Week 4

Ch 5: Higher-Order Functions
Ch 6: Modules

University of the Fraser Valley

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COMP 481: Functional and Logic Programming

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Overview

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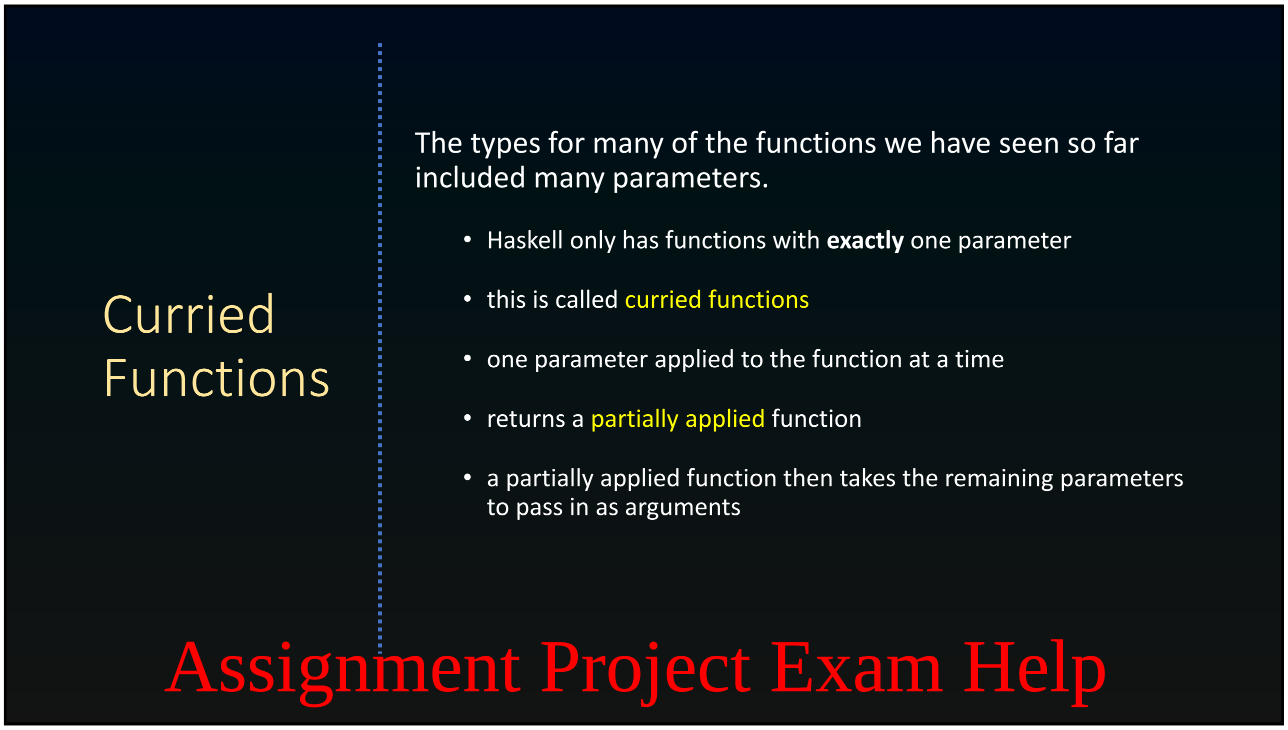

Curried Functions

The types for many of the functions we have seen so far included many parameters.

- Haskell only has functions with **exactly** one parameter
- this is called curried functions
- one parameter applied to the function at a time
- returns a partially applied function
- a partially applied function then takes the remaining parameters to pass in as arguments

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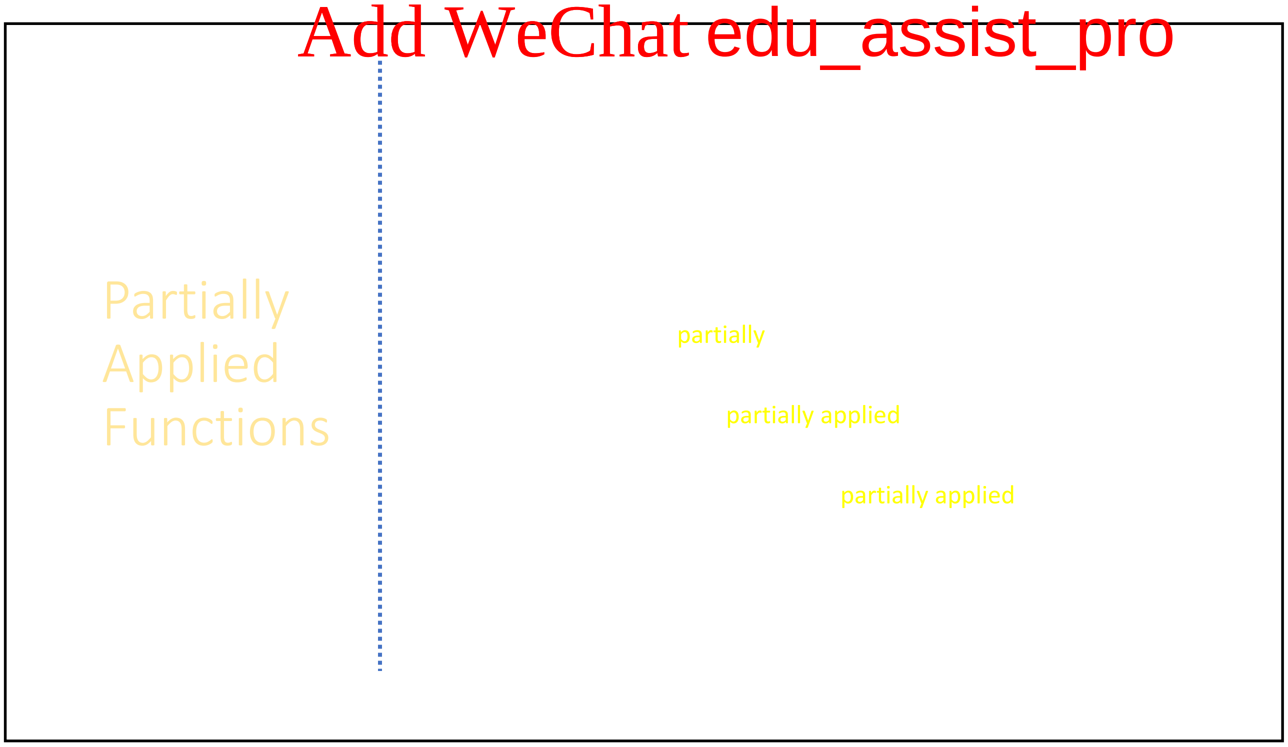

Partially Applied Functions

partially

partially applied

partially applied

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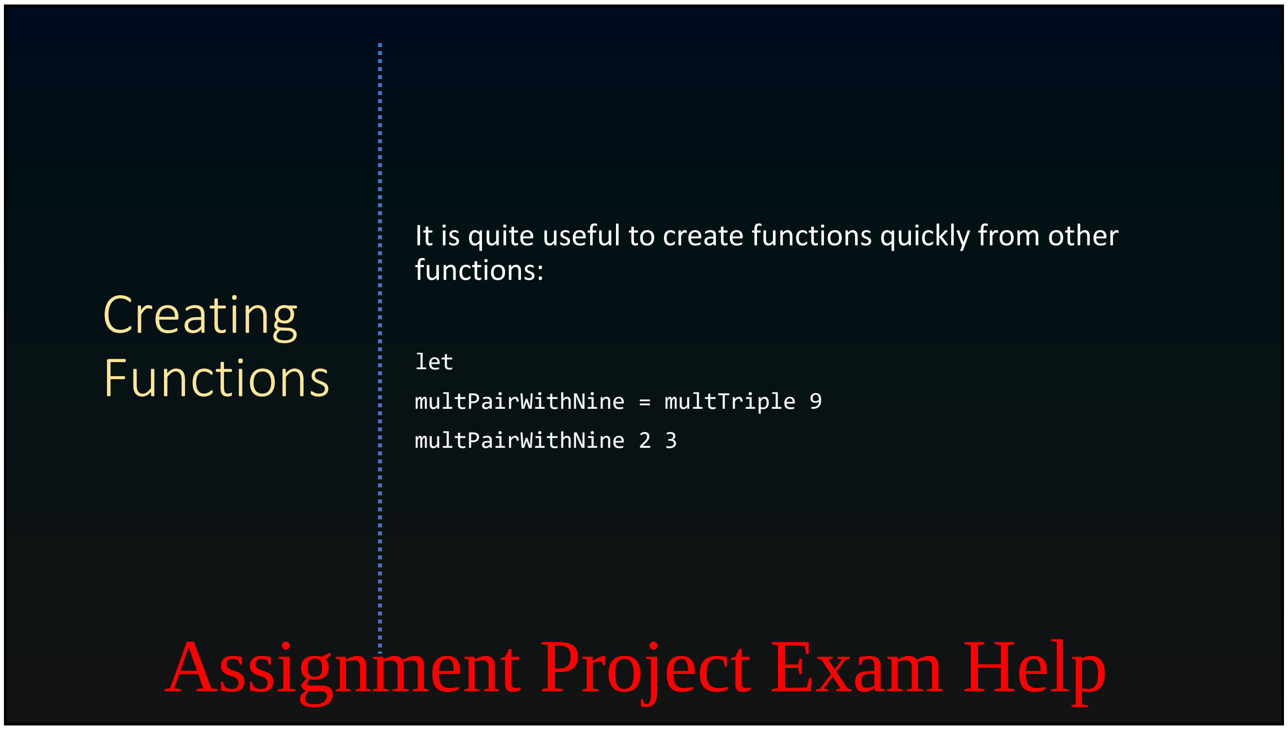

