

COMP2610 / COMP6261 - Information Theory

Lecture 4: Bayesian Inference

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July 31, 2018

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- Examples of joint, marginal and conditional distributions

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- What, if anything, does $p(X = x|Y = y)$ tell you about $p(Y = y|X = x)$?
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Review Exercise

Suppose we have binary random variables X, Y such that

$$p(X = 1) = 0.6$$

$$p(Y = 1 | X = 0) = 0.7$$

$$p(Y = 1 | X = 1) = 0.8$$

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Then,

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$$p(X = 1 | Y = 1) =$$

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Review Exercise

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Then,

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$$p(X = 1|Y = 1) = \frac{p(Y = 1|X = 1)p(X = 1)}{p(Y = 1)}$$

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$$= \frac{p(Y = 1|X = 1)p(X = 1) + (p(Y = 1|X = 0)p(X = 0))}{p(Y = 1)}$$

Review Exercise

Suppose we have binary random variables X, Y such that

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$$= \frac{p(Y = 1|X = 1)p(X = 1)}{p(Y = 1|X = 1)p(X = 1) + (p(Y = 1|X = 0)p(X = 0))}$$

$$= \frac{(0.8)(0.6)}{(0.8)(0.6) + (0.7)(0.4)}$$

$$\approx 0.63$$

This time

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- More examples on Bayes' theorem:
 - ▶ Eating hamburgers

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- Are there notions of probability beyond frequ

Outline

1 Bayes' Rule: Examples

- Eating Hamburgers
- Detecting Terrorists
- Th

2 Mom

3 The meaning of Probability

4 Wrapping Up

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1 Bayes' Rule: Examples

- Eating Hamburgers
- Detecting Terrorists
- The Monty Hall Problem

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2 Mom

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3 The meaning of Probability

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4 Wrapping Up

Bayesian Inference:

Example 1 (Barber, BRML, 2011)

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- 90% of people with McD syndrome are frequent hamburger eaters

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Bayesian Inference:

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Proportion of hamburger eaters is about 50

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- 90% of people with McD syndrome are frequent hamburger eaters

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Proportion of hamburger eaters is about 50

What is the probability that a hamburger eater will have McD syndrome?

Bayesian Inference:

Example 1: Formalization

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Let $McD \in \{0, 1\}$ be the variable denoting having the McD syndrome and $H \in \{0,$

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Bayesian Inference:

Example 1: Formalization

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Let $McD \in \{0, 1\}$ be the variable denoting having the McD syndrome and $H \in \{0, 1\}$ be the variable denoting having the Huntington's disease.

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We need to compute $p(McD = 1 | H = 1)$,
the probability of having the McD syndrome given that the patient has Huntington's disease.

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Bayesian Inference:

Example 1: Formalization

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We need to compute $p(McD = 1 | H = 1)$,
the probability of having the McD syndrome given having the Huntington's disease.

Any ballpark estimates of this probability?

Bayesian Inference:

Example 1: Solution

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= . ×

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Bayesian Inference:

Example 1: Solution

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$\frac{1}{n} \sum_{i=1}^n \log \pi(y_i | \theta)$
 $= \frac{1}{n} \sum_{i=1}^n \log \pi(y_i | \theta)$

Repeat the above computation if the proportion of h
rather small. (say in France) 0.001.

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Example 2: Detecting Terrorists:

From understandinguncertainty.org

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- Scanner detects true terrorists with 95% accuracy

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Example 2: Detecting Terrorists:

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- The shifty looking man sitting next to you tests p

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- There is 1 terrorist on your plane with 100 passengers

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- The shiftier looking man sitting next to you tests positive

What are the chances of this man being a terrorist?

Example 2: Detecting Terrorists:

Simple Solution Using “Natural Frequencies” (David Spiegelhalter)

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The chances of the man being a terrori $\approx \bar{6}$

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The chances of the man being a terrori $\approx \frac{1}{6}$

- Relation to disease example
- Consequences when catching criminals

Example 2: Detecting Terrorists:

Formalization with Actual Probabilities

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Let $T \in \{0, 1\}$ denote the variable regarding whether the person is a terrorist a

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$$p(T = 1) = 0.01$$

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Example 2: Detecting Terrorists:

Formalization with Actual Probabilities

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Let $T \in \{0, 1\}$ denote the variable regarding whether the person is a terrorist a

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$$p(T = 1) = 0.01$$

We want to compute $p(T = 1 | S = 1)$, the terrorist given that he has tested positive.

