

COMP4418: Knowledge Representation and Reasoning

Nonmonotonic Reasoning

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Nonmonotonic Reasoning

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- Suppose you are told “Tweety is a bird”

- What conclusions would you draw?



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- What conclusions would you draw now? Do they differ from the conclusions that you would draw with the additional information? If, what way(s)?

- Nonmonotonic reasoning is an attempt at *commonsense* reasoning

1 Nonmonotonicity

2 Closed World Assumption

3

4

5 Default Logic

6 Nonmonotonic Consequence

■ KLM Systems

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Nonmonotonic Reasoning

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- In classical logic the more facts (premises) we have, the more conclusions we can draw
- This property is known as *Monotonicity*

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(where C_n denotes classical consequence)

- However, the previous example shows reasoning in this manner
- Might a nonmonotonic logic—one that lacks the Monotonicity property—provide a more effective way of reasoning?

Why Nonmonotonicity?

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- Problems with the classical approach to consequence
 - It is usually not possible to write down all we would like to

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- Sometimes we would like to represent something that is not *entirely* true in our knowledge
- Nonmonotonic reasoning is concerned with getting around these shortcomings

Makinson's Classification

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Makinson has suggested the following classification of non

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- Additional rules

David Makinson, *Bridges from Classic Logic*, Texts in Computing, Volume 5, King's Publications, 2005.

Nonmonotonicity

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- Classical logic satisfies the following property
- Monotonicity: If $\Delta \subseteq \Gamma$, then $Cn(\Delta) \subseteq Cn(\Gamma)$

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- More information can lead us to retract previous conclusions

- We shall adopt the following notation

- \vdash classical consequence relation
- \vdash nonmonotonic consequence relation

Consequence Operation Cn

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Other properties of consequence operation Cn

Inclusion $\Delta \subseteq Cn(\Delta)$

Cum

$Cn(\Delta)$

Com

such

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Disjunction in the Premises

$$Cn(\Delta \cup \{\alpha\}) \cap Cn(\Delta \cup \{\beta\}) = Cn(\Delta \cup \{\alpha \vee \beta\})$$

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Note: $\Delta \vdash \phi$ iff $\phi \in Cn(\Delta)$

alternatively: $Cn(\Delta) = \{\phi : \Delta \vdash \phi\}$

Example

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Suppose I tell you 'Tweety is a bird'

You

I then tell you

You

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$bird(Tweety) \vdash flies(Tweety)$
 $bird(Tweety) \wedge \neg emu(Tweety) \vdash \neg flies(Tweety)$

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The Closed World Assumption

- A *complete* theory is one in which for every ground atom in the language, either the atom or its negation appears in the theory



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base

- In other words, if we have no evidence as to whether a ground atom P is true or false, we assume that it is false

- Given a base set of formulae Δ
assumption set

$\neg P \in \Delta_{asm}$ iff for ground atom P , $\Delta \not\models P$

- $CWA(\Delta) = Cn\{\Delta \cup \Delta_{asm}\}$

Example

$$\Delta = \{P(a), P(b), P(a) \rightarrow Q(a)\}$$

$$\Delta_{\text{casm}} = \{\neg Q(b)\}$$

Theorem. The CWA applied to a consistent set of formulae Δ is inconsistent iff there are positive ground literals L_1, \dots, L_n such

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language could contain other constants

the *Domain Closure Assumption*

- Another common assumption is the *Unique Name Assumption* (UNA).

*If two ground terms can't be proved equal,
assume that they are not.*

Predicate Completion

Goal: The only objects that satisfy a predicate are those that must



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- Can add the *only if* part:

$$\forall x. P(x) \Rightarrow x = a$$

- Giving:

$$\forall x. P(x) \leftrightarrow x = a$$

Predicate Completion

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- **Definition:** A clause is *solitary* in a predicate P if