Creating Functions

It is quite useful to create functions quickly from other functions:

```
let
multPairWithNine = multTriple 9
multPairWithNine 2 3
```

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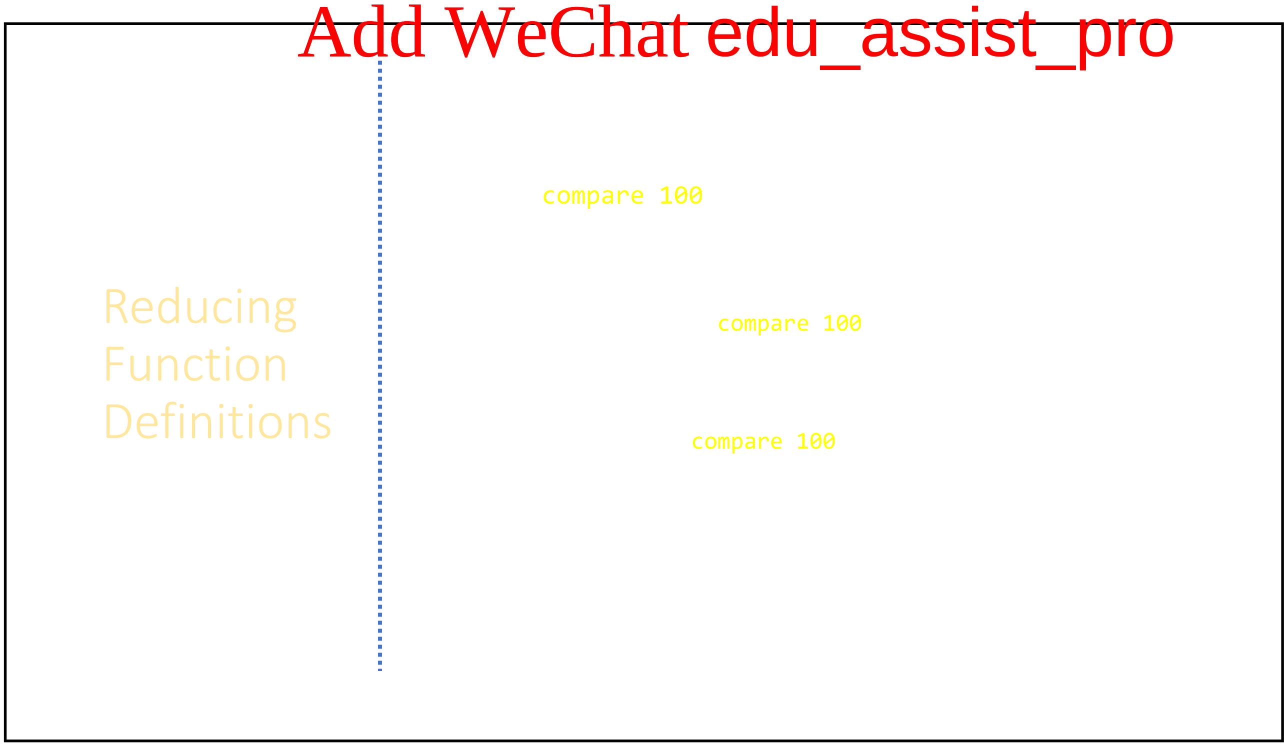

Reducing Function Definitions

```
compare 100
```

```
compare 100
```

```
compare 100
```

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Parentheses Around Functions

Sections are functions surrounded by parentheses:

```
(/10) 20
```

```
isUpper :: Char -> Bool
isUpper = (`elem` ['A'..'Z'])
```

The minus sign has multiple uses, where `(-1)` means a negative number, not partially applied subtraction:

- partially applied subtraction function is `(−)`
- OR e.g.: `(subtract 1)`
  "subtract 1 from the next parameter"

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Cannot Print Functions

```
No instance for (Show (Int -> Int)) arising from a use of `print’
(maybe you haven't applied a function to enough arguments?)
In a stmt of an interactive GHCi command: print it
```

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Functions as Parameters

Haskell can define functions that take other functions as parameters.

```
let
applyTwice :: (a -> a) -> a -> a
applyTwice f x = f (f x)
```

Try the examples of using the `applyTwice` function:

```
applyTwice (+3) 10
applyTwice (++ " HAHA") "HEY"
applyTwice ("HAHA " ++) "HEY"
applyTwice (multTriple 2 2) 9
applyTwice (3:) [1]
```

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zipWith'

```
f x y f x y
```

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Flipping Parameters

Consider the two ways to implement the standard library `flip` function:

```
flip' :: (a -> b -> c) -> (b -> a -> c)
flip' f = g
    where g x y = f y x

flip' :: (a -> b -> c) -> (b -> a -> c)
flip' f x y = f y x
```

Note that because functions are curried, the second set of parentheses in the type description is not needed.

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zipWith and flip'

flip'

flip'

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Processing Lists

Most of the advantages of functional programming use operations on lists:

```
map' :: (a -> b) -> [a] -> [b]
map' _ [] = []
map' f (x:xs) = f x : map' f xs
```

```
map' (replicate 3) [3..6]
```

```
map' (map' (^2)) [[1,2],[3,4,5,6],[7,8]]
```

```
map' fst [(1,2),(3,4),(5,6),(7,8),(9,10)]
```

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map

map

map

map

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Filtering Lists

A predicate is a function:

- that inputs something
- and results in `True` or `False`

The `filter` function applies a predicate function

- to the elements of a list
- and creates a new list with only those elements that return `True` by the predicate.

```
filter' :: (a -> Bool) -> [a] -> [a]
filter' _ [] = []
filter' p (x:xs)
 | p x    = x : filter' p xs
 | True   = filter' p xs
```

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filter Examples

filter

filter

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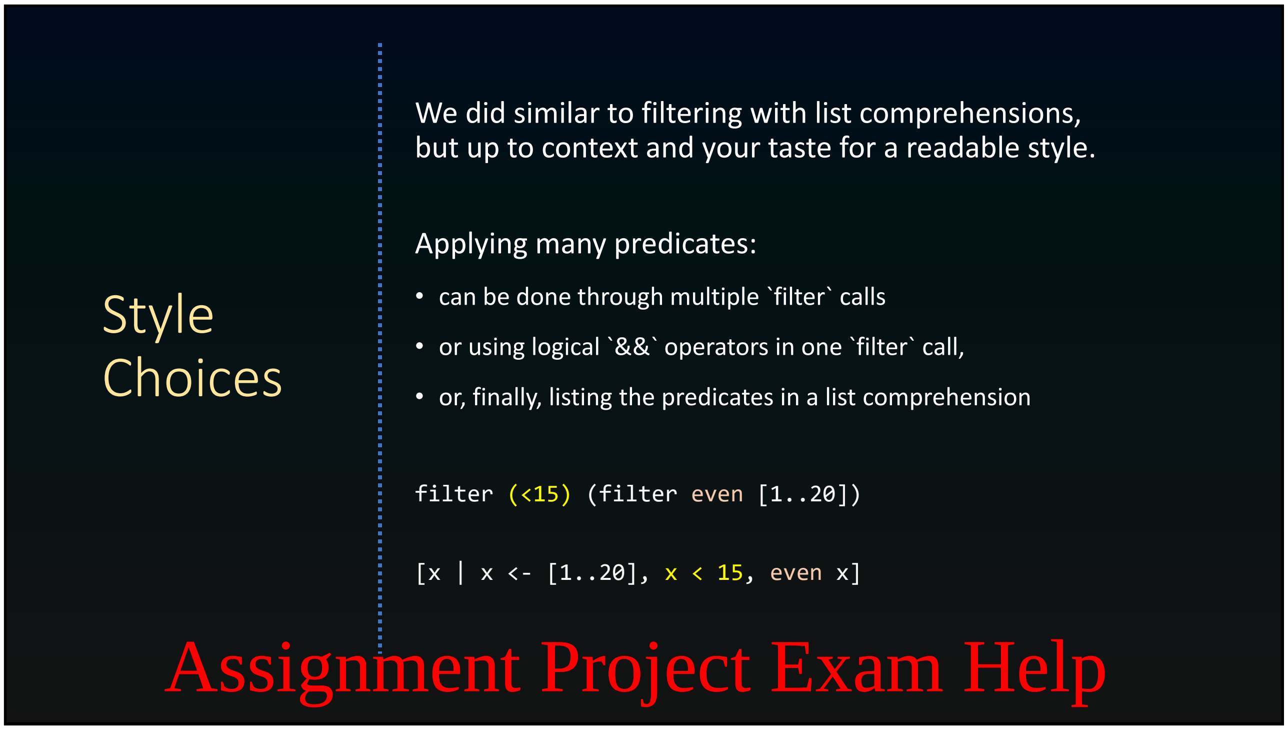

Style Choices

We did similar to filtering with list comprehensions, but up to context and your taste for a readable style.

