COMP2610 / COMP6261 - Information Theory

Lecture 3: Probability Theory and Bayes' Rule

Australian National University

July 30, 2018

Last time

- A general communication system

- Wh

- Basics of probability theory

- Joint, marginal and conditional distributions

Review Exercise

Suppose I go through the records for $N = 1000$ students, checking their admission status, $A = \{0, 1\}$, and whether they are "brilliant" or not, $B = \{0, 1\}$

(Aside: "Brilliance" is a dodgy concept, and does not predict scientific achievement as well as persistence and combinatorial ability; see e.g. Dean Simonton, *Scientific Genius: A Psychology of Science*, Cambridge University Press, 2009; this is just a toy example!)

Say that th

$A = 0$	680	10
$A = 1$	220	90

Then:

$$\boxed{\begin{aligned} &p(A = 1, B = 0) \\ &p(B = 1) \\ &p(A = 0) \\ &p(B = 1 | A = 1) \\ &p(A = 0 | B = 0) \end{aligned}}$$

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This time

- More on joint, marginal and conditional distributions

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- What, if anything, does $p(X = x | Y = y)$ tell us about $p(Y = y | X = x)$?

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- What, if anything, does $p(X = x|Y =$ $p(Y = y|X = x)$?

Philosophically related to “How do we know / learn ab

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I am *not* providing a general answer; but keep it in mind!

Outline

Document Modelling Example

Suppose we have a large document of English text, represented as a sequence of characters:

$$X_1 X_2 X_3 \ldots X_N$$

- e.g. `hello␣how␣are␣you`

Treat eac

variable

`X = 'h', Y = 'e'`

`X = 'e', Y = 'l'`

`X = 'l', Y = 'l'`

$\vdots$

Document Modelling: Marginal and Joint Distributions

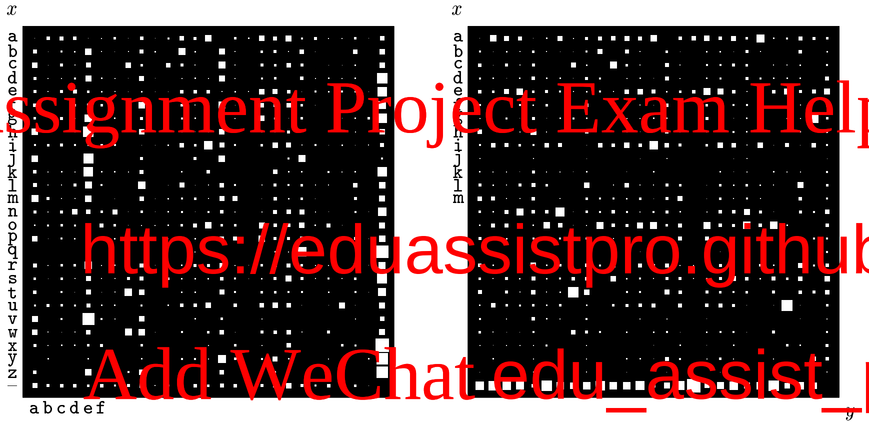
i	a_i	p_i
1	a	0.0575
2	b	0.0128
3	c	0.0263
4	d	0.0285
5	e	0.0913
6	f	0.0173
7	g	0.0133
8	h	0.0313
9	i	0.0599
10	j	0.0006
11	k	0.0084
26	z	0.0007
27	–	0.1928

Unigram / Monogram

Bigram

Marginal and joint distributions for English alphabet, estimated from the "FAQ manual for Linux". Figure from Mackay (ITILA, 2003); areas of squares proportional to probability (the right way to do it!).

Document Modelling: Conditional Distributions



Conditional distributions for English alphabet, estimated from the "FAQ manual for Linux". Are these distributions "symmetric"? Figure from Mackay (ITILA, 2003)

$P(X = x|Y = y) = P(Y = y|X = x)$? $P(X = x|Y = y) = P(X = y|Y = x)$?.

Recap: Sum and Product Rules

Sum rule:

Product rule:

$$p(X = x_i, Y = y_j) = p(Y = y_j$$

Relating the Marginal, Conditional and Joint

Suppose we knew $p(X = x, Y = y)$ for all values of x, y. Could we compute all of $p(X = x | Y = y)$, $p(X = x)$ and $p(Y = y)$?

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	$B = 0$	$B = 1$
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A	
A	

These have the same marginals, but different joint distributions

Joint as the “Master” Distribution

In genera
of margin

The joint di
dependence

Statistical Independence

Suppose that both boxes (red and blue) contain the same proportion of apples and oranges.

If fruit is sel

Statistical Independence

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The probability of selecting an apple (or an orange) i box that is chosen

We may study the properties of F and B separately: this often simplifies analysis

Statistical Independence: Definition

Definition: Independent Variables

Two varia ... $\perp\!\!\!\perp Y$, if
and only if t
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This definition generalises to more than two variabl

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Are the variables in the language example statistic

A Note on Notation

When we write

$$p(X, Y) = p(X)p(Y)$$

we have n

This state

$$p(X = x, Y = y) = p(X$$

for every possible x and y

This notation is sometimes called implied universality

Conditional independence

We may also consider random variables that are conditionally independent given some other variable

Definition: Conditionally Independent Variables

Two varia … enoted $X \perp\!\!\!\perp Y|$

$$p(X, Y|Z) = p(X|Z)p(Y|Z)$$

Intuitively, Z is a common cause for X and

Example: X = whether I have a cold

Y = whether I have a headache

Z = whether I have the flu

1 More on Joint, Marginal and Conditional Distributions

2 Stati

3 Baye

4 Wrapping up

Revisiting the Product Rule

The product rule tells us:

$$p(X, Y) = p(Y\ X)p(X)$$

This can e

probability:

$$p(Y|X) = \frac{p(X}{p(}$$

Can we use these to relate $p(X|Y)$ and $p(Y|X)$?

