COMP2610 – Information Theory

Lecture 12: The Source Coding Theorem

Australian National University

28 August 2018

Last time

Basic goal of compression

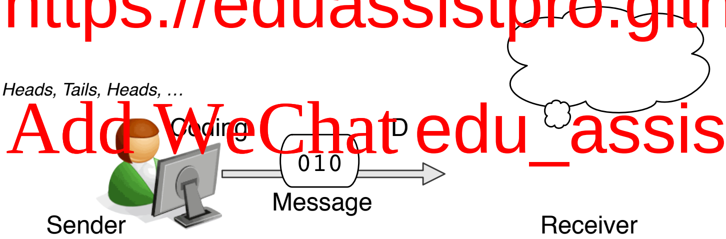
Key concepts: codes and their types, raw bit content, essential bit content

Informal statement of source coding theorem

A General Communication Game (Recap)

Data compression is the process of replacing a message with a smaller message which can be reliably converted back to the original.

- Want small messages on average when outcomes are from a fixed, kno



Definitions (Recap)

Source Code

Given an ensemble X, the function $c : \mathcal{A}_X \to \mathcal{B}$ is a **source code** for X.
The number of symbols in $c(x)$ is the **length** $l(x)$ of the codeword for x.
The **extension** of c is defined by $c(x_1 \ldots x_n) = c(x_1) \ldots c(x_n)$

Definitions (Recap)

Source Code

Given an ensemble X, the function $c : \mathcal{A}_X \to \mathcal{B}$ is a **source code** for X.
The number of symbols in $c(x)$ is the **length** $l(x)$ of the codeword for x.
The **extension** of c is defined by $c(x_1 \ldots x_n) = c(x_1) \ldots c(x_n)$

Smalles [illegible]

Let X be [illegible] bset of $\mathcal{A}_X$ such that

$$P(x \in S_\delta) \geq$$ [illegible]

Definitions (Recap)

Source Code

Given an ensemble X, the function $c : \mathcal{A}_X \to \mathcal{B}$ is a **source code** for X.
The number of symbols in $c(x)$ is the **length** $l(x)$ of the codeword for x.
The **extension** of c is defined by $c(x_1 \ldots x_n) = c(x_1) \ldots c(x_n)$

Smalles [illegible]

Let X be [illegible] bset of $\mathcal{A}_X$ such that

$$P(x \in S_\delta) \geq \text{[illegible]}$$

Essential Bit Content

Let X be an ensemble then for $\delta \geq 0$ the **essential bit content** of X is

$$H_\delta(X) \stackrel{\text{def}}{=} \log_2 |S_\delta|$$

Essential Bit Content (Recap)

Intuitively, construct S_δ by removing elements of X in ascending order of probability, till we have reached the $1-\delta$ threshold

$\mathbf{x}$	$P(\mathbf{x})$
a	
b	
c	
d	3/16
e	1/64
f	1/64
g	1/64
h	1/64

- $a_i)$
  $) \geq 1-\delta$

Essential Bit Content (Recap)

Intuitively, construct S_δ by removing elements of X in ascending order of probability, till we have reached the $1 - \delta$ threshold

$\mathbf{x}$	$P(\mathbf{x})$
a	[illegible]
b	[illegible]
c	[illegible]
d	3/16
e	1/64
f	1/64
g	1/64

- $a_i)$

  $) \geq 1 - \delta$

$\delta = 1/64$: $S_\delta = \{\mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{d}, \mathtt{e}, \mathtt{f}, \mathtt{g}$

Essential Bit Content (Recap)

Intuitively, construct S_δ by removing elements of X in ascending order of probability, till we have reached the $1-\delta$ threshold

$\mathbf{x}$	$P(\mathbf{x})$
a	
b	
c	
d	3/16

- $a_i)$

$) \geq 1 - \delta$

$\delta = 1/64$: $S_\delta = \{\mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{d}, \mathtt{e}, \mathtt{f}, \mathtt{g}$

$\delta = 1/16$: $S_\delta = \{\mathtt{a},$

Essential Bit Content (Recap)

Intuitively, construct S_δ by removing elements of X in ascending order of probability, till we have reached the $1-\delta$ threshold