Applying many predicates:

- can be done through multiple `filter` calls
- or using logical `&&` operators in one `filter` call,
- or, finally, listing the predicates in a list comprehension

```
filter (<15) (filter even [1..20])

[x | x <- [1..20], x < 15, even x]
```

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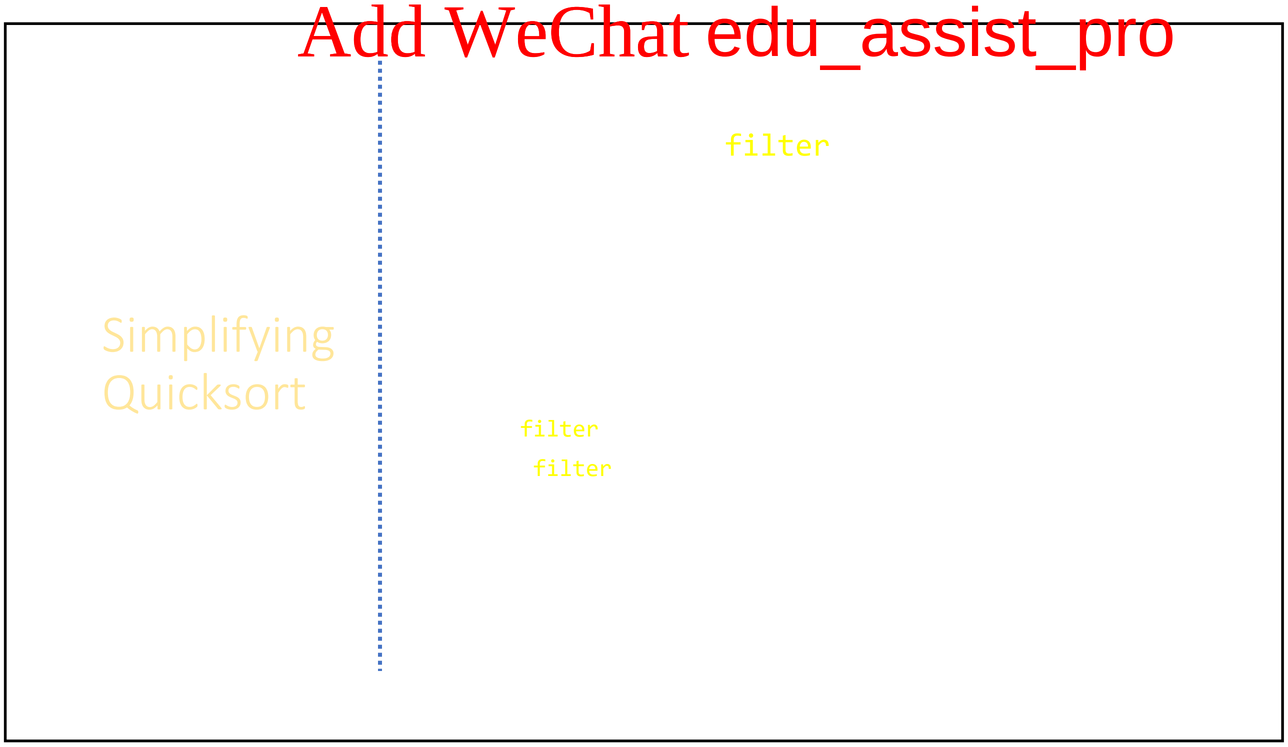

Simplifying Quicksort

```
filter
filter
 filter
```

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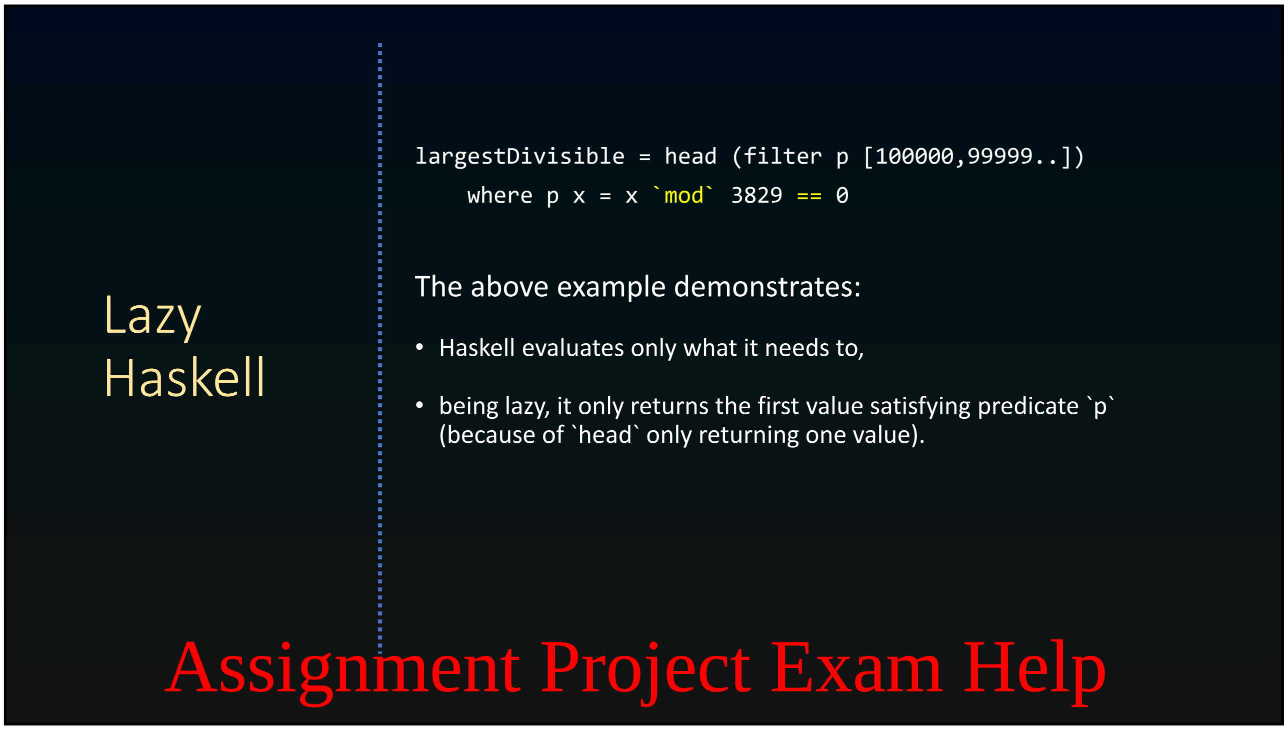

Lazy Haskell

```haskell
largestDivisible = head (filter p [100000,99999..])
    where p x = x `mod` 3829 == 0
```

The above example demonstrates:

- Haskell evaluates only what it needs to,
- being lazy, it only returns the first value satisfying predicate `p` (because of `head` only returning one value).

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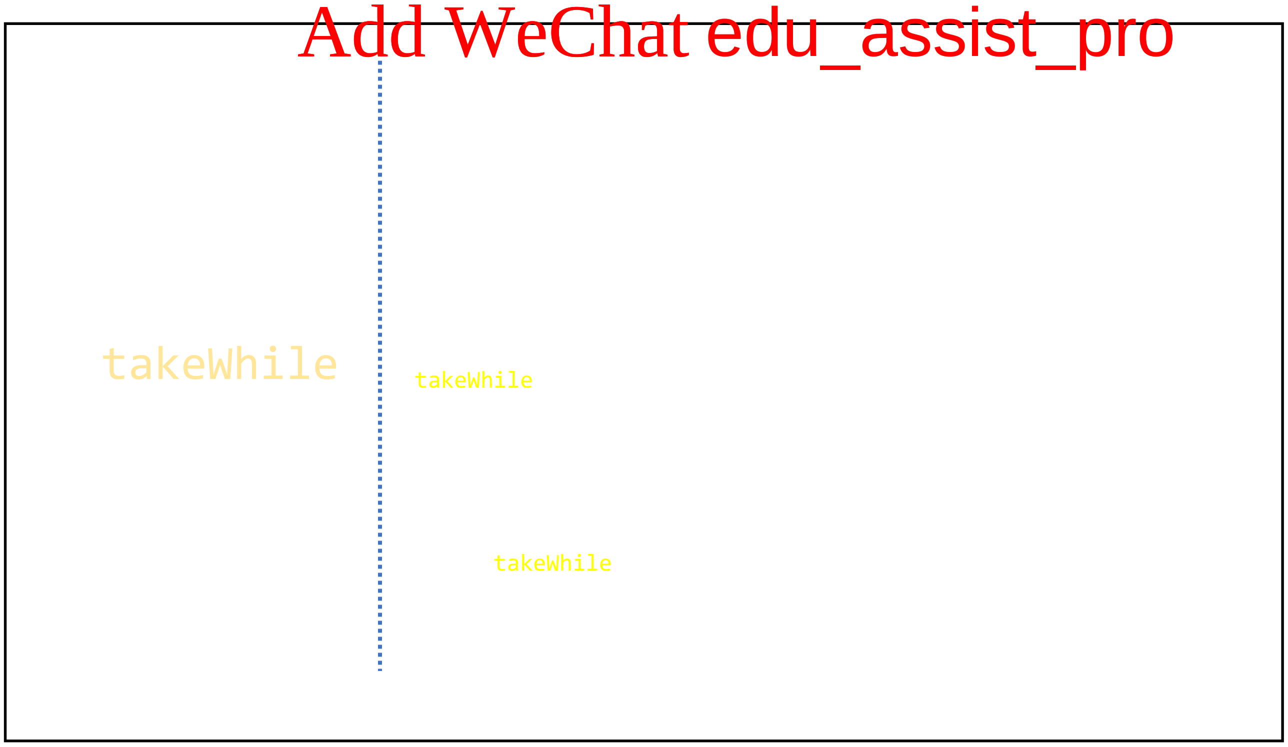

takeWhile

takeWhile

takeWhile

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Alternative to Nesting

We can rearrange how function composition is written using the concept of piping:

```
let a `pipe` b = flip ($) a b

:{
    (map (^2)) [1..]
    `pipe`
    (filter (odd))
    `pipe`
    (takeWhile (<10000))
    `pipe`
    (sum)
:}
```

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But Wait, There's More…

`odd (m^2)`

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Collatz Chains

- begin with any natural number (1 or larger) for a_0
- if a_{n-1} is 1, stop, otherwise:

$$\begin{cases} \frac{1}{2}a_{n-1}, & a_{n-1} \text{ even} \\ 3a_n + 1, & a_{n-1} \text{ odd} \end{cases}$$

- repeat with the resulting number

Does the sequence that forms a chain always end in the number 1?

