

COMP 250

INTRODUCTORY SCIENCE

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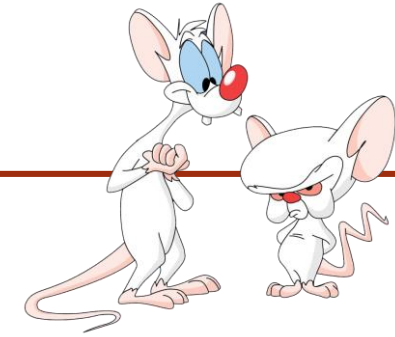
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Week 12-3:
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Giulia Alberini, Fall 2020

Slides adapted from Michael Langer's

WHAT ARE WE GOING TO DO IN THIS VIDEO?



- How to build a heap naively: best and worst case
- write `removeMin()`
- Faster algorithm for building a h

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HOW TO BUILD A HEAP?

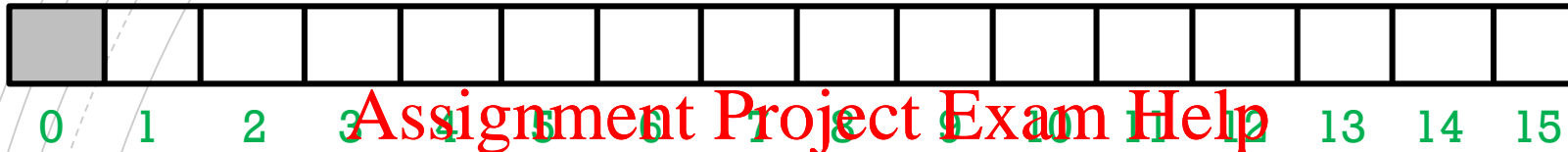
Suppose we have a list with n elements, we can create an empty heap and use `add()` to add one element at a time to the heap:

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```
buildHeap(list) {  
    create new heap array  
    for (k = 0; k < list.size()  
        add( list[k] ) // add the element to the heap  
}
```

Note that you could write the `buildHeap` algorithm slightly differently by putting all the list elements into the array at the beginning, and then 'upheaping' each one.

BEST CASE OF BUILDHEAP IS ... ?



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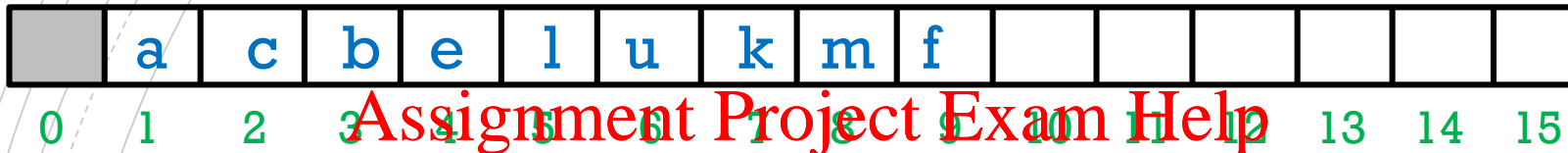
Suppose we want to add some elements to an empty heap:

a c b e l u k m f

How many swaps do we need to add each element?

In the best case, ...

BEST CASE OF BUILDHEAP IS $O(n)$



