COMP2610 – Information Theory
Lecture 12: The Source Coding Theorem

Australian National University

28 August 2018

Last time

Basic goal of compression

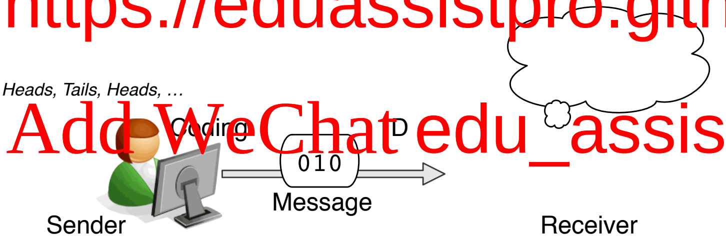
Key concepts: codes and their types, raw bit content, essential bit content

Informal statement of source coding theorem

A General Communication Game (Recap)

Data compression is the process of replacing a message with a smaller message which can be reliably converted back to the original.

- Want small messages on average when outcomes are from a fixed, kno



Definitions (Recap)

Source Code

Given an ensemble X, the function $c : \mathcal{A}_X \to \mathcal{B}$ is a **source code** for X.
The number of symbols in $c(x)$ is the **length** $l(x)$ of the codeword for x.
The **extension** of c is defined by $c(x_1 \ldots x_n) = c(x_1) \ldots c(x_n)$

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Smalles [illegible]

Let X be [illegible] bset of $\mathcal{A}_X$ such that

$$P(x \in S_\delta) \geq [illegible]$$

Essential Bit Content

Let X be an ensemble then for $\delta \geq 0$ the **essential bit content** of X is

$$H_\delta(X) \stackrel{\text{def}}{=} \log_2 |S_\delta|$$

Essential Bit Content (Recap)

Intuitively, construct S_δ by removing elements of X in ascending order of probability, till we have reached the $1-\delta$ threshold

x	$P(\mathbf{x})$
a	[illegible]
b	[illegible]
c	[illegible]
d	3/16
e	1/64
f	1/64
g	1/64
h	1/64

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$\delta = 3/4$: $S_\delta = \{$a

Lossy Coding (Recap)

Consider a coin with $P(Heads) = 0.9$

If we are ha

sequenc

There are

So, we can just code those, and **ignore** th

- Coding 10 outcomes with 2% failure doable wi
  bits/outcome

This time

Recap: typical sets

Formal statement of source coding theorem

Proof of source coding theorem

The Source Coding Theorem

(Theorem 4.1 in MacKay)

Our aim this week is to understand this:

The Source Coding Theorem

Let X be an ensemble with entropy $H = H(X)$ bits. Given $\epsilon > 0$ and $0 < \delta <$ $> N_0$

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https://eduassistpro.github.i

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$H_\delta(X^N)$ measures the *fewest* number of bits needed to uniformly code *smallest* set of N-outcome sequence S_δ with $P(x \in S_\delta) \geq 1 - \delta$.

1. Introduction
   - Quick Review

2. Extended Ensembles
   - De
   - Es
   - Th

3. The Source Coding Theorem
   - Typical Sets
   - Statement of the Theorem

Extended Ensembles (Review)

Instead of coding single outcomes, we now consider coding blocks and sequences of blocks

Example (Coin Flips):

hhhh blocks)

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The **extended ensemble** of blocks of size es from X^N are denoted $\mathbf{x} = (x_1, x_2, \ldots, x_N)$. The **probability** of $\mathbf{x}$ is defined to be $P(\mathbf{x}) = P(x_1)P(x_2)\ldots P(x_N)$.

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What is the entropy of X^N?

Extended Ensembles (Review)

Example: Bent Coin

Let X be an ensemble with outcomes $\mathcal{A}_X = \{\mathtt{h}, \mathtt{t}\}$ with $p_{\mathtt{h}} = 0.9$ and $p_{\mathtt{t}} = 0.1$.

Consider X^4 – i.e., 4 flips of the coin.

