Assignment Project Exam Help

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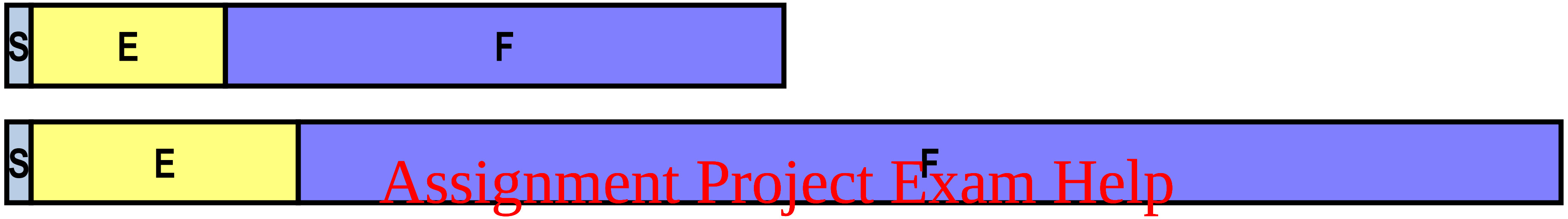
# Outline

- Special "numbers" revisited
- Rounding
- FP add/sub
- FP on MIPS
- Integer multiplication & division

# IEEE 754 Floating Point Review






| Precision | Sign | [illegible] | [illegible] n (F) | Bias |
|---|---|---|---|---|
| Float | 1 bit | 8 bits | [illegible] | 127 |
| Double | 1 bit | 11 bits | [illegible] | 1023 |



$$(-1)^S \times (1+F) \times 2^{(E\text{-bias})}$$

- Numbers in *normalized* form, i.e., 1.xxxx…
- The standard also defines special symbols

# Special Numbers Reviewed

- Special symbols (single precision)

| Exponent | Fraction | Object represented |
|---|---|---|
| 0 | [illegible] | 0 |
| 0 | Nonzero | [illegible]alized number |
| 1-254 | Anything | ± floating point number |
| 255 | 0 | ± infinity |
| 255 | Nonzero | NaN (Not a Number) |

# Representation for Not a Number

- What do I get if I calculate `sqrt(-4.0)` or `0/0`?



  - If infinity is not an er e either.
  - Called Not a Numbe 
  - Exponent = 255, Significand nonze 
- Why is this useful?
  - Hope NaNs help with debugging?
  - They contaminate: op(NaN,X) = NaN

# Representation for Denorms (1/2)

- Problem: There's a gap among representable FP numbers around 0
  - Smallest representa

    a = 1.0000000000000



  - Second smallest representable po ber:

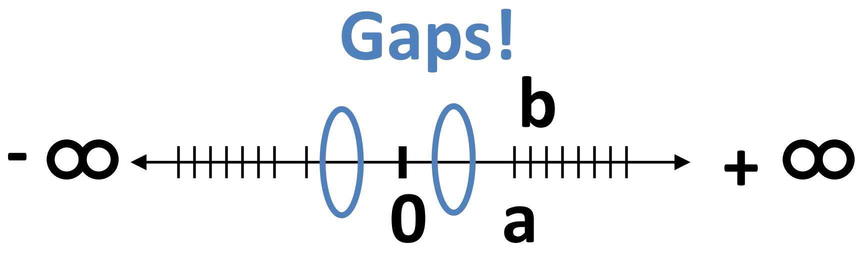


    $b = 1.00000000000000000000001_2 * 2^{-126} = 2^{-126} + 2^{-149}$

  $a - 0 = 2^{-126}$

  $b - a = 2^{-149}$

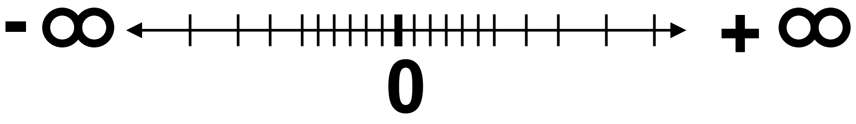
**Normalization and implicit 1 is to blame!**

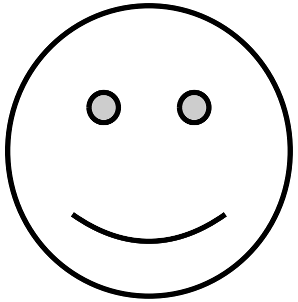


# Representation for Denorms (2/2)

- Solution: special symbol in exponent field
  - Use 0 in exponent field, nonzero for fraction
  - Denormalized numb
    - Has no leading 1
    - **Has implicit exponent = -126 (i.e., d t bias)**
  - Smallest positive *float*: 2e-149
  - 2nd smallest positive *float*: 2e-148





# Small numbers and Denormalized

$1.00000000000000000000010_2$ x 2^-126

$1.00000000000000000000001_2$ x 2^-126

$1.00000000000000000000000_2$ x 2^-126



$0.11111111111111111111111_2$ x 2^-126   Denormalized!

