

COMP2610/6261 - Information Theory

Lecture 21: Hamming Codes & Coding Review

Assignment Project Exam Help

<https://eduassistpro.github.io>



Australian
National
University

Add WeChat edu_assist_pro

16 October, 2018

Today's plan

Noisy Channel Coding Theorem proves there exists codes with rate $R < C$ with arbitrarily low probability of error.

Assignment Project Exam Help

<https://eduassistpro.github.io>

Add WeChat edu_assist_pr

Today's plan

Noisy Channel Coding Theorem proves there exists codes with rate $R < C$ with arbitrarily low probability of error.

But proof
able to act

<https://eduassistpro.github.io>

Add WeChat edu_assist_pr

Today's plan

Noisy Channel Coding Theorem proves there exists codes with rate $R < C$ with arbitrarily low probability of error.

But proof

able to act

What about

<https://eduassistpro.github.io>

Add WeChat edu_assist_pr

Today's plan

Noisy Channel Coding Theorem proves there exists codes with rate $R < C$ with arbitrarily low probability of error.

But proof

able to act

What about

We will focus

- repetition codes
- Hamming codes

We will sketch what can be said about the rate and reliability of the latter

1 Repetition Codes

Assignment Project Exam Help

2 The $(7,4)$ Hamming code

- Co
- De
- Sy
- M
- Error Probabilities

<https://eduassistpro.github.io>

Add WeChat edu_assist_pr

3 Coding: Review

Repetition Codes

Assignment Project Exam Help

Simplest channel code: add *redundancy* by repeating every bit of the message (say) 3 times:

<https://eduassistpro.github.io>

0

0 0 0

1

1 1 1

Add WeChat edu_assist_pro

This *repetition* code is called R_3 .

Repetition Codes for the BSC

Example

On a binary symmetric channel with flip probability f , receiver sees

$r = t + \eta$
Assignment Project Exam Help

where η is a *noise* vector

- $p(\eta$

<https://eduassistpro.github.io>

Add WeChat edu_assist_pr

Repetition Codes for the BSC

Example

On a binary symmetric channel with flip probability f , receiver sees

Assignment Project Exam Help

where η is a *noise* vector

- $p(\eta$

Example

s	0	0	1	0	1	1	0
t	000	000	111	000	000	000	000
η	000	001	000	000	101	000	000
<hr/>							
r	000	001	111	000	010	111	000

Note that elements of η are not replicated like those of t

- noise acts independently on every bit

Beyond Repetition Codes

Goal: Communication with small probability of error and high rate:

- Repetition codes introduce redundancy on a per-bit basis
- Can we improve on this?

Assignment Project Exam Help

<https://eduassistpro.github.io>

Add WeChat edu_assist_pr

Beyond Repetition Codes

Goal: Communication with small probability of error and high rate:

- Repetition codes introduce redundancy on a per-bit basis
- Can we improve on this?

Assignment Project Exam Help

<https://eduassistpro.github.io>

Add WeChat edu_assist_pr

Beyond Repetition Codes

Goal: Communication with small probability of error and high rate:

- Repetition codes introduce redundancy on a per-bit basis
- Can we improve on this?

Idea: Introduce redundancy to **blocks** of data instead

Assignment Project Exam Help

<https://eduassistpro.github.io>

Add WeChat edu_assist_pr

Beyond Repetition Codes

Goal: Communication with small probability of error and high rate:

- Repetition codes introduce redundancy on a per-bit basis
- Can we improve on this?

Idea: Introduce redundancy to **blocks** of data instead

Block Co

A block code maps a message k bits into a length- N sequence of transmitted bits t .

- Introduce redundancy: $N > K$
- Focus on *linear codes*

We will introduce a simple type of block code called the (7,4) Hamming code

An Example

The (7, 4) Hamming Code

Consider $K = 4$, and a source message $\mathbf{s} = 1\ 0\ 0\ 0$

The repetition code R_2 produces

The (7,4) Hamming code produces

- Redundancy, but not repetition
- How are these magic bits computed?

