Algorithms

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University of New South Wales Sydney

1. INTRODUCTION

What is this course about?

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The word "algorithm" comes by corruption of the name of *Muhammad ibn Musa al-Khwarizmi*, a Persian scientist 780-850, who wrote an important book on algebra, *"Al-kitab al-mukhtasar fi hisab al-gabr wal-muqabala"*. You are encouraged to read about him in Wikipedia.

In this course we will deal only with sequential deterministic algorith

- the ste

- the action of each step gives the same result whe executed for the same input

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- particular algorithms will be mostly used to ill techniques
- emphasis on development of your algorithm design **skills**

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paperback edition available at the UNSW book store

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- not so good: somewhat formalistic and written in a rather dry style.

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The hard part: how can a thief split the pile into two equal parts? Remarkably, this turns out that, most likely, there is no more efficient algorithm than the brute force: we consider all partitions of the pile and see if there is one which results in two equal parts.

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Examples of Algorithms

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- But what if the remaining two thieves choose the same pile?

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- Let the thieves be T_1, T_2, T_3;

Assignment Project Exam Help

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Algorithm:

T_1 makes a pile P_1 which he believes is $1/3$ of the whole loot;
T_1 proceeds to ask T_2 if T_2 agrees that $P_1 \leq 1/3$;
If T_2 says YES, **then** T_1 asks T_3 if T_3 agrees that $P_1 \leq 1/3$;

  If T_3 says YES, **then** T_1 takes P_1;
    T_2 and T_3 split the rest as in Problem 1.
  Else if T_3 says NO, **then** T_3 takes P_1;

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Hint: there is a *nested recursion* happening even with 3 thieves!

The role of proofs in algorithm design

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Example: MergeSort

Merge-Sort(A,p,r) *sorting A[p..r]*

1. **if** $p < r$
2. **then** $q \leftarrow \lfloor \frac{p+r}{2} \rfloor$
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5. The above is essentially a proof by induction, but we will never bother formalising proofs of (essentially) obvious facts.

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- For that reason we will **NEVER** p textbook sometimes does just that, by sometimes formulating and proving trivial little lemmas, being too pedantic!). We will prove only what is genuinely nontrivial.
- **However, <u>BE VERY CAREFUL</u> what you call trivial!!**

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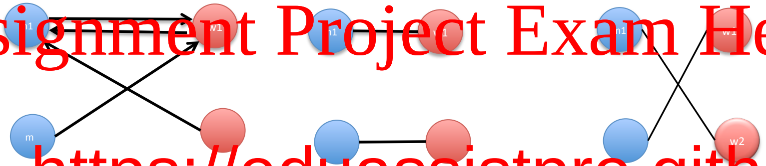
for two pairs $p = (m, w)$ and $p' = (m', w')$:

- man m prefers woman w' to woman w, **and**
- woman w' prefers man m to man m'.

Stable Matching Problem: examples

Case1:

Preferences: | stable matching | not stable



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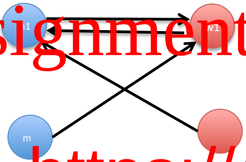

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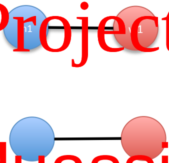

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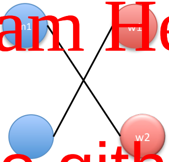

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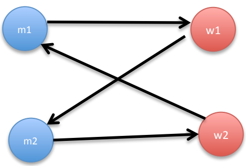
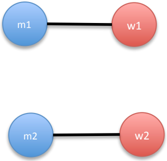
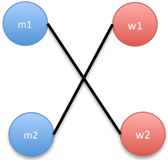


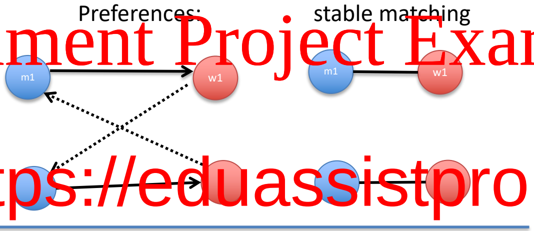
Case2:
stable matchin





Is there always a stable matching for any preferences of two pairs?

Case1: two men like two different women (or vice versa)



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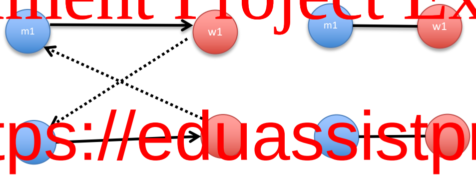
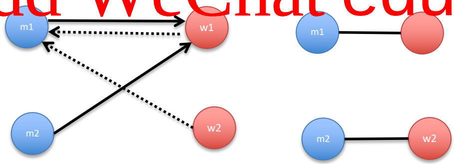


Case2: men like the same woman and

Preferences: stab



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Else m is lower on her preference list than m';

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Stable Matching Problem: Gale - Shapley algorithm

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- **Contradiction!**

A Puzzle!!!

Why puzzles? It is a fun way to practice problem solving!

Problem : Tom and his wife Mary went to a party where nine more couples were present.

- Not
  eac

- People who knew each other from before did no
- Later that evening Tom got bored, so he walke
  all other guests (including his wife) how many
  shaken that evening, and got 19 different answers.

- How many hands did Mary shake?
- How many hands did Tom shake?

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That's All, Folks!!