$0.11111111111111111111110_2$



$0.11111111111111111111101_2$ x 2^-126

…



$0.00000000000000000000011_2$ x 2^-126

$0.00000000000000000000010_2$ x 2^-126

$0.00000000000000000000001_2$ x 2^-126

Next smaller number is zero

# Rounding

- When we perform math on real numbers, we must worry about rounding to fit the result in the significant field.



  – The FP hardware car recision, and then rounds to get the proper val



  – Rounding also occurs when conve ble to a single precision value, or converting a floating point number to an integer

# IEEE Has Four Rounding Modes

1. Round towards +infinity
   - ALWAYS round “up”: 2.001 -> 3
   - -2.001 -> -2 ceiling($x$) or $\lceil x \rceil$
2. Round towards -infi
   - ALWAYS round “do
   - -1.999 -> -2 $x$) or $\lfloor x \rfloor$
3. Truncate
   - Just drop the last bits (round towards 0)
4. Round to (nearest) even
   - Normal rounding, almost

# Round to Even

- Round like you learned in grade school
- ***Except*** if the value is right on the borderline, in which case we round to the nearest EVEN number
  - 2.5 -> 2
  - 3.5 -> 4
- Insures ***fairness***
  - This way, half the time we round up on tie, the other half time we round down
- This is the default rounding mode

# FP Addition and Subtraction 1/2

- ***Much*** more difficult than with integers
- Cannot just add significands
- Recall how we do it:



1. De-normalize to ma

2. Add significands to get resulting on

3. Normalize and check for under/ov
4. Round if needed (may need to goto 3)

- Note: If signs differ, perform a subtract instead
  - Subtract is similar except for step 2

# FP Addition and Subtraction 2/2

- Problems in implementing FP add/sub:
  – If signs differ for add (or same for sub), what is the sign of the result?



- Question:
  – How do we integrat  arithmetic unit?
  – Answer: We don’t!

# MIPS Floating Point Architecture (1/4)

- Separate floating point instructions:
  - Single Precision:
    **`add.s, sub.s, mul.s, div.s`**
  - Double Precision:
    **`add.d, sub.d, mul.d, di`**
- These instructions are ***far more complicated*** than their integer counterparts, so they can take much longer to execute.

# MIPS Floating Point Architecture (2/4)

- Observations
  - It’s inefficient to have different instructions take vastly differing amounts of time.
  - Generally, a <u>particul</u> <u>t change from FP to int</u>, or vice versa, within a program. So o pe of instruction will be used on it.
  - Some programs <u>do no floating point calculations</u>
  - It takes lots of hardware relative to integers to make Floating Point fast

# MIPS Floating Point Architecture (3/4)


- Pre 1990 Solution:
  - separate chip to do floating point (FP)
- Coprocessor 1: FP chi
  - Contains 32 32-bit reg
  - Usually registers specified in FP instr er to this set
  - Separate load and store: **lwc1** and **swc1** ("load word coprocessor 1", "store …")
  - Double Precision: by convention, even/odd pair contain one DP FP number: `$f0`/`$f1`, `$f2`/`$f3`, … , `$f30`/`$f31` where the ***even register*** is the name

# MIPS Floating Point Architecture (4/4)

- Pre 1990 Computers contains multiple separate chips:
  - Processor: handles all the normal stuff
  - Coprocessor 1: hand
  - more coprocessors? r)
- Today, FP coprocessor integrat PU, or specialized or inexpensive chips may leave out FP HW
- Instructions to move data between main processor and coprocessors, e.g., mfc0, mtc0, mfc1, mtc1

# Some More Example FP Instructions

```
abs.s $f0, $f2  # f0 = abs( f2 );
neg.s $f0, $f2  # f0 = - f2;
sqrt.s $f0, $f2 # f0 = sqrt( f2 );

c.lt.s $f0, $f2 # is
bc1t   label    # branch on co        rue
```



See 4th edition text 3.5 and App. B for a complete list of floating point instructions

# Copying, Conversion, Rounding

```
mfc1 $t0, $f0     # copy $f0 to $t0
mtc1 $t0, $f0     # copy $t0 to $f0

cvt.d.s $f0 $f2   # f0f1 gets float f2 converted to double
cvt.d.w $f0 $f2   # f0f1 ge                             double

cvt.s.d $f0 $f2   # f0 gets double f2f3 c               float
cvt.s.w $f0 $f2   # f0 gets int f2 conver               t

ceil.w.s  $f0 $f2 # round to next higher integer
floor.w.s $f0 $f2 # round down to next lower integer
trunc.w.s $f0 $f2 # round towards zero
round.w.s $f0 $f2 # round to closest integer
```

