COMP30026 Models of Computation

Propositional Logic

Lecture Week 2 Part 1 (Zo

Semester 2, 2021

This Lecture is Being Recorded

Our Goal for the Next Few Lectures

- Introduce/recapitulate propositional logic
- Use it as a vehicle for launching more generally applicable logic co
- Us

If you are familiar with propositional logic, some of this will be old hat.

But pay attention anyway, because the concepts a e introduce now will serve as a blueprint for similar (but more complex and powerful) concepts and methods for predicate logic.

Administrivia: Grok

Solutions to module 1 problems are now available—Study them!
The first serious assessment tasks are not far away.

Do not neg

Of all Grok es in all 8 modules, and 46 have completed the 6 mandatory modules.

415 have at least one green diamond. But 105 have not d module 1 yet. And more than 60 are still to do th !

If you don't have a Grok account, please identify yourself asap.

Propositional = Boolean Logic

Philosophers have been interested in the "rules of reasoning" for thousands of years. Aristotle's syllogisms had particular importance for European scholars.

George B

father of modern logic. Boole took an algebraic view of logic, pointing out that there are important abstract analogies between certain arithmetic operations and the logical connectives.

Intro Puzzle

Huey, Dewey and Louie are being questioned by their uncle. Here is what they say:

> *Hu*
>
> *so is t*
>
> *De*
>
> *Lo*

Their uncle, knowing that they are cub scouts, reali ey cannot tell a lie.

Has he got sufficient information to decide who (if any) are guilty?

(Classical) Propositional Logic: Syntax

We shall build propositional formulas from this set of symbols:

$$\underbrace{A, B, C, \ldots Z}_{\text{prop. letters}}, \underbrace{\neg, \wedge, \vee, \Rightarrow, \Leftrightarrow, \oplus}_{\text{connectives}}, \mathbf{f}, \mathbf{t}, (,).$$

Well-for

$$
\begin{array}{ll}
 & \ldots \\
\mid & (\neg wff) \\
\mid & (wff \wedge wff) \\
\mid & (wff \vee wff) \\
\mid & (wff \Rightarrow wff) \\
\mid & (wff \Leftrightarrow wff) \\
\mid & (wff \oplus wff)
\end{array}
$$

Propositional Logic: Notational Conveniences

We shall drop outermost parentheses.

We shall assume that $\neg$ binds tighter than $\wedge$ and $\vee$.

These bin d $\Leftrightarrow$.

This allo

$$(((\neg Q) \wedge P) \Rightarrow (P \vee$$

as

$$\neg Q \wedge P \Rightarrow P \vee ($$

Note: O'Donnell et al. (and Makinson) use $\rightarrow$ instead of $\Rightarrow$, and $\leftrightarrow$ instead of $\Leftrightarrow$. Makinson also uses 0 for **f** and 1 for **t**. On a whiteboard I tend to use 0 and 1, as they are faster to write.

Propositional Logic: Semantics

A proposition is false (**f**) or true (**t**).

A truth assignment maps each propositional letter to **t** or **f**.

We can giv [illegible] es:

f	**f**	**t**	**f**	**f**	**t**	**t**	**f**
f	**t**	**t**	**f**	**t**			
t	**f**	[illegible]	**f**	**t**			
t	**t**	**f**	**t**	**t**			

This gives meaning to all propositional formulas, as we let A and B stand for the values of arbitrary (compound) propositions.

Connectives Defined in Haskell

Haskell has a type `Bool`, and some connectives are pre-defined:

```
data Bool = False | True
```



```
not :: B
not Tr
not Fa
```



```
(&&) :: Bool -> Bool -> Bool
False && _ = False
True  && x = x
```



```
(||) : Bool -> Bool -> Bool
False || x = x
True  || _ = True
```

$P \wedge Q$ is the conjunction of P and Q.

$P \vee Q$ i

An "or" in E

I'll eat if there is peanut butter or jam in the fridge.

Other times it translates to exclusive or:

Would you like the ice cream or the crème br

The Conditional

The proposition $P \Rightarrow Q$ is best read “if P then Q” (or sometimes “P only if Q” or “Q whenever P”). Usually, “implies” is misleading.

1. If the volume is increased, the pressure

A		
f	[illegible]	[illegible]
f	[illegible]	[illegible]
t	**f**	**f**
t	**t**	**t**

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A		
f		
f		
t	**f**	**f**
t	**t**	**t**

1. If the volume is increased, the pressure

3. Melbourn
different st
is in Queensl

We talk about material implication.

Note that $A \Rightarrow B$ has the same truth table as $\neg A \vee B$.

More Connectives in Haskell


```
infix 0 ==>
infix 0 <=>
infix 1 <+>

(==>
Fals
True  ==

(<=>) :: Bool -> Bool -> Bool
x <=> y  =  x == y

(<+>) :: Bool -> Bool -> Bool
x <+> y  =  x /= y
```

Poll 1

Which of these claims hold?

1. P
2. $(P$ … $(Q \wedge R)$
3. $(P \Rightarrow R) \wedge (Q \Rightarrow R)$ has the same truth t … $\Rightarrow R$

We can also define $\downarrow$, or "nor", as well as $\uparrow$, or "nand".

t	**f**	**f**	
t	**t**	**f**	

"Nand" is sometimes called Sheffer's stroke.

Some Ternary Connectives

A	B	C	if A then B else C	*median*(A, B, C)
f	**f**	**f**	**f**	**f**
t	**f**	**f**	**f**	
t	**f**	**t**	**f**	
t	**t**	**f**	**t**	
t	**t**	**t**	**t**	**t**

On Boolean Short-Circuit Definitions

Most programming languages offer the Boolean connectives 'and' and 'or', but usually these are not commutative!

In C, Hask
has a beha .

One eval
version avoids the runtime error, because conjun strict
function in typical programming languages: If the nt is
false, the second won't be evaluated.

To model the behaviour properly, we really need three-valued propositional logic, the third truth value being "undefined".

Knights and Knaves Puzzle

On the island of Knights and Knaves, everyone is a knight or knave. Knights always tell the truth. Knaves always lie.

Today th

You are a d ... you know about each of these three houses.

- **In house 1:** Husband: We are both kna

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- **In house 2:** Wife: At least one of us is a knav
- **In house 3:** Husband: If I am a knight then so is my wife.