StudentID	Name
457	Lisa



RA Queries – Exercises (Difference 2)

- Given the following relation schemas:

STUDENT={StudentID, Name, DoB}

ENROL={StudentID, CourseNo, Semester, EnrolDate}

- Query 3:** Which students have ~~only~~ enrolled in the course COMP2400?
Show their IDs and nam

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StudentID	Na	
456	Io	991
457	Lisa	18-Oct-1993
458	Mike	16-May-1990

ENROL			
StudentID	CourseNo	Semester	EnrolDate
456	COMP2400	2010 S2	02-Jul-2010
457	COMP2400	2010 S2	08-Jul-2010
458	COMP2400	2010 S2	23-Jun-2010
458	COMP2600	2010 S2	05-Aug-2010



RA Queries – Exercises (Difference 2)

- Given the following relation schemas:

STUDENT={StudentID, Name, DoB}

ENROL={StudentID, CourseNo, Semester, EnrolDate}

- Query 3:** Which students have **only** enrolled in the course COMP2400?
Show their IDs and names

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Hints:

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- (1) Students who have enrolled in the course COMP2400.
- (2) Students who have enrolled in a course but not COMP2400.

Answer: Students in the result (1) but not in the result (2).



RA Queries – Exercises (Difference 2)

- Given the following relation schemas:

$$\text{STUDENT} = \{\text{StudentID}, \text{Name}, \text{DoB}\}$$
$$\text{ENROL} = \{\text{StudentID}, \text{CourseNo}, \text{Semester}, \text{EnrolDate}\}$$

- Query 3:** Which students have **only** enrolled in the course COMP2400?

Show their IDs and nam

(1) Students who have <https://eduassistpro.github.io/>

$$R_1 := \pi_{\text{StudentID}}(\sigma_{\text{CourseNo} = 'COMP2400'}(\text{ENROL}))$$

(2) Students who have enrolled in a course but not COMP2400.

$$R_2 := \pi_{\text{StudentID}}(\sigma_{\text{CourseNo} \neq 'COMP2400'}(\text{ENROL}))$$

Answer: Students in **the result (1) but not in the result (2)**.

$$\pi_{\text{StudentID}, \text{Name}}((R_1 - R_2) \bowtie \text{STUDENT}) =$$
$$\begin{aligned} & \pi_{\text{StudentID}, \text{Name}}((\pi_{\text{StudentID}}(\sigma_{\text{CourseNo} = 'COMP2400'}(\text{ENROL})) \\ & - \pi_{\text{StudentID}}(\sigma_{\text{CourseNo} \neq 'COMP2400'}(\text{ENROL}))) \bowtie \text{STUDENT}) \end{aligned}$$



RA Queries – Exercises (Difference 2)

- **Query 3:** Which students have **only** enrolled in the course COMP2400?
Show their IDs and names.
- If evaluating our query over the following relations, what will be the result?
 - $R_1 := \pi_{StudentID}(\sigma_{CourseNo='COMP2400'}(ENROL))$
 - $R_2 := \pi_{StudentID}(\sigma_{CourseNo \neq 'COMP2400'}(ENROL))$
 - $\pi_{StudentID, Name}(($

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StudentID	Name	EnrolDate
456	Tom	1991-01-01
457	Lisa	18-Oct-1993
458	Mike	16-May-1990

ENROL			
StudentID	CourseNo	Semester	EnrolDate
456	COMP2400	2010 S2	02-Jul-2010
457	COMP2400	2010 S2	08-Jul-2010
458	COMP2400	2010 S2	23-Jun-2010
458	COMP2600	2010 S2	05-Aug-2010



RA Queries – Exercises (Difference 2)

- **Query 3:** Which students have **only** enrolled in the course COMP2400? Show their IDs and names.
- If evaluating our query over the following relations, what will be the result?
 - $R_1 := \pi_{StudentID}(\sigma_{CourseNo='COMP2400'}(ENROL))$
 - $R_2 := \pi_{StudentID}(\sigma_{CourseNo \neq 'COMP2400'}(ENROL))$
 - $\pi_{StudentID, Name}(($

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StudentID
456
457
458

458

$\pi_{StudentID, Name}((R_1 - R_2) \bowtie STUDENT)$

StudentID	Name
456	Tom
457	Lisa



More Hints for Writing RA Queries

- Pay attention to keywords like **not**, **never**, **only**, **always**, **exactly**, etc. which often indicates the use of **difference** in the corresponding RA queries.

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- To show “never”:

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- Find all the (comb

- Use difference to subtract those that ha

- To show “only” and “always”:

- Find all the (combinations of) tuples that are involved.

- Use difference to subtract those that didn’t always occur.



Equivalence of RA and SQL Queries (1)

- Each RA query can be easily re-written in SQL, or vice versa.

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- **Selection:** σ_φ

SELECT DISTINCT <https://eduassistpro.github.io/>

- **Projection:** $\pi_{A_1, \dots, A_n}(R)$ corresponds to

SELECT DISTINCT A_1, \dots, A_n FROM R ;

- **Renaming:** $\rho_{S(B_1, \dots, B_n)}(R)$ (with attributes A_1, \dots, A_n in R) corresponds to

SELECT A_1 AS B_1, \dots, A_n AS B_n FROM R AS S ;



Equivalence of RA and SQL Queries (2)

- **Union:** $R_1 \cup R_2$ corresponds to

```
SELECT * FROM  $R_1$  UNION SELECT * FROM  $R_2$ 
```

- **Intersection:** $R_1 \cap R_2$ corresponds to

```
SELECT * FROM  $R_1$  INTERSECT https://eduassistpro.github.io/
```

- **Difference:** $R_1 - R_2$ (with attribute n) corresponds to

```
SELECT * FROM  $R_1$  EXCEPT SELECT * FROM  $R_2$ 
```

```
SELECT DISTINCT * FROM  $R_1$  WHERE NOT EXISTS
```

```
(SELECT * FROM  $R_2$ 
```

```
WHERE  $R_1.A_1=R_2.A_1$  AND ... AND  $R_1.A_n=R_2.A_n$ )
```

SQL eliminates duplicate tuples in the resulting relations of set operations UNION, INTERSECT and EXCEPT.



Equivalence of RA and SQL Queries (3)

- **Cartesian Product:** $R_1 \times R_2$ corresponds to

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- **Join:** $R_1 \bowtie_{\varphi}$ <https://eduassistpro.github.io/>

SELECT DISTINCT * FROM R_1 **Add WeChat edu_assist² ON** φ ;

(φ may contain $=, <, \leq, >, \geq, \neq$)

- **Natural-Join:** $R_1 \bowtie R_2$ corresponds to

SELECT DISTINCT * FROM R_1 NATURAL JOIN R_2 ;

Outer joins are not considered in the traditional relational algebra, as well as aggregation.



Summary

- RA is a **procedural query language** defined in the relational model.

An RA query itself suggests a procedure for constructing the result (i.e., implement the query).

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- RA is **not used as a query**!

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- RA is **used for the internal representation** a **of SQL queries** in relational DBMSs, which is a basis of query optimisation techniques.

- Thus, to understand how SQL queries are processed and how they can be optimised, we **first need to understand relational algebra**.