$\mathtt{t}\}$

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What is the

- Four heads? $P(\mathtt{hhhh}) = (0.9) \approx 0.656$
- Four tails? $P(\mathtt{tttt}) = (0.1)^4 = 0.00$

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What is the entropy and raw bit content of

- The outcome set size is $|\mathcal{A}_{X^4}| = |\{0000, 0001, 0010, \ldots, 1111\}| = 16$
- Raw bit content: $H_0(X^4) = \log_2 |\mathcal{A}_{X^4}| = 4$
- Entropy: $H(X^4) = 4H(X) = 4.\,(-0.9 \log_2 0.9 - 0.1 \log_2 0.1) = 1.88$

Essential Bit Content of Extended Ensembles

What if we use a lossy uniform code on the extended ensemble?

$\mathbf{x}$	$P(\mathbf{x})$	$\mathbf{x}$	$P(\mathbf{x})$
hhhh	0.656	thht	0.008
hhht			
hhth			
hthh			
thhh			
htht	0.008	ttht	0.001
htth	0.008	ttth	0.001
hhtt	0.008	tttt	0.000

$$\delta = 0 \text{ gives } H_\delta\left(X^4\right) = \log_2 16 = 4$$

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$\mathbf{x}$	$P(\mathbf{x})$	$\mathbf{x}$	$P(\mathbf{x})$
hhhh	0.656	thht	0.008
hhht			
hhth			
hthh			
thhh			
htht	0.008	ttht	0.001
htth	0.008	ttth	0.001
hhtt	0.008		

$$\delta = 0.0001 \text{ gives } H_\delta\left(X^4\right) = \log_2 15 = 3.91$$

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What if we use a lossy uniform code on the extended ensemble?

$\mathbf{x}$	$P(\mathbf{x})$	$\mathbf{x}$	$P(\mathbf{x})$
hhhh	0.656	thht	0.008
hhht			
hhth			
hthh			
thhh			
htht	0.008		
htth	0.008		
hhtt	0.008		

$$\delta = 0.005 \text{ gives } H_\delta\left(X^4\right) = \log_2 11 = 3.46$$

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What if we use a lossy uniform code on the extended ensemble?

$\mathbf{x}$	$P(\mathbf{x})$	$\mathbf{x}$	$P(\mathbf{x})$
hhhh	0.656		
hhht			
hhth			
hthh			
thhh			

$\delta = 0.05$ gives $H_\delta\left(X^4\right) = \log_2 5 = 2.32$

Essential Bit Content of Extended Ensembles

What if we use a lossy uniform code on the extended ensemble?

x	$P(\mathbf{x})$	**x**	$P(\mathbf{x})$
hhhh	0.656		
hhht			
hhth			

$$\delta = 0.25 \text{ gives } H_\delta\left(X^4\right) = \log_2 3 = 1.6$$

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What if we use a lossy uniform code on the extended ensemble?

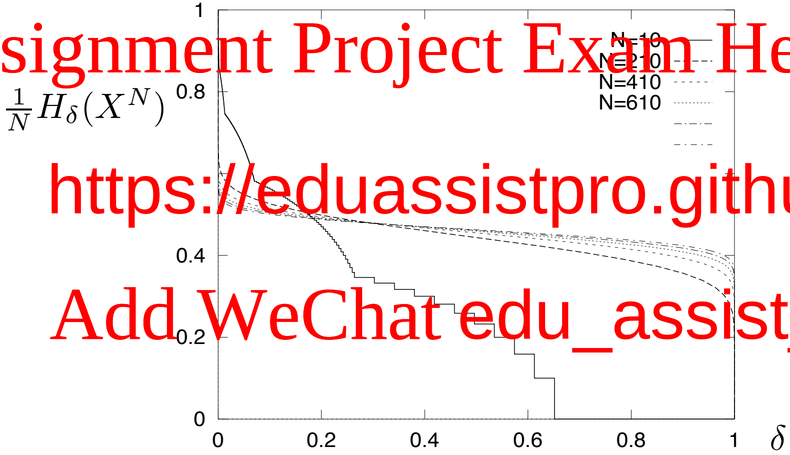
$\mathbf{x}$	$P(\mathbf{x})$	$\mathbf{x}$	$P(\mathbf{x})$
hhhh	0.656		
hhht	0.073		
hhth			

$\delta = 0.25$ gives $H_\delta\left(X^4\right) = \log_2 3 = 1.6$

Unlike entropy, $H_\delta(X^4) \neq 4H_\delta(X) = 0$

Essential Bit Content of Extended Ensembles

What happens as *N* increases?