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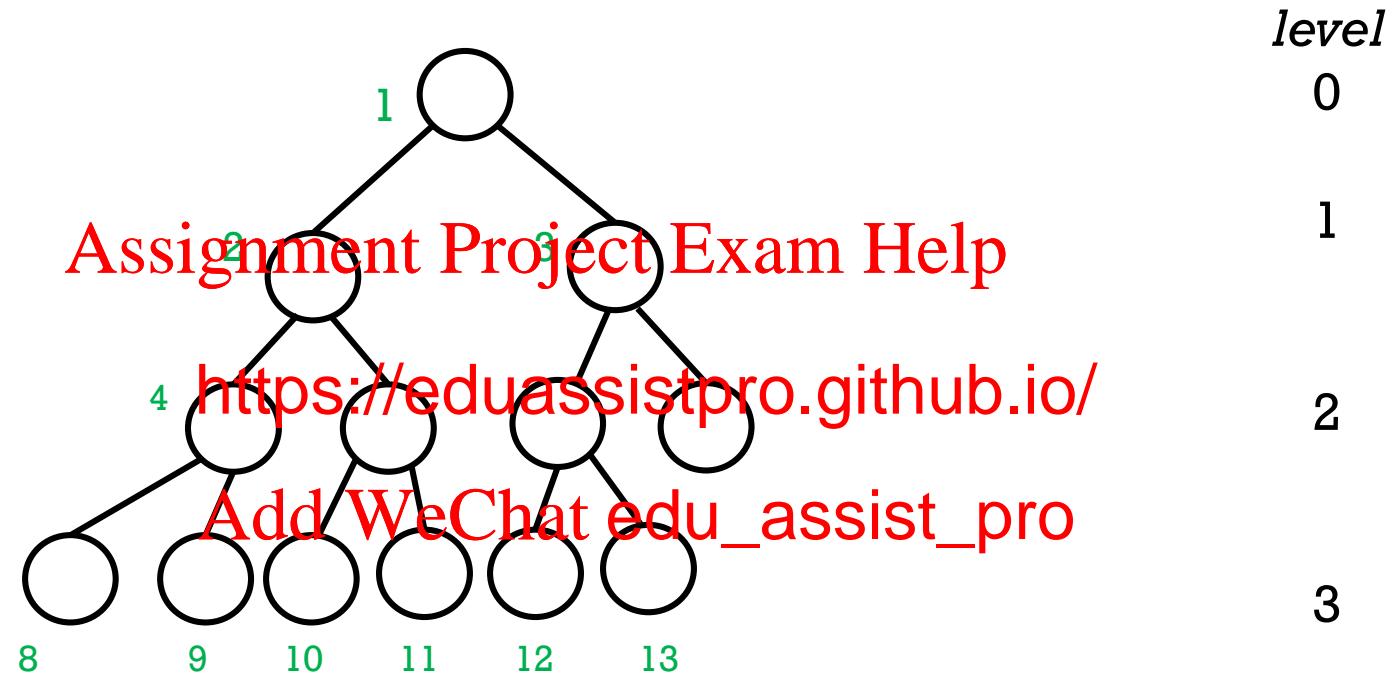
Suppose we want to add some element to an empty heap:

a c b e l u k m f

How many swaps do we need to add each element?

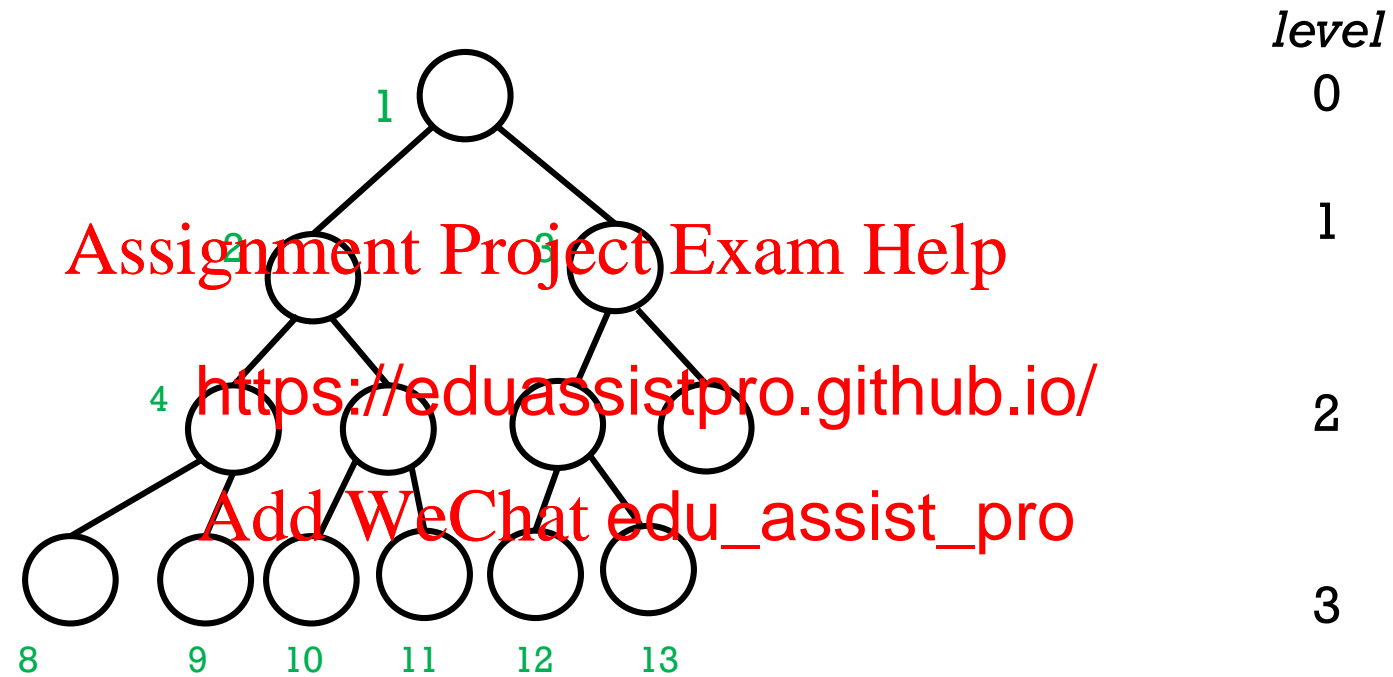
In the best case, the order of elements that we add is already a heap, and no swaps are necessary.

WORST CASE OF BUILDHEAP IS ... ?



How many swaps do we need to add the i -th element?

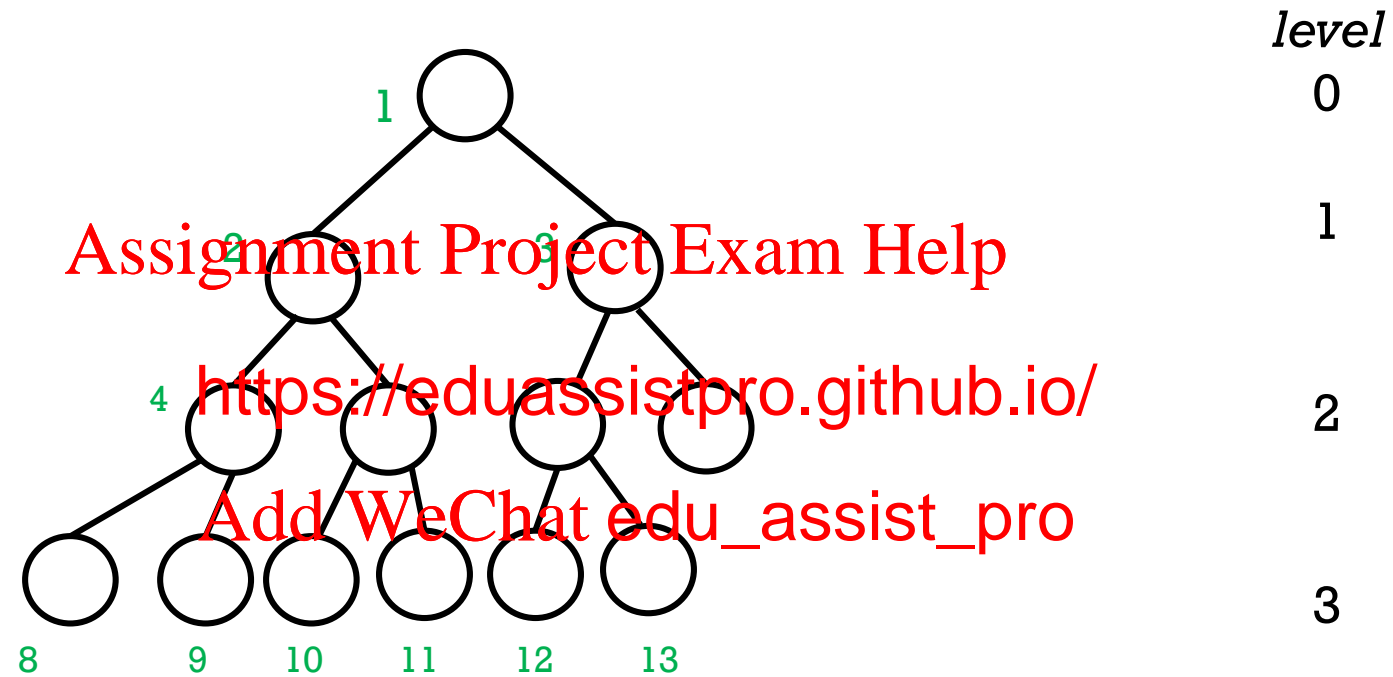
WORST CASE OF BUILDHEAP IS ... ?



How many swaps do we need to add the i -th element?
Element i gets added to some $level$, such that:

$$2^{level} \leq i < 2^{level+1}$$

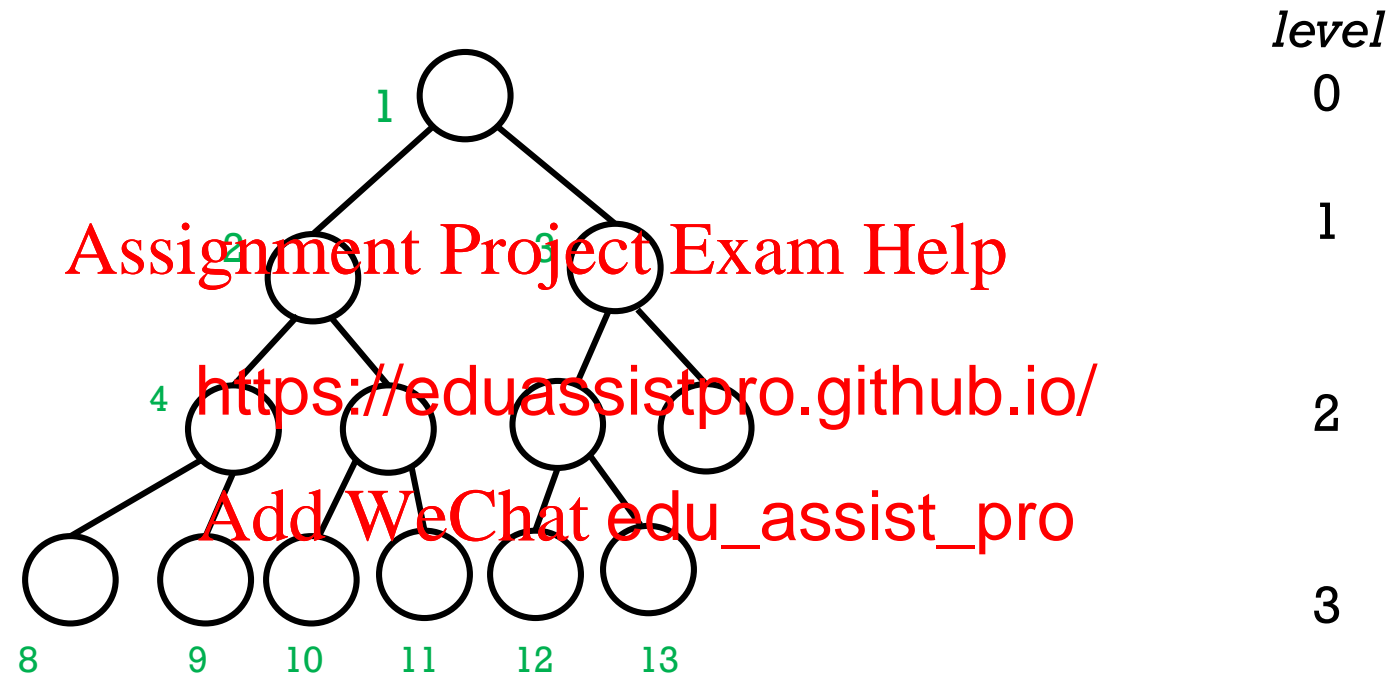
