

Declarative language

Before building system

before there can be learning, reasoning, planning, explanation ...

need to be able to express knowledge

Want a precise declarative language

- declarative: believe P = hold P to be true
cannot believe P without some sense of what it would mean
- precise
 - what strings of symbols mean
 - what it means for a sentence to be true (but without having to specify the world)

What does it mean to have a language?

- syntax
- semantics
- pragmatics

Here: language of first-order logic

again: not the only choice

Alphabet

Logical symbols:

- Punctuation: $(,), .$
- Connectives: $\neg, \wedge, \vee, \forall, \exists, =$
- Variables: $x, x_1, x_2, \dots, x', x'', \dots, y, \dots, z, \dots$

Fixed meaning and use

like keywords in a programming language

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Non-logical symbols <https://eduassistpro.github.io/>

- Predicate symbols (like Dog)
- Function symbols (like bestFrie

Domain-dependent meaning and use

like identifiers in a programming language

Have arity: number of arguments

arity 0 predicates: propositional symbols

arity 0 functions: constant symbols

Assume infinite supply of every arity

Note: not treating $=$ as a predicate

Grammar

Expressions: terms and formulas (wffs)

Terms

1. Every variable is a term.
2. If t_1, t_2, \dots, t_n are terms and f is a function of arity n , then $f(t_1, t_2, \dots, t_n)$ is a term.

Atomic wffs

1. If t_1, t_2, \dots, t_n are terms and P is a predicate of arity n , then $P(t_1, t_2, \dots, t_n)$ is an atomic wff.

2. If t_1 and t_2 are terms, then $t_1 = t_2$ is an atomic wff.

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Wffs

1. Every atomic wff is a wff.
2. If α and β are wffs, and v is a variable, then $\neg\alpha$, $(\alpha \wedge \beta)$, $(\alpha \vee \beta)$, $\exists v.\alpha$, $\forall v.\alpha$ are wffs.

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The propositional subset:

No terms

Atomic wffs: only predicates of 0-arity

No variables and no quantifiers

$$(p \wedge \neg(q \vee r))$$

Notation

Occasionally add or omit $(,), .$

Use $[,]$ and $\{, \}$ also.

Abbreviations:

$(\alpha \supset \beta)$ for $(\neg \alpha \vee \beta)$

$(\alpha \equiv \beta)$ for $((\alpha \supset \beta) \wedge (\beta \supset \alpha))$

Non-logical symbols:

Predicates: Person, Happy, OlderThan

Function

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Lexical scope for variables

$P(x) \wedge \exists x[P(x) \vee Q(x)]$

free

bound

occurrences of variables

Sentence: wff with no free variables (closed)

Substitution: $\alpha[v/t]$ means α with all free occurrences of v replaced by term t (also α^v_t).

Semantics

How to interpret sentences?

- what do sentences claim about the world?
- what does believing one amount to?

Without answers, cannot use sentences to represent knowledge

Problem:

cannot fully specify interpretation of sentences because non-logical symbols reach outside the language

So: <https://powcoder.com>

make clear dependence of interpretation on non-logical symbols

Logical interpretation:

specification of how to understand predicate and function symbols

Can be complex!

DemocraticCountry,
IsABetterJudgeOfCharacterThan,
favouriteIceCreamFlavourOf,
puddleOfWater27

Simple case

There are objects

some satisfy predicate P ; some do not

Each interpretation settles extension of P

borderline cases ruled in separate interpretations

Each interpretation assigns to function f a mapping from objects to objects

functions always well-defined and single-valued

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Main assumption:

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this is all you need to know about the non-logical symbols to understand which sentences of FOL are true or false

In other words, given a specification of

- what objects there are
- which of them satisfy P
- what mapping is denoted by f

it will be possible to say which sentences of FOL are true and which are not

Interpretations

Two parts: $I = \langle D, \Phi \rangle$

D is the domain of discourse

__can be any set

not just formal / mathematical objects

e.g. people, tables, numbers, sentences, chunks of peanut butter, situations, the universe

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Φ is an int <https://eduassistpro.github.io/>

If P is a predicate symbol of ar

$\Phi(P) \subseteq [D \times D \times \dots \times D]$

an n -ary relation over D

Can view interpretation of predicates

in terms of characteristic function

$$\Phi(P) \in [D \times D \times \dots \times D \rightarrow \{0, 1\}]$$

If f is a function symbol of arity n ,

$$\Phi(f) \in [D \times D \times \dots \times D \rightarrow D]$$

an n -ary function over D

For constants, $\Phi(c) \in D$

Denotation

In terms of interpretation I , terms will denote elements of D .

will write element as $I\|t\|$

For terms with variables, denotation depends on the values of variables.

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will write

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Rules of interpretation:

1. $I, \mu \|v\| = \mu(v).$
2. $I, \mu \|f(t_1, t_2, \dots, t_n)\| = H(d_1, d_2, \dots, d_n)$
where $H = \Phi(f)$
and $d_i = I, \mu \|t_i\|$, recursively

Satisfaction

In terms of I , wffs will be true for some values of the free variables and false for others

will write as $I, \mu \models \alpha$ “ α is satisfied by I and μ ”

where $\mu \in [Variables \rightarrow D]$, as before

or $I \models \alpha$, when α is a sentence

or $I \models S$, when S is a set of sentences

(all sentences in S are true in I).

