School of Computing and Information Systems
COMP30026 Models of Computation Problem Set 5

23–27 August 2021

Content: interpretations in predicate logic, clausal form

P5.1 For each of the following predicate logic formulas, give an interpretation that makes the formula true, and one that makes it false:

(a) $\forall x \forall y (P(x, y))$

(b) $\forall x \exists y (P(x, y) \land P(y, x))$

(c) $(\forall x \exists y P(x, y)) \land (\forall x \exists y P(y, x))$

P5.2 Show that $\forall x(P(x)) \models \exists y(P(y))$ holds, by supposing that an interpretation $\mathcal{I}$ makes $\forall x(P(x))$ true (so $\mathcal{I} \models \forall x(P(x))$), and explaining why it must make $\exists y(P(y))$ true. Does $\exists x(P(x)) \models \forall y(P(y))$ also hold? Recall that part of our definition of *interpretation* is that the *domain* is non-empty.

P5.3 Turn the closed formula $\forall x \forall y \exists z \Big(P(x) \Rightarrow \forall y \forall z (Q(y, z)) \Big)$ into a simpler, equivalent formula of the form $\varphi \Rightarrow \psi$.

P5.4 Determine whether nvert the formula to clausal form.

P5.5 Turn the closed formula $\neg \forall x \, \exists y \, \Big[\forall z \, \Big(Q(x, z) \land \Big]$ into clausal form.