1 Repetition Codes

Assignment Project Exam Help

2 The $(7,4)$ Hamming code

- Co
- De
- Sy
- M
- Error Probabilities

<https://eduassistpro.github.io>

Add WeChat edu_assist_pr

3 Coding: Review

The (7,4) Hamming code

Coding

Consider $K = 4$, $N = 7$ and $\mathbf{s} = 1\ 0\ 0\ 0$

Assignment Project Exam Help

<https://eduassistpro.github.io>

Add WeChat edu_assist_pr

The (7,4) Hamming code

Coding

Consider $K = 4$, $N = 7$ and $\mathbf{s} = 1\ 0\ 0\ 0$

Assignment Project Exam Help

<https://eduassistpro.github.io>

Add WeChat edu_assist_pr

The (7,4) Hamming code

Coding

Copy the source bits into the the first 4 target bits:

Assignment Project Exam Help

<https://eduassistpro.github.io>

Add WeChat edu_assist_pr

The (7,4) Hamming code

Coding

Set *parity-check* bits so that the number of ones within each circle is even:

Assignment Project Exam Help

<https://eduassistpro.github.io>

Add WeChat edu_assist_pr

So we have $\mathbf{s} = 1\ 0\ 0\ 0 \xrightarrow{\text{encoder}} \mathbf{t} = 1\ 0\ 0\ 0\ 1\ 0\ 1$

The (7,4) Hamming code

Coding

Assignment Project Exam Help

Algebraically, we have set:

<https://eduassistpro.github.io>

$$t_6 = s_2 \oplus s_3 \oplus s$$

$$t_7 = s_1 \oplus s_3 \oplus s$$

Add WeChat edu_assist_pr

where we use modulo-2 arithmetic

The (7,4) Hamming code

Coding

In matrix form:

Assignment Project Exam Help

$$\left[\begin{array}{cccc|cc} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{array} \right]$$

<https://eduassistpro.github.io>

Add WeChat edu_assist_pro

where $\mathbf{s} = [s_1 \ s_2 \ s_3 \ s_4]^T$

\mathbf{G} is called the *Generator matrix* of the code.

The Hamming code is linear!

The (7,4) Hamming code:

Codewords

Assignment Project Exam Help

Each (unique) sequence that can be transmitted is called a *codeword*.

Codeword

<https://eduassistpro.github.io>

1010 1010010

1110

?

For the (7,4) Hamming code we have a total of 16 code

Add WeChat edu_assist_pr

The (7,4) Hamming code

Codewords

Assignment Project Exam Help

Write

$$\mathbf{G}^T = \mathbf{G}_1 \cdot \mathbf{G}_2 \cdot \mathbf{G}_3 \cdot \mathbf{G}_4.$$

where ea

Then, the t

<https://eduassistpro.github.io>

Add WeChat edu_assist_pr

$$\begin{aligned} \mathbf{t} &= \mathbf{G}^T \mathbf{s} \\ &= [\mathbf{G}_1 \quad \mathbf{G}_2 \quad \mathbf{G}_3 \quad \mathbf{G}_4] \begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{bmatrix} \\ &= s_1 \mathbf{G}_1 + \dots + s_4 \mathbf{G}_4. \end{aligned}$$

The (7,4) Hamming code:

Codewords

There are 2^7 possible transmitted bit strings

- There are $2^7 - 2^4$ other bit strings that immediately imply corruption

- If we see

<https://eduassistpro.github.io>

Any two codewords differ in at least three bits

- Each original bit belongs to at least two codes

- Useful in constructing reliable decoders

Add WeChat edu_assist_pro

1 Repetition Codes

Assignment Project Exam Help

2 The $(7,4)$ Hamming code

- Co

- De

- Sy

- M

- Error Probabilities

<https://eduassistpro.github.io>

Add WeChat edu_assist_pr

3 Coding: Review

The (7,4) Hamming code:

Decoding

We can encode a length-4 sequence \mathbf{s} into a length-7 sequence \mathbf{t} using 3 parity check bits

Assignment Project Exam Help

<https://eduassistpro.github.io>

Add WeChat edu_assist_pr

The (7,4) Hamming code:

Decoding

We can encode a length-4 sequence \mathbf{s} into a length-7 sequence \mathbf{t} using 3 parity check bits

\mathbf{t} can be corrupted by noise which can flip *any* of the 7 bits (including the parity che

