COMP9101

Curtis Millar
CSE, UNSW (and Data61)
29 July 2020

What is Intuitionistic Logic?

What is Intuitionistic Logic?

- Classical lo

What is Intuitionistic Logic?

- Classical lo
- Intuitionis $p \vee \neg p$ or equivalent

What is Intuitionistic Logic?

- Classical lo
- Intuitionis $p \vee \neg p$ pr equivalent

- In classical logic more can be proven but less can be expresse

What is Intuitionistic Logic?

- Classical lo
- Intuitionis $p \vee \neg p$ pr equivalent

- In classical logic more can be proven but less can be expresse
- Intuitionistic proof of an existence statement gives a wit

Example of Existence in the Classical Sense

- Let $\mathbb{Q}$ be the set of rational numbers and $\mathbb{I}$ be the set of irrational numbers.
- Consider th y

Example of Existence in the Classical Sense

- Let $\mathbb{Q}$ be the set of rational numbers and $\mathbb{I}$ be the set of irrational numbers.
- Consider th y
- Proof:

Example of Existence in the Classical Sense

- Let $\mathbb{Q}$ be the set of rational numbers and $\mathbb{I}$ be the set of irrational numbers.
- Consider th y
- Proof:
  - Consi

Example of Existence in the Classical Sense

- Let $\mathbb{Q}$ be the set of rational numbers and $\mathbb{I}$ be the set of irrational numbers.
- Consider th y
- Proof:
  - Consi
  1. If $\sqrt{2}$
  2. Otherwise, if $\sqrt{2}^{\sqrt{2}} \in \mathbb{I}$

Example of Existence in the Classical Sense

- Let $\mathbb{Q}$ be the set of rational numbers and $\mathbb{I}$ be the set of irrational numbers.
- Consider th y
- Proof:
  - Consi
  1. If $\sqrt{2}$
     - Pick $x = \sqrt{2}$ and $y = \sqrt{2}$
     - Then $x^y = \sqrt{2}^{\sqrt{2}}$ so $x^y \in \mathbb{Q}$
  2. Otherwise if $\sqrt{2}^{\sqrt{2}} \in \mathbb{I}$
     -
     -

Example of Existence in the Classical Sense

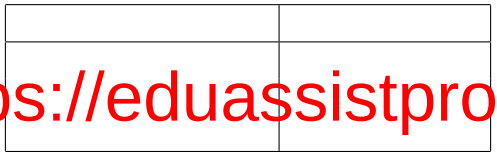
- Let $\mathbb{Q}$ be the set of rational numbers and $\mathbb{I}$ be the set of irrational numbers.
- Consider th y
- Proof:
  - Consi

1. If $\sqrt{2}$
   - Pick $x = \sqrt{2}$ and $y = \sqrt{2}$
   - Then $x^y = \sqrt{2}^{\sqrt{2}}$ so $x^y \in \mathbb{Q}$
2. Otherwise if $\sqrt{2}^{\sqrt{2}} \in \mathbb{I}$
   - Pick $x = \sqrt{2}^{\sqrt{2}}$ and $y = \sqrt{2}$
   - Then $x^y = (\sqrt{2}^{\sqrt{2}})^{\sqrt{2}} = \sqrt{2}^2 = 2$ so $x^y \in \mathbb{Q}$

Recall: The Curry-Howard Isomorphism

This correspondence goes by many names, but is usually attributed to Haskell Curry and William Howard.

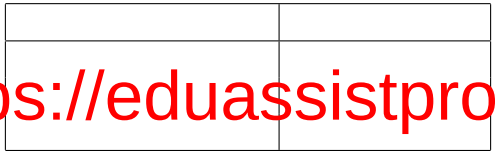
It is a *very deep* result.



Recall: The Curry-Howard Isomorphism

This correspondence goes by many names, but is usually attributed to Haskell Curry and William Howard.

It is a *very deep* result.



It turns out, no matter what logic you want to define, there is alway λ-calculus, and vice versa.

Constructive Logic	Ty
Classical Logic	Continuations
Modal Logic	Monads
Linear Logic	Linear Types, Session Types
Separation Logic	Region Types

Translating

We can translate logical connectives to types and back:

True	()
False	

We can also translate our *equational reasoning* *plification* on proofs!