- this is an open problem no one has solved yet
- the largest value known to stop at 1 is 2^100000 - 1 (as of 2018)
    - https://ieeexplore.ieee.org/document/8560077

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Collatz Chain

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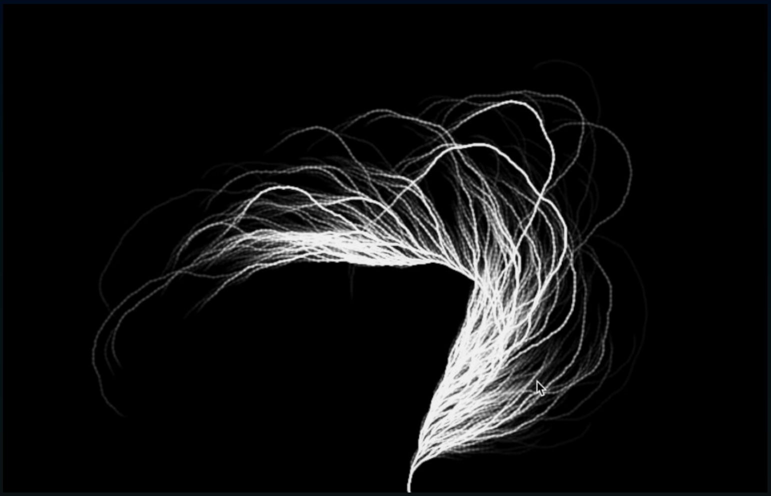
Collatz Conjecture



- copyright: (YouTube) Coding Train
- Coding in the Cabana 2: Collatz Conjecture
- https://youtu.be/EYLWxwo1Ed8?t=1263

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How Many Long Chains?

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Making More Functions

Partially applied functions can be used to create functions that take multiple parameters.

Combine this with Haskell's laziness:

```
listOfFuns = map (*) [0..]
(listOfFuns !! 4) 5
```

- the first line returns a function for each element
- the last line pulls element at index `4` to apply its function `(4*)` to the value `5`

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Lambda Functions

lambda

```
(filter (\xs -> length xs > 15)
```

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Function Styles

Use partial application to avoid using lambdas when it is not even necessary, as seen in equivalent examples:

```
map (\x -> x + 3) [1, 2, 3]
map (+3) [1, 2, 3]
```

A few more involved examples with two parameters:

```
zipWith (\a b -> a + b) [6,5,4,3,2,1] [1,2,3,4,5,6]
```

Another example with pattern matching:

```
map (\(a,b) -> a + b) [(1,2),(3,5),(6,3),(2,6),(2,5)]
```

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Cases, and Runtime Error

`case`

`case` `of`

`(1, 2)`

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Emphasized Currying

This example overemphasizes currying by way of unnecessary lambda functions:

```
addThree' :: Int -> Int -> Int -> Int
addThree' x y z = x + y + z

addThree' :: Int -> Int -> Int -> Int
addThree' = \x -> \y -> \z -> x + y + z
```

Note that lambdas written without parentheses assume everything to the right of `\` belongs to the lambda.

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`flip` Implementation

`flip`

`flip`

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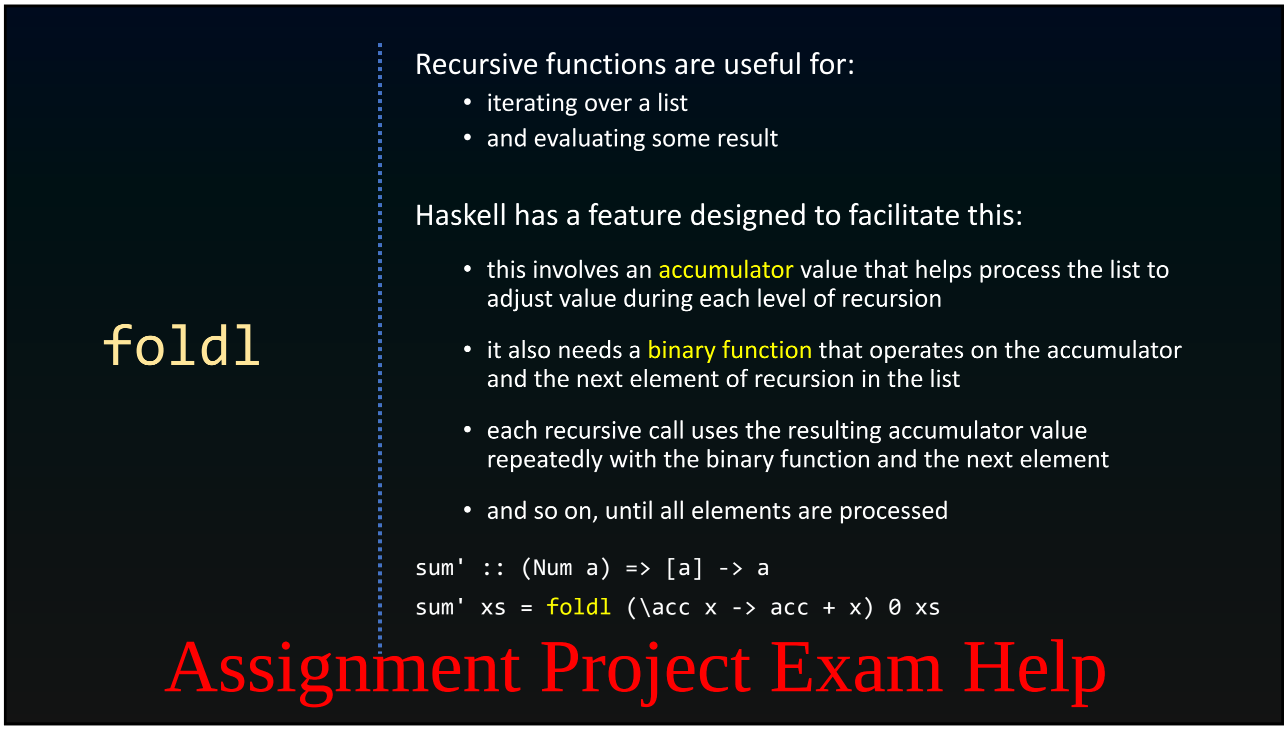

foldl

Recursive functions are useful for:

- iterating over a list
- and evaluating some result

Haskell has a feature designed to facilitate this:

- this involves an accumulator value that helps process the list to adjust value during each level of recursion
- it also needs a binary function that operates on the accumulator and the next element of recursion in the list
- each recursive call uses the resulting accumulator value repeatedly with the binary function and the next element
- and so on, until all elements are processed

```
sum' :: (Num a) => [a] -> a
sum' xs = foldl (\acc x -> acc + x) 0 xs
```

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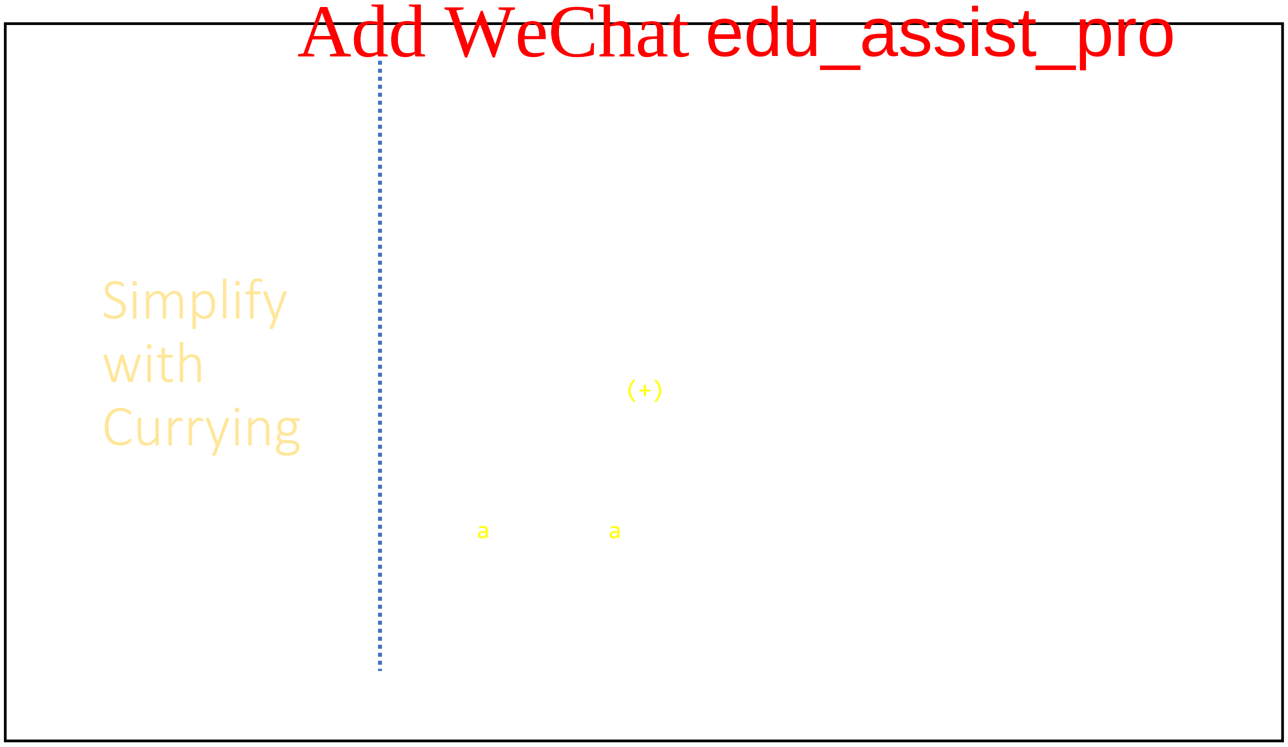

