

Assignment Project Exam Help

COMP2610/6261 - Information Theory
Lecture 19: Block Codes and the Coding Theorem

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9 October, 2018

Channel Capacity: Recap

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The largest possible reduction in uncertainty achievable across a channel is its **capacity**

Channel

The capacity

of a channel is the maximum amount of information it can transmit per second, given its input and output for any choice of input ensemble. That is,

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$$C = \max_{\mathbf{P}_X} I(X; Y)$$

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② The N

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③ Extended Channels

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2

The N

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3

Extended Channels

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Suppose we know we have to communicate over some channel Q and we want build s over Q .

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Block Codes

We now consider codes that make repeated use of a noisy channel to communicate a predefined set of messages $S = \{1, 2, \dots, S\}$

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Block Codes

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Recall a ge

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Equivalently, each $s \in S = \{1, 2, \dots, S\}$ is

represented by a sequence of symbols $x \in \mathcal{X}^V$

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Block Codes

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Recall a ge

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Equivalently, each $s \in S = \{1, 2, \dots, S\}$ is represented by a sequence of symbols $x^{(s)} \in \mathcal{X}^N$

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Thus, we can imagine there being S unique codewords $\{\mathbf{x}^{(1)}, \dots, \mathbf{x}^{(S)}\}$, where each codeword has block length N

Block Codes: Example

Suppose $S = \{1, 2, 3, 4\}$

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Message ID s	Message encoding
----------------	------------------

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Block size $N = 2$

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Codewords $\mathbf{x}^{(1)} = 00$, $\mathbf{x}^{(2)} = 01$, and so on

Block Codes: Formally

We formalise the preceding with the following notion:

(N,K) Block Code

Given a channel Q with inputs \mathcal{X} and outputs \mathcal{Y} , an integer $N > 0$, and $K > 0$, an

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where each $\mathbf{x}^{(s)} \in \mathcal{X}^N$ consists of N symb

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The code is parameterised by the length of the block, messages that are encoded

- We parametrise by $K = \log_2 S$ for mathematical convenience
- Doesn't have to be an integer

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An (N, K) block code makes N uses of a channel to transmit one of S possible

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Rate of an (N, K) Block Code

The **rate** of an (N, K) block code is $\frac{\log_2 S}{N}$

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Block Codes: Examples

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Examples (for Binary Symmetric Channel Q)

- A $(1, 1)$ block code: $\mathcal{C} = \{0, 1\}$ — Rate: 1

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- A $(3, \log_2 3)$ block code: $\mathcal{C} = \{001, 0$

$$\overline{3}$$

0.53

A $n \times N$ block code sends each message $s \in S = \{1, 2, \dots, 2^k\}$ over a channel Q as $\mathbf{x}^s \in \mathcal{X}^N$

The recei

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The recei

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Even if $\mathcal{X} = \mathcal{Y}$, the decoder must allow for any codewords $\{\mathbf{x}^{(1)}, \dots, \mathbf{x}^{(2^K)}\}$

Decoding Block Codes: Formally

Block Decoder

A decoder for a (N, K) block code is a mapping that associates each $\mathbf{y} \in \mathcal{Y}^N$ with an $\hat{s} \in \{1, 2, \dots, 2^K\}$.

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Ideally, w

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Optimal Decoder

An optimal decoder for a code S channel $\mathcal{Y} \rightarrow \mathcal{S}$ such that $P(\hat{s}|\mathbf{y})$ is maximal.

That is, $\text{dec}_{opt}(\mathbf{y}) = \arg \max_s P(s|\mathbf{y}) = \arg \max_s P(\mathbf{y}|s) \cdot P(s)$

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Example The $(2, 1)$ block code = 000, 111 and majority vote decoder
 $d : \{0, 1\}$

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$$d(111) = d(110) = d(10)$$

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3 Extended Channels

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Ideally, we would like to have high rates for our channel code

- Lo

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- If N is small, we may be more easily “confuse

How to measure reliability?
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Reliability

Want an *encoder/decoder* pair to **reliably** send a messages over channel Q .



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Reliability

Want an *encoder/decoder* pair to **reliably** send a messages over channel Q .



