Can you use a hashtable to implement skipTo()?

Better th xt()

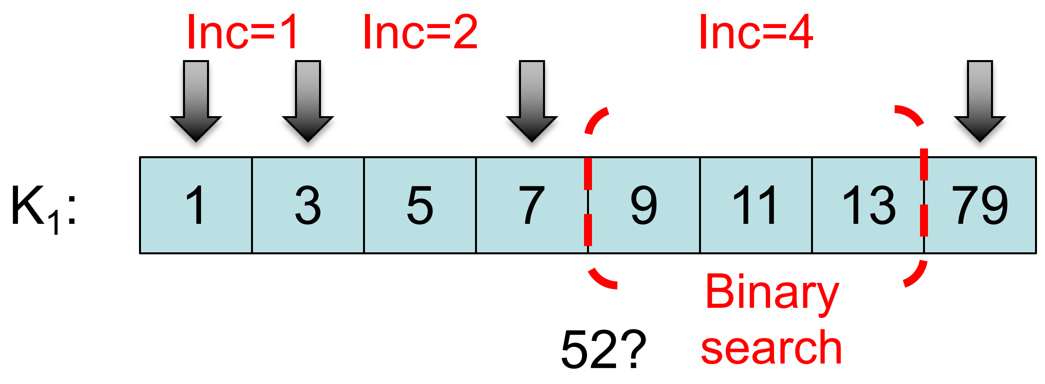
- What's the worst case for sequential merge-based intersection?
- {52, 1} ➔ move K_2's cursor
  - To the positi **skipTo**(52)
  - Essentially, $_2$[i] >= 52 (K2's list is sorted).
  - Takes many sequential call of
  - Could use binary search in the rest of the list
  - Cost: $\lceil \log_2(N_{remainder}) \rceil$

K_2:	1	3	5	…	…	…	…	79
K_1:	52	54	56	58				

skip

- Galloping search (gambler's strategy)
  - [Stage 1] Doubling the search range until you overshoot
  - [Stage 2] Perform binary search in the last range

- Performance analysis (worst case)
  - Let the desti s away.
  - ≈ $\log_2 n$ probes in Stage 1 + ≈ s in Stage 2
  - Total = $2\lceil \log_2 (n+1) \rceil = O(\log_2$



Tota

- Galloping search (gambler's strategy)
  - Cost of the i-th probe: ≈ $2 \log_2(n_i)$
  - Total cost of K_1 probes: ≈ $2 \log_2(\prod_i^{|K_1|} n_i)$

  ≤ $2 \log_2($ (($\sum$ log$_2$($|K_2|/|K_1|$)
- Asymptotic r merge when $|K_2|/|K_1| = O(1)$, resemb search when $|K_1| = O(1)$

What about list intersection using binary search?

Multiple Ter junctive Queries

- K_1 AND K_2 AND … AND K_n
- SvS does not perform well if none of the associate
- In addition, it is bloc
- Can you design non-blocking multiple sorted array intersection algorithm?

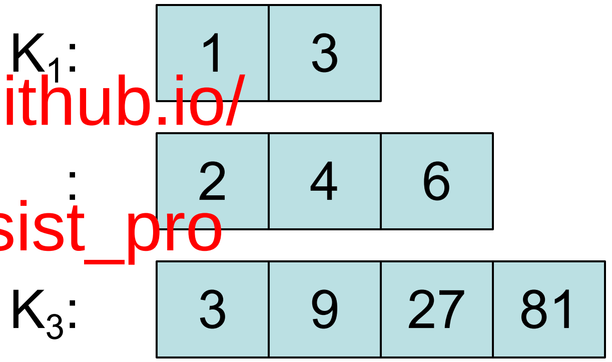
Gener n

- Generalize the 2-way intersection algorithm

- 2-way:

  - {1, 2} ➔ move k_1's c

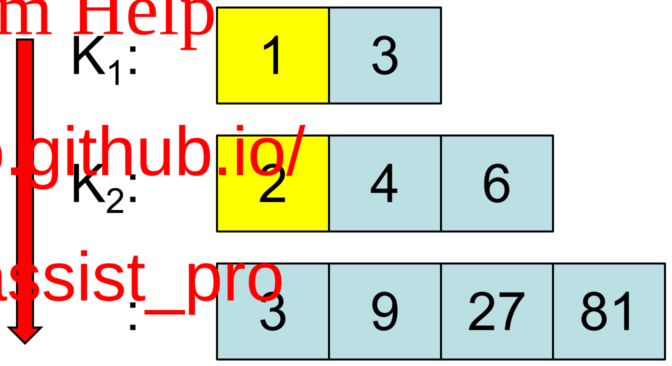
  - skipTo(2)
- 3-way:
  - {1, 2, 3} ➔ move k_1,k_2's cursor
  - skipTo(3)