Constructors and Eliminators for Sums


```
data TrafficLight = Red | Amber | Green
```

Example (Tra


```
Red     ≃ Left ()
Amber   ≃ Right (Le
Green   ≃ Right (Ri
```

Type Correctness

$$\frac{}{\Gamma \vdash () :: ()}\,() \qquad \frac{\Gamma \quad e :: A}{} \qquad \frac{\Gamma \quad e :: B}{\texttt{Either}\ A\ B}\,\mathrm{S}_R$$

Type Correctness

$$\dfrac{\dfrac{}{\Gamma \vdash () :: ()}\,()}{\Gamma \vdash \texttt{Left}\ () :: \texttt{Either}\ () \ ()}\,S_R$$

$$\dfrac{\dfrac{\dfrac{}{\Gamma \vdash () :: ()}\,()}{\Gamma \vdash \texttt{Left}\ () :: \texttt{Either}\ () \ ()}\,S_R}{\Gamma \vdash \texttt{Right}\ (\texttt{Left}\ ()) :: \texttt{Either}\ () \ (\texttt{Either}\ () \ ())}\,S_R$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma \vdash () :: ()}\,()}{\Gamma \vdash \texttt{Left}\ () :: \texttt{Either}\ () \ ()}\,S_R}{\Gamma \vdash \texttt{Right}\ (\texttt{Left}\ ()) :: \texttt{Either}\ () \ (\texttt{Either}\ () \ ())}\,S_R}{\Gamma \vdash \texttt{Right}\ (\texttt{Right}\ (\texttt{Left}\ ())) :: \texttt{Either}\ () \ (\texttt{Either}\ () \ (\texttt{Either}\ () \ ()))}\,S_R$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma \vdash () :: ()}\,()}{\Gamma \vdash \texttt{Left}\ () :: \texttt{Either}\ () \ ()}\,S_R}{\Gamma \vdash \texttt{Right}\ (\texttt{Left}\ ()) :: \texttt{Either}\ () \ (\texttt{Either}\ () \ ())}\,S_R}{\Gamma \vdash \texttt{Right}\ (\texttt{Right}\ (\texttt{Left}\ ())) :: \texttt{Either}\ () \ (\texttt{Either}\ () \ (\texttt{Either}\ () \ ()))}\,S_R}{\Gamma \vdash \texttt{Left}\ (\texttt{Right}\ (\texttt{Right}\ (\texttt{Left}\ ()))) :: \texttt{Either}\ (\texttt{Either}\ () \ (\texttt{Either}\ () \ (\texttt{Either}\ () \ ()))) \ ()}\,S_R$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma \vdash () :: ()}\,()}{\Gamma \vdash \texttt{Left}\ () :: \texttt{Either}\ () \ ()}\,S_R}{\Gamma \vdash \texttt{Right}\ (\texttt{Left}\ ()) :: \texttt{Either}\ () \ (\texttt{Either}\ () \ ())}\,S_R}{\Gamma \vdash \texttt{Right}\ (\texttt{Right}\ (\texttt{Left}\ ())) :: \texttt{Either}\ () \ (\texttt{Either}\ () \ (\texttt{Either}\ () \ ()))}\,S_R}{\Gamma \vdash \texttt{Left}\ (\texttt{Right}\ (\texttt{Right}\ (\texttt{Left}\ ()))) :: \texttt{Either}\ (\texttt{Either}\ () \ (\texttt{Either}\ () \ (\texttt{Either}\ () \ ()))) \ ()}\,S_R}{\Gamma \vdash \texttt{Right}\ (\texttt{Left}\ (\texttt{Right}\ (\texttt{Right}\ (\texttt{Left}\ ())))) :: \texttt{Either}\ () \ (\texttt{Either}\ (\texttt{Either}\ () \ (\texttt{Either}\ () \ (\texttt{Either}\ () \ ()))) \ ())}\,S_R$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma \vdash () :: ()}\,()}{\Gamma \vdash \texttt{Left}\ () :: \texttt{Either}\ () \ ()}\,S_R}{\Gamma \vdash \texttt{Right}\ (\texttt{Left}\ ()) :: \texttt{Either}\ () \ (\texttt{Either}\ () \ ())}\,S_R}{\Gamma \vdash \texttt{Right}\ (\texttt{Right}\ (\texttt{Left}\ ())) :: \texttt{Either}\ () \ (\texttt{Either}\ () \ (\texttt{Either}\ () \ ()))}\,S_R}{\Gamma \vdash \texttt{Left}\ (\texttt{Right}\ (\texttt{Right}\ (\texttt{Left}\ ()))) :: \texttt{Either}\ (\texttt{Either}\ () \ (\texttt{Either}\ () \ (\texttt{Either}\ () \ ()))) \ ()}\,S_R}{\Gamma \vdash \texttt{Right}\ (\texttt{Left}\ (\texttt{Right}\ (\texttt{Right}\ (\texttt{Left}\ ())))) :: \texttt{Either}\ () \ (\texttt{Either}\ (\texttt{Either}\ () \ (\texttt{Either}\ () \ (\texttt{Either}\ () \ ()))) \ ())}\,S_R}{\Gamma \vdash \texttt{Left}\ (\texttt{Left}\ (\texttt{Right}\ (\texttt{Left}\ (\texttt{Right}\ (\texttt{Right}\ (\texttt{Left}\ ())))))) :: \texttt{Either}\ (\texttt{Either}\ (\texttt{Either}\ () \ (\texttt{Either}\ () \ (\texttt{Either}\ () \ ()))) \ ()) \ ()}\,S_R$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma \vdash () :: ()}\,()}{\Gamma \vdash \texttt{Left}\ () :: \texttt{Either}\ () \ ()}\,S_R}{\Gamma \vdash \texttt{Right}\ (\texttt{Left}\ ()) :: \texttt{Either}\ () \ (\texttt{Either}\ () \ ())}\,S_R}{\Gamma \vdash \texttt{Right}\ (\texttt{Right}\ (\texttt{Left}\ ())) :: \texttt{Either}\ () \ (\texttt{Either}\ () \ (\texttt{Either}\ () \ ()))}\,S_R}{\Gamma \vdash \texttt{Left}\ (\texttt{Right}\ (\texttt{Right}\ (\texttt{Left}\ ()))) :: \texttt{Either}\ (\texttt{Either}\ () \ (\texttt{Either}\ () \ (\texttt{Either}\ () \ ()))) \ ()}\,S_R}{\Gamma \vdash \texttt{Right}\ (\texttt{Left}\ (\texttt{Right}\ (\texttt{Right}\ (\texttt{Left}\ ())))) :: \texttt{Either}\ () \ (\texttt{Either}\ (\texttt{Either}\ () \ (\texttt{Either}\ () \ (\texttt{Either}\ () \ ()))) \ ())}\,S_R}{\Gamma \vdash \texttt{Left}\ (\texttt{Left}\ (\texttt{Right}\ (\texttt{Left}\ (\texttt{Right}\ (\texttt{Right}\ (\texttt{Left}\ ())))))) :: \texttt{Either}\ (\texttt{Either}\ (\texttt{Either}\ () \ (\texttt{Either}\ () \ (\texttt{Either}\ () \ ()))) \ ()) \ ()}\,S_R}{\Gamma \vdash \texttt{Right}\ (\texttt{Left}\ (\texttt{Left}\ (\texttt{Right}\ (\texttt{Left}\ (\texttt{Right}\ (\texttt{Right}\ (\texttt{Left}\ ()))))))) :: \texttt{Either}\ () \ (\texttt{Either}\ (\texttt{Either}\ (\texttt{Either}\ () \ (\texttt{Either}\ () \ (\texttt{Either}\ () \ ()))) \ ()) \ ())}\,S_R$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma \vdash () :: ()}\,()}{\Gamma \vdash \texttt{Left}\ () :: \texttt{Either}\ () \ ()}\,S_R}{\Gamma \vdash \texttt{Right}\ (\texttt{Left}\ ()) :: \texttt{Either}\ () \ (\texttt{Either}\ () \ ())}\,S_R}{\Gamma \vdash \texttt{Right}\ (\texttt{Right}\ (\texttt{Left}\ ())) :: \texttt{Either}\ () \ (\texttt{Either}\ () \ (\texttt{Either}\ () \ ()))}\,S_R}{\Gamma \vdash \texttt{Left}\ (\texttt{Right}\ (\texttt{Right}\ (\texttt{Left}\ ()))) :: \texttt{Either}\ (\texttt{Either}\ () \ (\texttt{Either}\ () \ (\texttt{Either}\ () \ ()))) \ ()}\,S_R}{\Gamma \vdash \texttt{Right}\ (\texttt{Left}\ (\texttt{Right}\ (\texttt{Right}\ (\texttt{Left}\ ())))) :: \texttt{Either}\ () \ (\texttt{Either}\ (\texttt{Either}\ () \ (\texttt{Either}\ () \ (\texttt{Either}\ () \ ()))) \ ())}\,S_R}{\Gamma \vdash \texttt{Left}\ (\texttt{Left}\ (\texttt{Right}\ (\texttt{Left}\ (\texttt{Right}\ (\texttt{Right}\ (\texttt{Left}\ ())))))) :: \texttt{Either}\ (\texttt{Either}\ (\texttt{Either}\ () \ (\texttt{Either}\ () \ (\texttt{Either}\ () \ ()))) \ ()) \ ()}\,S_R}{\Gamma \vdash \texttt{Right}\ (\texttt{Left}\ (\texttt{Left}\ (\texttt{Right}\ (\texttt{Left}\ (\texttt{Right}\ (\texttt{Right}\ (\texttt{Left}\ ()))))))) :: \texttt{Either}\ () \ (\texttt{Either}\ (\texttt{Either}\ (\texttt{Either}\ () \ (\texttt{Either}\ () \ (\texttt{Either}\ () \ ()))) \ ()) \ ())}\,S_R}{\Gamma \vdash \texttt{Left}\ (\texttt{Right}\ (\texttt{Left}\ (\texttt{Left}\ (\texttt{Right}\ (\texttt{Left}\ (\texttt{Right}\ (\texttt{Right}\ (\texttt{Left}\ ()))))))))) :: \texttt{Either}\ (\texttt{Either}\ (\texttt{Either}\ (\texttt{Either}\ () \ (\texttt{Either}\ () \ (\texttt{Either}\ () \ ()))) \ ()) \ ()) \ ()}\,S_R$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma \vdash () :: ()}\,()}{\Gamma \vdash \texttt{Left}\ () :: \texttt{Either}\ () \ ()}\,S_R}{\Gamma \vdash \texttt{Right}\ (\texttt{Left}\ ()) :: \texttt{Either}\ () \ (\texttt{Either}\ () \ ())}\,S_R}{\Gamma \vdash \texttt{Right}\ (\texttt{Right}\ (\texttt{Left}\ ())) :: \texttt{Either}\ () \ (\texttt{Either}\ () \ (\texttt{Either}\ () \ ()))}\,S_R}{\Gamma \vdash \texttt{Left}\ (\texttt{Right}\ (\texttt{Right}\ (\texttt{Left}\ ()))) :: \texttt{Either}\ (\texttt{Either}\ () \ (\texttt{Either}\ () \ (\texttt{Either}\ () \ ()))) \ ()}\,S_R}{\Gamma \vdash \texttt{Right}\ (\texttt{Left}\ (\texttt{Right}\ (\texttt{Right}\ (\texttt{Left}\ ())))) :: \texttt{Either}\ () \ (\texttt{Either}\ (\texttt{Either}\ () \ (\texttt{Either}\ () \ (\texttt{Either}\ () \ ()))) \ ())}\,S_R}{\Gamma \vdash \texttt{Left}\ (\texttt{Left}\ (\texttt{Right}\ (\texttt{Left}\ (\texttt{Right}\ (\texttt{Right}\ (\texttt{Left}\ ())))))) :: \texttt{Either}\ (\texttt{Either}\ (\texttt{Either}\ () \ (\texttt{Either}\ () \ (\texttt{Either}\ () \ ()))) \ ()) \ ()}\,S_R}{\Gamma \vdash \texttt{Right}\ (\texttt{Left}\ (\texttt{Left}\ (\texttt{Right}\ (\texttt{Left}\ (\texttt{Right}\ (\texttt{Right}\ (\texttt{Left}\ ()))))))) :: \texttt{Either}\ () \ (\texttt{Either}\ (\texttt{Either}\ (\texttt{Either}\ () \ (\texttt{Either}\ () \ (\texttt{Either}\ () \ ()))) \ ()) \ ())}\,S_R}{\Gamma \vdash \texttt{Left}\ (\texttt{Right}\ (\texttt{Left}\ (\texttt{Left}\ (\texttt{Right}\ (\texttt{Left}\ (\texttt{Right}\ (\texttt{Right}\ (\texttt{Left}\ ()))))))))) :: \texttt{Either}\ (\texttt{Either}\ (\texttt{Either}\ (\texttt{Either}\ () \ (\texttt{Either}\ () \ (\texttt{Either}\ () \ ()))) \ ()) \ ()) \ ()}\,S_R}{\Gamma \vdash \texttt{Right}\ (\texttt{Right}\ (\texttt{Left}\ (\texttt{Right}\ (\texttt{Left}\ (\texttt{Left}\ (\texttt{Right}\ (\texttt{Left}\ (\texttt{Right}\ (\texttt{Right}\ (\texttt{Left}\ ()))))))))))) :: \texttt{Either}\ () \ (\texttt{Either}\ () \ (\texttt{Either}\ (\texttt{Either}\ (\texttt{Either}\ (\texttt{Either}\ () \ (\texttt{Either}\ () \ (\texttt{Either}\ () \ ()))) \ ()) \ ()) \ ()))}\,S_R$$