WORST CASE OF BUILDHEAP IS ... ?



$$2^{level} \leq i < 2^{level+1}$$
$$level \leq \log_2 i < level + 1$$

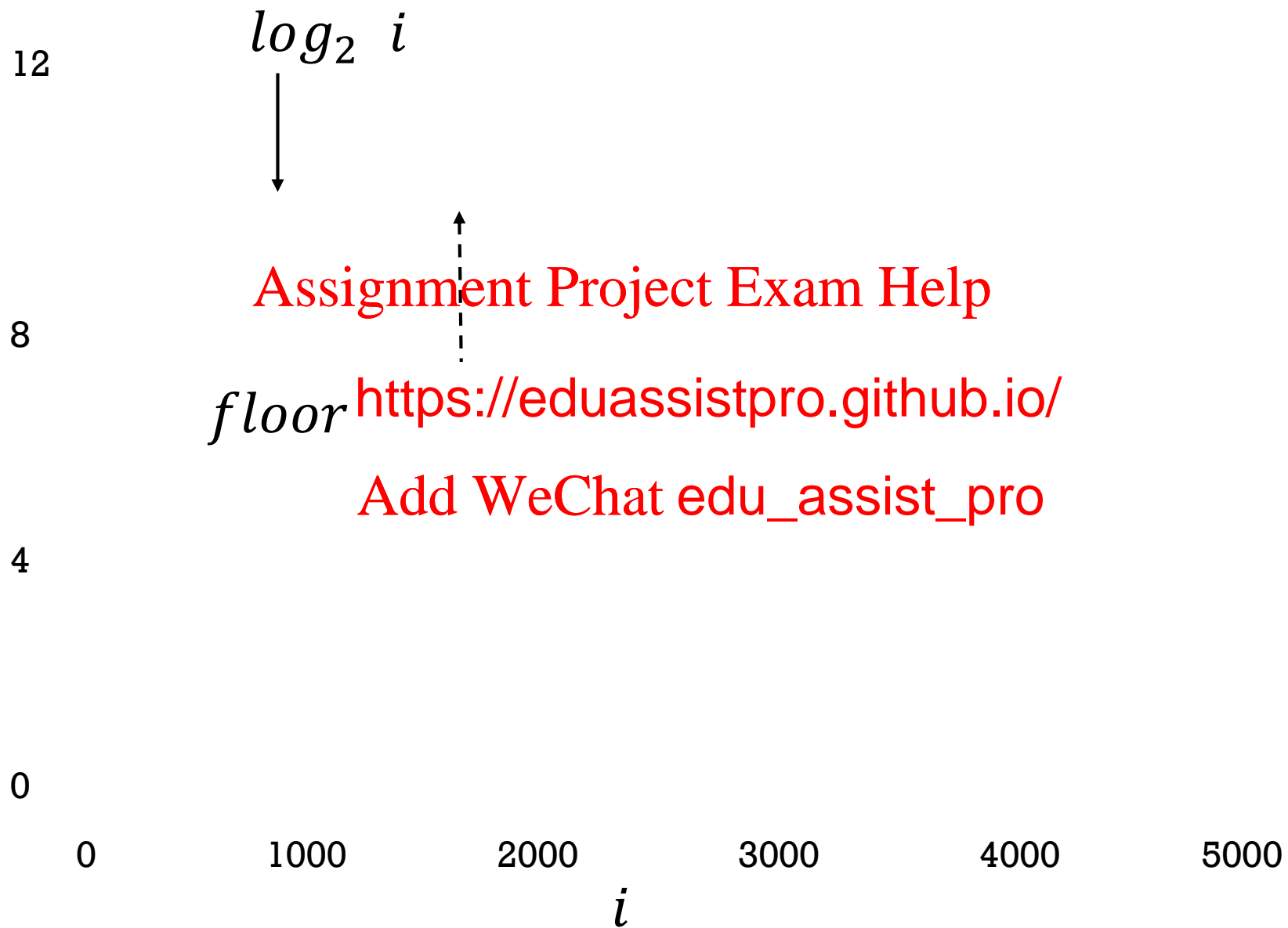
Thus, $level = \text{floor}(\log_2 i)$

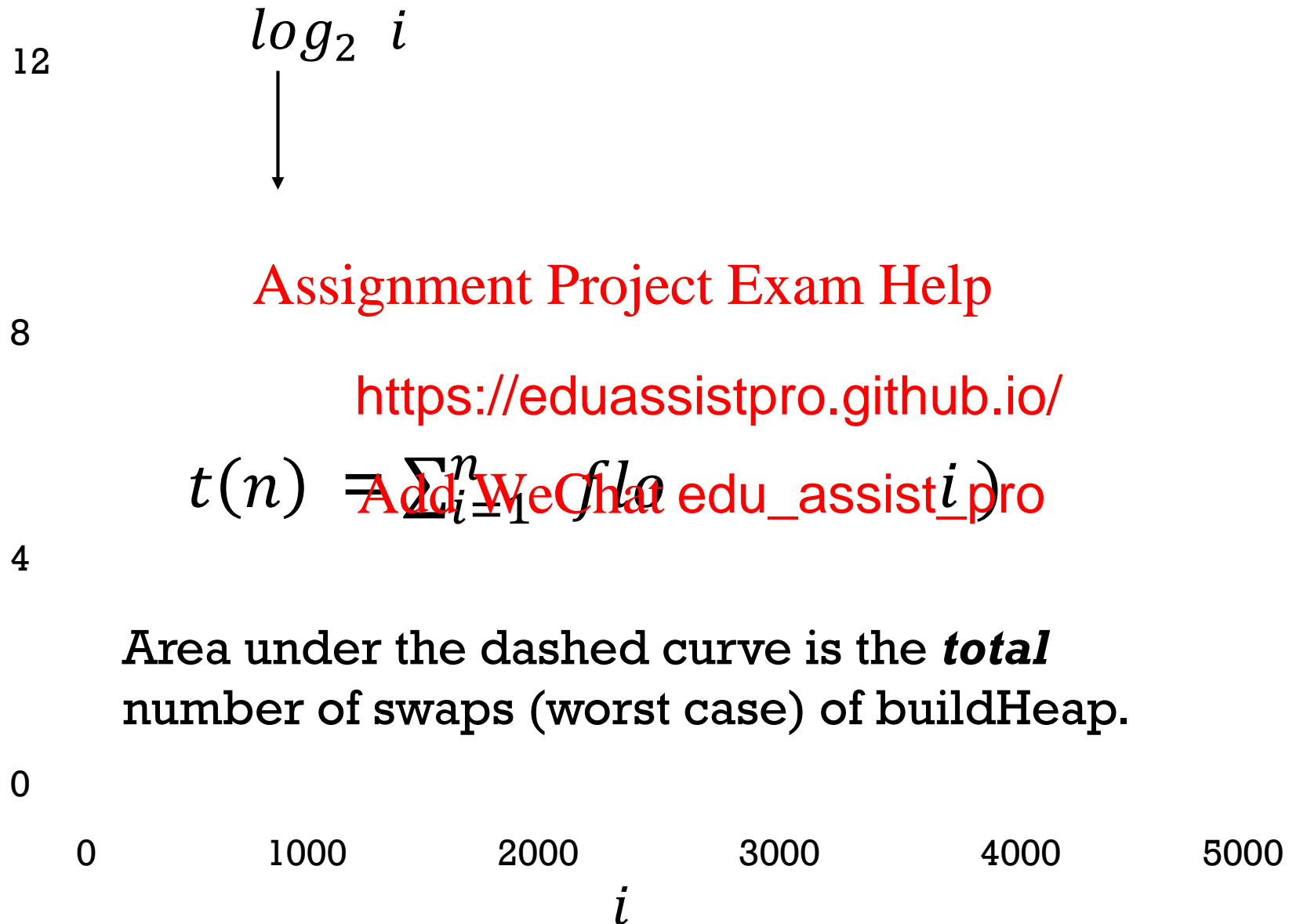
WORST CASE OF BUILDHEAP IS ... ?

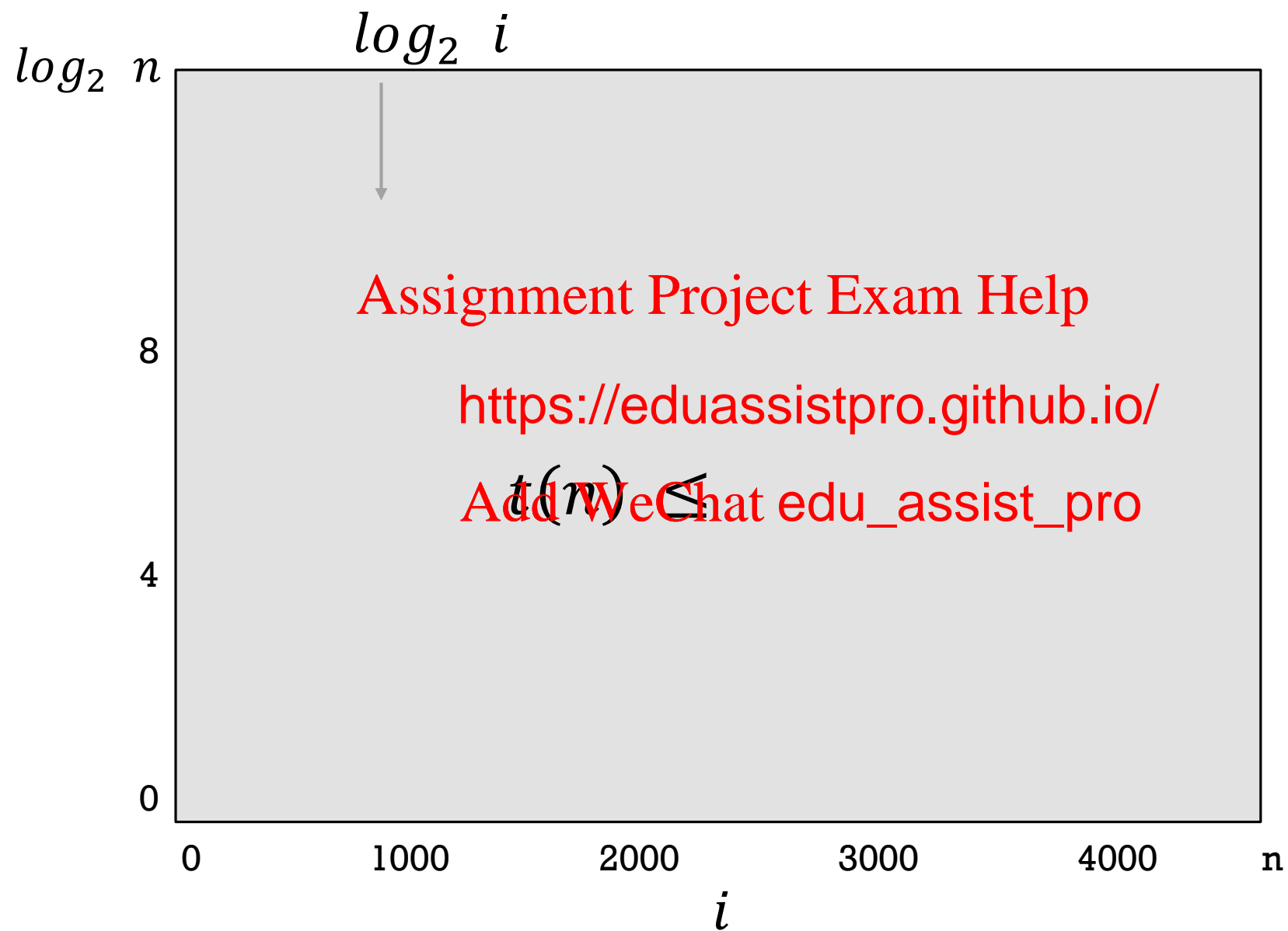


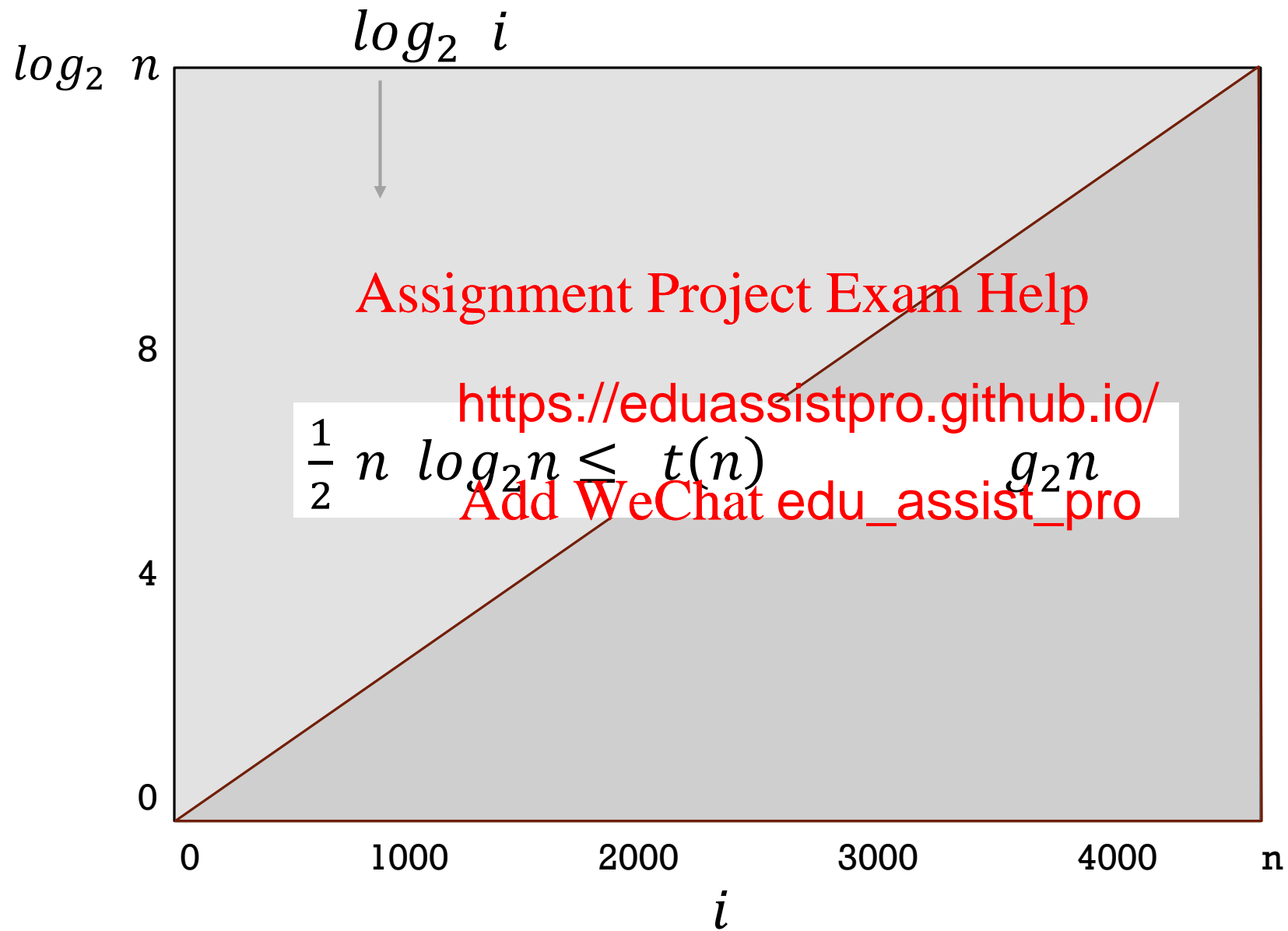
Suppose there are n elements to add, then in the worst case the number of swaps needed to add all the elements is:

$$t(n) = \sum_{i=1}^n \text{floor}(\log_2 i)$$

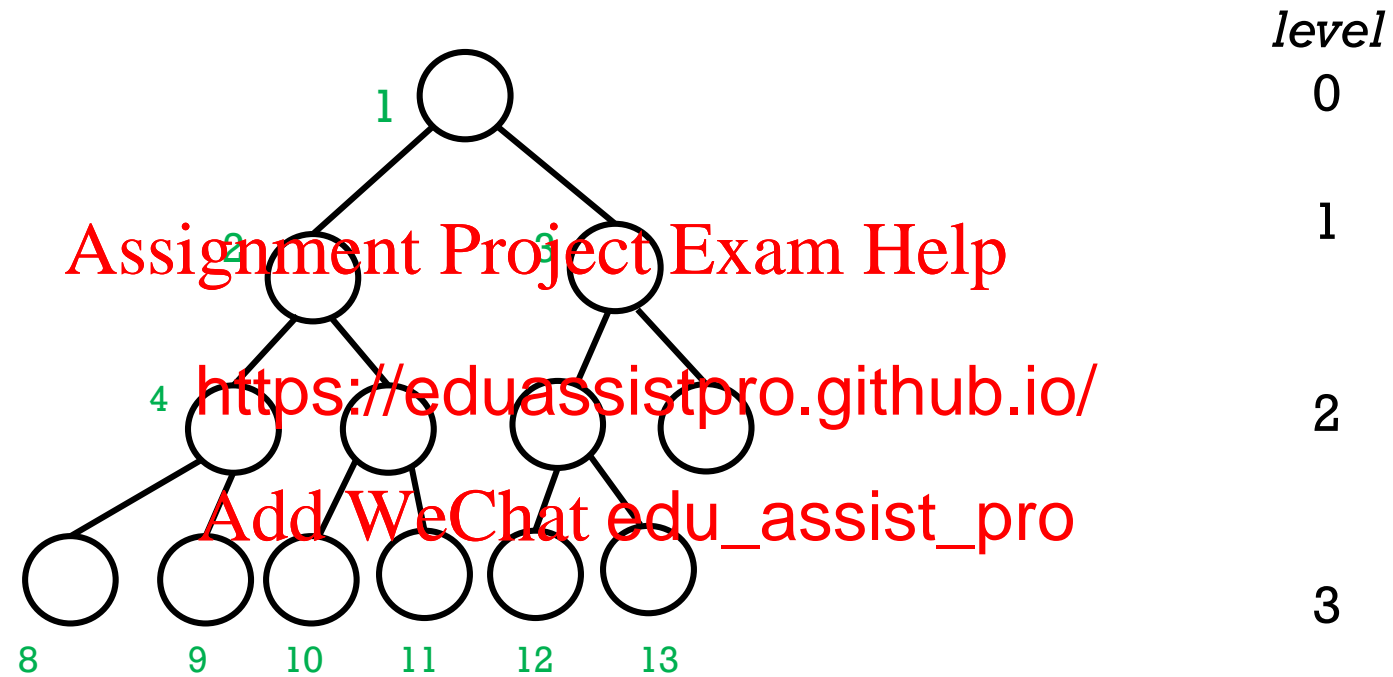








WORST CASE OF BUILDHEAP IS $O(n * \log_2 n)$



Thus, in the worst case scenario for buildHeap() is $O(n * \log n)$

