COMP2610/COMP6261 - Information Theory

Tutorial 8: Source Coding

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Week 10 (9th – 13th Oct), Semester 2, 2017

1. *Optimal Coding and Huffman Codes*

   Consider the ensemble X with probabilities $\mathcal{P}_X = \mathbf{p} = \{\frac{1}{2}, \frac{1}{4}, \frac{31}{128}, \frac{1}{128}\}$ and the code $C = \{0, 11, 100, 101\}$.

   (a) What is the entropy $H(X)$?

   (b) What is the expected length $L(C, X)$? Is C an optimal code for X?

   (c) What are the code lengths for X? Construct a prefix Shannon code C_S for X. Compute the expected code length $L(C_S, X)$.

   (d) What are the probabilities $\mathbf{q} = \{q_1, q_2, q_3, q_4\}$ for the code lengths of C?

   (e) Compute the relative entropy $D(\mathbf{p}\|\mathbf{q})$. What do you notice about $D(\mathbf{p}\|\mathbf{q})$, $H(X)$, $L(C, X)$, and
   ~~$L(C_S, X)$~~

   (f) Construct a Huffman code C_H for X. How does its code lengths compare to C and C_S? How do their expected cod

2. *Binary Represent*

   Express the followi

   (a) 4.25_{10}

   (b) 8.1_{10}

3. *Shannon-Fano-Elias Coding*

   Let X be an ensemble with alphabet $\mathcal{A}_X = \{x_1, x_2, x_3, x_4\}$ and probabilities $p_X = (0.25, 0.5, 0.125, 0.125)$.

   (a) Compute the cumulative distribution function $F(x_i)$ for each symbol x_i.

   (b) Compute the symbol intervals $[F(x_{i-1}), F(x_i))$ for each symbol x_i. (Recall that we assume for convenience that $F(x_0) = 0$.)

   (c) Compute the modified cumulative distribution function $\bar{F}(x_i)$ for each symbol x_i.

   (d) Compute the Shannon-Fano-Elias codewords for each symbol x_i.

   (e) Decode the string 10001.

   (f) Suppose you only use the first $\lceil \log \frac{1}{p(x)} \rceil$ bits to code the above ensemble. Will the result be a prefix code?

4. *Arithmetic Coding*

   (a) For the same ensemble as the previous section, compute an arithmetic code for the sequence c. Assume here that d = □, i.e. that the symbol d denotes the end of stream. Assume also that at each iteration, the probability of the various outcomes is unchanged.

   (b) Compute an arithmetic code for the sequence ca, with the same setup as the previous part. Does the codeword for ca start with that for c?

   (c) Compute an arithmetic code for the sequence ca using adaptive probabilities, assuming an initial set of virtual counts $\mathbf{m} = (1, 1, 1)$ and a constant end-of-stream probability $p(\mathsf{d}) = 0.25$.