Probabi

Given a ch

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$$p_B = P(s_{out} \neq s_{in}) = \sum_{s_{in}} P(\quad / \quad)$$

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and its **maximum probability of (block) error**

$$p_{BM} = \max_{s_{in}} P(s_{out} \neq s_{in} | s_{in})$$

Reliability

Want an *encoder/decoder* pair to **reliably** send a messages over channel Q .



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and its **maximum probability of (block) error**

$$p_{BM} = \max_{\mathbf{s}_{in}} P(\mathbf{s}_{out} \neq \mathbf{s}_{in} | \mathbf{s}_{in})$$

As $P(\mathbf{s}_{out} \neq \mathbf{s}_{in} | \mathbf{s}_{in}) \leq p_{BM}$ for all \mathbf{s}_{in} we get $p_B \leq \sum_{\mathbf{s}_{in}} p_{BM} P(\mathbf{s}_{in}) = p_{BM}$ and so if $p_{BM} \rightarrow 0$ then $p_B \rightarrow 0$.

Reliability: Example

Suppose $s \in \{a, b\}$ and we encode by $a \rightarrow 000$ and $b \rightarrow 111$.

To decode we count the number of 1s and 0s and set all bits to the majority count to determine s .

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000, 001, 010, 100 $\rightarrow a$ and 111, 110, 101, 011 $\rightarrow b$

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Reliability: Example

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$$\underline{\underline{000, 001, 010, 100}} \xrightarrow{a} \text{ and } \underline{\underline{111, 110, 101, 011}} \xrightarrow{b}$$

If the chan

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$$B = (in \neq out)$$

$$= P(y \in B | 000) p_a + P(y \in B | 111) p_b$$

$$= f^3 + 3f^2(1-f).$$

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$$= f^3 + 3f^2(1-f).$$

In fact,

$$p_{BM} = \max(P(y \in B | 000), P(y \in A | 111)) = f^3 + 3f^2(1-f).$$

Achievable Rates

Ideally, we would like to consider rates of transmission for which we can guarantee small maximum probability of block error

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Even more ideally, we would like rates for which we can guarantee arbitrary

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Achievable Rate

A rate R over a channel Q is said to be ac-

is a (N, K) block code and decoder such that its $\max_{\mathbf{s}_{in}} P(\mathbf{s}_{out} \neq \mathbf{s}_{in} | \mathbf{s}_{in}) < \epsilon$

maximum probability of block error satisfies

$$P_{BM} = \max_{\mathbf{s}_{in}} P(\mathbf{s}_{out} \neq \mathbf{s}_{in} | \mathbf{s}_{in}) < \epsilon$$

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Achievable rates sound nice in theory, but surely they cannot exist?

- Surely we will have to drive $R \rightarrow 0$ to get small error probability?

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Achievable rates sound nice in theory, but surely they cannot exist?

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Remark

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Noisy-Channel Coding Theorem (Brief)

If Q is a channel with capacity C then the rate

if $R \leq C$, that is, the rate is no greater than the channel

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The Noisy-Channel Coding Theorem

Example

Example:

- In last lecture: RSCC with $\gamma = 0.15$ has capacity $C = 0.39$ bits

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- Suppose $\gamma = 0.25$

- The <https://eduassistpro.github.io> (N, K) code with $K/N \geq 0.25$

Indeed, we showed the code $S = \{000, 1\}$

has probability of error $0.028 < 0.15$ for Q

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The Noisy-Channel Coding Theorem

Example

Example:

- In last lecture: RSC Q with $\gamma = 0.15$ has capacity $C = 0.39$ bits

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- Suppose $\epsilon = 0.25$

- The <https://eduassistpro.github.io> website provides an (N, K) code with $K/N \geq 0.25$

Indeed, we showed the code $S = \{000, 111\}$

has probability of error $0.028 < 0.15$ for Q

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- For $N = 3$ there is a $(3, 1)$ code meeting the requirements.
- However, there is *no code* with same ϵ and rate $1/2 > 0.39 = C$.

The Noisy Typewriter Channel

This channel simulates a noisy “typewriter”. Inputs and outputs are 26 letters A through Z plus space. With probability $\frac{1}{3}$, each letter is either: unchanged; changed to the next letter; changed to the previous letter.