Simplify with Currying

(+)

a a

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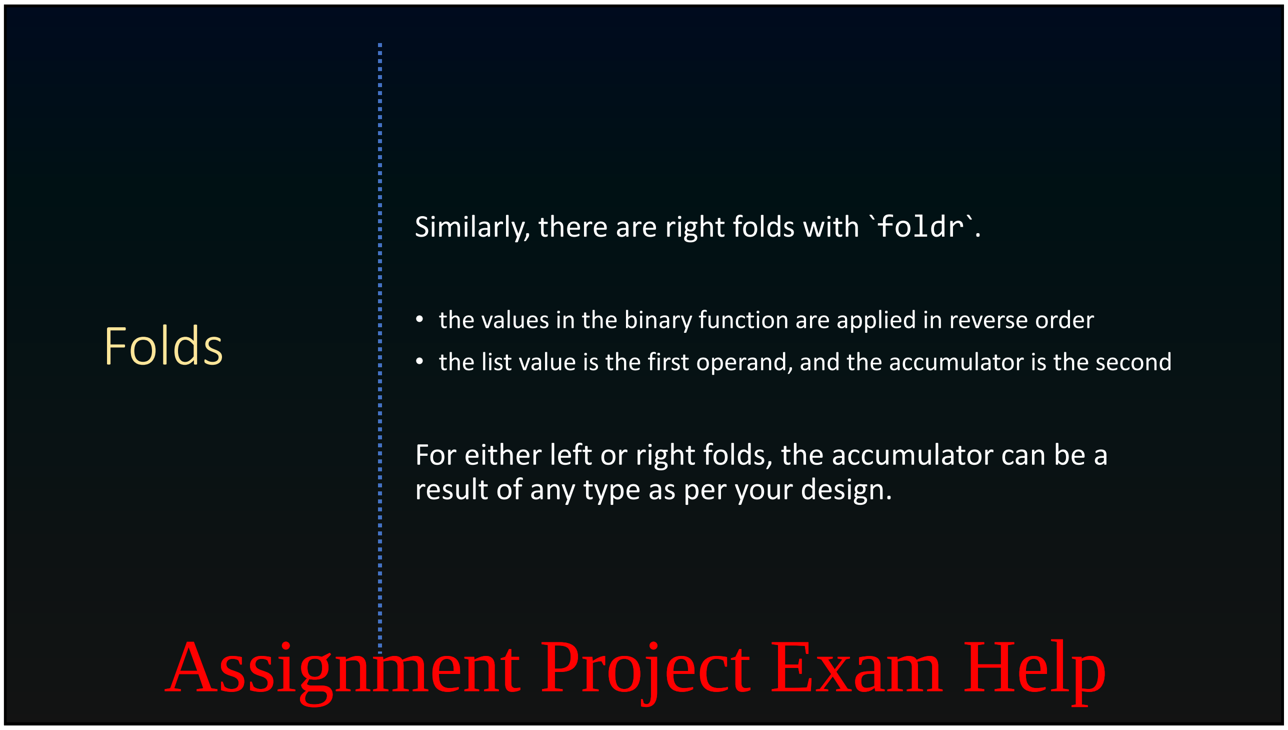

Folds

Similarly, there are right folds with `` `foldr` ``.

- the values in the binary function are applied in reverse order
- the list value is the first operand, and the accumulator is the second

For either left or right folds, the accumulator can be a result of any type as per your design.

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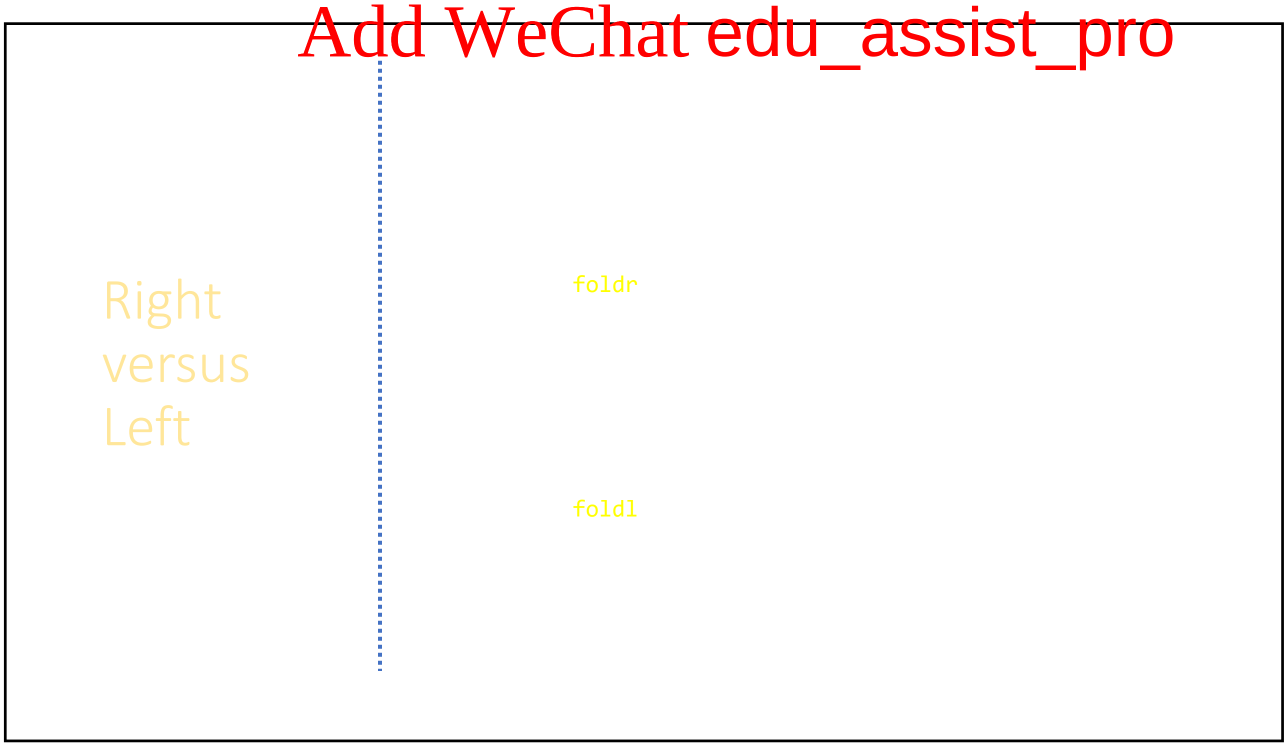

Right versus Left

`foldr`

`foldl`

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`++` versus `:`

The `:` operation is more efficient than `++` operation, so we mostly use folding on the right for processing lists.

Right folds work on infinite lists, whereas left folds do not.

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elem

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First Element Starts Accumulation

`foldl1` and `foldr1` functions assume the accumulator is the value of the first item in the list that they process.

Another implementation of `max`:

```
max' :: (Ord a) => [a] -> a
max' = foldl1 max
```

- partial application to help create functions
- the difficulty is in knowing that `foldl1` takes two parameters
- the second parameter is a list

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Empty List or Not

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— Break —

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```
reverse'
```
and
```
product'
```

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filter

Another implementation of `` `filter` ``:

```
filterr :: (a -> Bool) -> [a] -> [a]
filterr p = foldr (\x acc -> if p x then x : acc else acc) []
```

Remember, the accumulator parameter is always ordered on the side you are folding.

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last

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Right Fold Nesting Operations

Suppose we want to apply a right fold with binary function `f` on the list `[3,4,5,6]`.

- this can be seen as the expression

`f 3 (f 4 (f 5 (f 6 acc)))`

- the value `acc` is the starting accumulator value

- if `f` is replaced with `+` and `acc` starts with `0`,
- then the expression would be

`3 + (4 + (5 + (6 + 0)))`.

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Left Fold Nesting Operations

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Infinite? Use `foldr`

The `and'` function will combine Boolean elements of a list together with the `&&` operator.