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add()

removeMin()

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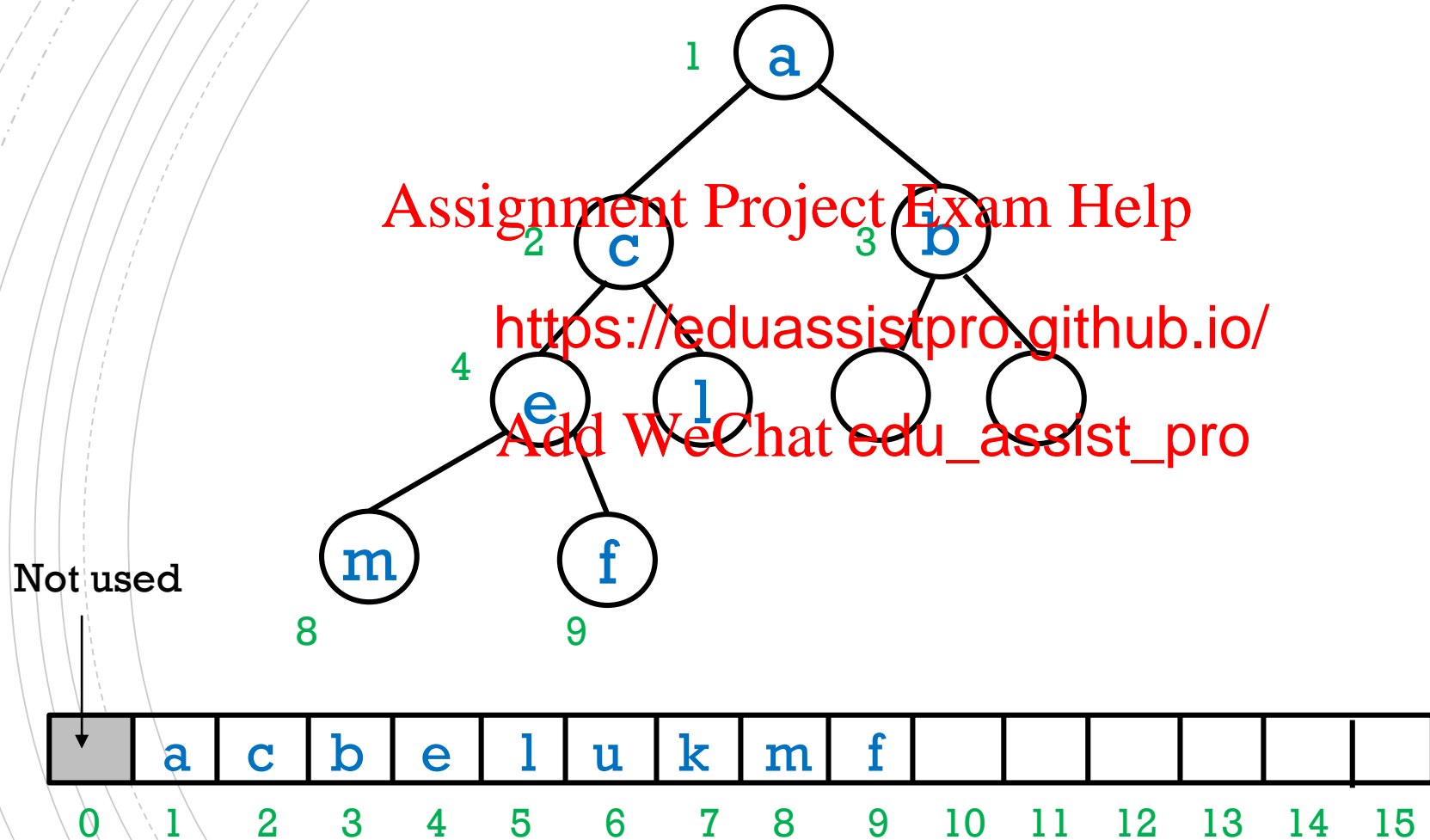
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“upHeap”

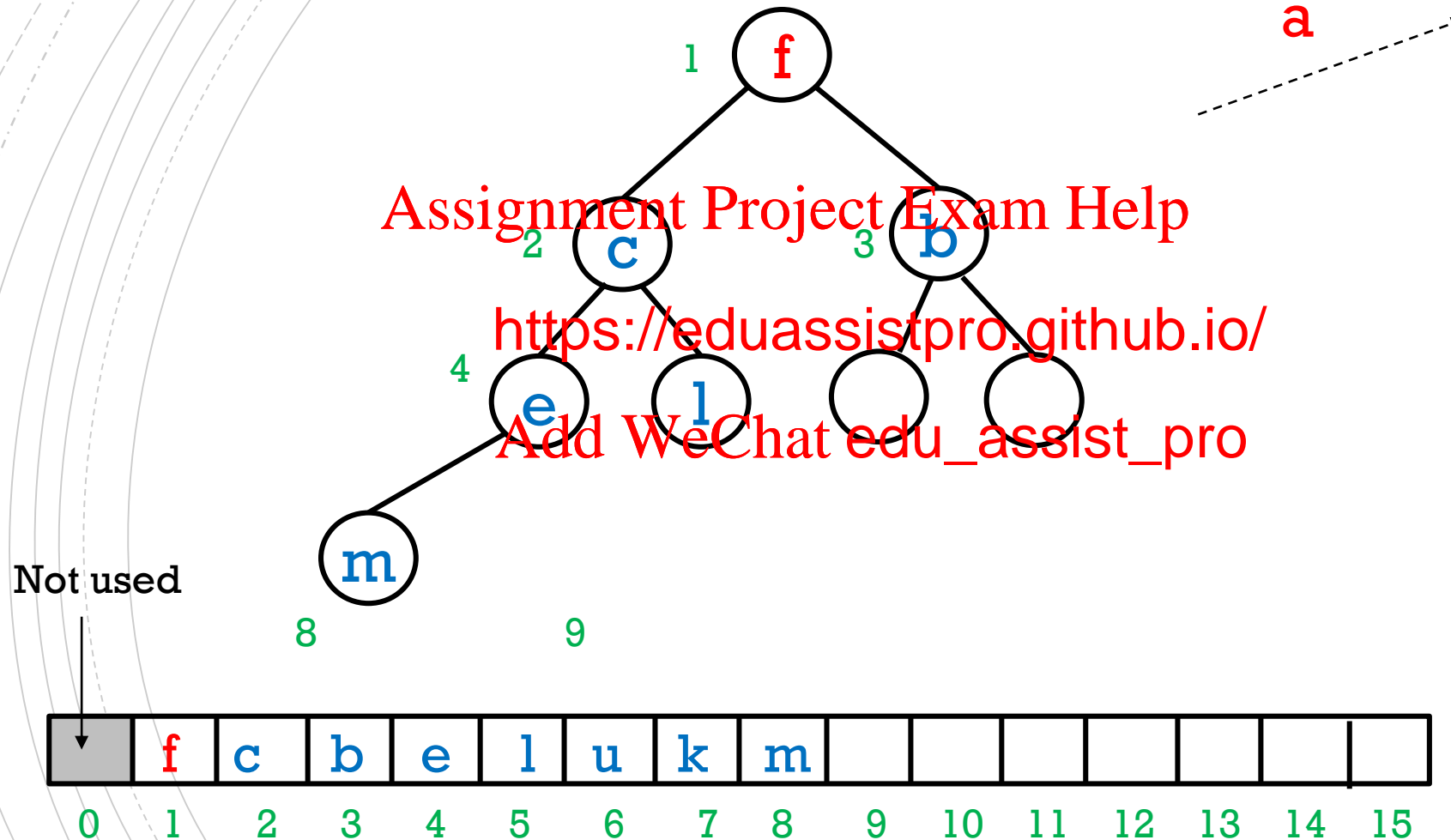
“downHeap”



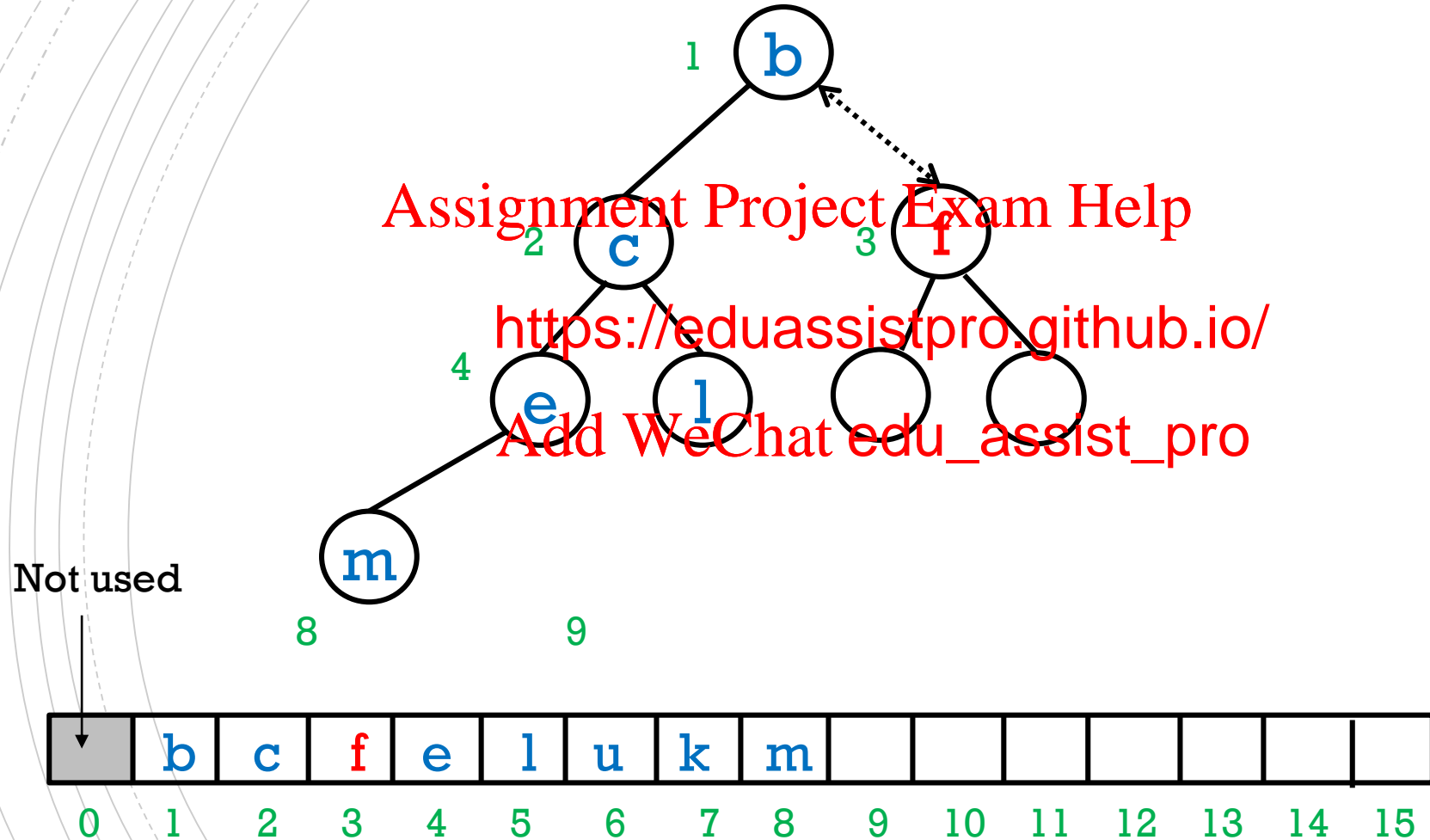
E.G. removeMin()



E.G. removeMin ()



E.G. removeMin()



REMOVEDMIN() - IMPLEMENTATION

Let `heap` be the underlying array, and let `size` be the number of elements in the heap.

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```
removeMin ( ) {  
    tmpElement = heap[size]  
    heap[1] = heap[size]  
    heap[size] = null // not necessary  
  
    return tmpElement  
}
```