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<https://eduassistpro.github.io>

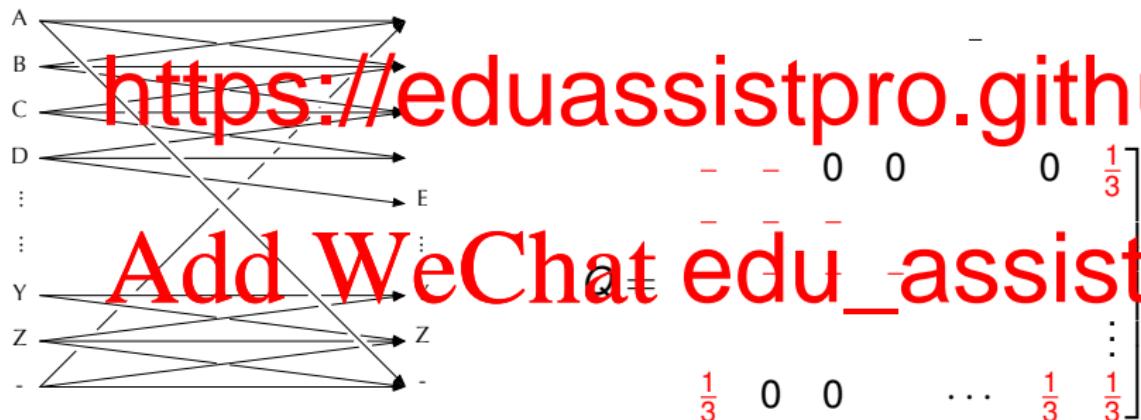
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Inputs = A, B, . . . , Z, - ;

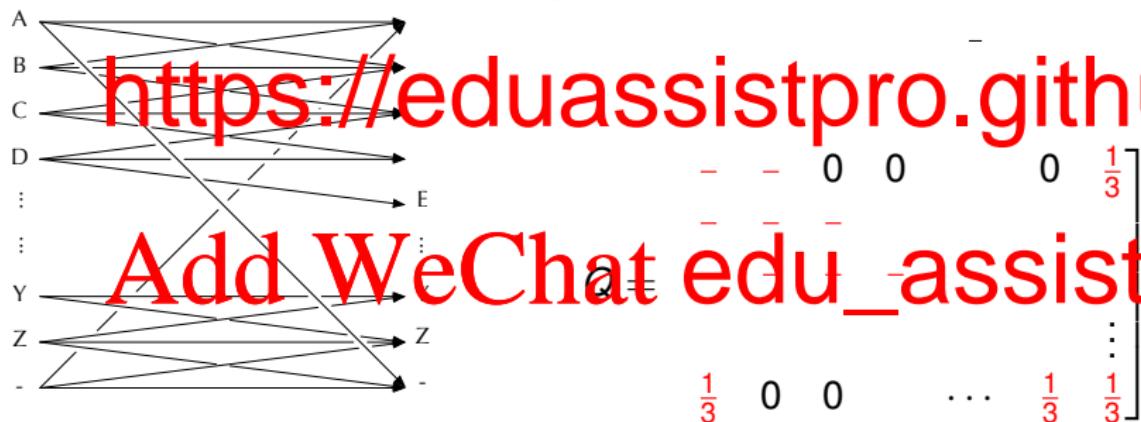


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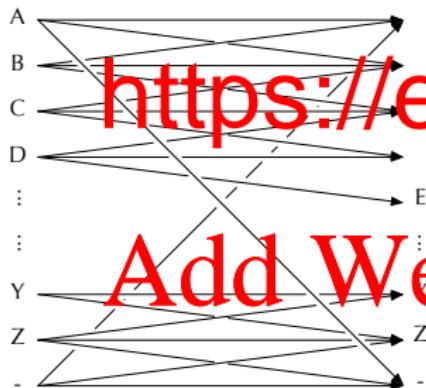
The transition matrix for this channel has a **diagonal structure**: all of the probability mass is concentrated around the diagonal.

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The transition matrix for this channel has a **diagonal structure**: all of the probability mass is concentrated around the diagonal.