```
and' :: [Bool] -> Bool
and' = foldr (&&) True
```

Take special care to use the `foldr` function, and not the `foldl` function, since the input could be an infinite list.

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Short Circuiting

short circuits evaluation

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scan

`scanl`/`scanr` functions are similar to `foldl`/`foldr`

- they return a list with all of the intermediate accumulator values from processing the input list
- of course, there are also the `scanl1` and `scanr1` functions

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scan

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Example with Piping

```
sqrtSums :: Int
sqrtSums = length (takeWhile (<1000)
    (scanl1 (+) (map sqrt [1..])))
```

Equivalently:

```
sqrtSums :: Int
sqrtSums =
 (map sqrt) [1..]
 `pipe` (scanl1 (+))
 `pipe` (takeWhile (<1000))
 `pipe` (length)
```

```
sum (map sqrt [1..130])
sum (map sqrt [1..131])
```

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Using $ to Apply Functions

```
) -> a
```

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Replace $ for Parentheses

Observe how $ can clean up nesting a bit:

```
sum (filter (> 10) (map (*2) [2..10]))

sum $ filter (> 10) (map (*2) [2..10])

sum $ filter (> 10) $ map (*2) [2..10]
```

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Alternative Syntax

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Apply Functions with $ as a Parameter

Another important use of $ is to tell Haskell to immediately apply some function:

```
map ($ 3) [(4+), (10*), (^2), sqrt]
```

Note that the `($ 3)` function takes some other function as input and applies that function to `3`.

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Function Composition

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Right Associative

- function composition is right-associative
- this is similar to our right folds, with

```
f (g (h x))
```

equivalently

```
(f . g . h) x
```

```
map (negate . sum . tail) [[1..5],[3..6],[1..7]]
```

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Currying and Multiline: . and $

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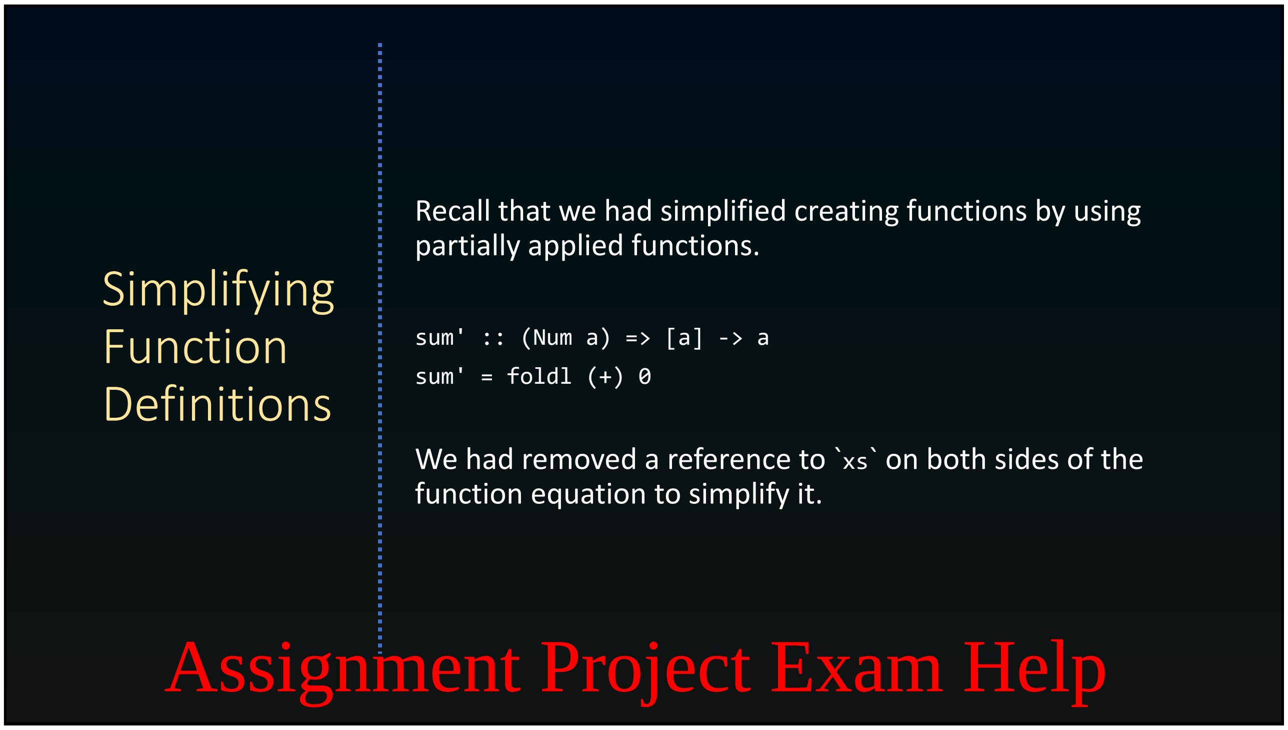

Simplifying Function Definitions

Recall that we had simplified creating functions by using partially applied functions.

```
sum' :: (Num a) => [a] -> a
sum' = foldl (+) 0
```

We had removed a reference to `xs` on both sides of the function equation to simplify it.

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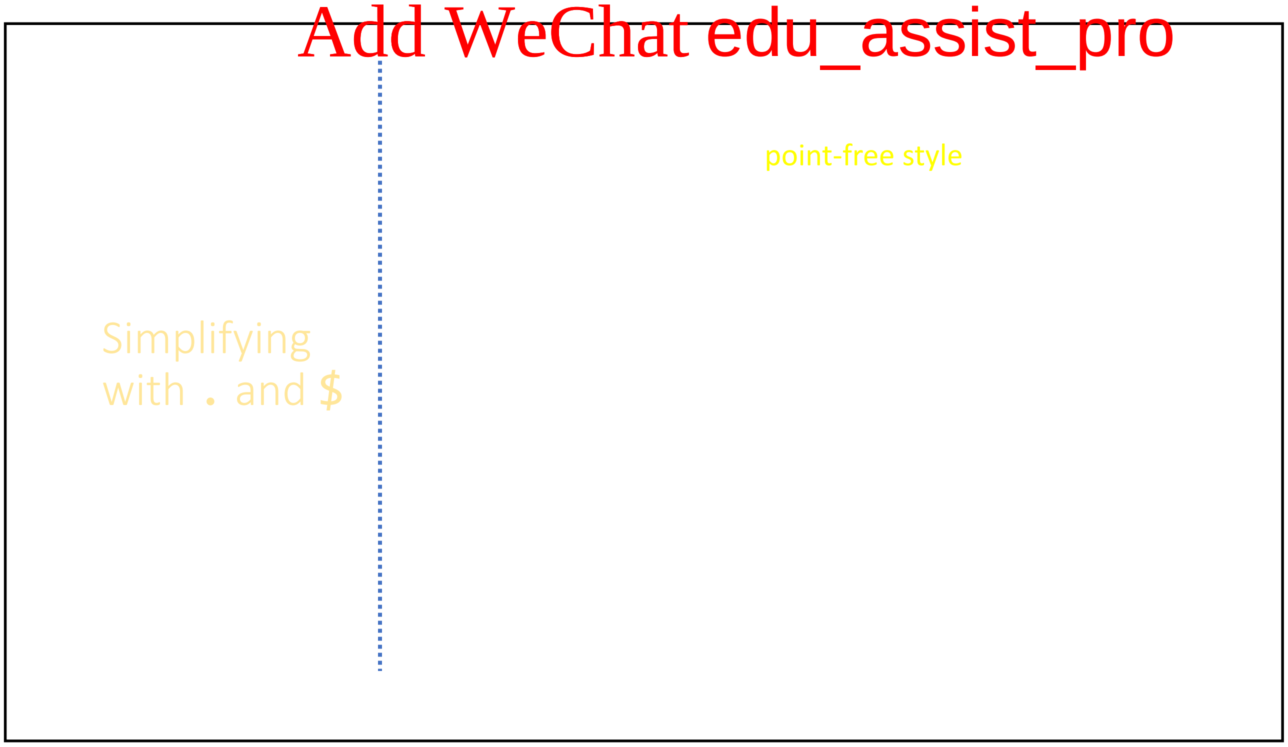

Simplifying with . and $

point-free style

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Point-free Form

Now we rewrite `sqrtSum`

- sum of square roots for the first `n` integers reaches a threshold of `1000` in point-free form:

```
sqrtSum :: Integer
sqrtSum = length . takeWhile (<1000) . scanl1 (+) $ map
(sqrt) [1..]
```

Alexis King demonstrates extreme consideration of reduction optimizations in realistic code:

- https://www.youtube.com/watch?v=yRVjR9XcuPU
- current research!

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— Break —

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— Chapter 6 —

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Modules

exported

reuse

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import

The Haskell standard library is separated by modules, e.g.:

- managing lists
- concurrent programming
- complex numbers
- and more…

The type classes, types, and functions we have used are part of the default imported Prelude module.

To import modules:

```
import Data.List
```

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Data.List Module Functions

nub

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Import Variations

To import in an interactive ghci session:

```
:m + Data.List
```

We can specify which functions we want to use:

```
import Data.List (nub, sort)
```

Or those we do not want, to avoid naming conflicts:

```
import Data.List hiding (nub)
```

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Naming Conflicts