$\mathbf{x}$	$P(\mathbf{x})$
a	

- $a_i)$

  $) \geq 1 - \delta$

  $\delta = 1/64$: $S_\delta = \{\mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{d}, \mathtt{e}, \mathtt{f}, \mathtt{g}$

  $\delta = 1/16$: $S_\delta = \{\mathtt{a},$

  $\delta = 3/4$: $S_\delta = \{\mathtt{a}$

Lossy Coding (Recap)

Consider a coin with $P(Heads) = 0.9$

If we are ha
sequenc

There are

So, we can just code those, and **ignore** th

- Coding 10 outcomes with 2% failure doable wi
  bits/outcome

This time

Recap: typical sets

Formal statement of source coding theorem

Proof of source coding theorem

The Source Coding Theorem

(Theorem 4.1 in MacKay)

Our aim this week is to understand this:

The Source Coding Theorem

Let X be an ensemble with entropy $H = H(X)$ bits. Given $\epsilon > 0$ and $0 < \delta <$ ... $> N_0$

The Source Coding Theorem
(Theorem 4.1 in MacKay)

Our aim this week is to understand this:

The Source Coding Theorem

Let X be an ensemble with entropy $H = H(X)$ bits. Given $\epsilon > 0$ and $0 < \delta <$ … $> N_0$

In English:

- Given outcomes drawn from X …

The Source Coding Theorem

(Theorem 4.1 in MacKay)

Our aim this week is to understand this:

The Source Coding Theorem

Let X be an ensemble with entropy $H = H(X)$ bits. Given $\epsilon > 0$ and $0 < \delta <$ … $> N_0$

In English:

- Given outcomes drawn from X …
- … no matter what *reliability* $1 - \delta$ an

The Source Coding Theorem

(Theorem 4.1 in MacKay)

Our aim this week is to understand this:

The Source Coding Theorem

Let X be an ensemble with entropy $H = H(X)$ bits. Given $\epsilon > 0$ and $0 < \delta <$ $> N_0$

$-$

In English:

- Given outcomes drawn from X ...
- ... no matter what *reliability* $1 - \delta$ an
- ... there is always a length N_0 so sequences X^N longer than this ...

The Source Coding Theorem

(Theorem 4.1 in MacKay)

Our aim this week is to understand this:

The Source Coding Theorem

Let X be an ensemble with entropy $H = H(X)$ bits. Given $\epsilon > 0$ and $0 < \delta <$ ... $> N_0$

In English:

- Given outcomes drawn from X ...
- ... no matter what *reliability* $1 - \delta$ an
- ... there is always a length N_0 so sequences X^N longer than this ...
- ... have an average essential bit content $\frac{1}{N}H_\delta(X^N)$ within ϵ of $H(X)$

The Source Coding Theorem

(Theorem 4.1 in MacKay)

Our aim this week is to understand this:

The Source Coding Theorem

Let X be an ensemble with entropy $H = H(X)$ bits. Given $\epsilon > 0$ and $0 < \delta <$ ... $> N_0$

In English:

- Given outcomes drawn from X ...
- ... no matter what *reliability* $1 - \delta$ an
- ... there is always a length N_0 so sequences X^N longer than this ...
- ... have an average essential bit content $\frac{1}{N} H_\delta(X^N)$ within ϵ of $H(X)$

$H_\delta(X^N)$ measures the *fewest* number of bits needed to uniformly code *smallest* set of N-outcome sequence S_δ with $P(x \in S_\delta) \geq 1 - \delta$.

1. Introduction
   - Quick Review

2. Extended Ensembles
   - De
   - Es
   - Th

3. The Source Coding Theorem
   - Typical Sets
   - Statement of the Theorem

Instead of coding single outcomes, we now consider coding blocks and sequences of blocks

Example (Coin Flips):

hhhh blocks)

blocks)

blocks)

Extended Ensembles (Review)

Instead of coding single outcomes, we now consider coding blocks and sequences of blocks

Example (Coin Flips):

hhhh blocks)

blocks)

blocks)

Extended Ensemble

The **extended ensemble** of blocks of size es from X^N are denoted $\mathbf{x} = (x_1, x_2, \ldots, x_N)$. The **probability** of $\mathbf{x}$ is defined to be $P(\mathbf{x}) = P(x_1)P(x_2)\ldots P(x_N)$.