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Rules of in

1. $I, \mu \models P$ <https://eduassistpro.github.io/>

where $R = \Phi(P)$

and $d_i \in I, \mu \models t_i$ as on [Add WeChat edu_assist_pro](#)

2. $I, \mu \models (t_1 = t_2)$ iff $I, \mu \models t_1$ is the same as $I, \mu \models t_2$

3. $I, \mu \models \neg \alpha$ iff $I, \mu \not\models \alpha$

4. $I, \mu \models (\alpha \wedge \beta)$ iff $I, \mu \models \alpha$ and $I, \mu \models \beta$

5. $I, \mu \models (\alpha \vee \beta)$ iff $I, \mu \models \alpha$ or $I, \mu \models \beta$

6. $I, \mu \models \exists v. \alpha$ iff for some $d \in D$, $I, \mu\{d;v\} \models \alpha$

7. $I, \mu \models \forall v. \alpha$ iff for all $d \in D$, $I, \mu\{d;v\} \models \alpha$

where $\mu\{d;v\}$ is just like μ , except on v , where $\mu(v)=d$.

For propositional subset:

$I \models p$ iff $\Phi(p) = 1$ and the rest as above

Logical consequence

Semantic rules of interpretation tell us how to understand all wffs in terms of specification for non-logical symbols.

But some connections among sentences are independent of non-logical symbols involved.

e.g. If α is true under I , then so is $\neg(\beta \wedge \neg\alpha)$, no matter what I is, why α is true, what β is, ...

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a function of logical symbols only

S entails consequence of S :

$S \models \alpha$ iff for every I , if $I \models S$ then $I \models \alpha$.

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In other words: for no I , $I \models S \cup \{\neg\alpha\}$.

Say that $S \cup \{\neg\alpha\}$ is unsatisfiable

Special case: S is empty

$\models \alpha$ iff for every I , $I \models \alpha$. Say α is valid.

Note: $\{\alpha_1, \alpha_2, \dots, \alpha_n\} \models \alpha$ iff $\models (\alpha_1 \wedge \alpha_2 \wedge \dots \wedge \alpha_n) \supset \alpha$
finite entailment reduces to validity

Why do we care?

We do not have access to user-intended interpretation of non-logical symbols

But, with entailment, we know that if S is true in the intended interpretation, then so is α .

If the user's view has the world satisfying S , then it must also satisfy α .

There may be other sentences true also; but α is logically guaranteed.

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So what ab

Dog(fido) <https://eduassistpro.github.io/>

Not entailment!

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There are logical interpret

$$\Phi(\text{Dog}) \not\subseteq \Phi(\text{Mammal})$$

Key idea of KR:

include such connections explicitly in S

$$\forall x[\text{Dog}(x) \supset \text{Mammal}(x)]$$

$$\text{Get: } S \cup \{\text{Dog}(\text{fido})\} \models \text{Mammal}(\text{fido})$$

The rest is just the details...

Knowledge Bases

KB is set of sentences

explicit statement of sentences believed (including assumed connections among non-logical symbols)

$KB \models \alpha$ α is a further consequence of what is believed

- explicit knowledge: KB
- implicit knowledge: KB

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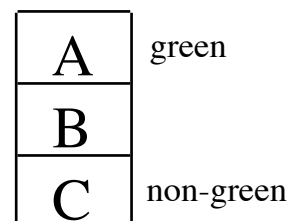
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Often non trivial: explicit

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Example:

Three blocks stacked.
Top one is green.
Bottom one is not green.



Is there a green block directly on top of a non-green block?

A formalization

$S = \{ \text{On}(a,b), \text{On}(b,c), \text{Green}(a), \neg \text{Green}(c) \}$
all that is required

$\alpha = \exists x \exists y [\text{Green}(x) \wedge \neg \text{Green}(y) \wedge \text{On}(x,y)]$

Claim: $S \models \alpha$

Proof:

Let I be any interpretation such that $I \models S$.

Case 1: $I \models \alpha$
 $\therefore I \models \alpha$

Case 2: $I \not\models \alpha$.

$\therefore I \models \neg \alpha$

$\therefore I \models \text{Green}(a) \wedge \neg \text{Green}(b) \wedge \text{On}(a,b)$.

$\therefore I \models \alpha$

Either way, for any I , if $I \models S$ then $I \models \alpha$.

So $S \models \alpha$. QED

Knowledge-based system

Start with (large) KB representing what is explicitly known

e.g. what the system has been told

Want to influence behaviour based on what is implicit in the KB (or as close as possible)

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Requires <https://eduassistpro.github.io/>
deductive inference:

process of calculating ant [Add WeChat edu_assist_pro](#)

i.e given KB and any α , determine if $KB \models \alpha$

Process is sound if whenever it produces α , then $KB \models \alpha$

does not allow for plausible assumptions that may be true in intended interpretation

Process is complete if whenever $KB \models \alpha$, it produces α

does not allow for process to miss some α or be unable to determine the status of α