P , it

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P

$Q(a) \vee R(a) \vee P(b)$ is solitary i

- Completion of a predicate is only define clauses solitary in that predicate

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Predicate Completion

- Each clause can be written:

$$\forall t. Q_1 \wedge \dots \wedge Q_n \rightarrow P(t) \quad (P \text{ not contained in } Q_i)$$

$$\forall y. \forall x. (x = t) \wedge Q_1 \wedge \dots \wedge Q_m \rightarrow P(x)$$

m of

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$$\forall x. E_1 \rightarrow P(x)$$

$$\dots$$

$$\forall x. E_n \rightarrow P(x)$$

- Grouping these together we get:

$$\forall x. E_1 \vee \dots \vee E_n \rightarrow P(x)$$

- Completion becomes: $\forall x. P(x) \leftrightarrow E_1 \vee \dots \vee E_n$
and we can add this to the original set of formulae

Example

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- Suppose $\Delta = \{\forall x. \text{Emu}(x) \rightarrow \text{Bird}(x),$

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$$\forall x. (\text{Emu}(x) \vee x = \text{Tweety})$$

- Predicate completion of P in Δ

$$\Delta \cup \{\forall x. \text{Bird}(x) \rightarrow \text{Emu}(x)$$

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Circumscription

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- Idea: Make extension of predicate as small as possible
- Example:

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- Want to be able to conclude *Flie*
 $\neg \text{Flies Sam}$
- Accept interpretations where *Ab* possible
- That is, we *minimise abnormality*

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Circumscription

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- Given interpretations $I_1 = \langle D, I_1 \rangle$, $I_2 = \langle D, I_2 \rangle$, $I_1 \leq I_2$ iff for every predicate $P \in \mathbf{P}$, $I_1[P] \subseteq I_2[P]$.

- , either

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- Now consider

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$\forall x. Bird(x) \wedge \neg Ab(x)$
 $\forall x. Emu(x) \rightarrow Bird(x)$
 $Bird(Tweety)$

Reiter's Default Logic (1980)

- Add default rules of the form $\frac{\alpha:\beta}{\gamma}$

■ “If α can be proven and consistent to assume β , then conclude γ ”

- $\frac{\alpha:\beta}{\gamma}$

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D – set of defaults; W – set of facts

- *Extension* of default theory contains conclusions as possible and must be closed under classical consequence

- Concluding whether formula ϕ follows from $\langle D, W \rangle$

- Sceptical inference: ϕ occurs in *every* extension of $\langle D, W \rangle$
Credulous inference: ϕ occurs in *some* extension of $\langle D, W \rangle$

Examples

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- $W = \{ \}; D = \{ \frac{p}{\neg p} \}$ – no extensions
- $W = \{ p \vee r \}; D = \{ \frac{p:q}{q}, \frac{r:q}{q} \}$ – one extension $\{ p \vee r \}$

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- $D = \{ \frac{bird(x):flies(x)}{flies(x)} \}$ – one extension

What if we add $\frac{emu(x):\neg flies(x)}{\neg flies(x)}$?

- Poole (1988) achieves a similar effect (in general) by changing the way the underlying logic is used rather than introducing a new element into the syntax

Default Theories—Properties

Observation: Every normal default theory (default rules are all normal) has an extension

Observation: If a normal default theory has several extensions

Obs
 D is l

Theorem: (Semi-monotonicity)

Given two normal default theories $\langle D$

that $D \subseteq D'$ then, for any extension \mathcal{E}

extension $\mathcal{E}(D', W)$ where $\mathcal{E}(D, W)$

(The addition of normal default rules does not lead to the retraction of consequences.)

Nonmonotonic Consequence

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- Abstract study and analysis of nonmonotonic consequence relation \vdash in terms of general properties Kraus, Lehmann

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Left Logical Equivalence If $\vdash \phi$

Right Weakening If $\vdash \psi \rightarrow \chi$ and

And If $\phi \vdash \psi$ and $\phi \vdash \chi$, \vdash

$\vdash \chi$

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- Plus many more!

KLM Systems

- Kraus, Lehman and Magidor (1991) study various classes of nonmonotonic consequence relations

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- This has been extended since. A good reference for this line of work is Schlechta (1997)

Summary

- Nonmonotonic reasoning attempts to capture a form of commonsense reasoning
- Nonmonotonic reasoning often deals with inferences
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- Can introduce abstract study of nonmonotonic consequence relations in same way as consequence relations
- Similar links exist with conditionals
- One area where nonmonotonic reasoning is important is reasoning about action (dynamic systems)