COMPSCI4

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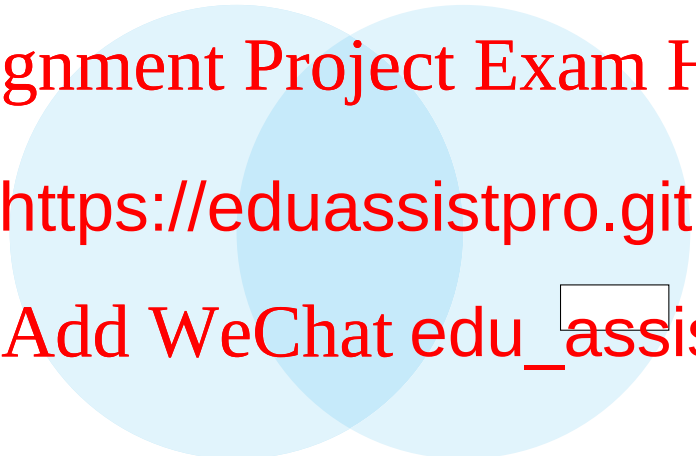
Term 2, 2020

Methods of Assurance



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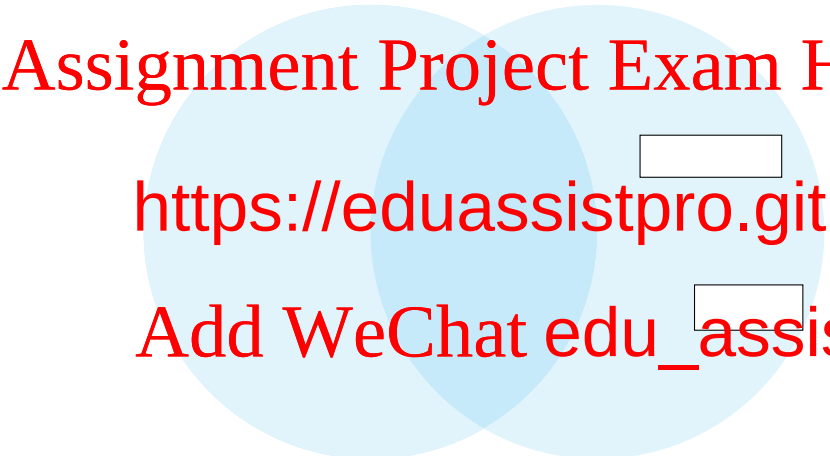

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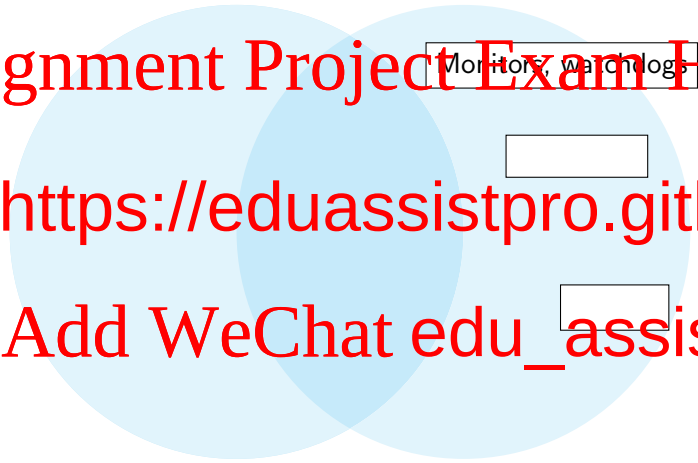

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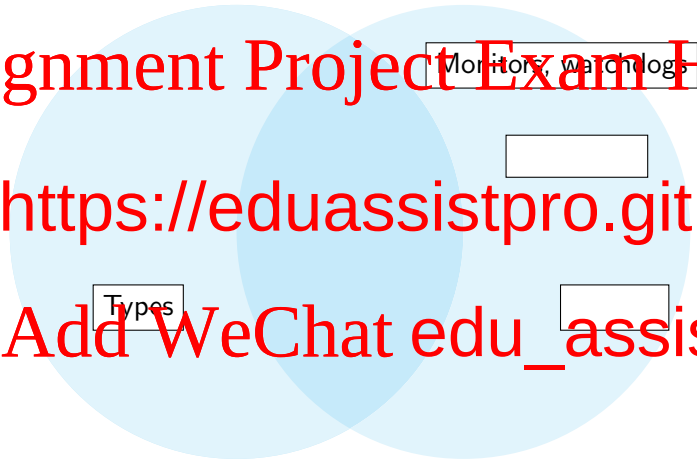

