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COMPSCI 141

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Term 2, 2020

Motivation

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We've already see

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How do we come up with correctness properties in the first place?

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Structure of a Module

A Haskell program will usually be made up of many modules, each of which exports one or more *data types*.
Typically a module for a data type `X` will also provide a set of functions, called *operations*, on

- `X`
- `X`
- to update the data type:

A lot of software can be designed with this structure.

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Example (Data Types)

A dictionary data type, with empty, insert and lookup.

Data Invariants

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One source of properties is *data invariants*.

Data Invariants

Data invariants a

Whenever we use a

invariants are mai

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Example

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- That a list of words in a dictionary is always in sorted order.
- That a binary tree satisfies the search tree properties.
- That a date value will never be invalid (e.g. 31/13/2019).

Properties for Data Invariants

For a given data type `X`, we define a *wellformedness predicate*

$$\texttt{wf} :: \texttt{X} \to \texttt{Bool}$$

For a given value x.

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Properties

For each operation, if all input values of type `X` satisfy `wf`.

In other words, for each constructor operation c $\texttt{wf}\,(c\ \cdots)$, and for each update operation $u :: \texttt{X} \to \texttt{X}$ we must show $\texttt{wf}\ x \implies \texttt{wf}(u\ x)$

Demo: Dictionary example, sorted order.

Stopping External Tampering

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Even with our sorted dictionary example, there's nothing to stop a malicious or clueless programmer fro

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Example

The malicious programmer could just add a word directly to the dictionary, unsorted, bypassing our carefully written `insert` function.

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We want to prevent this sort of thing from happening.

Abstract Data Types

An *abstract* data type (ADT) is a data type where the implementation details of the type and its associated operations are hidden.

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Abstract Data Types

An *abstract* data type (ADT) is a data type where the implementation details of the type and its associated operations are hidden.

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```
newtype Dict
type Word = Strin
type Definit
emptyDict   :: Dict
insertWord  :: Word -> Definition -> Dict -> Dict
lookup      :: Word -> Dict -> Maybe Definition
```

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If we don't have access to the implementation of the provided operations, which we know preserve our data inv invariants cannot be violated if this module is correct.

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Demo: In Haskell, we make ADTs with module headers.

Abstract? Data Types

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In general, *abs*

The inverse of abs

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Abstract data typ

implementation details are hidden, and we no longer have to re

level of implementation.

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Validation

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Suppose we had a `sendEmail` function:

```
sendEmail :: String -- email address
          -
          -
```

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It is possible to mix th t, it's possible that the given email address is not valid.

Question

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Suppose that we wanted to make it impossible to call t first checking that the email address was valid.
How would we accomplish this?

Validation ADTs

We could define a tiny ADT for validated email addresses, where the data invariant is that the contained email address is valid:

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```
module EmailADT(Email, checkEmail, sendEmail)
    newtype E

    checkEm
    checkEm
                | otherwise      = Nothing
```

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Then, change the type of sendEmail:

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```
    sendEmail :: Email -> String -> IO()
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The only way (outside of the `EmailADT` module) to create a value of type `Email` is to use `checkEmail`.

`checkEmail` is an example of what we call a *smart constructor*: a constructor that enforces data invariants.

Reasoning about ADTs

Consider the following, more traditional example of an ADT interface, the unbounded queue:

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```
data Queue

emptyQueue :: Q
enqueue :: Int -> Q
front   :: Queue -> Int
dequeue :: Queue -> Queue -- partial
size    :: Queue -> Int
```

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We could try to come up with properties that relate these functio without reference to their implementation, such as:

```
dequeue (enqueue x emptyQueue) == emptyQueue
```

However these do not capture functional correctness (usually).

Models for ADTs

We could imagine a simple implementation for queues, just in terms of lists:

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Models for ADTs

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```
emptyQueueL = []
enqueueL a  = (++ [a])
frontL      = head
dequeueL    = tail
sizeL       = length
```

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However!

This is a dead simple implementation, and trivial to see that it is cor
better queue implementation, it should always give the same results as this simple one. Therefore: This implementation serves as a functional correctness specification for our `Queue` type!