Extended Ensembles (Review)

Instead of coding single outcomes, we now consider coding blocks and sequences of blocks

Example (Coin Flips):

hhhh blocks)

blocks)

blocks)

Extended Ensemble

The **extended ensemble** of blocks of size es from X^N are denoted $\mathbf{x} = (x_1, x_2, \ldots, x_N)$. The **probability** of $\mathbf{x}$ is defined to be $P(\mathbf{x}) = P(x_1)P(x_2)\ldots P(x_N)$.

What is the entropy of X^N?

Extended Ensembles (Review)

Example: Bent Coin

Let X be an ensemble with outcomes $\mathcal{A}_X = \{\mathtt{h}, \mathtt{t}\}$ with $p_{\mathtt{h}} = 0.9$ and $p_{\mathtt{t}} = 0.1$.

Consider X^4 – i.e., 4 flips of the coin.

$\mathtt{t}\}$

Extended Ensembles (Review)

Example: Bent Coin

Let X be an ensemble with outcomes $\mathcal{A}_X = \{\mathtt{h}, \mathtt{t}\}$ with $p_{\mathtt{h}} = 0.9$ and $p_{\mathtt{t}} = 0.1$.

Consider X^4 – i.e., 4 flips of the coin.

$\mathtt{t}\}$

What is the

- Four heads? $P(\mathtt{hhhh}) = (0.9) \approx 0.656$
- Four tails? $P(\mathtt{tttt}) = (0.1)^4 = 0.00$

Extended Ensembles (Review)

Example: Bent Coin

Let X be an ensemble with outcomes $\mathcal{A}_X = \{\mathtt{h}, \mathtt{t}\}$ with $p_\mathtt{h} = 0.9$ and $p_\mathtt{t} = 0.1$.

Consider X^4 – i.e., 4 flips of the coin.

$\mathtt{t}\}$

What is the

- Four heads? $P(\mathtt{hhhh}) = (0.9) \approx 0.656$
- Four tails? $P(\mathtt{tttt}) = (0.1)^4 = 0.00$

What is the entropy and raw bit content of

- The outcome set size is $|\mathcal{A}_{X^4}| = |\{0000, 0001, 0010, \ldots, 1111\}| = 16$
- Raw bit content: $H_0(X^4) = \log_2 |\mathcal{A}_{X^4}| = 4$
- Entropy: $H(X^4) = 4H(X) = 4.\,(-0.9 \log_2 0.9 - 0.1 \log_2 0.1) = 1.88$

Essential Bit Content of Extended Ensembles

What if we use a lossy uniform code on the extended ensemble?

$\mathbf{x}$	$P(\mathbf{x})$	$\mathbf{x}$	$P(\mathbf{x})$
hhhh	0.656	thht	0.008
hhht			
hhth			
hthh			
thhh			
htht	0.008	ttht	0.001
htth	0.008	ttth	0.001
hhtt	0.008	tttt	0.000

$$\delta = 0 \text{ gives } H_\delta\left(X^4\right) = \log_2 16 = 4$$

Essential Bit Content of Extended Ensembles

What if we use a lossy uniform code on the extended ensemble?

$\mathbf{x}$	$P(\mathbf{x})$	$\mathbf{x}$	$P(\mathbf{x})$
hhhh	0.656	thht	0.008
hhht			
hhth			
hthh			
thhh			
htht	0.008	ttht	0.001
htth	0.008	ttth	0.001
hhtt	0.008		

$$\delta = 0.0001 \text{ gives } H_\delta\left(X^4\right) = \log_2 15 = 3.91$$

Essential Bit Content of Extended Ensembles

What if we use a lossy uniform code on the extended ensemble?

$\mathbf{x}$	$P(\mathbf{x})$	$\mathbf{x}$	$P(\mathbf{x})$
`hhhh`	0.656	`thht`	0.008
`hhht`			
`hhth`			
`hthh`			
`thhh`			
`htht`	0.008		
`htth`	0.008		
`hhtt`	0.008		

$$\delta = 0.005 \text{ gives } H_\delta\left(X^4\right) = \log_2 11 = 3.46$$

Essential Bit Content of Extended Ensembles

What if we use a lossy uniform code on the extended ensemble?