Posterior Inference:

Example 1 (Mackay, 2003)

- Dicksy Sick had a test for a rare disease
  - Only 1% people of Dicksy's background have the disease

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  - It correctly identifies a healthy individual 96
    $p(\text{identifies healthy} \mid \text{healthy}) = 96$

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- Dicksy has tested positive (apparently sick)
- What is the probability of Dicksy having the disease?

Let $D \in \{0, 1\}$ denote whether Dicksy has the disease, and $T \in \{0, 1\}$ the outcome

Posterior Inference:

Example 1: Formalization

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$p(T = 0 | D = 1) = 0.05$

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$p(T = 0|D = 1) = 0.05$

We need to compute $p(D = 1|T = 1)$, *th*
the disease given that the test has resulted positive.

Posterior Inference:

Example 1: Solution

$$p(D = 1 \mid T = 1) = \frac{p(D = 1, T = 1)}{p(T = 1)} \quad \text{Def. conditional prob.}$$

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$$= \frac{p(T =}{p(T = 1|D = 1)p(D = 1) + p(T = 1|D = 0)p(D = 0)}$$

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\end{aligned}
$$

Despite testing positive and the high accuracy of the test, the probability of Dicksy having the disease is only 0.19!

Why is the Probability So Low?

A “Natural Frequency” Approach

In 100 people, only 1 is expected to have the disease ($p(D = 1) = 0.01$)

This sick person will most likely test positive ($p(T = 1\ D = 1) = 0.95$)

But aroun
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So when the test is positive, the chance of being sick is

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(Aside: If you can correctly perform the calculation on the previous slide
most medical doctors! See Gird Gigerenzer and Adrian Edwards, Si
from innumeracy to insight, *British Medical Journal*, 327(7417), 741–744, 27 September 2003; Gird Gigerenzer, *Reckoning with risk: Learning to live with uncertainty*, Penguin, 2002.

Moral of the story — if you get sick, don’t delegate conditional probability computations to your doctor!)

Bayes' Theorem

We have implicitly used the following (at first glance remarkable) fact:

Bayes' Theorem:

$$
\begin{aligned}
p(Z|X) &= \frac{p(Z,X)}{p(X)} \\
&= \frac{p(X|Z)p(Z)}{p(X)} \\
&= \frac{p(X|Z)p(Z)}{\sum_{Z'} p(X|Z')p(Z')}
\end{aligned}
$$

If we can express what knowledge of X (test) tells us about Z (disease), then we can express what knowledge of Z tells us about X

The Bayesian Inference Framework

Bayesian Inference

Bayesian inference provides a mathematical framework explaining how to change our (prior) beliefs in the light of new evidence.

$$\underbrace{}_{\text{posterior}} \qquad \frac{}{\underbrace{p(X)}_{\text{evid}}}.$$

Prior: Belief that someone is sick

Likelihood: Probability of testing positive given you are sick

Posterior: Probability of being sick given you test positive

Posterior Inference:

Example 2 (Bishop, 2006)

Recall our fruit-box example:

- The proportion of oranges and apples are given by

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- A piece of fruit has been picked up and it turned out to be an orange.
- What is the probability that it came from the red box?

Let $B \in \{r, b$ denote the selected box and F a, o the selected fruit.

Posterior Inference:

Example 2: Formalization

Let $B \in \{r, b$ denote the selected box and F a, o the selected fruit.

10

/1

$p(F = a|B = b) = 3/4$

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Example 2: Solution

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$p(F = o|B = r)p(B =$ $= b)$

$= \frac{2}{3}$

Posterior Inference:

Example 2: Solution

We simply use Bayes' rule:

$$p(B = r \quad \frac{\qquad}{p(F = o | B = r) p(B = \qquad = b)}$$

$$= \frac{2}{3}$$

and therefore $p(B = b | F = o) = 1/3$.

Posterior Inference:

Example 2: Interpretation of the Solution

- If we hadn't been told any information about the fruit picked, the blue box is more likely to be selected than the red box

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  - *Because the red box contains more orange*
- In fact, the proportion of oranges is so much hig this is strong evidence that the orange came fro

  - *So after picking up the orange the red box is mu been selected than the blue one*

Summary

- Recap on joint, marginal and conditional distributions

- Inte

- Stat

- Bayes' rule: combination of prior, likelihood to

- Reading: Mackay § 2.1, § 2.2 and § 2.3

Homework Exercise

Suppose we know that random variables X, Y satisfy

$$p(X|Y) = p(Y|X)$$

What can y Y?

If X and ?

Repeat the above questions for the statement

$$\frac{p(X|Y)}{p(Y|X)} = \frac{p}{p(Y)}$$

Next time

- More examples on Bayes' theorem:
  - Eating hamburgers
  -
  - The Monty Hall problem
  - Document modelling
- Are there notions of probability beyond frequency counting?