Type Correctness

$$\frac{}{\Gamma \vdash () :: ()}\,() \qquad \frac{\Gamma \quad e :: A}{\qquad}\qquad \frac{\Gamma \quad e :: B}{\texttt{Either}\ A\ B}\,S_R$$

$$\frac{\dfrac{}{() :: ()}\,()}{\texttt{Right}\ () :: \texttt{Either}\ ()\ (}$$

$$\frac{}{\texttt{Right}\ (\texttt{Right}\ ()) :: \texttt{Either}\ ()\ (\texttt{Either}\ ()\ ())}\,R$$

Examples

```
prop_or_false :: a -> (Either a Void)
prop_or_false a = Left a

prop_or_true :: a -> (Either a ())
prop_or_tru

prop_and_tr
prop_and_true a = (a, ())

prop_double_neg_intro :: a -> (a -> Void) -> Void
prop_double_neg_intro a f = f a

prop_triple_neg_elim ::
  (((a-> Void) -> Void) -> Void) -> a -> Void
prop_triple_neg_elim f a = f (\g ->  g a)
```

Wrap-up

1. Assignment 2 is before my next lecture (7th August).

Wrap-up

1. Assignment 2 is before my next lecture (8th August).
2. There is a quiz for this week, but no exercise.

Wrap-up

1. Assignment 2 is before my next lecture (8th August).
2. There is a quiz for this week, but no exercise.
3. Next week' ems and a **revision le**

Wrap-up

1. Assignment 2 is before my next lecture (8th August).
2. There is a quiz for this week, but no exercise.
3. Next week' ems and a **revision le**

4. There will be a specific questions
5. If you enjoyed the course and want to do more in this directio topics, taste of research projects, and consider attendi COMP4161.

Wrap-up

1. Assignment 2 is before my next lecture (8th August).
2. There is a quiz for this week, but no exercise.
3. Next week' ems and a **revision le**

4. There will be a specific questions
5. If you enjoyed the course and want to do more in this directi topics, taste of research projects, and consider attendi COMP4161.

6. Fill in the myExperience reports, it is important for us to receive your feedback.

Consultations

- Consultations will be made on request. Ask on piazza or email `cs3141@c`

Consultations

- Consultations will be made on request. Ask on piazza or email cs3141@c
- If there is a co number for

Consultations

- Consultations will be made on request. Ask on piazza or email cs3141@c
- If there is a co
number for
- Will be in the Thursday lecture slot, 9am to 11am on Blackb

Consultations

- Consultations will be made on request. Ask on piazza or email `cs3141@c`
- If there is a con

number for
- Will be in the Thursday lecture slot, 9am to 11am on Blackb
- Make sure to join the queue on Hopper. Be ready to share you

(`ghci` or `stack repl`) and editor set up.