eliminator = $\text{Max}_{1<=i<=n}(k_i.\text{cursor})$

Optim

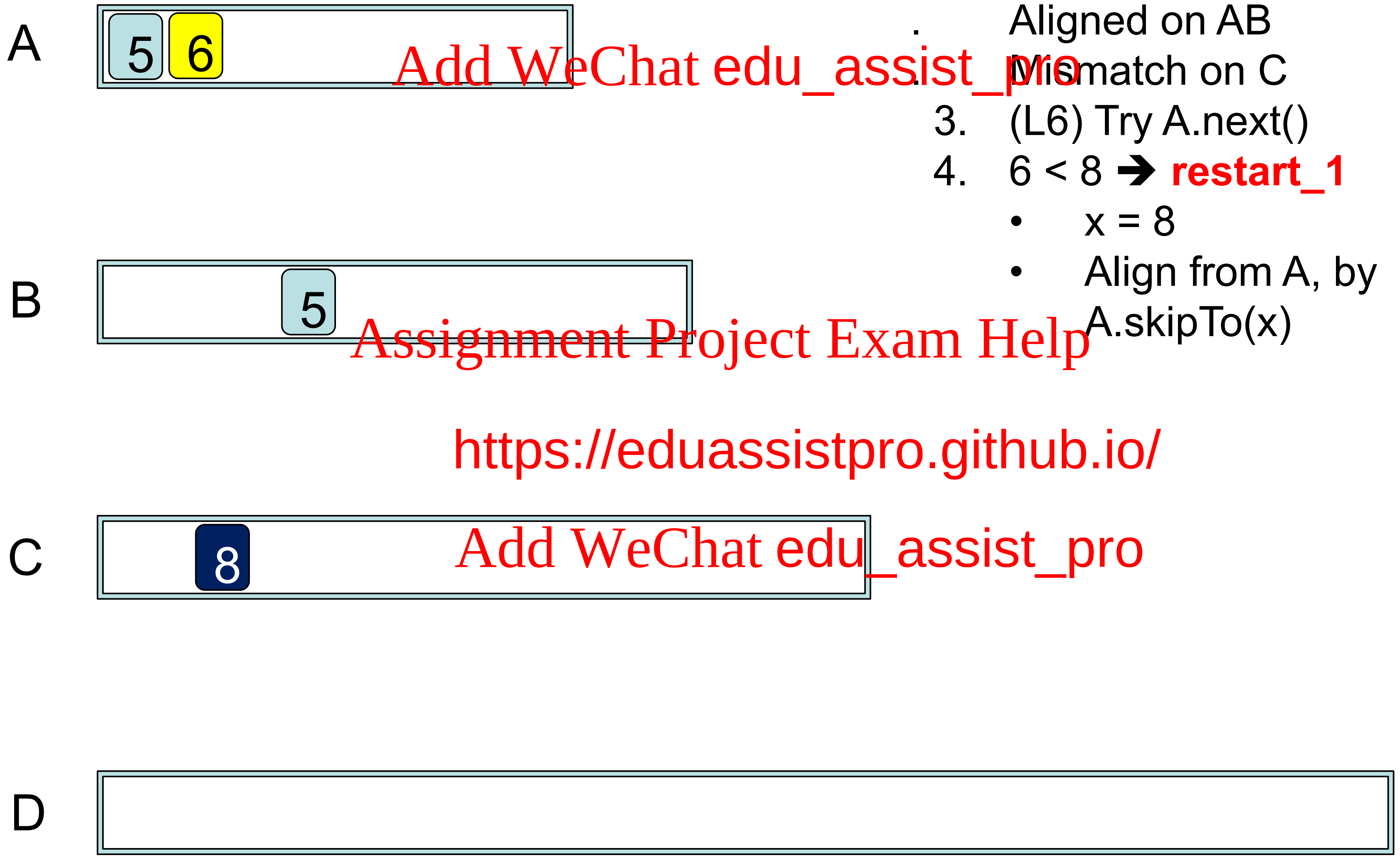
- Mismatch found even before accessing K_3's cursor
- Choice 1: c cursors of other list
- Choice 2: settle the dispute within the first two lists ➔ max algorithm [Culpepper & Moffat, 2010]
  - Better locality of access ➔ fewer cache misses
  - Similar to SvS