Assignment Project Exam Help

<https://eduassistpro.github.io>

$$\begin{array}{r} \eta \\ \hline \mathbf{r} \quad 1 \ 1 \ 0 \ 0 \ 1 \ 0 \ 1 \end{array}$$

Add WeChat edu_assist_pr

The (7,4) Hamming code:

Decoding

We can encode a length-4 sequence \mathbf{s} into a length-7 sequence \mathbf{t} using 3 parity check bits

\mathbf{t} can be corrupted by noise which can flip *any* of the 7 bits (including the parity che

<https://eduassistpro.github.io>

$$\begin{array}{r} \eta \\ \hline \mathbf{r} \quad 1 \quad 1 \quad 0 \quad 0 \quad 1 \quad 0 \quad 1 \end{array}$$

Add WeChat edu_assist_pr

How should we decode \mathbf{r} ?

- We could do this exhaustively using the 16 codewords
- Assuming BSC, uniform $p(\mathbf{s})$: Get the most probable explanation
- Find \mathbf{s} such that $\|\mathbf{t}(\mathbf{s}) \ominus \mathbf{r}\|_1$ is minimum

The (7,4) Hamming code:

Decoding

We can encode a length-4 sequence \mathbf{s} into a length-7 sequence \mathbf{t} using 3 parity check bits

\mathbf{t} can be corrupted by noise which can flip *any* of the 7 bits (including the parity che

<https://eduassistpro.github.io>

$$\begin{array}{r} \eta \\ \hline \mathbf{r} \quad 1 \quad 1 \quad 0 \quad 0 \quad 1 \quad 0 \quad 1 \end{array}$$

Add WeChat edu_assist_pr

How should we decode \mathbf{r} ?

- We could do this exhaustively using the 16 codewords
- Assuming BSC, uniform $p(\mathbf{s})$: Get the most probable explanation
- Find \mathbf{s} such that $\|\mathbf{t}(\mathbf{s}) \ominus \mathbf{r}\|_1$ is minimum

We can get the most probable source vector in an more *efficient* way.

The (7,4) Hamming code:

Decoding Example 1

We have $\mathbf{s} = 1\ 0\ 0\ 0 \xrightarrow{\text{encoder}} \mathbf{t} = 1\ 0\ 0\ 0\ 1\ 0\ 1 \xrightarrow{\text{noise}} \mathbf{r} = 1\ \mathbf{1}\ 0\ 0\ 1\ 0\ 1$:

Assignment Project Exam Help

<https://eduassistpro.github.io>

Add WeChat edu_assist_pr

(1) Detect circles with wrong (odd) parity

- What bit is responsible for this?

The (7,4) Hamming code:

Decoding Example 1

We have $\mathbf{s} = 1\ 0\ 0\ 0 \xrightarrow{\text{encoder}} \mathbf{t} = 1\ 0\ 0\ 0\ 1\ 0\ 1 \xrightarrow{\text{noise}} \mathbf{r} = 1\ \mathbf{1}\ 0\ 0\ 1\ 0\ 1$:

Assignment Project Exam Help

<https://eduassistpro.github.io>

Add WeChat edu_assist_pr

(2) Detect culprit bit and flip it

- The decoded sequence is $\hat{\mathbf{s}} = 1\ 0\ 0\ 0$

The (7,4) Hamming code:

Decoding Example 2

We have $\mathbf{s} = 1\ 0\ 0\ 0 \xrightarrow{\text{encoder}} \mathbf{t} = 1\ 0\ 0\ 0\ 1\ 0\ 1 \xrightarrow{\text{noise}} \mathbf{r} = 1\ 0\ 0\ 0\ 0\ 0\ 1$:

Assignment Project Exam Help

<https://eduassistpro.github.io>

Add WeChat edu_assist_pr

(1) Detect circles with wrong (odd) parity

- What bit is responsible for this?