```
qualified
```

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Function Composition vs Module Accessor

The previous examples show how to avoid

- the `Data.Map.filter` and `Data.Map.null` functions
- do not conflict with the default `Prelude.filter` nor the `Prelude.null` functions.

Note that the dot operator is used here to access the function from the module and is not function composition:

- ***make sure*** to not use any whitespace before nor after the dot
- use a space before and after a function composition dot ` . `

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words

Working with Text

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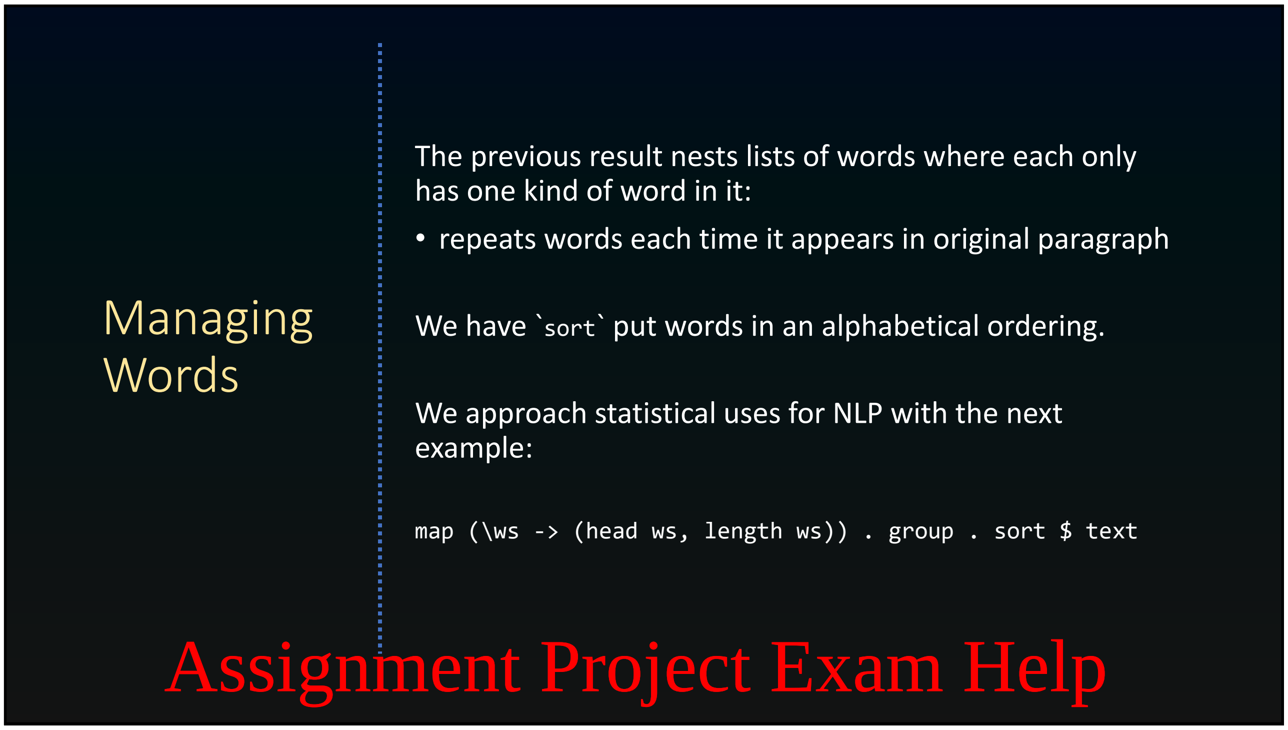

Managing Words

The previous result nests lists of words where each only has one kind of word in it:

- repeats words each time it appears in original paragraph

We have `` `sort` `` put words in an alphabetical ordering.

We approach statistical uses for NLP with the next example:

```
map (\ws -> (head ws, length ws)) . group . sort $ text
```

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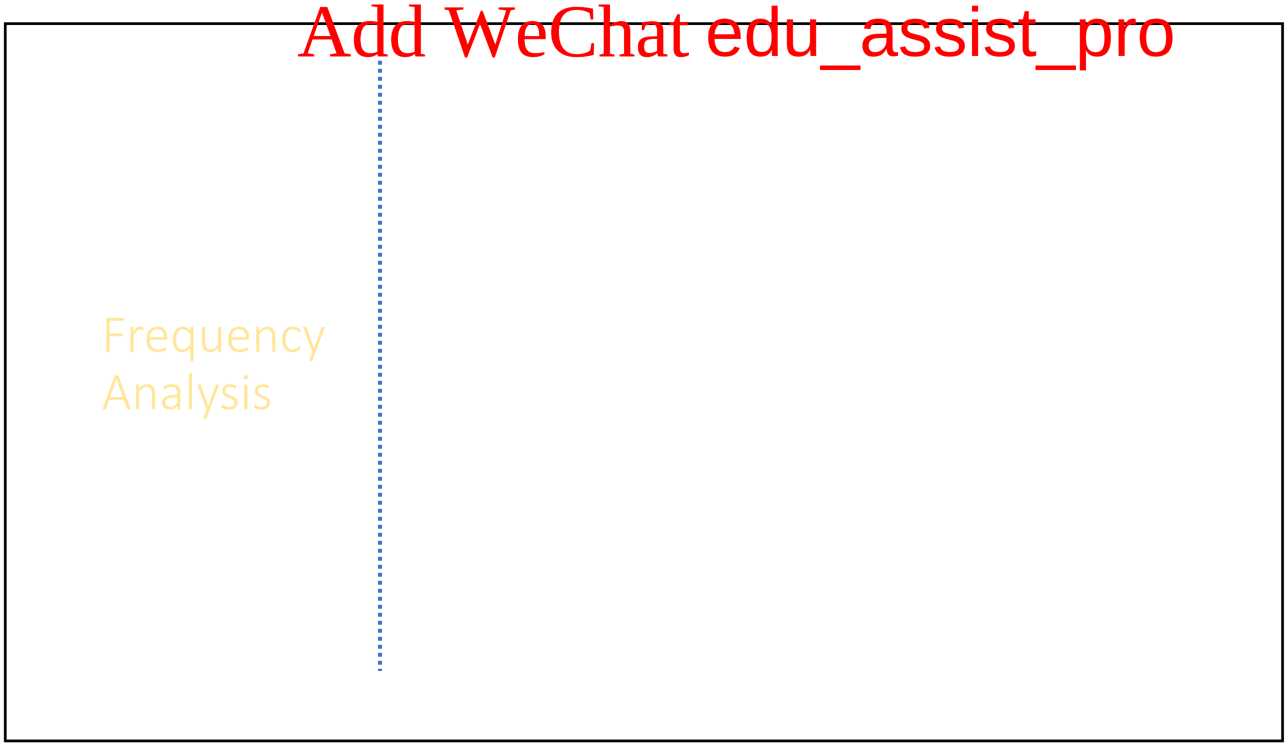

Frequency Analysis

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Try the following statements:

```
tails "party"
"ha" `isPrefixOf` "hawaii"
any (> 4) [1,2,3]
any (> 4) [1,2,3,4,5]
```

- `tails` folds a list of accumulated tail elements
- `isPrefixOf` tests for prefix of first argument at start of the second argument
- `any` will return `True` if at least one element of an input list satisfies the predicate function passed in

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The `Data.Char` module with the `ord` and `chr` functions converts back and forth between numbers/letters:

```
ord `a`
chr 97
```

A short function to do the Caesar Cipher for us:

```
import Data.Char

let
caesar :: Int -> String -> String
caesar offset msg =
    map (\c -> chr $ (ord c + offset) `mod` 26)) msg
```

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Careful with Left Folds

Possibility of stack overflows with using `foldl`:

```
foldl (+) 0 (replicate 100000000 1)
```

Order of operations plays out something like the following:

```
foldl (+) 0 [1,2,3] =
foldl (+) (0 + 1) [2,3] =
foldl (+) ((0 + 1) + 2) [3] =
foldl (+) (((0 + 1) + 2) + 3) [] =
((0 + 1) + 2) + 3 =
(1 + 2) + 3 =
3 + 3 =
6
```

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Avoiding Deferral

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No Stack Overflow

Then it would be safe to try:

```
foldl' (+) 0 (replicate 100000000 1)
```

There are similar versions for the other fold functions.

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find

Maybe a

Maybe a

Nothing

Just a

a

find

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Data.Char

We will use `find` to help us sum the digits of numbers.

- consider finding the smallest integer with digits that add up to `40`
- we also use `digitToInt` function from the `Data.Char` module:

```
digitToInt '2'
```

Convert hexadecimal digits to decimal:

```
import Data.Char
import Data.List

digitSum :: Int -> Int
digitSum = sum . map digitToInt . show
```

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Get What We Want Quickly

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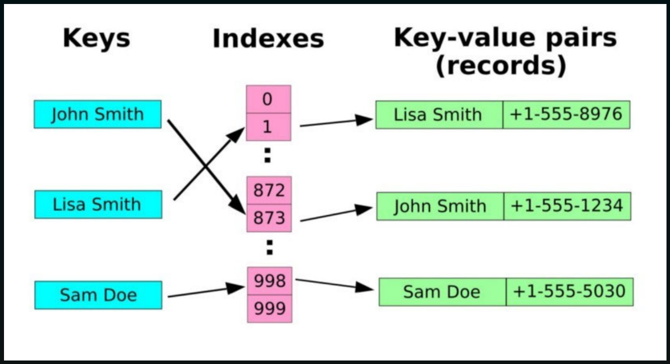
Map Data Structure

Data structure for a map is similar in Haskell to what they call an association list. (we are not implementing with hash table)



source: vinodronold (Medium, 2020)

https://medium.com/@vinodronold/hash-tables-implementation-in-peoplecode-231bca18442d

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Association List:

Word Keys and Phone Number Data

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Get Data from Our Association List

Function to extract one of the values matching input key:

```
:m Data.Maybe Data.List

let
findKey :: (Eq k) => k -> [(k, v)] -> v
findKey key xs =
    snd (
        fromJust $ find (\p -> (fst p) == key) xs
    )
```

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Alternative to Get Data

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Maybe

We use `Maybe` to deal with the case when there is no matching key (...but not quite like exceptions).