Example 2: Detecting Terrorists:

Solution with Bayes' Rule

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$p(T =$

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Example 2: Detecting Terrorists:

Solution with Bayes' Rule

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$p(T =$

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$\frac{(0.95)(0.01) + (0.05)}$

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Example 2: Detecting Terrorists:

Solution with Bayes' Rule

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$$p(T = 1) = \frac{(0.95)(0.01) + (0.05)(0.01)}{(0.95)(0.01) + (0.05)(0.01) + (0.05)(0.99)}$$

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Example 2: Detecting Terrorists:

Solution with Bayes' Rule

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$$p(T = 1) = \frac{(0.95)(0.01) + (0.05)(0.01)}{(0.95)(0.01) + (0.05)(0.01)}$$

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The probability of the man being a terrorist is \bar{p}

Example 2: Detecting Terrorists:

Posterior Versus Prior Belief

While the man has a low probability of being a terrorist, our belief has

increased compared to our prior

$$\frac{p(T = 1 | S = 1)}{p(T = 1)} = \frac{0.16}{0.01}$$

i.e. our belief

Since terrorists are so rare, a factor of 16 does not result

(absolute) probability or belief

(Aside: They are indeed very rare. For an intriguing (and surprising) example of the implications of inability to take account of actual base rates (in the example above we made the numbers up), and the effect on people's subsequent decisions, see Gerd Gigerenzer, Dread Risk, September 11, and Fatal Traffic Accidents, *Psychological Science* 15(4), 286–287, (2004); Gerd Gigerenzer, Out of the Frying Pan into the Fire: Behavioural Reactions to Terrorist Attacks, *Risk Analysis* 26(2), 347–351 (2006). His calculation (which of course is based on some assumptions) is that in the year following 9/11, 6 times the number of people who were killed as passengers *additionally* died on roads (that is the increase in road deaths due to people choosing to drive instead of flying)! He calls the reaction to very low probability events with a bad outcome “dread risk”.)

Example 3: The Monty Hall Problem

Problem Statement

- Three boxes, one with a prize and the other two are empty

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- Three boxes, one with a prize and the other two are empty
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- Three boxes, one with a prize and the other two are empty
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- You
- You select one of the boxes

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- Three boxes, one with a prize and the other two are empty
- Each box has equal probability of having the prize
- You choose one of the boxes
- The host, who knows the location of the prize, removes one of the other two boxes

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Problem Statement

- Three boxes, one with a prize and the other two are empty
- Each box has equal probability of having the prize
- You choose one of the boxes
- The host, who knows the location of the prize, removes one of the other two boxes

Should you switch to the other box? Would that increase your chances of winning the prize?

Example 3: The Monty Hall Problem:

Formalization

Let $C \in \{r, g, b\}$ denote the box that contains the prize where r, g, b refer to the identity of each box.

WLOG assume the following:

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$P(C=r) = \frac{1}{3}$ $P(C=g) = \frac{1}{3}$ $P(C=b) = \frac{1}{3}$

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$$p(C = r) = \frac{1}{3} \quad p(C = g) = \frac{1}{3} \quad p(C = b) = \frac{1}{3}$$
$$p(H = b | C = r) = \frac{1}{2} \quad p(H = b | C = g) = 0 \quad p(H = b | C = b) = 0$$

Example 3: The Monty Hall Problem:

Formalization

Let $C \in \{r, g, b\}$ denote the box that contains the prize where r, g, b refer to the identity of each box.

WLOG assume the following:

- You

- De

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$$\begin{aligned} p(C = r) &= \frac{1}{3} & p(C = g) &= \frac{1}{3} \\ p(H = b | C = r) &= \frac{1}{2} & p(H = b | C = g) &= 0 \end{aligned}$$

We want to compute $p(C = r | H = b)$ and $p(C = g | H = b)$ to decide if we should switch from our initial choice.

Example 3: The Monty Hall Problem:

Solution

We have that:

$$P(H = b) = \sum_{c \in \{r, g, b\}} P(H = b | C = c) P(C = c)$$

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$$p(C = r | H = b) = \frac{p(H = b | C = r) p(C = r)}{p(H = b)}$$

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Therefo

$$p(C = r | H = b) = \frac{p(H = b | C = r) p(C = r)}{p(H = b)} = \frac{1/3}{1/3}$$

Similarly, $p(C = g | H = b) = 2/3$.

Example 3: The Monty Hall Problem:

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We have that:

$$p(H = b) = \sum_{c \in \{r, g, b\}} p(H = b | C = c) p(C = c)$$

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$$p(C = r | H = b) = \frac{p(H = b | C = r) p(C = r)}{p(H = b)} = \frac{1/3}{1/3}$$

Similarly, $p(C = g | H = b) = 2/3$.

You should switch from your initial choice to the other box in order to increase your chances of winning the prize!

Example 3: The Monty Hall Problem:

Illustration of the Solution

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Illustration of the solution when you have initially selected box r .

Example 3: The Monty Hall Problem:

Another Perspective

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Switching is bad if, and only if, we initially picked the prize box (because if not, the other

We picked
host's action

Hence, with probability $2/3$ switching will reveal

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Example 3: The Monty Hall Problem:

Variants to Ponder

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Would switching be rational if

- The host only revealed a box when he knew we picked the right one?

- The one?

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- The host is himself unaware of the prize box, an random, which by chance does not have the prize?

1 Bayes' Rule: Examples

- Eating Hamburgers

- Detecting Spam

- The Monty Hall Problem

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The Expected Value of a Function of Two Discrete Random Variables

(Assuming you have met **Expectation** $E[X]$ and **Variance** $\text{Var}(X)$ before.)