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0 not used.

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REMOVEDMIN() - IMPLEMENTATION

Let `heap` be the underlying array, and let `size` be the number of elements in the heap.

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```
removeMin ( ) {  
    tmpElement = heap[size]  
    heap[1] = heap[size]  
    heap[size] = null // not necessary  
    size = size - 1  
    downHeap(1, size)  
    return tmpElement  
}
```

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0 not used.

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DOWNHEAP() - IMPLEMENTATION

```
downHeap( startIndex , maxIndex ){  
    i = startIndex  
    while (2*i <= maxIndex){ // if there is a left child  
        child = 2*i  
  
        }  
    }
```

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DOWNHEAP() - IMPLEMENTATION

```
downHeap( startIndex , maxIndex ){
    i = startIndex
    while (2*i <= maxIndex){ // if there is a left child
        child = 2*i
        if (child < maxIndex // a right sibling
            if (heap[child + 1] < heap[child] // rightchild < leftchild
                child = child + 1
            )
        }
    }
}
```

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DOWNHEAP() - IMPLEMENTATION

```
downHeap( startIndex , maxIndex ){
    i = startIndex
    while (2*i <= maxIndex){ // if there is a left child
        child = 2*i
        if (child < maxIndex // right sibling
            if (heap[child + 1] < heap[child] // rightchild < leftchild
                child = child + 1
        )
        if (heap[child] < heap[i]){ // Do we need to swap with child?
            swapElements(i , child)
            i = child
        } else
            break
    }
}
```

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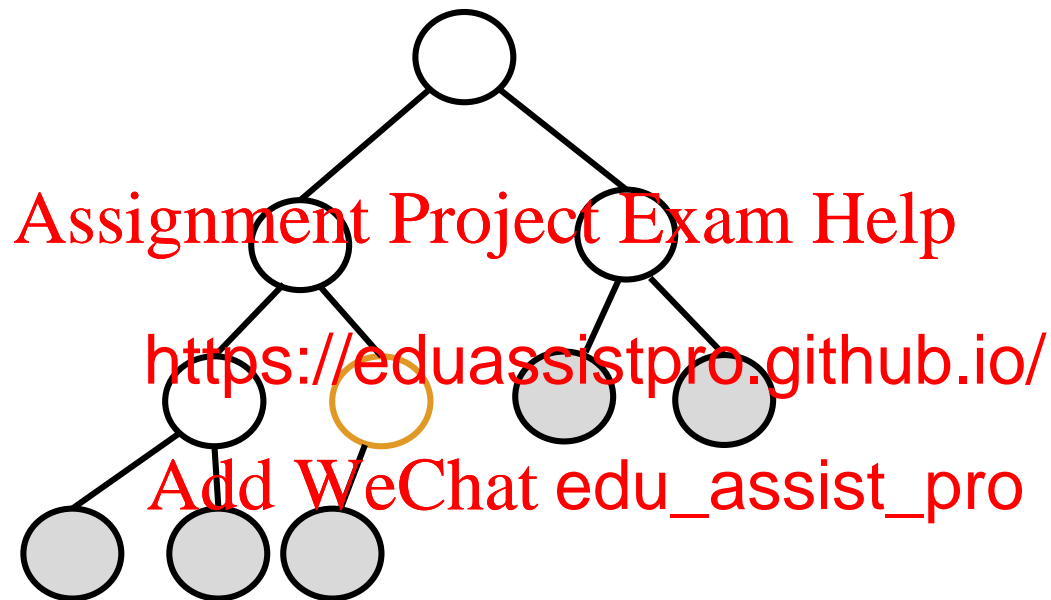
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BUILD A HEAP

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HOW TO BUILD A HEAP ? (FAST)



Observations:

- Half the nodes of a heap are leaves.
(Each leaf is a heap with one node)
- The last non-leaf node has index $\text{size}/2$.

HOW TO BUILD A HEAP ? (FAST)

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```
buildHeapFast() {
```

```
    // assume that the array has size elements
```

```
    for (k = size/2; k >= 1; k
```

```
        downHeap( k, size )
```

```
}
```

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EXAMPLE

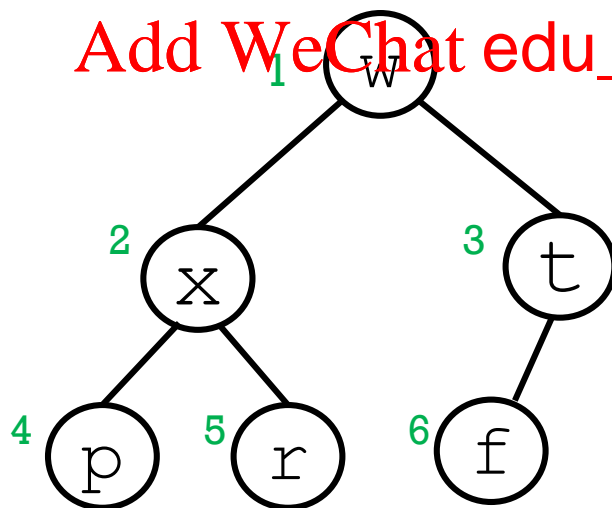
1 2 3 4 5 6

w x t p r f

$k = 3$

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EXAMPLE

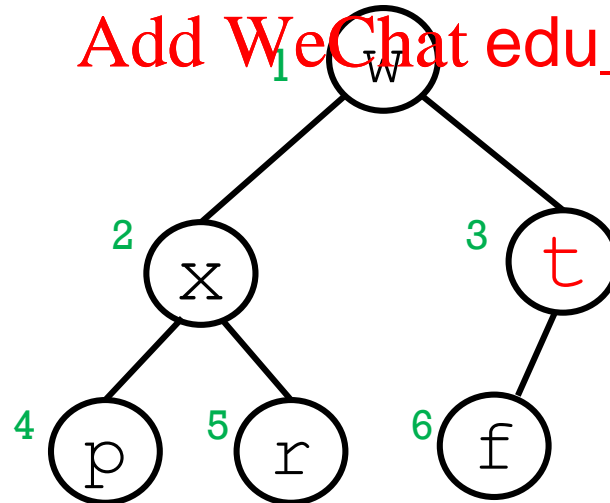
1 2 3 4 5 6

w x p r f

$k = 3$

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downHeap(3, 6)

EXAMPLE

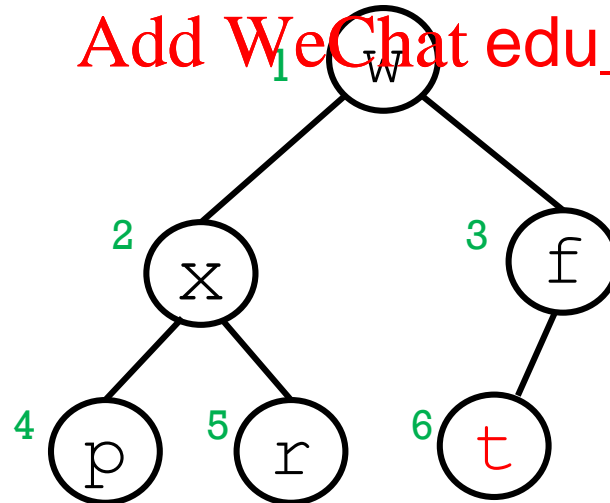
1 2 3 4 5 6

w x f p r

$k = 3$

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downHeap(3, 6)

EXAMPLE

1 2 3 4 5 6

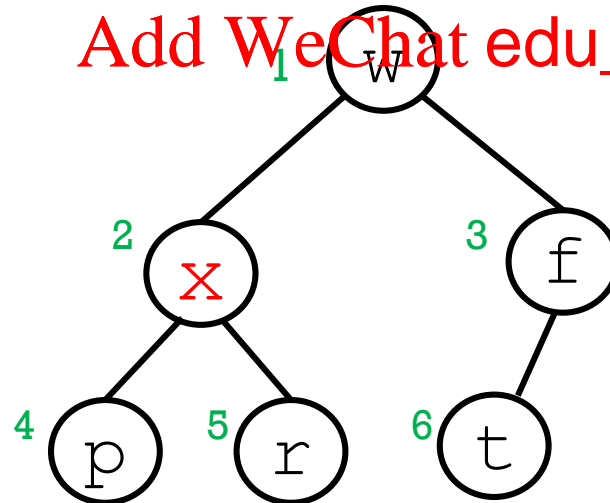
w ~~x~~ f p r t

$k = 2$

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downHeap(2, 6)



