COMP90038

Algorit plexity

Lecture 16: Time/Space Tradeo arch Revisited

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Recap

- BST have optimal performance when they are balanced.

- AVL Trees:
  - Self-balancing trees for , 0, or 1, for every sub-tree.
  - Rebalancing is achieved
  - It guarantees depth of a tree with n nodes)

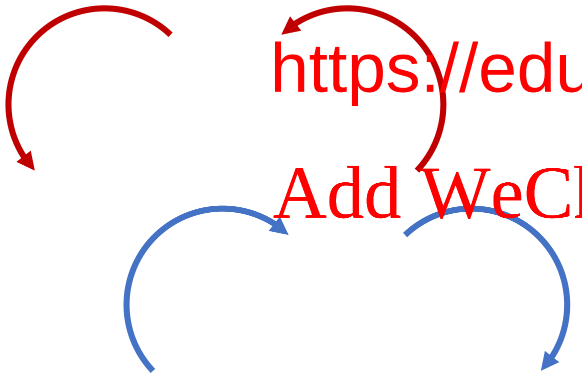
- 2–3 trees:
  - Trees that allow more than one item to be stored in a tree node.
  - This allows for a simple way of keeping search trees perfectly balanced.
  - Insertions, splits and promotions are used to grow and balance the tree.

AVL Trees: R-Rotation

AVL Trees: L-Rotation

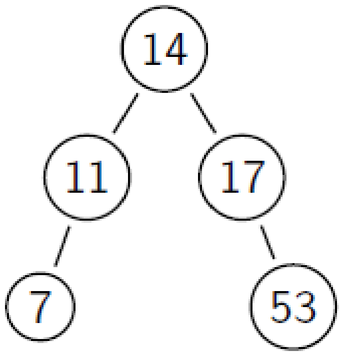
AVL Trees: LR-Rotation

AVL Trees: RL-Rotation



Example

- On the tree below, insert the elements {4, 13, 12}

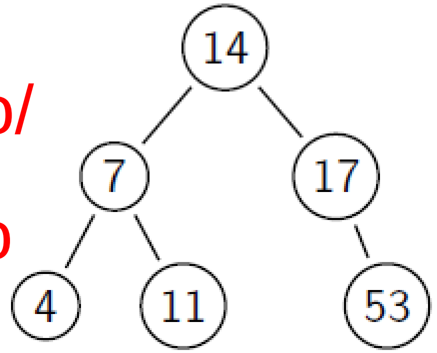


3

2 0 0 1 -1

1 0 0 0 1 0

0 0 0



- https://www.cs.usfca.edu/~galles/visualization/AVLtree.html

Example: Build a 2–3 Tree from {9, 5, 8, 3, 2, 4, 7}

2–3 Tree Analysis

- Worst case search time results when all nodes are 2-nodes. The relation between the number n of nodes and the height h is:

$$n = 1 + 2 + 4 + \ldots + 2^h = 2^{h+1} - 1$$

- That is, $\log_2(n+1) = h+1$.

- In the best case, all nodes are 3-nodes:

$$n = 2 + 2\times3 + 2\times3^2 + \ldots {}^{+1} - 1$$

- That is, $\log_3(n+1) = h+1$.

- Hence we have $\log_3(n+1) - 1 \le h \le \log_2(n+1) - 1$.

- Useful formula: $\displaystyle\sum_{i=0}^{n} a^i = \frac{a^{n+1}-1}{a-1}$ for $a \neq 1$

Spending Space to Save Time

- Often we can find ways of decreasing the time required to solve a problem, by using additional memory in a clever way.

- For example, in **Lecture 6 (Recursion)** we considered the simple recursive way of finding the *n*-th Fibonacci number a ithm uses exponential time.

function $\textsc{Fib}(n)$
    if $n = 0$ **then**
        return 1
    if $n = 1$ **then**
        return 1
    return $\textsc{Fib}(n-1) + \textsc{Fib}(n-2)$

Spending Space to Save Time

Spending Space to Save Time

- However, suppose the same algorithm uses a table to **tabulate** the function FIB() as we g ediate result FIB(i) has been found, it is not si ; the value is first placed in slot i of a table (an array). o FIB() first looks in this table to see if the required value is t only if it is not, the usual recursive process kicks in.

Fibonacci Numbers with Tabulation

- We assume that, from the outset, all entries of the table F are 0.

```
function Fib(n)
    if n = 0 or n = 1 then
        return 1
    result ← F[n]
    if result = 0 then
        result ← Fib(n − 1) + Fib(n − 2)
        F[n] ← result
    return result
```


- (I show this code just so that you can see the principle; in **Lecture 6** we already discovered a different linear-time algorithm, so here we don't really need tabulation.)

Sorting by Counting

- Suppose we need to sort large arrays, but we know that they will hold keys taken from a **small**, **fixed** set (so lots of duplicate keys).