# Dealing with Constants

`float a = 3.14;`

- Option 1
  - Declare constant 3.14 in data segment of memory
  - Load the address labe
  - Load to coprocessor

```
.data
PI: .float 3.14
.text
la   $t0 PI
lwc1 $f0 ($t0)
l.s  $f0 PI    # easiest
```



- Option 2
  - Compute hexadecimal IEEE representation for 3.14 (it is 48F5C3)
  - immediate
  - o coprocessor

```
lui  $t0 0x4048
ori  $t0 $t0 0xF5C3
mtc1 $t0 $f0
```

**Option 3, pseudoinstruction not available in MARS:**
`li.s $f0, 3.14`

# Floating Point Register Conventions

| | |
|---|---|
| **($f0, $f1), and ($f2, $f3)** | Function **return** registers used to return float and double values from function calls. |
| **($f12, $f13) and ($f14, $f15)** | Two pairs of registers used to pass float and double valued **arguments** to functions. Pai esized because the ues. To pass float values, only $f are used. |
| **$f4, $f6, $f8, $f10, $f16, $f18** | **Temporary** registers |
| **$f20, $f22, $f24, $f26, $f28, $f30** | **Save** registers whose values are **preserved** across function calls |



Unfortunately no nice names (e.g., $t#, $s#) like with the main registers)

*With double precision instructions, the high-order 32-bits are in the implied odd register.*

# Fahrenheit to Celsius

```
float f2c(float f) { return 5.0/9.0*(f-32.0); }
```

```
.text
f2c:
      la     $t0 co
      lwc1   $f16 (
      la     $t0 const9
      lwc1   $f18 ($t0)
      div.s  $f16 $f16 $f18    # f16 = 5.0/9.0
      la     $t0 const32
      lwc1   $f18 ($t0)
      sub.s  $f18 $f12 $f18    # f18 = fahr-32.0
      mul.s  $f0 $f16 $f18     # return f16*f18
      jr     $ra
```

```
.data
const5: .float 5.0
            loat 9.0
            loat 32.0
```

# Debugging FP Code in MARS

- MARS displays floating point registers in hexadecimal
- This makes debugging floating point code tricky…
  - Can use MARS “Floating Point Representation” tool to examine single precision
  - Alternatively **syscall** can le



| Service | Code in $v0 | Arguments |
|---|---|---|
| Print float | 2 | $f12 = float to print |
| Print double | 3 | $f12 = double to print |
| Print string | 4 | $a0 = address of null-terminated string to print |

```
.data
spaceString:    .asciiz " "
newlineString:  .asciiz "\n"
```

```
printSpace:
        li $v0, 4
        la $a0, spaceString
        syscall
        jr $ra

printNewLine:
        li $v0, 4
        la $a0, newlineString
        syscall
        jr $ra

printFloat: # in $f12
        li $v0, 2
        syscall
        jr $ra
```

```
# print( float vec[4] )
printFloatVector:
        addi $sp, $sp, -8
        sw $ra, 0($sp)
        sw $s0, 4($sp)
        move $s0, $a0
        lwc1 $f12, 0($s0)
        jal printFloat
        jal printSpace
        lwc1 $f12, 4($s0)
        jal printFloat
        jal printSpace
        lwc1 $f12, 8($s0)
        jal printFloat
        jal printSpace
        lwc1 $f12, 12($s0)
        jal printFloat
        jal printNewLine
        lw $ra, 0($sp)
        lw $s0, 4($sp)
        addi $sp, $sp, 4
        jr $ra
```

# REMEMBER: Floating Point Fallacy

- FP add, subtract associative? ***FALSE!***

  $x = -1.5 \times 10^{38}$ $y = 1.5 \times 10^{38}$ $z = 1.0$

  $x + (y + z) = -1.5\text{x}10^{38} + (1.5\text{x}10^{38} + 1.0)$

  $= -1.$

  $= \mathbf{0.0}$

  $(x + y) + z = (-1.5\text{x}10^{38} + 1.5\text{x}1$

  $= (0.0) + 1.0$

  $= \mathbf{1.0}$

- Floating Point add, subtract are not associative!
  – Floating point result ***approximates*** real result!

