Multi-agent Decision Making

COMP 4418 – Assignment 3

Due 25 Nov. 2020, 15:00
Total Marks: 50

Question 1 (10 marks) Consider a resource allocation setting in which indivisible items are to be allocated to agents, and agents have positive and additive utilities over the items. Prove or disprove the following:

1. If an allocation is proportional, it is envy-free.
2. If an allocation satisfies MmS fairness, it is proportional.
3. If an allocatio

Question 2 (10 marks) Consider a Shapley-Scarf housing market with a set of agents $N = \{0,1,2,3,4\}$, a set of items $O = \{o_0, o_1, o_2,$ tion ω : $N \to 2^O$ such that $\omega(i) = \{o_i\}$. The preferences of the ag right in decreasing order of preference.

$$0 : o_0, o_4, o_2, o_1, o_3$$
$$1 : o_0, o_2, o_4, o_1, o_3$$
$$2 : o_3, o_0, o_2, o_4, o_1$$
$$3 : o_0, o_2, o_3, o_1, o_4$$
$$4 : o_3, o_2, o_1, o_4, o_0$$

Find the outcome of the TTC (top trading cycles) algorithm. Can agent 4 misreport her preference to get a more preferred allocation? Prove or disprove that the outcome is individually rational.

Question 3 (10 marks) Consider Shapley-Scarf housing markets in which we are only allowed to obtain allocations in which at most two agents are a part of a trading cycle and each agent can be a part of at most of one trading cycle. Discuss at least three axiomatic properties that you consider to be desirable for the problem and explain why. Design a polynomial-time algorithm for the problem and prove it satisfies two of these properties.

Question 4 (10 marks) Consider the following market with 10 students

$$N = \{0,1,2,3,4,5,6,7,8,9\},$$

and three schools $C = \{c_0, c_1, c_2\}$. All the schools have the same capacity 4. The preferences of the students are as follows from left to right in decreasing order of preference.

$$\begin{aligned}
&\succ_0: c_0, c_2, c_1\\
&\succ_1: c_0, c_2, c_1\\
&\succ_2: c_0, c_2, c_1\\
&\succ_3: c_0, c_1, c_2\\
&\succ_4: c_2, c_0, c_1\\
&\succ_5: c_2, c_0, c_1\\
&\quad : c\ , c\ , c
\end{aligned}$$

The priorities of the schools are as follows from left to right in decreasing order of priority.

$$\begin{aligned}
&\succ_{c_0}: 9,4,1,5,3,8,6,7,2,0\\
&\succ_{c_1}: 5,4,3,0,6,2,7,1,9,8\\
&\succ_{c_2}: 2,6,9,7,3,5,4,8,1
\end{aligned}$$

Find the outcome matching of the student-proposing deferred a explain how you found the matching. Prove or disprove that the result example is Pareto

Question 5 (10 ma ave additive positive utilities over indivisible items. Consider the *ation* rule in which the policy of turns is $(1,2,\ldots,n)^*$ where the p cates the turns $1,2,\ldots,n$ repeat. Prove or disprove that rule

1. returns a Pareto optimal allocation.
2. returns a proportional allocation.
3. returns an EF1 allocation.

SUBMISSION

- You will need to answer the questions in a file named `assn3.pdf`. Submit using the command:

```
give cs4418 assn3 assn3.pdf
```

- Your answers are to be submitted in a single PDF file.
- The deadline for this submission is 25. Nov 2020, 15:00

Academic Honesty and Plagiarism

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- the submission of work derived from another person, or jointly written with someone else will, at the very least, result in automatic failure for COMP4418 with a mark of zero;
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