$\mathbf{x}$	$P(\mathbf{x})$	$\mathbf{x}$	$P(\mathbf{x})$
hhhh	0.656		
hhht			
hhth			
hthh			
thhh			

$\delta = 0.05$ gives $H_\delta\left(X^4\right) = \log_2 5 = 2.32$

Essential Bit Content of Extended Ensembles

What if we use a lossy uniform code on the extended ensemble?

$\mathbf{x}$	$P(\mathbf{x})$	$\mathbf{x}$	$P(\mathbf{x})$
hhhh	0.656		
hhht			
hhth			

$$\delta = 0.25 \text{ gives } H_\delta\left(X^4\right) = \log_2 3 = 1.6$$

Essential Bit Content of Extended Ensembles

What if we use a lossy uniform code on the extended ensemble?

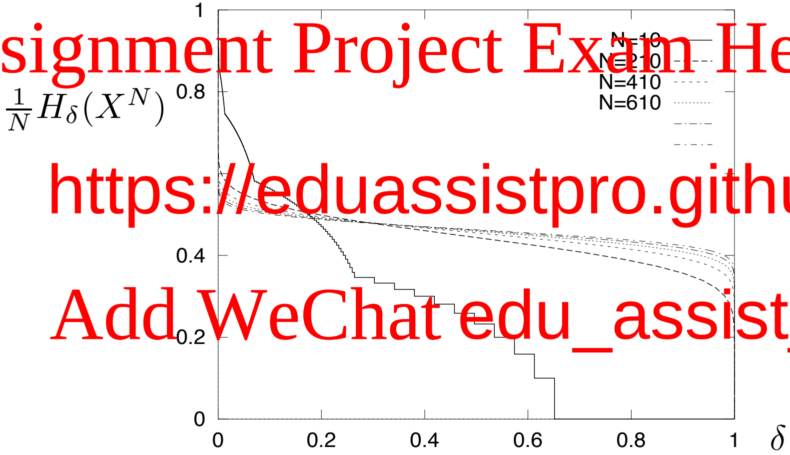
$\mathbf{x}$	$P(\mathbf{x})$	$\mathbf{x}$	$P(\mathbf{x})$
hhhh	0.656		
hhht	0.073		
hhth			

$\delta = 0.25$ gives $H_\delta\left(X^4\right) = \log_2 3 = 1.6$

Unlike entropy, $H_\delta(X^4) \neq 4H_\delta(X) = 0$

Essential Bit Content of Extended Ensembles

What happens as *N* increases?



Recall that the entropy of a single coin flip with $p_h = 0.9$ is $H(X) \approx 0.47$

Essential Bit Content of Extended Ensembles

Some Intuition

Why does the curve flatten for large N?

Recall tha d

typical

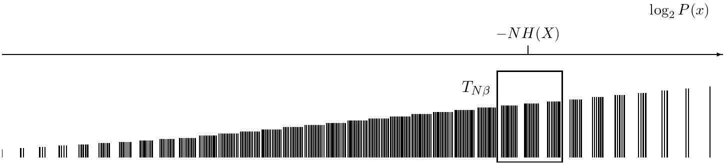
Such sequences occupy most of the probability ma

equally likely

As we increase δ, we will quickly encounter thes

small, roughly equal sized changes to $|S_\delta|$

$\mathbf{x}$	$\log_2(P(\mathbf{x}))$
...1..................1.....1....1.1.......1........1...........1.....................1.......11...	−50.1
[illegible]	−37.3
[illegible]	−65.9
[illegible]	−56.4
...11...........1...1.....1.1......1..........1....1...1....1............1.......................	−53.2
.............1......1.........1.1.......1..........1............1...1.....................1.......	−43.7
	−
.1..........1...1.1.............1.......11...........1.1...1..............1.............11..........	−56.4
.......1...1..1.....1..11.1.1.1...1.....................1............	
...........11.1......1....1..1...........................1.......1.	
[illegible]	
[illegible]	



Typical Sets and the AEP (Review)

Typical Set

For "closeness" $\beta > 0$ the typical set $T_{N\beta}$ for X^N is

—

The nam

occurences of symbol a_1, $p_2 N$ of a_2, . . ., p

Typical Sets and the AEP (Review)

Typical Set

For "closeness" $\beta > 0$ the typical set $T_{N\beta}$ for X^N is

$-$

The nam

occurences of symbol a_1, $p_2 N$ of a_2, . . ., p

Asymptotic Equipartition Property (Informa

As $N \to \infty$, $\log_2 P(x_1, \ldots, x_N)$ is close to $-NH(X)$ with high probability.