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Refinement Relations

The typical approach to connect our model queue to our `Queue` type is to define a relation, called a *refinement relation*, that relates a `Queue` to a list and tells us if the two structures represent the same queue conceptually:

```
rel :: Queue -> [In
```

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prop_empty_
```

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That any query functions for our two types produce equal result
such as for size:

```
prop_size_r fq lq = rel fq lq ==> size fq == size lq
```

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```
prop_size_r fq lq = rel fq lq ==> size fq = size lq
```

And that each of the queue operations preserves our refinement relation, for example for enqueue:

```
prop_enq_ref fq lq x =
  rel fq lq ==> rel (enqueue x fq) (enqueueL x lq)
```

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In Pictures

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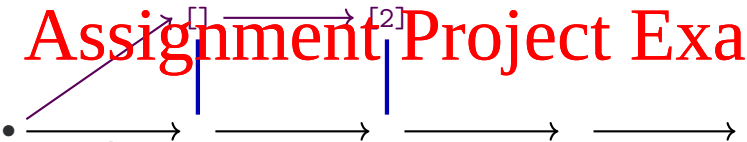
```
prop_empty_r = rel emptyQueue emptyQueue
```

In Pictures



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```
prop_enq ref fq lq x =
rel fq lq ==> ref (enqueue x fq) (enqueue x lq)
```

In Pictures

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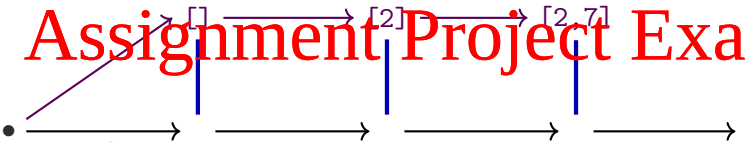

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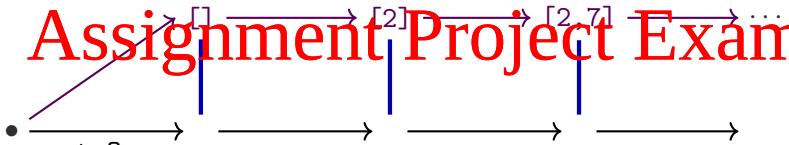

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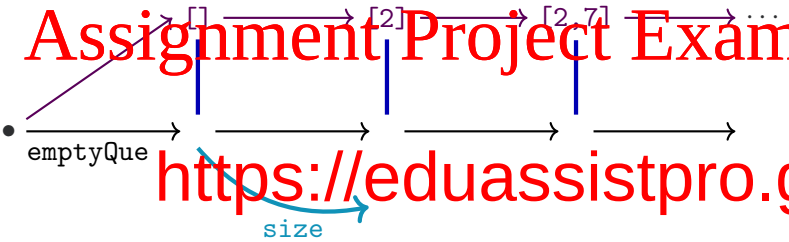

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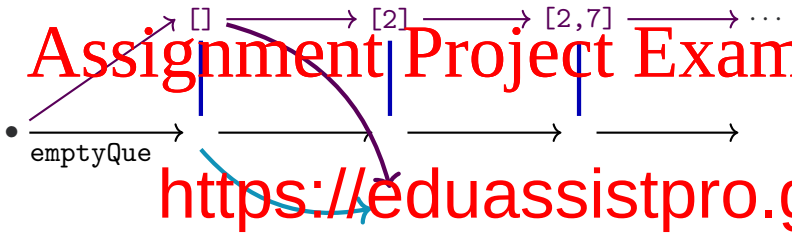

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```
prop_size_r fq lq = rel fq lq ==> size fq == sizeL lq
```

In Pictures

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```
prop_size_r fq lq = rel fq lq ==> size fq == sizeL lq
```

Whenever we use a Queue, we can reason as if it were a list!

Abstraction Functions

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These refinement relations are very difficult to use with QuickCheck because the `rel fq lq` preconditions are very hard to satisfy with randomly generated inputs.

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For this example, it
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Conceptually, our refinement relation is then just:

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```
\fq lq → absfun fq =
```

However, we can re-express our properties in a much more QC-friendly format (**Demo**)

Fast Queues

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Let's use test-driven development! We'll implement a fast Queue with amortised $\mathcal{O}(1)$ operations.