# Casting floats ↔ ints

- `(int)` *`floating point expression`*
  - Coerces and converts it to the nearest integer (C uses truncation)



```
i = (int) (3.14
```



- `(float)` *`expression`*



  - converts integer to nearest floating point

```
f = f + (float) i;
```

# int → float → int

```
if ( i == (int)((float) i) ) {
    printf("true");
}
```



- Does this al
  – No, it will ***not*** always pri
  – Large values of integers don't have exact floating point representations
- What about `double`?

# float → int → float

```
if ( f == (float)((int) f) ) {
    printf(“true”);
}
```



- Does this always print t
  - No, it will ***not*** always pri
  - Small floating point numbers (<1) don’t have integer representations
  - Same is true for large numbers
  - For other numbers, rounding errors

# MIPS Integer Multiplication

- Syntax of Multiplication (signed): **`MULT reg1 reg2`**
- Result of multiplying 32 bit registers has 64 bits



- MIPS splits 6 … cial registers



  – upper half i …
  – Registers hi and lo are sep … the 32 general purpose registers



  – Use **`MFHI reg`** to move from hi to register
  – Use **`MFLO reg`** to move from lo to another register
- Unusual syntax compared to other instructions!

# MIPS Integer Multiplication Example

**a = b * c;**

Let b be $s2; let c be $s3;



And let a be $s0 and $ bits)





```
mult $s2 $s3   # b*c
mfhi $s0       # get upper half of product
mflo $s1       # get lower half of product
```

- We often only care about the low half of the product!

# MIPS Integer Division

- Syntax of Division (signed): **`DIV reg1 reg2`**
  - Divides register 1 by register 2
  - Puts remainder of di
  - Puts quotient of divi
- Notice that this can be used to i t both the division operator (/) and modulo operator (%) in a high level language

# MIPS Integer Division Example

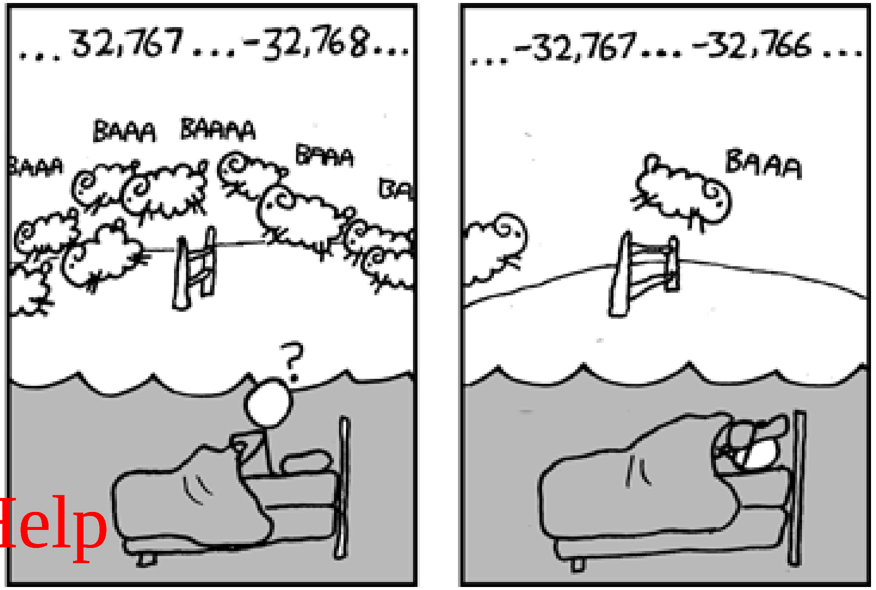
```
a = c / d;
b = c
```



| Variable | Register |
|---|---|
| a | $s0 |
| b | $s1 |
| c | $s2 |
| d | $s3 |

```
div  $s2 $s3   # lo=c/d, hi=c%d
mflo $s0       # get quotient
mfhi $s1       # get remainder
```

# Unsigned Instructions and Overflow




- MIPS has versions of **`mult`** and **`div`** for unsigned operands:

    **`multu, divu`**

    – Determines wheth … nd quotient are changed if the operands are sign … gned.

- Typically unsigned instructions … r overflow (e.g., add vs addu)
- MIPS ***does not*** check overflow or division by zero on ANY signed/unsigned multiply, divide instruction
    – Up to the software to check “hi”, “divisor”

# Things to Remember

- Integer multiplication and division:
  - **`mult, div, mfhi, mflo`**
- New MIPS registers (**`$f0-$f31`**) and instructions in two flavours
  - Single Precision **`.s`**
  - Double Precision **`.d`**
- FP add and subtract are ***not associative***…
- IEEE 754 NaN & Denorms (precision) review
- IEEE 754's Four different rounding modes

# Review and More Information

- Textbook
  - Section 3.5 Floating Point
    - We saw the representation and addition and multiplication algorithm material earlier in th
    - And now we have seen the Floating- tions