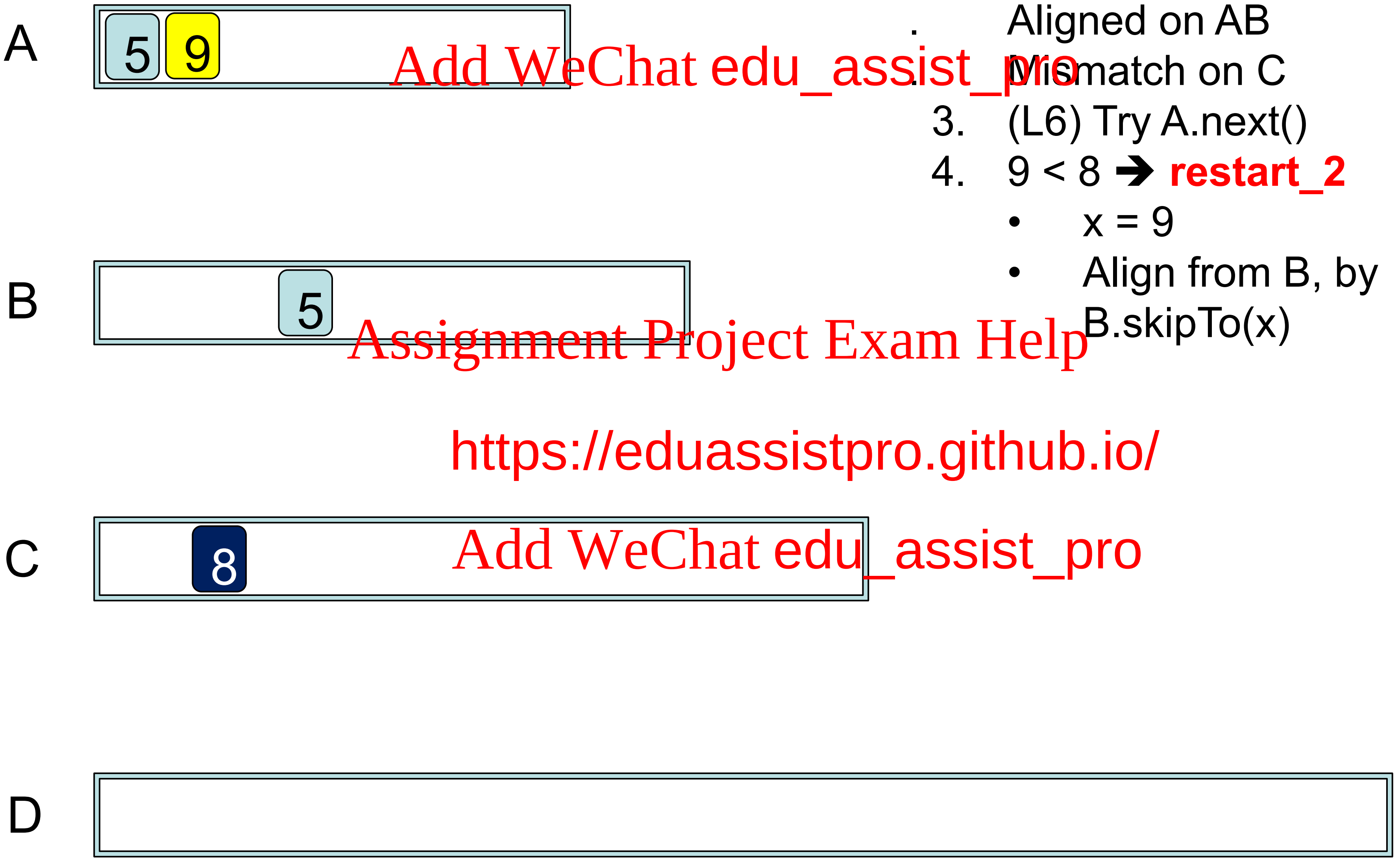
Pseudo-Code for the Max Algorithm (Wr

- Input: $K_1, K_2, \ldots, K_n$ in increasing size

```
(1)  x := K_1[1]; startAt := 2            //x is the eliminator
(2)  while x is defined do
(3)      for i = startAt to n do
(4)          y := K_i.skipTo(x)
(5)          if y > x th[illegible]
(6)              x := K_1                  //res           //restart_2
(7)              if y > x then startAt := [illegible] startAt := 2 end if
(8)              break                     //match in all  lists
(9)          elsif i = n then              //y = x
(10)             Output x
(11)             x := K_1.next()
(12)         end if
(13)     end for
(14) end while
```



1. Aligned on AB
2. Mismatch on C
3. (L6) Try A.next()
4. 6 < 8 ➔ **restart_1**
   - x = 8
   - Align from A, by A.skipTo(x)



1. Aligned on AB
2. Mismatch on C
3. (L6) Try A.next()
4. 9 < 8 ➔ **restart_2**
   - x = 9
   - Align from B, by B.skipTo(x)

The original code has a bug when in restart_1 cases

Pseudo-Code for the gorithm (Fixed)

- Input: $K_1, K_2, \ldots, K_n$ in increasing size

```
(1)  x := K_1[1]; startAt := 2
(2)  while x is defined do
(3)      for i = startAt to n do
(4)          y := K_i.skipTo(x)
(5)          if y > x th
(6)              x := K_1
(7)              if y > x then startAt :=          startAt := 2 end if
(8)              break
(9)          elsif i = n then
(10)             Output x
(11)             x := K_1.next()
(12)         end if
(13)     end for
(14) end while
```

```
(4.1) if i = 1 then
(4.2)     if y > x then
(4.3)         x := y
(4.4)         break
              if
(4
```

Refer

- J. Shane Culpepper, Alistair Moffat: Efficient set intersection for inverted indexing. ACM Trans. Inf.
- F.K. Hwan e algorithm for merging two disjoint lin red sets. SIAM J. Comput. 1 1 (1972), pp. 31–39.
- Stefan Buettcher, Charles L. A. Clarke, Gordon V. Cormack, Information Retrieval: Implementing and Evaluating Search Engines, 2010 [Chapter 5]