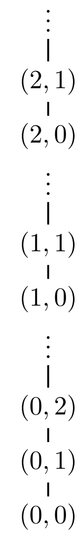
Selected Problem Set Solutions, Week 12

P12.1 See the Week 12 lecture slides.

P12.2 See the Week 12 lecture slides.

P12.3 (b) is not well-founded, as we can have infinite strictly decreasing sequences in $\mathbb{Q}$, such as $\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$. But (a), (c) and (d) are all well-founded. For (d) it may help to look at the Hasse diagram for $\mathbb{N} \times \mathbb{N}$ ordered by $\prec$ (shown here in the margin).



P12.4

```
hailstone :: Integer -> Int
hailstone 0 = 0
hailstone 1 = 0
hailstone n
  | even n = hailstone (n `div` 2) + 1
  | otherwise = hailstone (3*n+1) + 1
```


P12.5 Assume $\mathcal{B}$ is countable. Then we can enumerate $\mathcal{B}$:

$$\begin{array}{llllllll}
b_2 & 1 & \boxed{0} & 1 & 1 & 1 & 0 & 1 \quad \ldots \\
b_3 & 1 & 0 & \boxed{1} & & 1 & 0 & 1 \\
b_4 & 0 & 1 & 0 & \boxed{0} & & & \\
\vdots & & & & & & &
\end{array}$$

However, the infinite sequence which has

$$i\text{'th bit} = \begin{cases} 0 & \text{if the } i\text{th bit of } b_i \text{ is } 1 \\ 1 & \text{if the } i\text{th bit of } b_i \text{ is } 0 \end{cases}$$

is different from each of the b_i. Hence no enumeration can exist, and $\mathcal{B}$ is uncountable. This should not be surprising, because the set $\mathcal{B}$ is really the same as (or is isomorphic to) $\mathbb{N} \to \Sigma$.