```
findKey :: (Eq k) => k -> [(k, v)] -> Maybe v
findKey key [] = Nothing
findKey key ((k,v):xs)
 | key == k  = Just v
 | True      = findKey key xs

findKey :: (Eq k) => k -> [(k, v)] -> Maybe v
findKey key xs = foldr (\(k, v) acc ->
        if k == key
            then Just v
            else acc
    ) Nothing xs
```

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Context of List vs Elements

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Data.Map

The `Data.Map` module has association lists:

- are efficient with many functions for managing them.
- but, there are many naming clashes with `Prelude` and `Data.List`
- so import with qualified:

```
import qualified Data.Map as Map
```

The `fromList` function turns an association list into a Map:

```
Map.fromList [(3,"shoes"),(4,"trees"),(9,"bees")]
Map.fromList [("MS",1),("MS",2),("MS",3)]
```

See how extra duplicate-key pairs are discarded.

- the result is displayed as a list, but with prefix `fromList`

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Maps vs Association Lists

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Example Map

Setup the phone book we created earlier as a map:

```
import qualified Data.Map as Map

let
phoneBook :: Map.Map String String
phoneBook = Map.fromList
 [("betty", "555-2938")
 ,("bonnie", "452-2928")
 ,("patsy", "493-2928")
 ,("lucille", "205-2928")
 ,("wendy", "939-8282")
 ,("penny", "853-2492")
 ]
```

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Create New Maps

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Map.size

A basic function to get the number of pairs in a map:

- `` `:t Map.size` `` has type `` `Map.size :: Map.Map k a -> Int` ``

Go ahead and give the function a try.

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Convert Map Types

digitToInt Data.Char

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Organizing Data

There could be duplicate phone numbers in our collection for a person with the same name.

- instead, we can accumulate values that correspond to the same key
- we can accumulate the values in a way we specify as parameter of the `fromListWith` function (which we see soon)

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Repeated Keys with More Data

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Collect by Key

```
findKey "patsy" phoneBook
findKey "wendy" phoneBook
findKey "betty" phoneBook
```

We could arrange multiple phone numbers per name:

```
phoneBookToMap :: (Ord k) => [(k, a)] -> Map.Map k [a]
phoneBookToMap xs =
    Map.fromListWith (++) $ map (\(k, v) -> (k, [v])) xs

Map.lookup "patsy" $ phoneBookToMap phoneBook
Map.lookup "wendy" $ phoneBookToMap phoneBook
Map.lookup "betty" $ phoneBookToMap phoneBook
```

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Querying Maps

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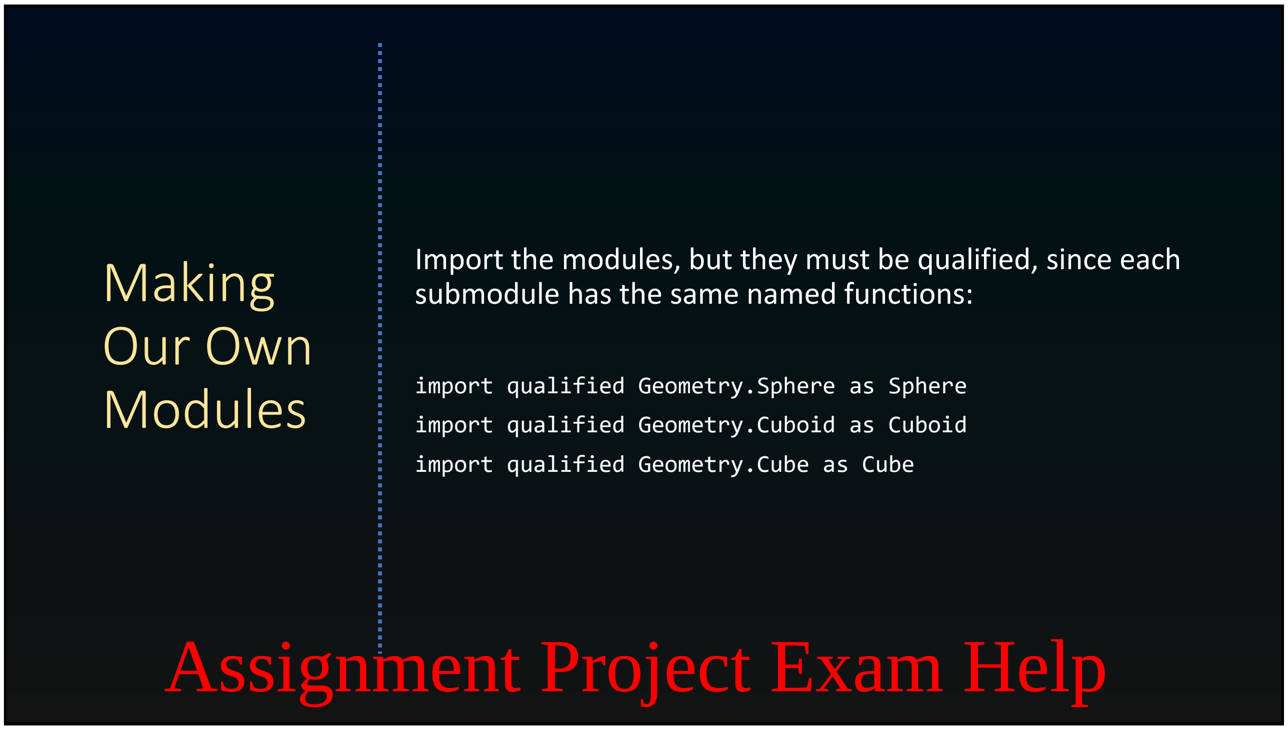

Making Our Own Modules

Import the modules, but they must be qualified, since each submodule has the same named functions:

```
import qualified Geometry.Sphere as Sphere
import qualified Geometry.Cuboid as Cuboid
import qualified Geometry.Cube as Cube
```

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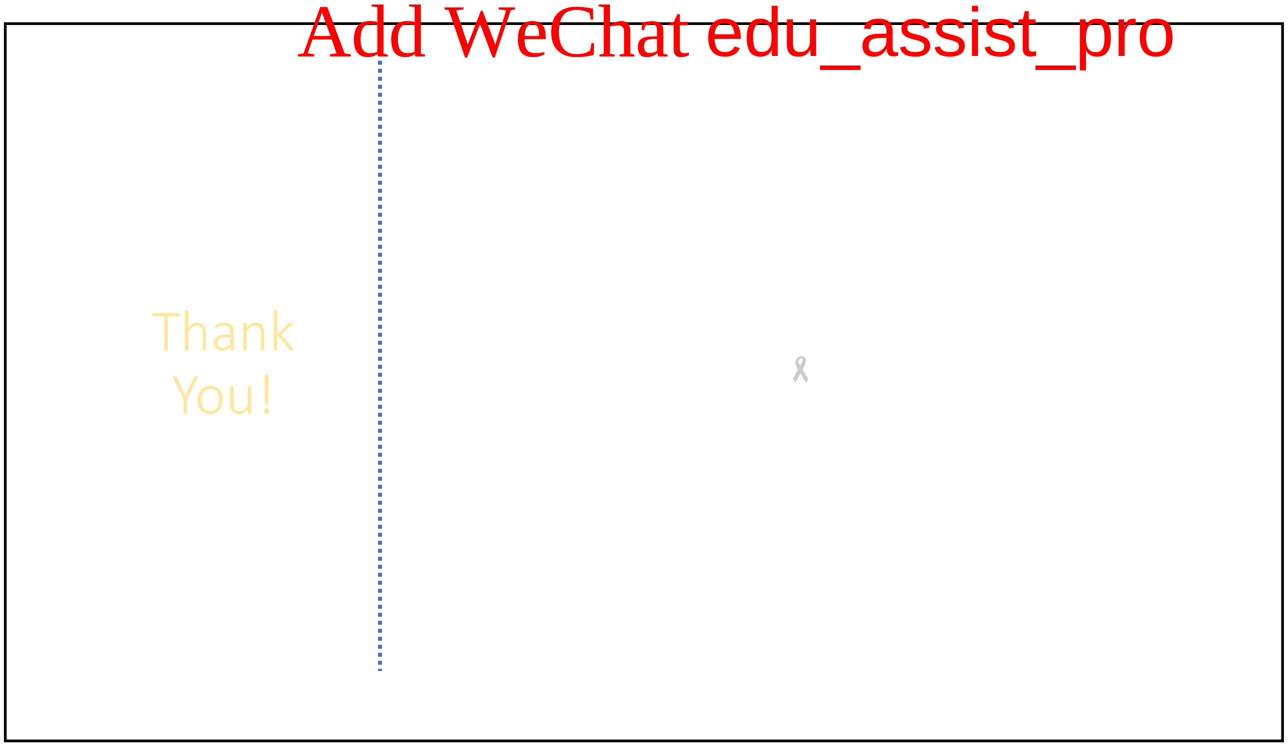

Thank You!

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