Introduction to Database Systems – Part 2

What are the Math Concepts behind Databases?

- Set
- T
- Cartesian Product of Sets
- Relation

Set Notation

Set Notation

- We need set notation to represent formal definitions in this course.
- A
- T
  - The elements in a set have no order.
    e.g., $\{1, 2, 3\} = \{2, 3, 1\}$
  - Each element can not be in the set more than o
    e.g., {Monday, Monday, Tuesday, Wednesday, Thursday, Friday, Friday} is Not a set. Note that **Multisets** allow to have duplicate elements.

Set Notation

- **Two ways of specifying a set**
  1. $\{x_1, \ldots, x_n\}$ (i.e., list all the elements in a set)

     - $\{\}$ or $\emptyset$, i.e., the *empty* set.

  2. $\{x \mid \phi\}$ (i.e., describe the elements that sat

     - $\{x \mid x$ is a student currently enrolled in COMP7240$\}$
     - $\{x \mid x$ is an integer and $x > 0\}$

Set Operations

- **Membership**: $x \in A$ if x is in the set A; $x \notin A$ if x is not in the set A.

Set Operations

- **Equality**: If A and B have the same elements, we write $A = B$; otherwise we write $A \neq B$.
  - x x is an integer, $x > 1$ and $x < 6$ = $2, 3, 4, 5$
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Set Operations

- **Subset**: A is called a **subset** of B if every element of A is in B and we write $A \subseteq B$.
- **Proper subset**: A is called a **proper subset** of B if $A \subseteq B$ and A and B are n

Set Operations

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Set Operations

- Union: $A \cup B$ for the set containing everything in A and everything in B

Set Operations

- **Intersection**: $A \cap B$ for the set of elements that are in both A and B
  - 3, 4, 5 3, 5, 7, 9 = 3, 5 .

Set Operations

- **Difference**: $A - B$ is the elements from A that are *not* in B.
  - 3, 4, 5 − 3, 5, 7, 9 = 4 .

Set Operations – Exercise

- Let $A = \{1,2,3\}$ and $B = \{true, false\}$.
- W

3. $\{2,3\} \subseteq A \cup B$ — Yes! $A \cup B$
4. $2 \in A \cap B$ — No! $A \cap B$
5. $2 \in A - \{1,3,5\}$ — Yes! $A - \{$
6. $\{1,4\} \subseteq A - B$ — No! $A - B = \{1,2,3\}$
7. $\emptyset \cap B = \emptyset$ — Yes! $\emptyset = \{\}$, the empty set

Tuple Notation

Tuple Notation

- A **tuple** is an ordered list of n elements.
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  - $(1, 2, 3) \neq (2, 3, 1)$ (i.e., the order does
- The same element *can* be in a tuple twice.
  - (Monday, Monday, Tuesday, Wednesday, Thursday, Friday, Friday) is a tuple.
- Ordered pairs are special cases of tuples.

Cartesian Product of Sets

Cartesian Product of Sets

Cartesian Product of Sets

Cartesian Product of Sets

- The Cartesian product operation takes an ordered list of sets, and returns a set of tuples.
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  element from the second set, ...
- For example, $A \times B = \{(a, b) \mid a \in A$ and
  If $A = \{2, 3\}$ and $B = \{Clubs, Diamonds,$
  Then $A \times B = \{(2, Clubs), (2, Diamonds), (2, Hearts), (2, Spades),$
  $(3, Clubs), (3, Diamonds), (3, Hearts), (3, Spades)\}$.
  $(2, Clubs) \in A \times B$, $(Spades, 3) \notin A \times B$, $(4, Hearts) \notin A \times B$
  $\{(3, Clubs), (3, Diamonds), (3, Hearts), (3, Spades)\} \subseteq A \times B$

Relation Notation

Relation Notation

Relation Notation

- A **relation** is a subset of a Cartesian product of sets.
- **E**

- Let $R = \{(a, b) | a \in X, b \in Y$ and a is a city in $b\}$.
- It is easy to see that R is a relation
  - $R \subseteq X \times Y$.
  - $(Canberra, Australia) \in R$, $(Paris, France) \in R$ but $(Tokyo, France) \notin R$, $(France, Japan) \notin R$

Relation Notation

- A **relation** is a subset of a Cartesian product of sets.
- **E**

- $= \{(\ , \ ) | \ \in \mathbb{Z}, \ \in \mathbb{Z} \ \ \ \ <$
- It is easy to see that R is a relation.

  - $R \subseteq \mathbb{Z} \times \mathbb{Z}$.
  - $(0, 1) \in R$, $(-4, -2) \in R$
    but $(0, 0) \notin R$, $(100, -2) \notin R$.