Recall that the entropy of a single coin flip with $p_{\mathrm{h}} = 0.9$ is $H(X) \approx 0.47$

Essential Bit Content of Extended Ensembles

Some Intuition

Why does the curve flatten for large N?

Recall tha d

typical

Such sequences occupy most of the probability ma

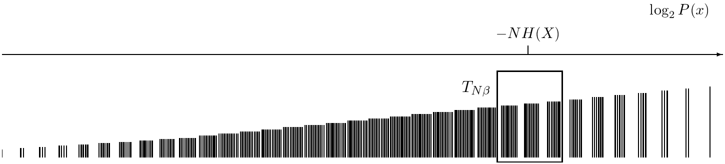
equally likely

As we increase δ, we will quickly encounter thes

small, roughly equal sized changes to $|S_\delta|$

Typical Sets and the AEP (Review)

$\mathbf{x}$	$\log_2(P(\mathbf{x}))$
...1..................1.....1....1.1.......1........1...........1...................1.......11...	-50.1
......................1.....1.....1.......1....1.........1.................................1...	-37.3
[illegible]	-65.9
[illegible]	-56.4
...11...........1...1.....1.1.......1..........1....1...1.....1.............1..........................	-53.2
..............1......1.........1.1.......1..........1...........1...1.....................1.......	-43.7
	$-$
.1..........1...1.1..............1.......11...........1.1...1...............1.............11..........	-56.4
......1...1..1.....1..11.1.1.1...1.....................1............	
...........11.1......1....1..1............................1.......1.	
....	
1111111111111111111111111111111111111111111111111111111111	



Typical Sets and the AEP (Review)

Typical Set

For "closeness" $\beta > 0$ the typical set $T_{N\beta}$ for X^N is

—

The nam

occurences of symbol a_1, $p_2 N$ of a_2, ..., p

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Asymptotic Equipartition Property (Informa

As $N \to \infty$, $\log_2 P(x_1, \ldots, x_N)$ is close to $-NH(X)$ with high probability.

For large block sizes "almost all sequences are typical" (i.e., in $T_{N\beta}$).

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The Source Coding Theorem

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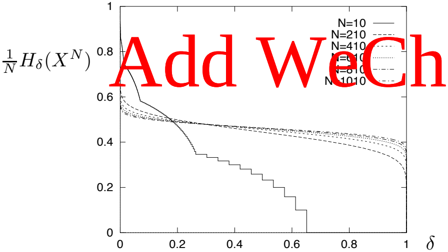
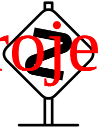
Let X be an ensemble with entropy $H = H(X)$ bits. Given $\epsilon > 0$ and $0 < \delta < 1$, there exists a positive integer N_0 such that for all $N > N_0$

—



- Given a tiny probability of error … e … H as
- Even if we allow a large probability of error, we **cannot** compress more than H bits per outcome for large sequences.

Warning: proof ahead



I don't exp

- I pre
- And it is a remarkable and fundamental result
- You are expected to **understand** and **be able to apply** the theorem

Proof of the SCT

The absolute value of a difference being bounded (e.g., $|x - y| \leq \epsilon$) says two things:

1. When $x - y$ is positive, it says $x - y < \epsilon$ which means $x < y + \epsilon$
2. When $x - y$ is negative, it says $-(x - y) < \epsilon$ which means $x < y - \epsilon$