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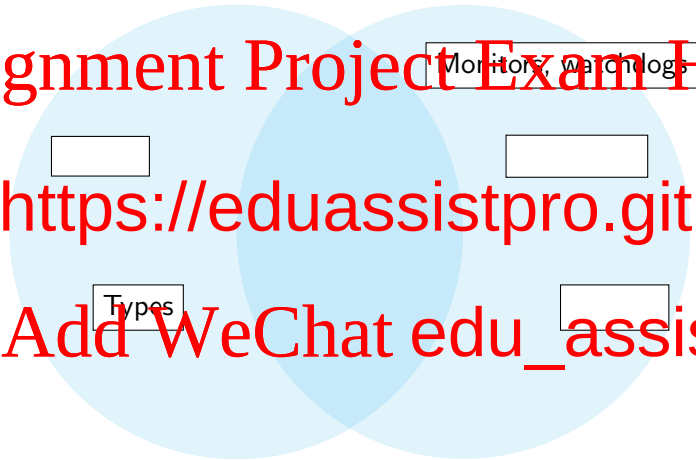

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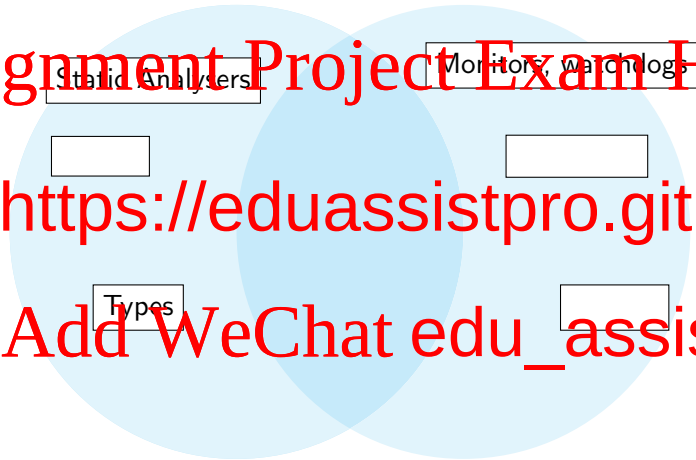

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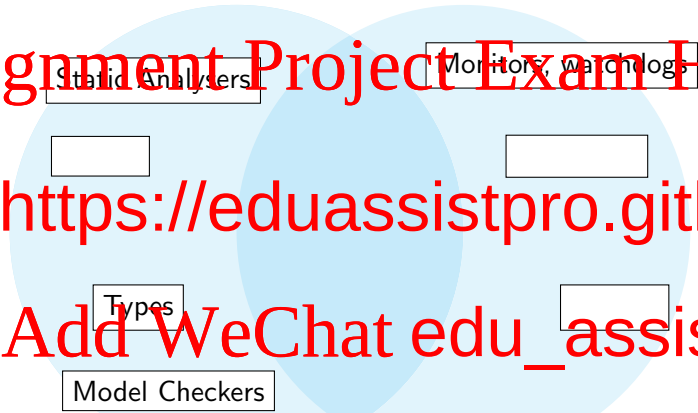

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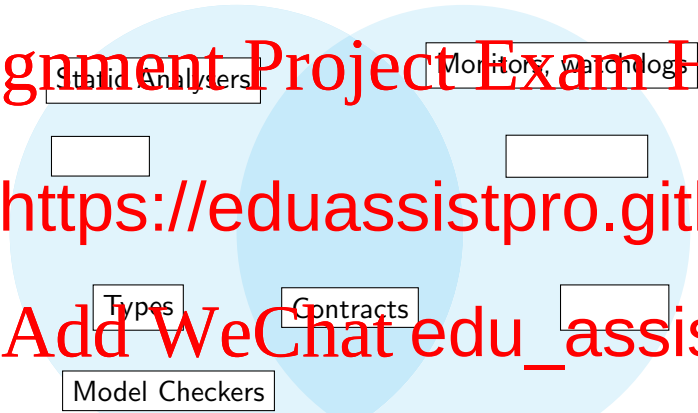

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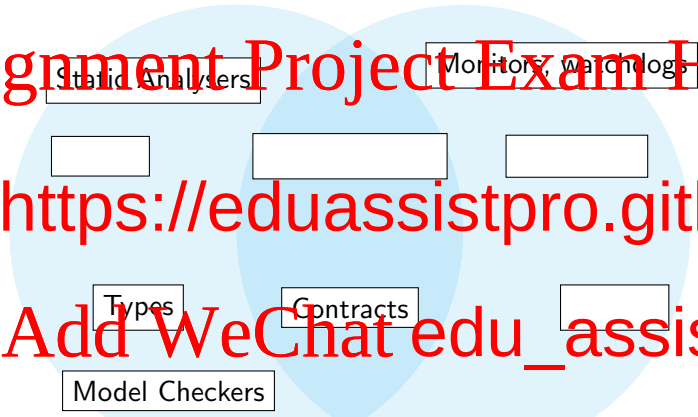

Static means of assurance analyse a program without running it.

