School of Computer Science and En
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9. STRING MATCHING ALGORITHMS

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- We now show how hashing can be combined with re[illegible] an efficient string matching algorithm.

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- Next we choose a large prime number p such that $(d+1)\,p$ still fits into a single register and define the hash value of B as $H(B) = h(B) \bmod p$.

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- This is where recursion comes into play: we do not have compute the hash value $H(A_{s+1})$ of $A_{s+1} = a_{s+1} a_{s+2} \ldots a_{s+m}$ "from scratch", but we can compute it efficiently from the hash value $H(A_s)$ of $A_s = a_s a_{s+1} \ldots a_{s+m-1}$ as follows.

Since

$$H(A_s) = (d^{m-1}a_s + d^{m-2}a_{s+1} + \dots d^1 a_{s+m-2} + a_{s+m-1}) \bmod p$$

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$d \cdot H(A_s))$ m

$= (d^m a_s + d^{m-1} a_{s+1} + \ldots d \cdot a_{s+m-1}) \bmod p$

$= (d^m a_s + (d^{m-1} a_{s+1} + \ldots d^2 a_{s+m-2} + d\, a_{s+m}$ $) \bmod p$

$= (d^m a_s + H(A_{s+1}) - a_{s+m}) \bmod p$

- Consequently,

$$H(A_{s+1}) = (d \cdot H(A_s) - d^m a_s + a_{s+m}) \bmod p.$$

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- Thus, for every s except $s = 0$ the value of $H(A_s)$ can be computed in constant time independent of the length of the strings A and B.

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- However, as always when we use hashing, we cann … case performance.
- So we now look for algorithms whose worst case performance can be guaranteed.

- A string matching finite automaton for a string S with k symbols has $k+1$ many states $0, 1, \ldots k$ which correspond to the number of characters matched thus far and a transition function $\delta(s, c)$ where s is a state and c is a character red at the moment.

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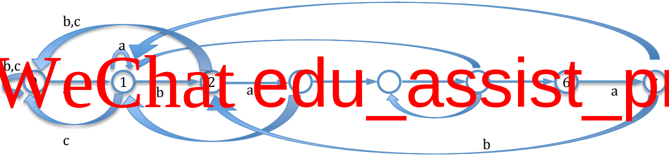
- We first look at the case when such $\delta(s, c)$ is given by a pre-constructed table.
- To make things easier to describe, we consider the string $S = ababaca$. The table defin

state	a			
0	**1**	0	0	a
1	1	**2**	0	b
2	**3**	0	0	a
3	1	**4**	0	b
4	**5**	0	0	a
5	1	4	**6**	c
6	**7**	0	0	a
7	1	2	0	



state transition diagram for string *ababaca*

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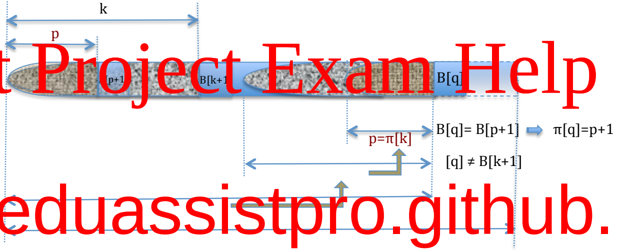
- We do that by matching the string against itself: we can recursively compute a function $\pi(k)$ which for each k returns the largest integer m such that the prefix B_m of B is a proper suffix of B_k.



function Compute − Prefix − Function(B)
  $m \leftarrow \text{length}[B]$
  let $\pi[1..m]$ be a new array
  $\pi[1] = 0$
  $k = 0$
  for q
    w
      $B[k+1] \neq B[q]$
    $k = \pi[k]$
    if $B[k+1] == B[q]$
      $k = k + 1$
    $\pi[q] = k$
  end for
  return π
end function

Assume that len… we have already f… k; to compute $\pi[q]$ we check if $B[q] = B[k+1]$; if true then $\pi[q] = k+1$; if not true then we find $\pi[k] = p$; if now $B[q] = B[p+1]$ then $\pi[q] = p+1$.

- We can now do our search for string B in a longer string A:


```
function KMP − Matcher(A, B)
    n    length[A]
    m
    π
    q
    fo
        while q > 0 and B[q + 1] ≠ A[i]
        q = π[q]
        if  B[q + 1] == A[i]
            q = q + 1
        if  q == m
            print pattern occurs with shift i − m
            q = π[q]
    end for
end function
```

Looking for imperfect matches

- Sometimes we are not interested in finding just the prefect matches, but also in matches that might have a few errors, such as a few insertions, deletions and replacements.

- So ass
  $A =$
  $B =$ nterested
  in fin

- Idea: split B into $k + 1$ consecutive subsequen
  length. Then any match in A with at most
  subsequence which is a perfect match for a subsequenc k
  for all perfect matches for all of $k + 1$ subseq e
  test by brute force if the remaining parts of B have sufficient number of matches in the appropriate parts of A.

PUZZLE!!

On a rectangular table there are 25 non-overlapping round coins of equal size

coin with

falling off t

within the table). Show that it is possible to complete

with 100 coins (of course with overlapping of coins).