Noisy Channel Coding Theorem

NCCT

Any rate $R < C$ is achievable for Q (i.e., for any tolerance $\epsilon > 0$, an (N, K) code with rate $K/N \geq R$ exists with max. block error $p_{BM} < \epsilon$)

Consider

For noisy

- Th
- For any $\epsilon > 0$ and $R < C$
we can choose $N = 1$.
- ... and code messages
using $\mathcal{C} = \{B, E, \dots, Z\}$



Noisy Channel Coding Theorem

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Since $|\mathcal{C}| = 9$ we have $K = \log_2 9$ so $K/N = \log_2 9 \geq R$ for any $R < C$,
and \mathcal{C} has zero error so $p_{BM} = 0 < \epsilon$

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② The N

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③ Extended Channels

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The main “trick” to minimising p_{BM} is to construct a (N, K) block code with (almost)

- A code with low error probability

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The main “trick” to minimising p_{BM} is to construct a (N, K) block code with (almost)

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This is possible because extended channels look like

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Extended Channels

When used N times, a channel Q from \mathcal{X} to \mathcal{Y} can be seen as an *extended channel* taking “symbols” from \mathcal{X}^N to “symbols” in \mathcal{Y}^N .

Extended Channel

The N^{th} ~~extended channel~~ of Q from \mathcal{X} to \mathcal{Y} is a channel from \mathcal{X}^N to \mathcal{Y}^N
with trans

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$n=1$

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$n=1$

Example: BSC Q with $i = 0, 1$ from $\mathcal{X} = \{0, 1\}$
extended channel from $\mathcal{X}^2 = \{00, 01, 10, 11\}$

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$$Q_2 = \begin{bmatrix} 0.81 & 0.09 & 0.09 & 0.01 \\ 0.09 & 0.81 & 0.01 & 0.09 \\ 0.09 & 0.01 & 0.81 & 0.09 \\ 0.01 & 0.09 & 0.09 & 0.81 \end{bmatrix}$$

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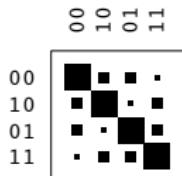
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Extended Channels and the Noisy Typewriter

As N increases, any extended channel looks like the noisy typewriter!

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0 1

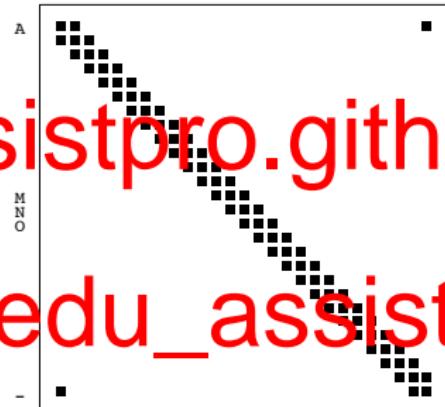
<https://eduassistpro.github.io>

1

$N = 1$

Extended Binary Symmetric Channel

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z -



Noisy Typewriter Channel

Extended Channels and the Noisy Typewriter

As N increases, any extended channel looks like the noisy typewriter!

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0 0 1 0 1
0 1 0 1

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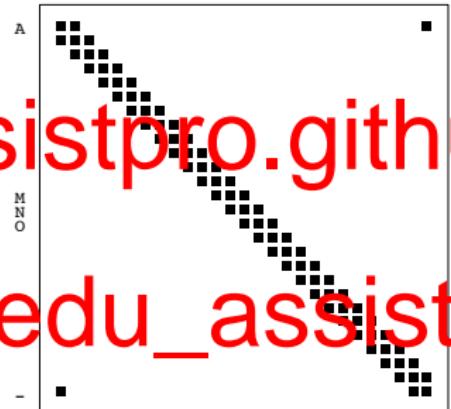
01

11

$N = 2$

Extended Binary Symmetric Channel

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z



Noisy Typewriter Channel

Extended Channels and the Noisy Typewriter

As N increases, any extended channel looks like the noisy typewriter!