Static vs. Dynamic

- Static checks can be exhaustive.

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Exhaustivity

An exhaustive check...
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An exhaustive check...
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- **However**, some properties cannot be checked statica
problem), or are intractable to feasibly check statically (
- Dynamic checks cannot be exhaustive, but can be used to c
where static methods are unsuitable.

Compiler Integration

Most static and all dynamic methods of assurance are not integrated into the compilation process.

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Types

Because types are integrated into the compiler, they cannot di code. This means that type signatures are a kind of machine-che for your code.

Types

Types are the most widely used kind of formal verification in programming today.

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This week, we'll look at techniques to encode various correctn
Haskell's type system.

Phantom Types

Definition

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- We can use this parameter to track what data invariants h about a value.
- We can use this parameter to track information about the representation (e.g. units of measure).
- We can use this parameter to enforce an ordering of operations performed on these values (*type state*).

Validation

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data UG -- empty type
data PG
data StudentID x = SID Int
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data Either a b = Left a | Right b
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(Recalling the following definition of Either)


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data Either a b = Left a | Right b
```

And then define functions:

```
enrolInCOMP3141 :: StudentID UG -> IO ()
lookupTranscript :: StudentID x -> IO String
```

Units of Measure

In 1999, software confusing units of measure (pounds and newtons) caused a Mars orbiter to burn up on atmospheric entry.


```
data Kilometres
data Miles
data Value x = MkInt Int
sydneyToMel
losAngelesToSanFran = (U 383 :: Value Miles)
```

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In addition to tagging values, we can also enforce constraints on

```
data Square a
area :: Value m -> Value m -> Value (Square m)
area (U x) (U y) = U (x * y)
```

Note the arguments to `area` must have the same units.

Type State

Example

A Socket can either be ready to retrieve data, or busy. If the socket is busy, the user must first use the `wait` operation, which blocks until the socket is ready. If the socket is ready, the user ca ke the socket busy again.

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```
data Busy
data Ready
newtype Socket s = Socket
```

```
wait :: Socket Busy -> IO (Socket Ready)
```

```
send :: Socket Ready -> String -> IO (Socket Busy)
```

What assumptions are we making here?

Linearity and Type State

The previous code assumed that we didn't re-use old Sockets:


```
send2 :: Socket Ready -> String -> String
      -> IO (Socket Busy)
send2 s x y = do s' <- send s x
```



```
                 pure s'''
```

Linearity and Type State

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But we can just re-use old values to send without waiting:

```
send2' s x y = do _ <- send s x
                  s' <- send s y
                  pure s'
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Linear type systems can solve this, but not in Haskell (yet).

Datatype Promotion

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Haskell types themselves have types, called kinds. Can we mak types more precise than *?

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Recall

Haskell types themselves have types, called kinds. Can we mak types more precise than *?

The DataKinds language extension lets us use data types

```
{-# LANGUAGE DataKinds, KindSignatures #-}
data Stream = UG | PG
data StudentID (x :: Stream) = SID Int
-- rest as before
```

Motivation: Evaluation


```
data Expr = BConst Bool
          | IConst Int
          | T
          | L
          | A
          | I

data Value = BVal Bool | IVal Int
```

Example

Define an expression evaluator:

```
eval :: Expr -> Value
```

Motivation: Partiality

Unfortunately, the eval function is partial, undefined for input expressions that are not well-typed, like:

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And (ICons 3) (BC
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With any partial function, we can make it total by either expanding the co-domain (e.g. with a `Maybe` type), or constraining the domain.

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Recall

With any partial function, we can make it total by either expanding the co-domain (e.g. with a `Maybe` type), or constraining the domain.

Can we use phantom types to constrain the domain of `eval` to only accept well-typed expressions?

Attempt: Phantom Types

Let's try adding a phantom parameter to `Expr`, and defining typed constructors with precise types:

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```
data Expr t = ...
bConst :: Bool -> Expr Bool
bConst = BConst
iConst :: Int -> Expr Int
iConst = IConst
times :: Expr Int -> Expr Int -> Expr Int
times = Times
less :: Expr Int -> Expr Int -> Expr Bool
less = Less
and :: Expr Bool -> Expr Bool -> Expr Bool
and = And
if'  :: Expr Bool -> Expr a -> Expr a -> Expr a
if' = If
```