The (7,4) Hamming code:

Decoding Example 2

We have $\mathbf{s} = 1\ 0\ 0\ 0 \xrightarrow{\text{encoder}} \mathbf{t} = 1\ 0\ 0\ 0\ 1\ 0\ 1 \xrightarrow{\text{noise}} \mathbf{r} = 1\ 0\ 0\ 0\ 0\ 0\ 1$:

Assignment Project Exam Help

<https://eduassistpro.github.io>

Add WeChat edu_assist_pr

(2) Detect culprit bit and flip it

- The decoded sequence is $\hat{\mathbf{s}} = 1\ 0\ 0\ 0$

The (7,4) Hamming code:

Decoding Example 3

We have $\mathbf{s} = 1\ 0\ 0\ 0 \xrightarrow{\text{encoder}} \mathbf{t} = 1\ 0\ 0\ 0\ 1\ 0\ 1 \xrightarrow{\text{noise}} \mathbf{r} = 1\ 0\ \mathbf{1}\ 0\ 1\ 0\ 1$:

Assignment Project Exam Help

<https://eduassistpro.github.io>

Add WeChat edu_assist_pr

(1) Detect circles with wrong (odd) parity

- What bit is responsible for this?

The (7,4) Hamming code:

Decoding Example 3

We have $\mathbf{s} = 1\ 0\ 0\ 0 \xrightarrow{\text{encoder}} \mathbf{t} = 1\ 0\ 0\ 0\ 1\ 0\ 1 \xrightarrow{\text{noise}} \mathbf{r} = 1\ 0\ 1\ 0\ 1\ 0\ 1$:

Assignment Project Exam Help

<https://eduassistpro.github.io>

Add WeChat edu_assist_pr

(2) Detect culprit bit and flip it

- The decoded sequence is $\hat{\mathbf{s}} = 1\ 0\ 0\ 0$

1 Repetition Codes

Assignment Project Exam Help

2 The $(7,4)$ Hamming code

- Co
- De
- Sy
- M
- Error Probabilities

<https://eduassistpro.github.io>

Add WeChat edu_assist_pr

3 Coding: Review

The (7,4) Hamming code:

Optimal Decoding Algorithm: Syndrome Decoding

Given $\mathbf{r} = r_1, \dots, r_7$, assume BSC with small noise level f :

1. Define the **syndrome** as the length-3 vector \mathbf{z} that describes the pattern of violations of the parity bits r_5, r_6, r_7 .



\mathbf{r}



<https://eduassistpro.github.io>

Add WeChat edu_assist_pr

The (7,4) Hamming code:

Optimal Decoding Algorithm: Syndrome Decoding

Given $\mathbf{r} = r_1, \dots, r_7$, assume BSC with small noise level f :

- 1 Define the **syndrome** as the length-3 vector \mathbf{z} that describes the pattern of violations of the parity bits r_5, r_6, r_7 .



\mathbf{r}



- 2 Ch

<https://eduassistpro.github.io>

Add WeChat edu_assist_pr

The (7,4) Hamming code:

Optimal Decoding Algorithm: Syndrome Decoding

Given $\mathbf{r} = r_1, \dots, r_7$, assume BSC with small noise level f :

- 1 Define the **syndrome** as the length-3 vector \mathbf{z} that describes the pattern of violations of the parity bits r_5, r_6, r_7 .



\mathbf{r}

- 2 Ch



- 3 Unflip the *single bit* responsible for this \mathbf{p}



This syndrome could have been caused by

The (7,4) Hamming code:

Optimal Decoding Algorithm: Syndrome Decoding

Given $\mathbf{r} = r_1, \dots, r_7$, assume BSC with small noise level f :

- 1 Define the **syndrome** as the length-3 vector \mathbf{z} that describes the pattern of violations of the parity bits r_5, r_6, r_7 .



\mathbf{r}



- 2 Ch

- 3 Unflip the *single bit* responsible for this p

- ▶ This syndrome could have been caused by

\mathbf{z}	0 0 0	0 0 1	0 1 0	0 1 1	1 0 0	1 0 1	1 1 0	1 1 1
Flip bit	none	r_7	r_6	r_4	r_5	r_1	r_2	r_3

The (7,4) Hamming code:

Optimal Decoding Algorithm: Syndrome Decoding

Given $\mathbf{r} = r_1, \dots, r_7$, assume BSC with small noise level f :

- 1 Define the **syndrome** as the length-3 vector \mathbf{z} that describes the pattern of violations of the parity bits r_5, r_6, r_7 .