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```
data Queue
       Int    -- size of the rear
```

We store the rear part of the queue in reverse order, to make enque

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```
       Int    -- size of the rear
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We store the rear part of the queue in reverse order, to make enque

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Thus, converting from our Queue to an abstract list re

```
toAbstract :: Queue -> [Int]
toAbstract (Q f sf r sr) = f ++ reverse r
```

Data Refinement

These kinds of properties establish what is known as a *data refinement* from the abstract, slow list model to the fast, complex Queue implementation.

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Refinement and Specifications

In general, all funct

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Warning

While abstraction can simplify proofs, abstraction does not reduce the fundamental complexity of verification, which is provably hard.

Data Invariants for Queue

In addition to the already-stated refinement properties, we also have some data invariants to maintain for a value `Q f sf rf ef`:

1. `length f == sf`

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Data Invariants for Queue

In addition to the already-stated refinement properties, we also have some data invariants to maintain for a value `Q f sf r sr`:

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2. `length r == sr`

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Data Invariants for Queue

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2. `length r == sr`
3. **importan**

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2. `length r == sr`
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We will ensure our ese invariants.

Thus, our `wellformed` predicate is used merely to enfo outputs of our operations:

```
prop_wf_empty = wellformed (emptyQueue)
prop_wf_enq q = wellformed (enqueue x q)
prop_wf_deq q = size q > 0 ==> wellformed (dequeue q)
```

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Implementing the Queue

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We will generally implement by:

- Dequeue fr
- Enqueue to t
- If necessary
  and appending it to the front.

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Implementing the Queue

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We will generally implement by:

- Dequeue fr
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- If necessary
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  This step is slow ($\mathcal{O}(n)$), but only happens every g
  average case amortised complexity of $\mathcal{O}(1)$ ti

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Amortised Cost

```
enqueue x (Q f sf r sr) = inv3 (Q f sf (x:r) (sr + 1))
```

When we enqueue each of [1..7] to the emptyQueue in turn:

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Q	[]	0	[]	0

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```
Q []          0 []      0
                        0 (
                        1
```

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	Q []	0	[]	0
				0 (
				1
→	Q [1, 2, 3]	3	[5	

Amortised Cost

```
enqueue x (Q f sf r sr) = inv3 (Q f sf (x:r) (sr + 1))
```

When we enqueue each of $[1..7]$ to the emptyQueue in turn:

	Q	[]	0	[]
$\rightarrow$	Q	$[1,2,3]$	3	[5
$\rightarrow$	Q	$[1,2,3]$	3	[6

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$\rightarrow$	Q	$[1,2,3,4,5,6,7]$	7	[]	0 (

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	Q []	0	[]	0
				0 (
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$\rightarrow$	Q [1, 2, 3]	3	[6	
$\rightarrow$	Q [1, 2, 3, 4, 5, 6, 7]	7	[]	0 (

Observe that the slow invariant-reestablishing step $(*)$ happens after 1 step, then 2, then 4...

Extended out, this averages out to $\mathcal{O}(1)$.

Another Example

Consider this ADT interface for a bag of numbers:

```
data Bag
emptyBag   :: Bag
addToBag   :: Int -> Ba
averageBag :: B
```

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Our conceptual a

```
emptyBagA = []

addToBagA x xs = x:xs

averageBagA [] = Nothing
averageBagA xs = Just (sum xs `div` length xs)
```

But do we need to keep track of all that information in our implementation?

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```

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But do we need to keep track of all that information in our implementation? No!

Concrete Implementation

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Our concrete version will just maintain two integers, the total and the count:

```
data Bag = B { total :: Int , count :: Int }
emptyBag :: Bag
emptyBag = B 0 0

addToBag :: Int -> Bag -> Bag
addToBag x (B t c) = B (x + t) (c + 1)

averageBag :: Bag -> Maybe Int
averageBag (B _ 0) = Nothing
averageBag (B t c) = Just (t `div` c)
```

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Refinement Functions

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Normally, writing an abstraction function (as we did for `Queue`) is a good way to express our refine

write such a functi

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```
toAbstract :: B
toAbstract (B t c) = ?????
```

Instead, we will go in the other direction, giving us a :

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```
toConc :: [Int] -> Bag
toConc xs = B (sum xs) (length xs)
```

Properties with Refinement Functions

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Refinement functions produce properties much like abstraction functions, only with the abstract and concrete layers swapped:

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```
prop_ref_em
   toConc em ==
```

```
prop_ref_add x ab =
   toConc (addToBagA x ab) == addToBag x (toConc ab)
```

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```
prop_ref_avg ab =
   averageBagA ab == averageBag (toConc ab)
```

Assignment 1 and Break

Assignment Project Exam Help

Assignment 1 has b

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Assignment 1 and Break

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Advice from Alu

The assignments do not involve much coding, but they do invol

Start early!

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Homework

Assignment Project Exam Help

1. Get started o
2. Next progr
3. Last week's
4. This week's quiz is also up, due the following Friday.

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