For large block sizes "almost all sequences are typical" (i.e., in $T_{N\beta}$).

1. Introduction
   - Quick Review

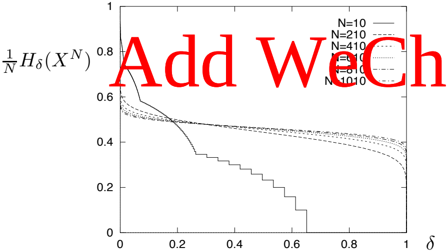
2. Extended Ensembles
   - De
   - Es
   - Th

3. The Source Coding Theorem
   - Typical Sets
   - Statement of the Theorem

The Source Coding Theorem

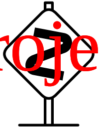
The Source Coding Theorem

Let X be an ensemble with entropy $H = H(X)$ bits. Given $\epsilon > 0$ and $0 < \delta < 1$, there exists a positive integer N_0 such that for all $N > N_0$



- Given a tiny probability of error
  e
  H as
- Even if we allow a large probability of error, we **cannot** compress more than H bits per outcome for large sequences.

Warning: proof ahead



I don't exp

- I pre

- And it is a remarkable and fundamental result
- You are expected to **understand** and **be able to apply** the theorem

Proof of the SCT

The absolute value of a difference being bounded (e.g., $|x - y| \leq \epsilon$) says two things:

1. When $x - y$ is positive, it says $x - y < \epsilon$ which means $x < y + \epsilon$
2. When $x - y$ is negative, it says $-(x - y) < \epsilon$ which means $x < y - \epsilon$

Proof of the SCT

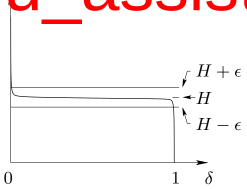
The absolute value of a difference being bounded (e.g., $|x - y| \leq \epsilon$) says two things:

1. When $x - y$ is positive, it says $x - y < \epsilon$ which means $x < y + \epsilon$
2. When $x - y$ is negative, it says $-(x - y) < \epsilon$ which means $x < y - \epsilon$

Using this
that for any ϵ and δ we can find N large eno

Part 1: $\frac{1}{N} H_\delta(X^N) < H + \epsilon$

Part 2: $\frac{1}{N} H_\delta(X^N) > H - \epsilon$



Proof the SCT

Idea

Proof Idea: As N increases

- $T_{N\beta}$ has $\sim 2^{NH(X)}$ elements
- alm

- S_δ and $T_{N\beta}$ increasingly overlap

- so $\log_2 |S_\delta| \sim NH$

Basically, we look to encode all typical sequences uniformly, and relate that to the essential bit content

Proof of the SCT (Part 1)

For $\epsilon > 0$ and $\delta > 0$, want N large enough so $\frac{1}{N} H_\delta(X^N) < H(X) + \epsilon$.

Proof of the SCT (Part 1)

For $\epsilon > 0$ and $\delta > 0$, want N large enough so $\frac{1}{N} H_\delta(X^N) < H(X) + \epsilon$.

Recall (see Lecture 10) for the *typical set* $T_{N\beta}$ we have for any N, β that

(1)

and, by the ... .

So for any ... $_{N\beta})$ ... $1 - \delta$.

Proof of the SCT (Part 1)

For $\epsilon > 0$ and $\delta > 0$, want *N* large enough so $\frac{1}{N}H_\delta(X^N) < H(X) + \epsilon$.

Recall (see Lecture 10) for the *typical set* $T_{N\beta}$ we have for any N, β that

(1)

and, by the

So for any $_{N\beta})$ $1 - \delta$.

Now recall the definition of the *smallest* δ-

smallest subset of outcomes such that *P*($T_{N\beta}|$.

Proof of the SCT (Part 1)

For $\epsilon > 0$ and $\delta > 0$, want N large enough so $\frac{1}{N}H_\delta(X^N) < H(X) + \epsilon$.

Recall (see Lecture 10) for the *typical set* $T_{N\beta}$ we have for any N, β that

$$|T_{N\beta}| \le 2^{N(H(X)+\beta)} \qquad (1)$$

and, by the … 1.
So for any … $) \ge 1 - \delta$

Now recall the definition of the *smallest* δ-…
smallest subset of outcomes such that $P($ … $T_{N\beta}|$.