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This makes invalid expressions into type errors (yay!):

```
-- Couldn't match Int and Bool
and (iConst 3) (bConst True)
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Bad News

Inside `eval`, the Haskell type checker cannot be sure that we us
constructors, so # e.g. the `IConst` case


```
eval :: Expr t -> t
eval (IConst i) = i -- type error
```

We are unable to tell that the type `t` is definitely `Int`.

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Phantom types aren't strong enough!

GADTs

Generalised Algebraic Datatypes (*GADTs*) is an extension to Haskell that, among other things, allows data types to be specified by writing the types of their constructors:


```
{-# LANGUAGE GA
-- Unary natura
data Nat = Z | S Nat
-- is the same as
data Nat :: * where
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When combined with the *type indexing* trick of phantom types, this becomes very powerful!

Expressions as a GADT

```
data Expr :: * -> * where
   BConst :: Bool -> Expr Bool
   IConst :: Int -> Expr Int
   Times :: Expr Int -> Expr Int -> Expr Int
   Less :: Expr I
   And :: Expr Bo
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Observation

There is now only one set of precisely-typed constructors.

Inside `eval` now, the Haskell type checker accepts our previously problematic case:

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eval :: Expr t -> t
eval (IConst i) = i -- OK now
```

GHC now knows that if we have `IConst`, the type `t` must be `Int`.

Lists

We could define our own list type using GADT syntax as follows:

```
data List (a :: *) :: * w
  Nil  :: List a
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We will constrain the domain of these functions by tracking the l
the type level.

Vectors

As before, define a natural number kind to use on the type level:

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data Nat = Z | S Nat
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Now our length-i

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data Vec (a :: *) :: Na
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Now `hd` and `tl` can be total:


```
hd :: Vec a (S n) -> a
hd (Cons x xs) = x
tl :: Vec a (S n) -> Vec a n
tl (Cons x xs) = xs
```

Vectors, continued

Our `map` for vectors is as follows:

```
mapVec :: (a -> b) -> Vec a n -> Vec b n
mapVec f Nil = Nil
mapVec f (Cons x xs
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Vectors, continued

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Properties

Using this type, it's impossible to write a `mapVec` that changes the length of the vector.

Properties are verified by the compiler!

Tradeoffs

The benefits of this extra static checking are obvious, however:

- It can be difficult to convince the Haskell type checker that your code is correct, even when it is.

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Pragmatism

We should use type-based encodings only when the assuranc
clarity disadvantages.
The typical use case for these richly-typed structures is to eliminate partial functions from our code base.
If we never use partial list functions, length-indexed vectors are not particularly useful.

Appending Vectors

Example (Problem)

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appendV :: Vec a m -> Vec a n -> Vec a ???
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plus :: Nat -> Nat -> Nat
plus Z y = y
plus (S x) y = S (plus x y)
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This function is not applicable to type-level `Nats`, though.

$\Rightarrow$ we need a type level function.

Type Families

Type-level functions, also called *type families*, are defined in Haskell like so:


```
{-# LANGUAGE Ty
type family Plu
  Plus Z     y = y
  Plus (S x) y = S (Plu
```

We can use our type family to define appendV:


```
appendV :: Vec a n -> Vec a m -> Vec a (Plus n m)
appendV Nil         ys = ys
appendV (Cons x xs) ys = Cons x (appendV xs ys)
```

Recursion

If we had implemented `Plus` by recursing on the second argument instead of the first:

```
{-# LANGUAGE TypeFamilies #-}
type family Plus' (x :: Nat) (y :: Nat) :: Nat where
  Plus' x Z     = x
  Plus' x (S y) = S (Pl
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Then our `app`

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appendV :: Vec a n -> V
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Why?

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```

Why?

Answer

Consider the `Nil` case. We know `m = Z`, and must show that our desired return type `Plus' Z n` equals our given return type `n`, but that fact is not immediately apparent from the equations.

Type-driven development

- This lecture is only a taste of the full power of type-based specifications.
- Language e
  and value lev s.
- Haskell is als

Next week: Fancy theory about types!

- Deep connections between types, logic and proof
- Algebraic type structure for generic algorithms and ref
- Using polymorphic types to infer properties for free.

Homework

1. Assignme
2. The last pro
3. This week's quiz is also up, due in Friday of Week 9.