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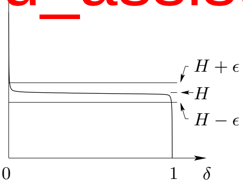
Using this

that for any ϵ and δ we can find N large eno

Part 1: $\frac{1}{N} H_\delta(X^N) < H + \epsilon$

Part 2: $\frac{1}{N} H_\delta(X^N) > H - \epsilon$



Proof the SCT

Idea

Proof Idea: As N increases

- $T_{N\beta}$ has $\sim 2^{NH(X)}$ elements
- alm

- S_δ and $T_{N\beta}$ increasingly overlap

- so $\log_2 |S_\delta| \sim NH$

Basically, we look to encode all typical sequences uniformly, and relate that to the essential bit content

Proof of the SCT (Part 1)

For $\epsilon > 0$ and $\delta > 0$, want N large enough so $\frac{1}{N}H_\delta(X^N) < H(X) + \epsilon$.

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$$(1)$$

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$$|T_{N\beta}| \leq 2^{N(H(X)+\beta)} \quad (1)$$

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$$H_\delta(X^N) = \log_2 |S_\delta| \leq \log_2 |T_{N\beta}| \leq N(H(X) + \beta)$$

Setting $\beta = \epsilon$ and dividing through by N gives result.

Proof of the SCT (Part 2)

For $\epsilon > 0$ and $\delta > 0$, want N large enough so $\frac{1}{N} H_\delta(X^N) > H(X) - \epsilon$.

Suppose this was not the case – that is, for every N we have

$$\frac{1}{N} H_\delta(X^N) \quad H(X) \quad \epsilon \qquad S_\delta \quad 2^{N(H(X)-\epsilon)}$$

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Let's look

$$P(x \in S_\delta) = P(x \in S_\delta \cap T_{N\beta}$$

$$\le |S_\delta| 2^{-N(H-\beta)} +$$

since every $x \in T_{N\beta}$ has $P(x) \le 2^{-N(H-\beta}$ $\delta \cap \overline{\ }_{N\beta} \subset \overline{\ }_{N\beta}$

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So

$$P(x \in S_\delta) \leq 2^{N(H-\epsilon)} 2^{-N(H-\beta)} + P(x \in \overline{T_{N\beta}})$$

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Let's look

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$$\leq |S_\delta| 2^{-N(H-\beta)} + \quad \overline{}$$

since every $x \in T_{N\beta}$ has $P(x) \leq 2^{-N(H-\beta}$

So

$$P(x \in S_\delta) \leq 2^{-N(\epsilon-\beta)} + P(x \in \overline{T_{N\beta}}) \to 0 \text{ as } N \to \infty$$

since $P(x \in T_{N\beta}) \to 1$.

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$$\in \quad \delta \qquad \in \quad \delta \cap \quad N\beta \qquad \delta \quad \overline{N\beta}$$

$$\le |S_\delta| 2^{-N(H-\beta)} +$$

since every $x \in T_{N\beta}$ has $P(x) \le 2^{-N(H-\beta}$

So

$$P(x \in S_\delta) \le 2^{-N(\epsilon-\beta)} + P(x \in \overline{T_{N\beta}}) \to 0 \text{ as } N \to \infty$$

since $P(x \in T_{N\beta}) \to 1$. But $P(x \in S_\delta) \ge 1 - \delta$, by defn. Contradiction

Interpretation of the SCT

The Source Coding Theorem

Let X be an ensemble with entropy $H = H(X)$ bits. Given $\epsilon > 0$ and $0 < \delta < 1$, there exists a positive integer N_0 such that for all $N > N_0$

If you want to uniformly code blocks of N s X

- If you use more than $NH(X)$ bits per blo almost no loss of information as $N \to$
- If you use less than $NH(X)$ bits per block you will almost certainly lose information as $N \to \infty$

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Making the error probability $\delta \approx 1$ doesn't re

- We're still "stuck with" coding the typical sequ

Assumes we deal with X^N

- If outcomes are dependent, entropy $H(X)$ need not be the limit
- We won't look at such extensions

Implications of SCT

How practical is it to perform coding inspired by the SCT?

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Can we de

- And

Next time

We move towards more practical compression ideas

Prefix and **Uniquely Decodeable** variable-length codes

The **Kraft Inequality**