The expected value of a function $g(X, Y)$ of two discrete random variables is defined

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In particular, the expected value of X is given

$$E[X] = \sum_x \sum_y xp(X, Y) \quad (2)$$

It should be noted that if we have already calculated the marginal distribution of X , then it is simpler to calculate $E[X]$ using this.

Covariance and the Correlation Coefficient

The covariance between X and Y , $\text{Cov}(X, Y)$ is given by

$$\text{Cov}(X, Y) = E(XY) - E(X)E(Y) \quad (3)$$

Note that

The coefficient

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$$\rho(X, Y) = \frac{\text{Cov}(X, Y)}{\sqrt{\text{Var}(X)}\sqrt{\text{Var}(Y)}} \quad (4)$$

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Always in $[-1, 1]$.

Example

Discrete random variables X and Y have the following joint distribution:

	$Y = 0$	$Y = 1$
$X = 0$	$\frac{0}{1}$	$\frac{1}{3}$
$X = 1$	$\frac{0}{1}$	$\frac{0}{1}$

Calculate

- 1 $p(X)$
- 2 marginal distributions of X and Y
- 3 expected values and variances of X
- 4 coefficient of correlation between X and Y

Are X and Y independent?

Example

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To calculate the probability of such an event, note that we sum over all the cells with

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$$+ p(X = 1, Y = -$$

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Example

Recall that

$$p(X = x) = \sum_y p(X = x, Y = y).$$

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Hence,

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Note that after obtaining $p(X = 0)$, we could calculate $p(X = 1)$ using the fact that

$$p(X = 1) = 1 - p(X = 0), \quad (5)$$

since X only takes the values 0 and 1.

Example

Similarly, Assignment Project Exam Help

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$$p(Y = 0) = \sum_{x=0} p(X = x, Y = 0) = \frac{1}{2} + 0 = \frac{1}{2}$$

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$$p(Y = 1) = 1 - p(Y = 0) = \frac{1}{2}$$

Example

We then calculate the expected values and variances of X and Y from these marginal distributions.

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$$E(X) = \sum_{y=-1}^1 y P(Y=y) = -\frac{1}{2} \times$$

Example

To calculate the variances of X and Y , $\text{Var}(X)$ and $\text{Var}(Y)$, we use the formula

$$\text{Var}(X) = E(X^2) - (E(X))^2$$

$$E(X^2) = \sum_{i=1}^n x_i^2 p(X = x_i) = (-1)^2 \cdot \frac{2}{3} + 0^2 \cdot \frac{1}{3} = \frac{2}{3}$$

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$$E(Y^2) = \sum_{j=-1}^1 y_j^2 p(Y = y_j) = (-1)^2 \cdot \frac{1}{3} + 0^2 \cdot \frac{2}{3} + 1^2 \cdot \frac{2}{3} = \frac{2}{3}$$

Thus we get

$$\text{Var}(X) = \frac{2}{3} - \left(\frac{2}{3}\right)^2 = \frac{2}{9}$$

$$\text{Var}(Y) = \frac{2}{3} - (0)^2 = \frac{2}{3}$$

Example

To calculate the correlation coefficient, we first calculate the covariance between X and Y . We have

$$\text{Cov}(X, Y) = E(XY) - E(X)E(Y).$$

where

$$E(XY)$$

$$= 0(-1)0 + 0(0)\frac{1}{3} + 0(1)0 + \dots - \frac{1}{3} = 0$$

Thus we get

$$\text{Cov}(X, Y) = E(XY) - E(X)E(Y) = 0 - \frac{1}{3} \times 0 = 0.$$

From the definition of the correlation coefficient,

$$\rho(X, Y) = 0.$$

Example - is X and Y independent

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We have t

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The meaning of Probability

Frequentist : Frequencies of random repeatable experiments

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The meaning of Probability

Frequentist : Frequencies of random repeatable experiments

- E.g. Prob. of biased coin landing "Heads"

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The meaning of Probability

Frequentist : Frequencies of random repeatable experiments

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Bayesian : Degrees of Belief

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The meaning of Probability

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Bayesian : Degrees of Belief



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The meaning of Probability

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Cox Axio

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- 2 $B(x) = f[B(\bar{x})]$
- 3 $B(x, y) = g[B(x|y), B(y)]$

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If a set of Beliefs satisfy these axioms they can be mapped onto probabilities satisfying the rules of probability.

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Image from <http://normaldeviate.wordpress.com/2012/11/09/anti-xkcd/>

In practice one needs to make use of both interpretations. Wise to be open to both. This is a huge topic which we can not get into further here. Note that Mackay was firmly in the Bayesian camp. . .

1 Bayes' Rule: Examples

- Eating Hamburgers

- Detecting Spam Emails

- The Monty Hall Problem

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2 Mom

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3 The meaning of Probability

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4 Wrapping Up

Summary

- Examples of application of Bayes' rule
- Formalization

- Intuition <https://eduassistpro.github.io/>

- Frequentist vs Bayesian probabilities

- Cox axioms

Next time

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- Working through some useful probability distributions

<https://eduassistpro.github.io>

- More on Bayesian inference

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