\mathbf{r}

- 2 Ch
- 3 Unflip the *single bit* responsible for this p

- ▶ This syndrome could have been caused by

\mathbf{z}	0 0 0	0 0 1	0 1 0	0 1 1	1 0 0	1 0 1	1 1 0	1 1 1
Flip bit	none	r_7	r_6	r_4	r_5	r_1	r_2	r_3

The optimal decoding algorithm unflips at most one bit

The (7,4) Hamming code:

Syndrome Decoding: Matrix Form

Recall that we just need to compare the expected parity bits with the actual ones we received:

Assignment Project Exam Help

$$z_1 = r_1 \oplus r_2 \oplus r_3 \ominus r_5$$

<https://eduassistpro.github.io>

Add WeChat edu_assist_pr

The (7,4) Hamming code:

Syndrome Decoding: Matrix Form

Recall that we just need to compare the expected parity bits with the actual ones we received:

Assignment Project Exam Help

$$z_1 = r_1 \oplus r_2 \oplus r_3 \ominus r_5$$

<https://eduassistpro.github.io>

but in modulo-2 arithmetic $-1 \equiv 1$ so we can re
have:

Add WeChat edu_assist_pro

$$\mathbf{z} = \mathbf{H}\mathbf{r} \text{ with } \mathbf{H} = \begin{bmatrix} \mathbf{P} & \mathbf{I}_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

The (7,4) Hamming code:

Syndrome Decoding: Matrix Form

Recall that we just need to compare the expected parity bits with the actual ones we received:

Assignment Project Exam Help

$$z_1 = r_1 \oplus r_2 \oplus r_3 \ominus r_5$$

<https://eduassistpro.github.io>

but in modulo-2 arithmetic $-1 \equiv 1$ so we can re
have:

Add WeChat edu_assist_pro

$$\mathbf{z} = \mathbf{H}\mathbf{r} \text{ with } \mathbf{H} = \begin{bmatrix} \mathbf{P} & \mathbf{I}_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Homework: What is the syndrome for a codeword?

1 Repetition Codes

Assignment Project Exam Help

2 The $(7,4)$ Hamming code

- Co
- De
- Sy
- M
- Error Probabilities

<https://eduassistpro.github.io>

Add WeChat edu_assist_pr

3 Coding: Review

The (7,4) Hamming code:

Optimal Decoding Algorithm: Syndrome Decoding

Assignment Project Exam Help

When the noise level f on the BSC is small, it may be reasonable that we see only a si

The syndr
this case

<https://eduassistpro.github.io>

- c.f. Noise flipping one bit in the repetition code

Add WeChat edu_assist_pr

But what happens if the noise flips more than one bit?

The (7,4) Hamming code:

Decoding Example 4: Flipping 2 Bits

We have $\mathbf{s} = 1\ 0\ 0\ 0 \xrightarrow{\text{encoder}} \mathbf{t} = 1\ 0\ 0\ 0\ 1\ 0\ 1 \xrightarrow{\text{noise}} \mathbf{r} = 1\ 0\ 1\ 0\ 1\ 0\ 0$:

Assignment Project Exam Help

<https://eduassistpro.github.io>

Add WeChat edu_assist_pr

(1) Detect circles with wrong (odd) parity

- What bit is responsible for this?

The (7,4) Hamming code:

Decoding Example 4: Flipping 2 Bits

We have $\mathbf{s} = 1\ 0\ 0\ 0 \xrightarrow{\text{encoder}} \mathbf{t} = 1\ 0\ 0\ 0\ 1\ 0\ 1 \xrightarrow{\text{noise}} \mathbf{r} = 1\ 0\ 1\ 0\ 1\ 0\ 0$:

Assignment Project Exam Help

<https://eduassistpro.github.io>

Add WeChat edu_assist_pr

(2) Detect culprit bit and flip it

- The decoded sequence is $\hat{\mathbf{s}} = 1\ 1\ 1\ 0$

- ▶ We have made 3 errors but only 2 involve the actual message

The (7,4) Hamming code:

Decoding Exercises

[Mackay, Ex 1.5]: Decode the following sequences using the syndrome decoding for the (7,4) Hamming code:

(a) $\mathbf{r} =$

(b) $\mathbf{r} =$ <https://eduassistpro.github.io>

(c) $\mathbf{r} = 0100111 \rightarrow \hat{\mathbf{s}} = ??$

(d) $\mathbf{r} = 1111111 \rightarrow \hat{\mathbf{s}} = ??$

Work out the answers on your own.

