COMP30026 Models of Computation

Logic Concepts

Lecture Week 2 Part 2 (Zo

Semester 2, 2021

Knights and Knaves Puzzle

On the island of Knights and Knaves, everyone is a knight or knave. Knights always tell the truth. Knaves always lie.

Today there is a census on the island!

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If you like these puzzles, Raymond Smullyan has written several books that you will like.

Before we get to more sophisticated logic and its applications, let us establis

“T

log

analysis and physics in the last.”

Validity and Satisfiability

A propositional formula is valid if no truth assignment makes it false. Otherwise it is non-valid.

It is uns

satisfiab

A valid propositional formula is a tauto

An unsatisfiable propositional formula is a

Tautology Example

$(\neg P \wedge Q) \Rightarrow (P \Rightarrow R)$ is valid:

P	Q	R	$\neg P \wedge Q$	$P \Rightarrow R$	$(\neg P \wedge Q) \Rightarrow (P \Rightarrow R)$
f	**t**	**t**	**t**	**t**	**t**
t	**f**	**f**	**f**	**f**	
t	**f**	**t**	**f**	**t**	
t	**t**	**f**	**f**	**f**	
t	**t**	**t**	**f**	**t**	**t**

Logically, "Valid" Means Vacuous

In ordinary discourse, we may praise somebody by saying "the point you make is valid."

But in fo

it is voi ement and valid.

"If Trump is sane then Trump is sane" tells us nothin ether Trump is sane—the statement is true whatever th

You don't even have to know who Trump is, or what it means to be sane, in order to agree: The statement is inherently true.

Validity Checking in Haskell

Given a truth table for a proposition P, it is easy to check if P is valid: The conjunction of all the entries in P's column must be true.

To check the validity of 3-place Haskell predicate:


```
vali
valid3 f
  = and [f p q r | p <- bs, q <- bs, r <- bs]
    where
      bs = [False, True]
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Poll 2: What does a function to check satisfiability look like?

$P \wedge Q \wedge (\neg Q \Leftrightarrow (\neg P \vee Q))$ is unsatisfiable.

Again, w ten
possible t

In this case
assignment maps either to **f**, the formula ev

And if P and Q are both mapped to **t**, t
$(\neg \mathbf{t} \Leftrightarrow (\neg \mathbf{t} \vee \mathbf{t}))$, which again evaluates to **f**.

Validity is preserved by substitution of propositional letters by formulas.

We saw that $(\neg P \land Q) \Rightarrow (P \Rightarrow R)$ is valid, and hence

is valid.

(Different letters can be replaced by the same form[ula] course, all occurrences of a letter must be replaced by the sam[e formu]la.)

A formula is unsatisfiable iff its negation is valid.

It follows that substitution also preserves unsatisfiability.

Poll 3

Does substitution preserve satisfiability?

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No — a cou

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Take P ... $Q \wedge \neg Q$.

The fact that P can be made true does not mea

make if false.

In fact, the typical propositional formula can be ma e and false; it will simultaneously be satisfiable and non-valid.

Models, Logical Consequence, and Equivalence

Let θ be a truth assignment and F be a propositional formula. If θ makes F true then θ is a model of F.

G is a l[illegible] el of G as well.

In that cas[illegible] F [illegible] G

If $F \models G$ and $G \models F$ both hold, that is, [illegible] e same models, then F and G are (logicall[illegible]

In that case we write $\boxed{F \equiv G}$

Poll 4

Of the following statements, which allow us to conclude $P \Rightarrow Q$?

- P
- $\neg P$
- Q
- P
- $(P$
- $\neg P \vee Q$
- $\neg Q \Rightarrow \neg P$
- $P \Rightarrow (Q \vee R)$
- $(P \Rightarrow Q) \vee R$

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- $(P \Rightarrow Q) \vee R$

Which of the statements are logical consequences of $P \Rightarrow Q$?

If $F \equiv$
of letter

Interchange of Equivalents

Replacing equals by equals yields equals. If

1. F is a sub-formula of H,
2. F
3. H'

then $H \equiv H'$.

Interchange of equivalents preserves not only logi e.

It also preserves logical consequence, validity, and unsatisfiability.

Unlike substitution, it even preserves satisfiability.

Some Equivalences

Absorption: $P \wedge P \equiv P$

$P \vee P \equiv P$

Comm

Associativity: $P \wedge (Q \wedge R) \equiv (P$

$P \vee (Q \vee R) \equiv (P$

Distributivity: $P \wedge (Q \vee R) \equiv (P$

$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$

More Equivalences

$\Leftrightarrow$ and $\oplus$ are also commutative and associative.

Double negation: $P \equiv \neg\neg P$

De Mor

Implication: $P \Rightarrow Q \equiv \neg P \vee Q$

Contraposition: $\neg P \Rightarrow \neg Q \equiv Q \Rightarrow$

$P \Rightarrow \neg Q \equiv Q \Rightarrow \neg$

$\neg P \Rightarrow Q \equiv \neg Q \Rightarrow P$

Biimplication: $P \Leftrightarrow Q \equiv (P \wedge Q) \vee (\neg P \wedge \neg Q)$

Quiz

Find an altnction.

Last Equivalences

Let $\bot$ be any unsatisfiable formula and let $\top$ be any valid formula.

Duality: $\neg\top \equiv \bot$

$\neg\bot \equiv \top$

Negati

Identit

$P \wedge \top \equiv P$

Dominance: $P \wedge \bot \equiv \bot$

$P \vee \top \equiv \top$

Contradiction: $P \wedge \neg P \equiv \bot$

Excluded middle: $P \vee \neg P \equiv \top$

Poll 5

Which of t

1. P

2. $(P$
3. $(P \Rightarrow R) \wedge (Q \Rightarrow R) \models (P \wedge Q)$

Next Week Same Time Tune In

Learn how symbolic manipulation beats truth tables.

We shall b

Mechanising deduction based on pro

Long before Covid-19, Mr. Smith and his wife invited four other couples. When everyone arrived, some of the people shook hands with some of the others. Of course, nobody shook hands with their partner,

person tw

After that, Mr. Smith asked everyone how many ti k somebody's hand. He received different answers f

How many times did Mrs. Smith shake hands?

Answers to the discussion board