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0000 0001 0010 0011 0100 0101 0110 0111
0100 0101 0110 0111 0000 0001 0010 0011
0110 0111 0000 0001 0010 0011 0100 0101
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0246 0247 0248 0249 0040 0041 0052 0053
0248 0249 0040 0041 0052 0053 0246 0247
0041 0042 0053 0054 0250 0251 0252 0253
0250 0251 0252 0253 0041 0042 0053 0054
0252 0253 0041 0042 0053 0054 0250 0251
0042 0043 0054 0055 0254 0255 0256 0257
0254 0255 0256 0257 0042 0043 0054 0055
0256 0257 0042 0043 0054 0055 0254 0255
0043 0044 0055 0056 0258 0259 0260 0261
0258 0259 0260 0261 0043 0044 0055 0056
0260 0261 0043 0044 0055 0056 0258 0259
0044 0045 0056 0057 0262 0263 0264 0265
0262 0263 0264 0265 0044 0045 0056 0057
0264 0265 0044 0045 0056 0057 0262 0263
0045 0046 0057 0058 0266 0267 0268 0269
0266 0267 0268 0269 0045 0046 0057 0058
0268 0269 0045 0046 0057 0058 0266 0267
0046 0047 0058 0059 0270 0271 0272 0273
0270 0271 0272 0273 0046 0047 0058 0059
0272 0273 0046 0047 0058 0059 0270 0271
0047 0048 0059 0060 0274 0275 0276 0277
0274 0275 0276 0277 0047 0048 0059 0060
0276 0277 0047 0048 0059 0060 0274 0275
0048 0049 0060 0061 0278 0279 0280 0281
0278 0279 0280 0281 0048 0049 0060 0061
0280 0281 0048 0049 0060 0061 0278 0279
0049 0050 0061 0062 0282 0283 0284 0285
0282 0283 0284 0285 0049 0050 0061 0062
0284 0285 0049 0050 0061 0062 0282 0283
0050 0051 0062 0063 0286 0287 0288 0289
0286 0287 0288 0289 0050 0051 0062 0063
0288 0289 0050 0051 0062 0063 0286 0287
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0290 0291 0292 0293 0051 0052 0063 0064
0292 0293 0051 0052 0063 0064 0290 0291
0052 0053 0064 0065 0294 0295 0296 0297
0294 0295 0296 0297 0052 0053 0064 0065
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0298 0299 0300 0301 0053 0054 0065 0066
0300 0301 0053 0054 0065 0066 0298 0299
0054 0055 0066 0067 0302 0303 0304 0305
0302 0303 0304 0305 0054 0055 0066 0067
0304 0305 0054 0055 0066 0067 0302 0303
0055 0056 0067 0068 0306 0307 0308 0309
0306 0307 0308 0309 0055 0056 0067 0068
0308 0309 0055 0056 0067 0068 0306 0307
0056 0057 0068 0069 0310 0311 0312 0313
0310 0311 0312 0313 0056 0057 0068 0069
0312 0313 0056 0057 0068 0069 0310 0311
0057 0058 0069 0070 0314 0315 0316 0317
0314 0315 0316 0317 0057 0058 0069 0070
0316 0317 0057 0058 0069 0070 0314 0315
0058 0059 0070 0071 0318 0319 0320 0321
0318 0319 0320 0321 0058 0059 0070 0071
0320 0321 0058 0059 0070 0071 0318 0319
0059 0060 0071 0072 0322 0323 0324 0325
0322 0323 0324 0325 0059 0060 0071 0072
0324 0325 0059 0060 0071 0072 0322 0323
0060 0061 0072 0073 0326 0327 0328 0329
0326 0327 0328 0329 0060 0061 0072 0073
0328 0329 0060 0061 0072 0073 0326 0327
0061 0062 0073 0074 0330 0331 0332 0333
0330 0331 0332 0333 0061 0062 0073 0074
0332 0333 0061 0062 0073 0074 0330 0331
0062 0063 0074 0075 0334 0335 0336 0337
0334 0335 0336 0337 0062 0063 0074 0075
0336 0337 0062 0063 0074 0075 0334 0335
0063 0064 0075 0076 0338 0339 0340 0341
0338 0339 0340 0341 0063 0064 0075 0076
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0342 0343 0344 0345 0064 0065 0076 0077
0344 0345 0064 0065 0076 0077 0342 0343
0065 0066 0077 0078 0346 0347 0348 0349
0346 0347 0348 0349 0065 0066 0077 0078
0348 0349 0065 0066 0077 0078 0346 0347
0066 0067 0078 0079 0350 0351 0352 0353
0350 0351 0352 0353 0066 0067 0078 0079
0352 0353 0066 0067 0078 0079 0350 0351
0067 0068 0079 0080 0354 0355 0356 0357
0354 0355 0356 0357 0067 0068 0079 0080
0356 0357 0067 0068 0079 0080 0354 0355
0068 0069 0080 0081 0358 0359 0360 0361
0358 0359 0360 0361 0068 0069 0080 0081
0360 0361 0068 0069 0080 0081 0358 0359
0069 0070 0081 0082 0362 0363 0364 0365
0362 0363 0364 0365 0069 0070 0081 0082
0364 0365 0069 0070 0081 0082 0362 0363
0070 0071 0082 0083 0366 0367 0368 0369
0366 0367 0368 0369 0070 0071 0082 0083
0368 0369 0070 0071 0082 0083 0366 0367
0071 0072 0083 0084 0370 0371 0372 0373
0370 0371 0372 0373 0071 0072 0083 0084
0372 0373 0071 0072 0083 0084 0370 0371
0072 0073 0084 0085 0374 0375 0376 0377
0374 0375 0376 0377 0072 0073 0084 0085
0376 0377 0072 0073 0084 0085 0374 0375
0073 0074 0085 0086 0378 0379 0380 0381
0378 0379 0380 0381 0073 0074 0085 0086
0380 0381 0073 0074 0085 0086 0378 0379
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0382 0383 0384 0385 0074 0075 0086 0087
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0078 0079 0090 0091 0398 0399 0400 0401
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0079 0080 0091 0092 0402 0403 0404 0405
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0412 0413 0081 0082 0093 0094 0410 0411
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0416 0417 0082 0083 0094 0095 0414 0415
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0420 0421 0083 0084 0095 0096 0418 0419
0084 0085 0096 0097 0422 0423 0424 0425
0422 0423 0424 0425 0084 0085 0096 0097
0424 0425 0084 0085 0096 0097 0422 0423
0085 0086 0097 0098 0426 0427 0428 0429
0426 0427 0428 0429 0085 0086 0097 0098
0428 0429 0085 0086 0097 0098 0426 0427
0086 0087 0098 0099 0430 0431 0432 0433
0430 0431 0432 0433 0086 0087 0098 0099
0432 0433 0086 0087 0098 0099 0430 0431
0087 0088 0099 0100 0434 0435 0436 0437
0434 0435 0436 0437 0087 0088 0099 0100
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0088 0089 0099 0100 0438 0439 0440 0441
0438 0439 0440 0441 0088 0089 0099 0100
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0089 0090 0099 0100 0442 0443 0444 0445
0442 0443 0444 0445 0089 0090 0099 0100
0444 0445 0089 0090 0099 0100 0442 0443
0090 0091 0099 0100 0446 0447 0448 0449
0446 0447 0448 0449 0090 0091 0099 0100
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0091 0092 0099 0100 0450 0451 0452 0453
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0092 0093 0099 0100 0454 0455 0456 0457
0454 0455 0456 0457 0092 0093 0099 0100
0456 0457 0092 0093 0099 0100 0454 0455
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0098 0099 0099 0100 0478 0479 0480 0481
0478 0479 0480 0481 0098 0099 0099 0100
0480 0481 0098 0099 0099 0100 0478 0479
0099 00100 0099 0100 0482 0483 0484 0485
0482 0