So, given any δ and β we can find an N la…

$$|S_\delta| \le |T_{N\beta}| \le 2^{N(H(X)+\beta)}$$

Proof of the SCT (Part 1)

For $\epsilon > 0$ and $\delta > 0$, want N large enough so $\frac{1}{N}H_\delta(X^N) < H(X) + \epsilon$.

Recall (see Lecture 10) for the *typical set* $T_{N\beta}$ we have for any N, β that

$$|T_{N\beta}| \le 2^{N(H(X)+\beta)} \tag{1}$$

and, by the ... 1.

So for any ... $\geq 1 - \delta$

Now recall the definition of the *smallest* δ-

smallest subset of outcomes such that $P($ ... $T_{N\beta}|$.

So, given any δ and β we can find an N la

$$\log_2 |S_\delta| \le \log_2 |T_{N\beta}| \le N(H(X) + \beta)$$

Proof of the SCT (Part 1)

For $\epsilon > 0$ and $\delta > 0$, want N large enough so $\frac{1}{N} H_\delta(X^N) < H(X) + \epsilon$.

Recall (see Lecture 10) for the *typical set* $T_{N\beta}$ we have for any N, β that

$$|T_{N\beta}| \leq 2^{N(H(X)+\beta)} \tag{1}$$

and, by the 1.

So for any $\geq 1 - \delta$.

Now recall the definition of the *smallest* δ-

smallest subset of outcomes such that $P($ $T_{N\beta}|$.

So, given any δ and β we can find an N la

$$H_\delta(X^N) = \log_2 |S_\delta| \leq \log_2 |T_{N\beta}| \leq N(H(X) + \beta)$$

Setting $\beta = \epsilon$ and dividing through by N gives result.

Proof of the SCT (Part 2)

For $\epsilon > 0$ and $\delta > 0$, want N large enough so $\frac{1}{N}H_\delta(X^N) > H(X) - \epsilon$.

Suppose this was not the case – that is, for every N we have

$$\frac{1}{}H_\delta(X^N) \qquad H(X) \qquad \epsilon \qquad\qquad S_\delta \qquad 2^{N(H(X)-\epsilon)}$$

Proof of the SCT (Part 2)

For $\epsilon > 0$ and $\delta > 0$, want N large enough so $\frac{1}{N}H_\delta(X^N) > H(X) - \epsilon$.

Suppose this was not the case – that is, for every N we have

$$\frac{1}{N}H_\delta(X^N) \quad H(X) \quad \epsilon \qquad S_\delta \quad 2^{N(H(X)-\epsilon)}$$

Let's look

$$P(x \in S_\delta) = P(x \in S_\delta \cap T_{N\beta}$$

$$\le |S_\delta| 2^{-N(H-\beta)} +$$

since every $x \in T_{N\beta}$ has $P(x) \le 2^{-N(H-\beta}$ $\delta \cap \overline{}_{N\beta} \subset \overline{}_{N\beta}$

Proof of the SCT (Part 2)

For $\epsilon > 0$ and $\delta > 0$, want N large enough so $\frac{1}{N} H_\delta(X^N) > H(X) - \epsilon$.

Suppose this was not the case – that is, for every N we have

$$\frac{1}{N} H_\delta(X^N) \quad H(X) \quad \epsilon \qquad S_\delta \quad 2^{N(H(X)-\epsilon)}$$

Let's look

$$P(x \in S_\delta) = P(x \in S_\delta \cap T_{N\beta}$$

$$\le |S_\delta| 2^{-N(H-\beta)} +$$

since every $x \in T_{N\beta}$ has $P(x) \le 2^{-N(H-\beta}$ $\quad \delta \cap \overline{\quad N\beta} \subset \overline{\quad N\beta}$

So

$$P(x \in S_\delta) \le 2^{N(H-\epsilon)} 2^{-N(H-\beta)} + P(x \in \overline{T_{N\beta}})$$

Proof of the SCT (Part 2)

For $\epsilon > 0$ and $\delta > 0$, want N large enough so $\frac{1}{N}H_\delta(X^N) > H(X) - \epsilon$.