EXAMPLE

1 2 3 4 5 6

w p f r t

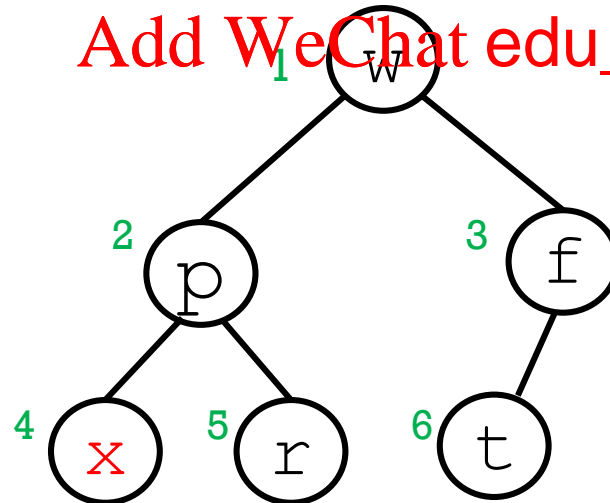
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$k = 2$

downHeap(2, 6)



EXAMPLE

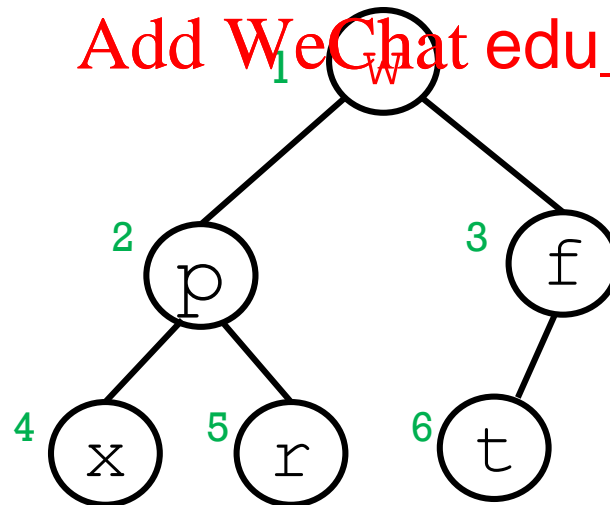
1 2 3 4 5 6

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$k = 1$

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EXAMPLE

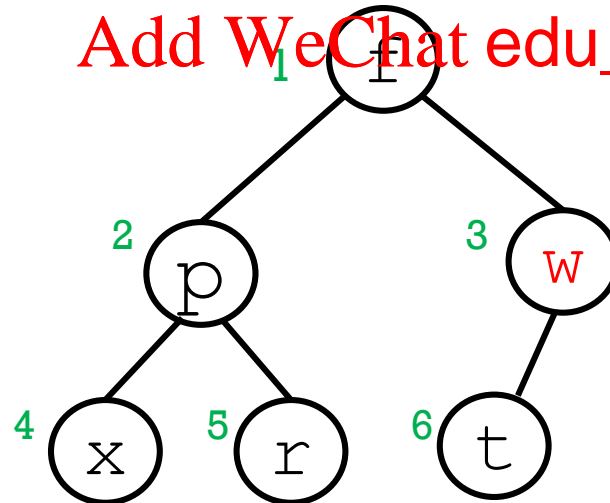
1 2 3 4 5 6

f Assignment Project Exam Help

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$k = 1$

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EXAMPLE

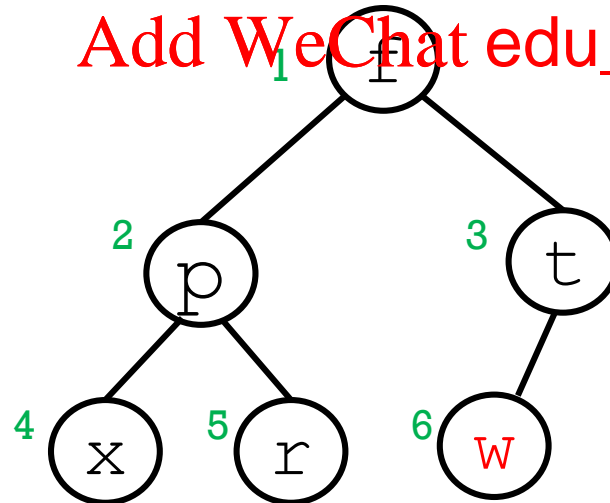
1 2 3 4 5 6

f p t x r w

$k = 1$

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BUILDHEAPFAST() – IMPLEMENTATION

```
buildHeapFast(list) {
```

```
    // copy elements from list to heap array
```

```
    for (k = size
```

```
        downHeap( k, size )
```

```
}
```

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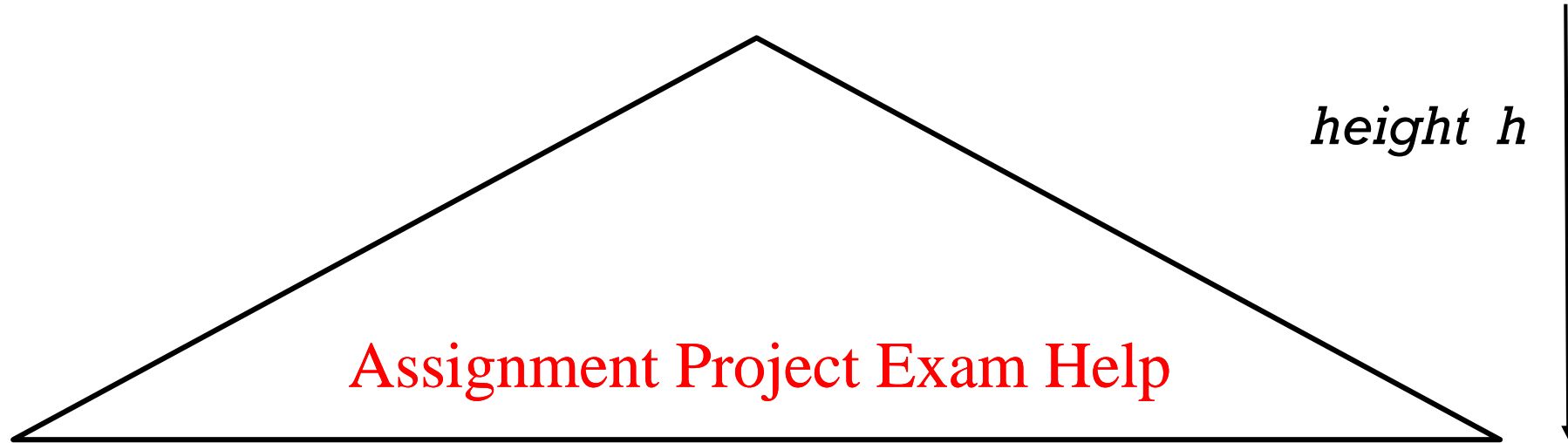
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Claim: this algorithm is $O(n)$.

What is the intuition for why this algorithm is so fast?

We tends to draw binary trees like this:



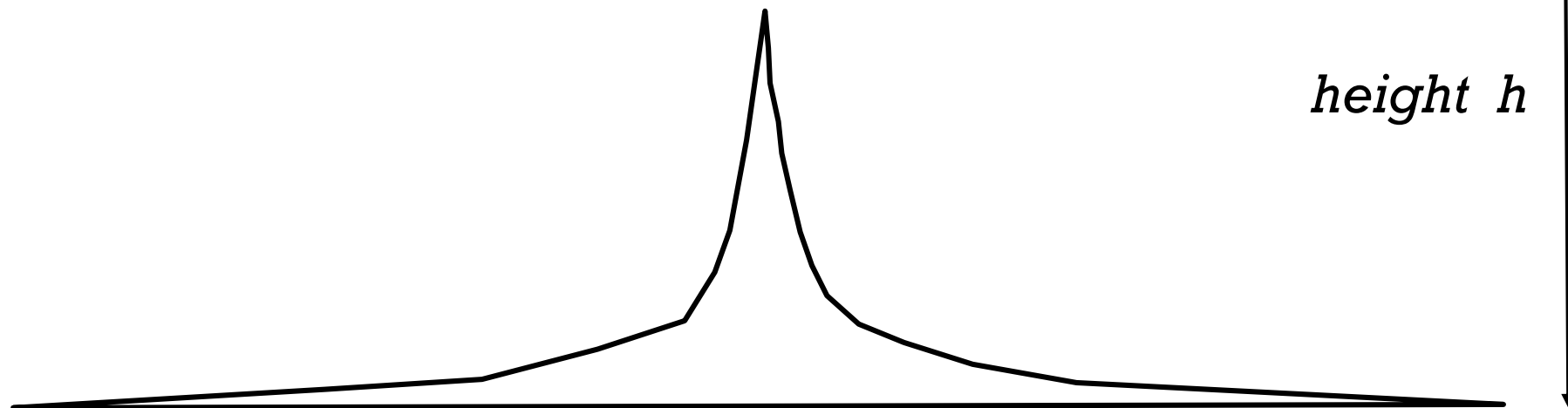
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But the number of nodes double

vel.