The (7,4) Hamming code: Solution

For each exercise we simply compute the *syndrome* and use the optimal decoding algorithm (Table above) to determine which bit we should unflip.

(a) $\mathbf{r} = 1101011 \rightarrow \mathbf{z}_1 = r_1 \oplus r_2 \oplus r_3 \oplus r_5 = 0 \quad \mathbf{z}_2 = r_2 \oplus r_3 \oplus r_4 \oplus r_6 = 1$
 $\hat{\mathbf{s}} =$

(b) $\mathbf{r} = 0110110 \rightarrow \mathbf{z} = 111$, we unflip r_3

(c) $\mathbf{r} = 0100111 \rightarrow \mathbf{z} = 001$, we unflip r_7

(d) $\mathbf{r} = 1111111 \rightarrow \mathbf{z} = 000$, we don't unflip any bit, $\hat{\mathbf{s}} = 1111$

The (7,4) Hamming code:

Zero-Syndrome Noise Vectors

Assignment Project Exam Help

[Mackay

(so that the

of these vectors are there?

<https://eduassistpro.github.io>

Add WeChat edu_assist_pr

Solution

By definition we have that the all-zero syndrome implies that the corresponding noise components should cancel out. For example for the first component we have:

$z_1 = r_1 \oplus t_2 \oplus t_3 \oplus t_5 = r_1 \oplus t_2 \oplus t_3 \oplus t_5 \oplus \eta_1 \oplus \eta_2 \oplus \eta_3 \oplus \eta_5$. But $t_i = s_i$ for $i = 1, \dots, 4$ and $t_5 = s_1 \oplus s_2 \oplus s_3$. Therefore

$z_1 = 2s_1$. Thus, we have:

<https://eduassistpro.github.io>

$$z_2 = \eta_2 \oplus \eta_3 \oplus \eta_4$$

Add WeChat [edu_assist_pro](https://eduassistpro.github.io)

which is equivalent to:

$$\eta_5 = \eta_1 \oplus \eta_2 \oplus \eta_3$$

$$\eta_6 = \eta_2 \oplus \eta_3 \oplus \eta_4$$

$$\eta_7 = \eta_1 \oplus \eta_3 \oplus \eta_4$$

Solution (cont.)

As η_5 is determined by η_1, η_2, η_3 we have $2^3 = 8$ possibilities here.

Now, for fixed η_1, η_2 (and η_3) in the previous step we only have two possibilities for η_4 , which determines η_6 .

We have now that all the variables are set and η_7 is fully determined by their values.

Thus, we have the syndrome.

The trivial noise vectors that yield this syndrome are: $\eta = 0000000$ and $\eta = 1111111$.

However, we can follow the above procedure and set all variables.

This is equivalent to having arbitrary settings for η_1, η_2, η_3 and η_4 which gives us 16 possible noise vectors which exactly correspond to the 16 codewords of the (7,4) Hamming code.

1 Repetition Codes

Assignment Project Exam Help

2 The $(7,4)$ Hamming code

- Co
- De
- Sy
- M

• Error Probabilities

<https://eduassistpro.github.io>

Add WeChat edu_assist_pr

3 Coding: Review

The (7,4) Hamming code:

Error Probabilities

Decoding Error : Occurs if at least one of the decoded bits \hat{s}_i does not match the corresponding source bit s_i for $i = 1, \dots, 4$.

Assignment Project Exam Help

<https://eduassistpro.github.io>

Add WeChat edu_assist_pr

The (7,4) Hamming code:

Error Probabilities

Decoding Error : Occurs if at least one of the decoded bits \hat{s}_i does not match the corresponding source bit s_i for $i = 1, \dots, 4$

$p(\text{Block E}$

/

<https://eduassistpro.github.io>

Add WeChat edu_assist_pr

The (7,4) Hamming code:

Error Probabilities

Decoding Error : Occurs if at least one of the decoded bits \hat{s}_i does not match the corresponding source bit s_i for $i = 1, \dots, 4$.