Suppose this was not the case – that is, for every N we have

$$\frac{1}{N}H_\delta(X^N) \quad H(X) \quad \epsilon \qquad S_\delta \quad 2^{N(H(X)-\epsilon)}$$

Let's look

$$P(x \in S_\delta) = P(x \in S_\delta \cap T_{N\beta}$$

$$\le |S_\delta| 2^{-N(H-\beta)} +$$

since every $x \in T_{N\beta}$ has $P(x) \le 2^{-N(H-\beta}$ $\delta \cap \overline{_{N\beta}} \subset \overline{_{N\beta}}$

So

$$P(x \in S_\delta) \le 2^{-N(H-H+\epsilon-\beta)} + P(x \in \overline{T_{N\beta}})$$

Proof of the SCT (Part 2)

For $\epsilon > 0$ and $\delta > 0$, want N large enough so $\frac{1}{N} H_\delta(X^N) > H(X) - \epsilon$.

Suppose this was not the case – that is, for every N we have

$$\frac{1}{N} H_\delta(X^N) \leq H(X) - \epsilon \iff |S_\delta| \leq 2^{N(H(X)-\epsilon)}$$

Let's look

$\in \quad \delta \qquad \in \quad \delta \cap \quad N\beta \qquad \delta \qquad \overline{N\beta}$

$$\leq |S_\delta| 2^{-N(H-\beta)} +$$

since every $x \in T_{N\beta}$ has $P(x) \leq 2^{-N(H-\beta}$

So

$$P(x \in S_\delta) \leq 2^{-N(\epsilon-\beta)} + P(x \in \overline{T_{N\beta}}) \to 0 \text{ as } N \to \infty$$

since $P(x \in T_{N\beta}) \to 1$.

Proof of the SCT (Part 2)

For $\epsilon > 0$ and $\delta > 0$, want N large enough so $\frac{1}{N} H_\delta(X^N) > H(X) - \epsilon$.

Suppose this was not the case – that is, for every N we have

$$\frac{1}{N} H_\delta(X^N) \leq H(X) - \epsilon \iff |S_\delta| \leq 2^{N(H(X)-\epsilon)}$$

Let's look

$\in \quad \delta \qquad \in \quad \delta \cap \quad N\beta \qquad \delta \quad \overline{N\beta}$

$$\leq |S_\delta| 2^{-N(H-\beta)} +$$

since every $x \in T_{N\beta}$ has $P(x) \leq 2^{-N(H-\beta}$

So

$$P(x \in S_\delta) \leq 2^{-N(\epsilon-\beta)} + P(x \in \overline{T_{N\beta}}) \to 0 \text{ as } N \to \infty$$

since $P(x \in T_{N\beta}) \to 1$. But $P(x \in S_\delta) \geq 1 - \delta$, by defn. Contradiction

Interpretation of the SCT

The Source Coding Theorem

Let X be an ensemble with entropy $H = H(X)$ bits. Given $\epsilon > 0$ and $0 < \delta < 1$, there exists a positive integer N_0 such that for all $N > N_0$

If you want to uniformly code blocks of N s X

- If you use more than $NH(X)$ bits per blo almost no loss of information as $N \to$

- If you use less than $NH(X)$ bits per block you will almost certainly lose information as $N \to \infty$

Interpretation of the SCT

The Source Coding Theorem

Let X be an ensemble with entropy $H = H(X)$ bits. Given $\epsilon > 0$ and $0 < \delta < 1$, there exists a positive integer N_0 such that for all $N > N_0$

Making the error probability $\delta \approx 1$ doesn't re

- We're still "stuck with" coding the typical sequ

Assumes we deal with X^N

- If outcomes are dependent, entropy $H(X)$ need not be the limit
- We won't look at such extensions

Implications of SCT

How practical is it to perform coding inspired by the SCT?

Implications of SCT

How practical is it to perform coding inspired by the SCT?

Not very!

- Theorem might require huge block sizes N_0
- We' N_0 $N_0 \; H(X)$

Implications of SCT

How practical is it to perform coding inspired by the SCT?

Not very!

- Theorem might require huge block sizes N_0
- We' N_0 $N_0 \, H(X)$

Can we de

- And

Next time

We move towards more practical compression ideas

Prefix and **Uniquely Decodeable** variable-length codes

The **Kraft Inequality**