Extended Channels and the Noisy Typewriter

As N increases, any extended channel looks like the noisy typewriter!

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0 1

<https://eduassistpro.github.io>

1



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$N = 1$

Extended Z Channel

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z



Noisy Typewriter Channel

Extended Channels and the Noisy Typewriter

As N increases, any extended channel looks like the noisy typewriter!

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0 0 1 0 1
0 1 0 1

<https://eduassistpro.github.io>

0 1

1 1

$N = 2$

Extended Z Channel

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z



Noisy Typewriter Channel

Extended Channels and the Noisy Typewriter

As N increases, any extended channel looks like the noisy typewriter!

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0000
0100
1100
1010
1110
0001
1001
1101
0011
1011
0111
1111



$N = 4$

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Extended Z Channel

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z -



Noisy Typewriter Channel

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Why does this happen?

Remem

typical

art looking

<https://eduassistpro.github.io>

For a given \mathbf{x} , the corresponding $p(\mathbf{y} \mid \mathbf{x})$ will also be concentrated on a few sequences

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Formalising this will require a notion of joint typicality

Summary and Reading

Main Points

- The Noisy Typewriter
- Extended Channels

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- Blo
- The

<https://eduassistpro.github.io>

Reading

- Mackay §9.6
- Cover & Thomas §7.5

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Next time: Detail of the NCCT, joint typicality, and a sketch of the proof!