$p(\text{Block Error})$

/

$p(\text{Bit Error})$

\overline{K}
 $k=1$

/

<https://eduassistpro.github.io>

Add WeChat edu_assist_pr

The (7,4) Hamming code:

Error Probabilities

Decoding Error : Occurs if at least one of the decoded bits \hat{s}_i does not match the corresponding source bit s_i for $i = 1, \dots, 4$.

$p(\text{Block Error})$

/

$p(\text{Bit Error})$

\bar{K}
 $k=1$

/

Rate

$$R = \frac{K}{N} = \frac{4}{7}$$

The (7,4) Hamming code:

Error Probabilities

Decoding Error : Occurs if at least one of the decoded bits \hat{s}_i does not match the corresponding source bit s_i for $i = 1, \dots, 4$

$p(\text{Block Error})$

/

$p(\text{Bit Error})$

\bar{K}
 $k=1$

/

Rate

$$R = \frac{K}{N} = \frac{4}{7}$$

What is the probability of block error for the (7,4) Hamming code with $f = 0.1$?

The (7,4) Hamming code:

Leading-Term Error Probabilities

Assignment Project Exam Help

Block Error: This occurs when 2 or more bits in the block of 7 are flipped

We can ap

<https://eduassistpro.github.io>

$$B = \sum_{m=2}^7 \binom{7}{m} 2^{-m} f$$

Add WeChat: edu_assist_pro

The (7,4) Hamming code:

Leading-Term Error Probabilities

Bit Error: Given that a block error occurs, the noise must corrupt 2 or more bits

The most p
errors in h

<https://eduassistpro.github.io>

Add WeChat edu_assist_pr

The (7,4) Hamming code:

Leading-Term Error Probabilities

Bit Error: Given that a block error occurs, the noise must corrupt 2 or more bits

The most p
errors in h

- $p(\hat{s}_i / \bar{7})$

<https://eduassistpro.github.io>

Add WeChat edu_assist_pr

The (7,4) Hamming code:

Leading-Term Error Probabilities

Bit Error: Given that a block error occurs, the noise must corrupt 2 or more bits

The most probable errors in the

- $p(\hat{s}_i / \bar{7})$

- All bits are equally likely to be corrupted (due to symmetry)

The (7,4) Hamming code:

Leading-Term Error Probabilities

Bit Error: Given that a block error occurs, the noise must corrupt 2 or more bits

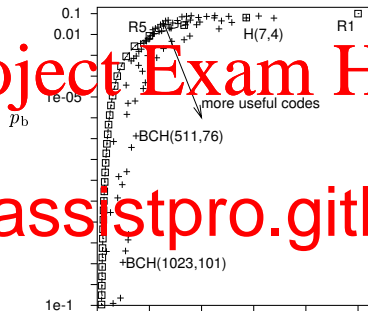
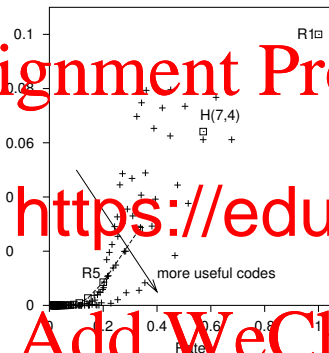
The most probable errors in the

- $p(\hat{s}_i / \bar{7})$

- All bits are equally likely to be corrupted (due to symmetry)

- $p_b \approx \frac{3}{7} p_B \approx 9f^2$

What Can Be Achieved with Hamming Codes?



- $H(7,4)$ improves p_b at a moderate rate $R = 4/7$
- BCH are a generalization of Hamming codes.

1 Repetition Codes

Assignment Project Exam Help

2 The $(7,4)$ Hamming code

- Co
- De
- Sy
- M
- Error Probabilities

<https://eduassistpro.github.io>

Add WeChat edu_assist_pr

3 Coding: Review

The Big Picture

Assignment Project Exam Help

SOURCE
CODING

Compressor

Decompressor

<https://eduassistpro.github.io>

Source Coding for Compression Channel

- Shrink sequences
- Identify and remove redundancy
- Size limited by entropy
- Source Coding Theorems (Block & Variable Length)
- Add known form of redundancy
- Rate limited by capacity
- Noisy-Channel Coding Theorem