So we should draw trees like thi

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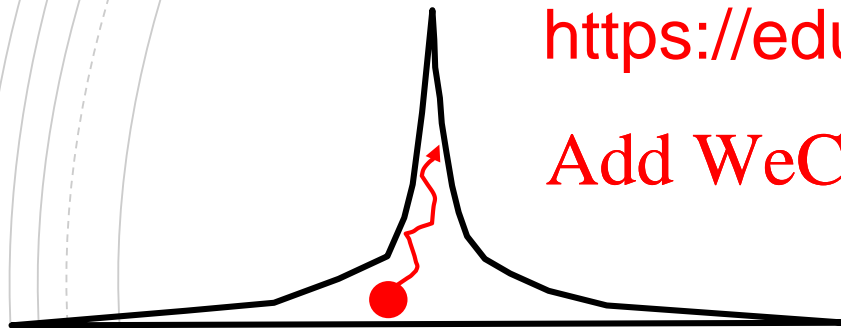


BUILDHEAP ALGORITHMS

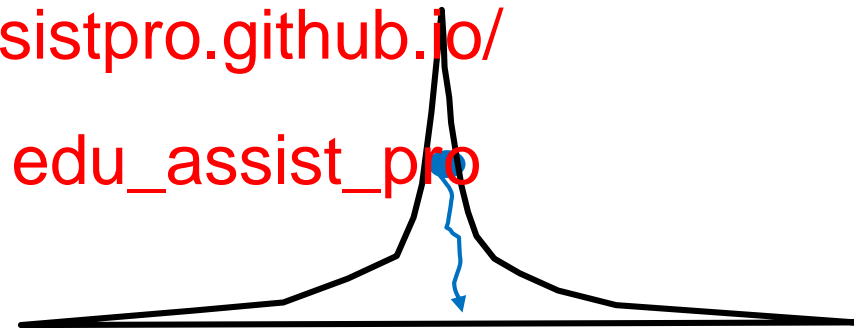
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Most nodes swap $\sim h$
times in worst case.



Few nodes swap $\sim h$
times in worst case.

HOW TO SHOW BUILDHEAPFAST IS $O(n)$?

The worst case number of swaps needed to downHeap node i is the height of that node.

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$t(n)$ = <https://eduassistpro.github.io/>

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$\frac{1}{2}$ of the nodes do no swaps.

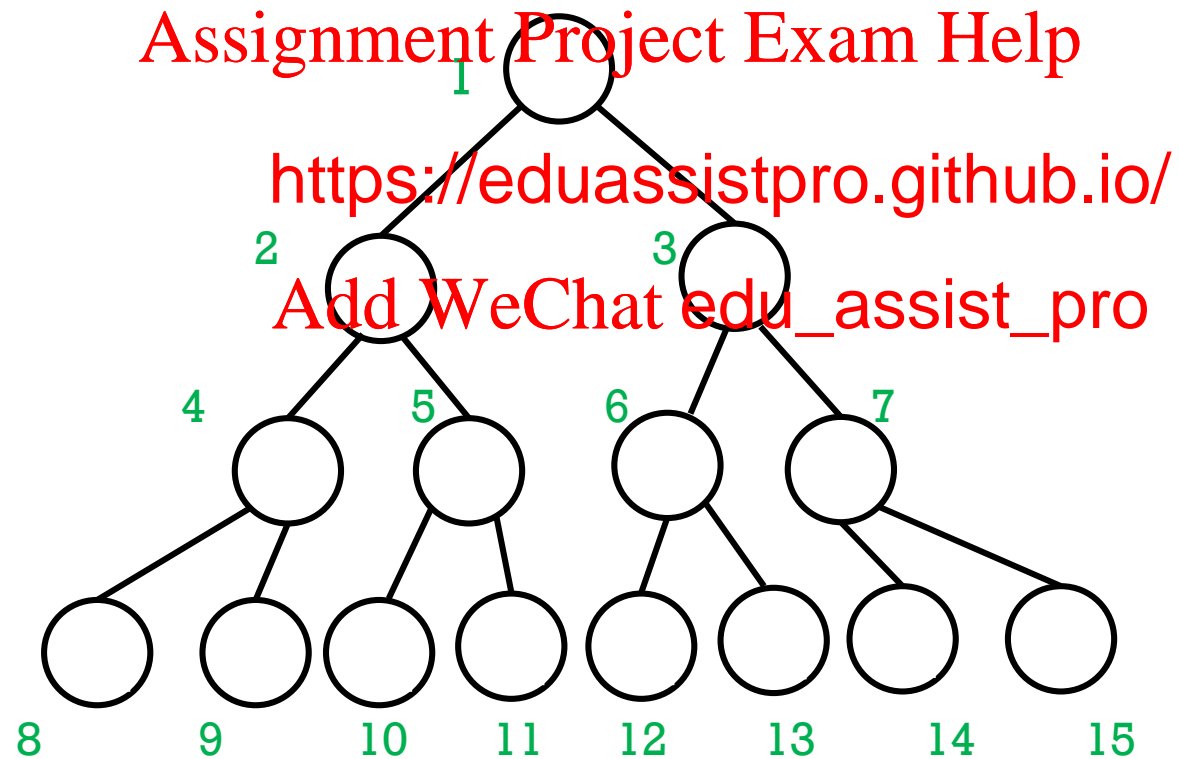
$\frac{1}{4}$ of the nodes do at most one swap.

$\frac{1}{8}$ of the nodes do at most two swaps....

ASSUME THE LAST LEVEL IS FULL

height

level



WORSE CASE OF BUILDHEAPFAST ?

- How many elements at *level* l ? ($l \in 0, \dots, h$)

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- What is the height <https://eduassistpro.github.io/>

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WORSE CASE OF BUILDHEAPFAST ?

- How many elements at *level* l ? ($l \in 0, \dots, h$)

➤ 2^l

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- What is the height of node i ? <https://eduassistpro.github.io/>

➤ $h - l$

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$$t(n) = \sum_{i=1}^n \text{height of node } i$$

= ?

WORSE CASE OF BUILDHEAPFAST ?

- How many elements at *level* l ? ($l \in 0, \dots, h$)

➤ 2^l

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- What is the height of node i ? <https://eduassistpro.github.io/>

➤ $h - l$

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$$t(n) = \sum_{i=1}^n \text{height of node } i$$

$$= \sum_{l=0}^h (h - l) 2^l$$

$$\begin{aligned}
 t_{worstcase}(h) &= \sum_{l=0}^h (h-l) 2^l \\
 &= h \sum_{l=0}^h 2^l - \sum_{l=0}^h l 2^l
 \end{aligned}$$

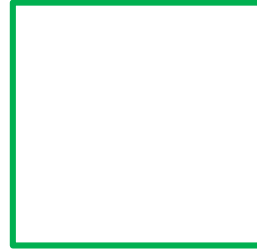
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(number of nodes)

(sum of node levels)



(See next slide)



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Second term index
goes to h-1 only

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Since $n = 2^{h+1} - 1$, we get :

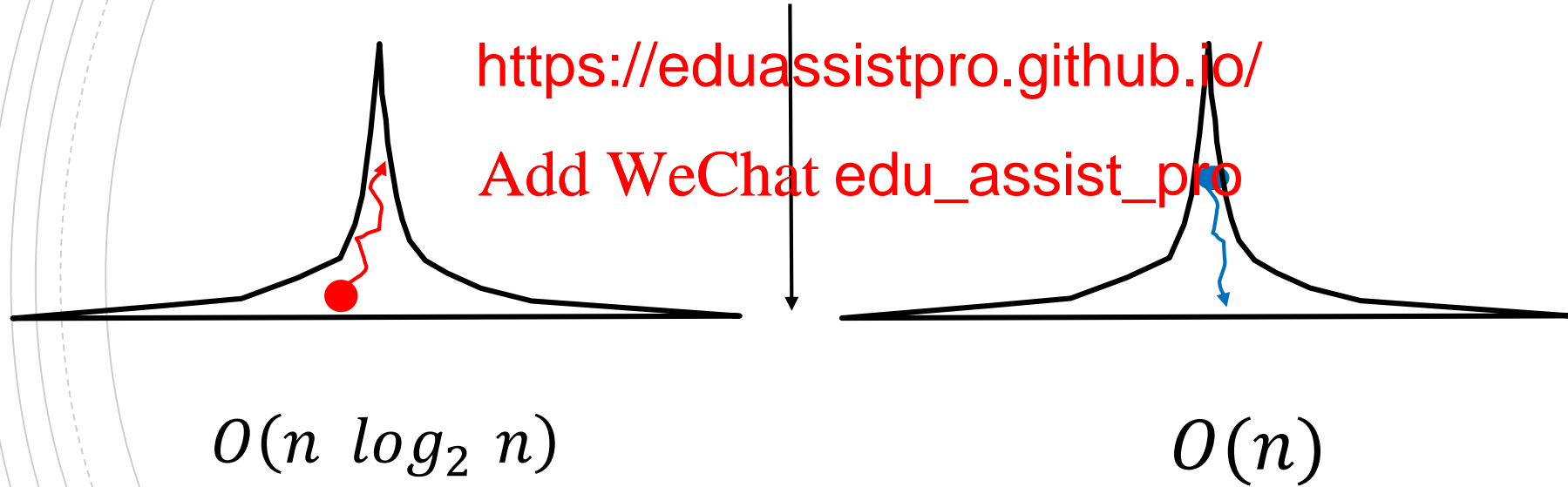
$$t_{worstcase}(n) = n - \log(n + 1)$$

SUMMARY: BUILDHEAP ALGORITHMS

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An orange paint roller with a red handle, positioned horizontally. The roller is in the process of painting a white background, with orange paint splatters and drips visible below the roller's path. The text "Coming Soon" is written in white on the orange surface of the roller.

Coming Soon

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In the next

- Hashing <https://eduassistpro.github.io/>
- Graphs Add WeChat edu_assist_pro