School of Computing and Information Systems
COMP30026 Models of Computation Tutorial Week 2

12–14 August 2020

Plan

This is the week when you need to get through Grok modules 2 and 3, if you have not already done that. Don't fall behind! We will often provide more exercises than can possibly be covered in a tutorial. That is so that those who want more practice can have that. Exercises that say "drill" will tend to cover old ground, rather than introduce new ideas.

The exercises

6. If any good questions or thoug … is a good time to share them. We have a quest … f defined below? Is it well-typed? Did somebo … n? Explain the function's behaviour in English.


```
f []  = 0
f [x] = x
f y   = 42
```


7. For each of the follow … ave the same truth table.
   (a) $\neg P \Rightarrow Q$ … d $Q \Rightarrow (P \Rightarrow R)$
   (b) $\neg P \Rightarrow Q$ … d $(P \Rightarrow Q) \Rightarrow R$
   (c) $\neg P \Rightarrow Q$ and $\neg Q \Rightarrow P$ … $\Rightarrow (Q \Rightarrow R)$
   (d) $(P \Rightarrow Q) \Rightarrow P$ and P … $\Rightarrow R) \wedge (Q \Rightarrow R)$

8. Find a formula that is equivalent to $(P \wedge \neg Q)$ … g fewer symbols.

9. Recall that $\oplus$ is the "exclusive or" connective. Show that $(P \oplus Q) \oplus Q$ is equivalent to P.

10. Show that $P \Leftrightarrow (Q \Leftrightarrow R) \equiv (P \Leftrightarrow Q) \Leftrightarrow R$. This tells us that we could instead write

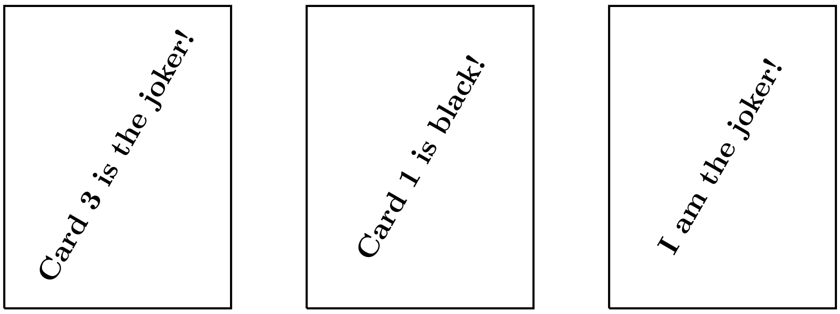
$$P \Leftrightarrow Q \Leftrightarrow R \qquad (1)$$

without introducing any ambiguity. Mind you, that may not be such a good idea, because many people (incorrectly) tend to read "$P \Leftrightarrow Q \Leftrightarrow R$" as

$$P, Q, \text{ and } R \text{ all have the same truth value} \qquad (2)$$

Show that (1) and (2) are incomparable, that is, neither is a logical consequence of the other.

11. Three playing cards lie face down on a table. One is red, one is black, and one is the joker. On the back of each card is written a sentence:



Card 1 Card 2 Card 3

The red card has a true sentence written on its back and the black card has a false sentence. Which card is red, which is bl

12. Consider the formula
negation?

13. Let F and G be propositional formulas. What is the difference between '$F \equiv G$' and '$F \Leftrightarrow G$' — do we really need both? Show that $F \equiv G$ iff $F \Leftrightarrow G$ is valid.

14. By negating a satisfiable proposition, can you get a tautology? A satisfiable proposition? A contradiction? Illustrate your affirmative answers.

15. (Drill.) Recall that $P \Leftrightarrow Q) \equiv (\neg P \Leftrightarrow \neg Q)$.

16. (Drill.) Is this claim co $P \vee Q) \Leftrightarrow Q$'. That is, do we have

$$((P \wedge Q) \Leftrightarrow P) \equiv$$

17. (Drill.) Find a formula equivalent to $P \Leftrightarrow (P$ wer symbols.

18. (Drill.) For each of the following propositional formulas, determine whether it is satisfiable, and if it is, whether it is a tautology:

   (a) $P \Leftrightarrow ((P \Rightarrow Q) \Rightarrow P)$
   (b) $(P \Rightarrow \neg Q) \wedge ((P \vee Q) \Rightarrow P)$
   (c) $((P \Rightarrow Q) \Rightarrow Q) \wedge (Q \oplus (P \Rightarrow Q))$