- For example, suppose all keys are single digits:

6 3 3 8 1 0 8 7 9 2 5 3 5 3 1 8 7 6 5 1 2 1 5 3

- Then we can, in a single linear scan, count the occurrenc n array *A* and store the result in a small table:

- Now use a second linear scan to make the counts **cumulative**:

key	0	1	2	3	4	5	6	7	8	9
Occ	1	5	7	12	12	16	18	20	23	24

-

Sorting by Counting

- We can now create a sorted array $S[1] \ldots S[n]$ of the items by simply slotting items into pre-determined slots in S (a third linear scan).

- Place the last record (with key 3) in $S[12]$ a[illegible]nt $Occ[3]$ (so that the next `3' will go into slot 11), and so on.

$$
\begin{aligned}
&\textbf{for } i \leftarrow n \textbf{ to } 1 \textbf{ do}\\
&\quad S[Occ[A[i]]] \leftarrow A[i]\\
&\quad Occ[A[i]] \leftarrow Occ[A[i]] - 1
\end{aligned}
$$

-

Sorting by Counting

- Note that this gives us a **linear-time** sorting algorithm (for the cost of some extra space).

- However, it only works i ave a small range of keys, known in advance.

- The method never performs a key-to-key comparison.

- The time complexity of **key-comparison based sorting** has been proven to be in $\Omega(n \log n)$.

String Matching Revisited

- In **Lecture 5 (Brute Force Methods)** we studied an approach to string search.

$$
\begin{array}{l}
\textbf{for } i \leftarrow 0 \text{ to } n - m \textbf{ do} \\
\quad j \leftarrow 0 \\
\quad \textbf{while } j < m \text{ and } p[j] = t[i + j] \textbf{ do} \\
\quad\quad j \leftarrow j + 1 \\
\quad \textbf{if } j = m \textbf{ then} \\
\quad\quad \textbf{return } i \\
\textbf{return } -1
\end{array}
$$

String Matching Revisited

String Matching Revisited

- "Strings" are usually built from a small, pre-determined alphabet.

- Most of the better alg re-processing of strings before the actual mat

- The pre-processing involves the construction of a small table (of predictable size).

- Levitin refers to this as "input enhancement".

Horspool's String Search Algorithm

- Comparing from right to left in the pattern.
- Very good for random text strings.

S T R I N G S E A R C H E X A M P
E X A M

- We can do better than just observing a misma
- Because the pattern has **no occurrence of I**, we might as well slide it 4 positions along.
- This decision is based only on knowing the pattern.

Horspool's String Search Algorithm

- Here we can slide the cause the last occurrence of E in the pattern is its first position

S	T	R	I	N	G	S	E	A	R	C	H	E	X	A	M	P
E	X	A	M													
				E	X	A	M									
							E	X	A	M						
											E	X	A	M		
												E	X	A	M	

Horspool's String Search Algorithm

- What happens when we have longer partial matches?

```
S E A R C H I N G
B I R C H
  B I R C H
          B I R C H
```


- The shift is determined by the last character in the pattern.

- Note that this is the same as the character in the text that we first matched against. Hence the skip is **always** determined by that character, whether it matched or not.

Horspool's String Search Algorithm

- Building (calculating) the shift table is easy.
- We assume indices start from 0.

- Let *alphasize* be the size

$$
\begin{array}{ll}
\textbf{function } \text{FINDSHIFTS}(P[\cdot], m) & \triangleright \text{ Pattern } P \text{ has length } m \\
\quad \textbf{for } i \leftarrow 0 \text{ to } alphasize - 1 \textbf{ do} & \\
\quad\quad Shift[i] \leftarrow m & \\
\quad \textbf{for } j \leftarrow 0 \text{ to } m - 2 \textbf{ do} & \\
\quad\quad Shift[P[j]] \leftarrow m - (j + 1) &
\end{array}
$$

Horspool's String Search Algorithm

Horspool's String Search Algorithm

- We can also consider posting a sentinel: Append the pattern *P* to the end of the text *T* so that a match is guaranteed.

Horspool's String Search Algorithm

- Unfortunately the wor orspool's algorithm is still $O(m \times n)$, like the

- However, in practice, for example, when used on English texts, it is linear-time, and fast.

Other Important String Search Algorithms

- Horspool's algorithm was inspired by the famous **Boyer-Moore** algorithm (**BM**), also covered in Levitin's book. The BM algorithm is very similar, but it has a more sophisticated shifting strategy, which makes it $O(m+n)$.

- Another famous string sea -Morris-Pratt algorithm (**KMP**), explained in the remainder of thes P is very good when the alphabet is small, say, we need to search long bit strings.

- Also, we shall soon meet the **Rabin-Karp** algorithm (**RK**), albeit briefly.

- While very interesting, **the BM, KMP, and RK algorithms are not examinable.**

Knuth-Morris-Pratt (Not Examinable)

- Suppose we are searching in strings that are built from a small alphabet, such as the binary digits 0 and 1, or the nucleobases.
- Consider the brute-force approach for this example.

- Every “false start” contains a lot of information.
- Again, we hope to **pre-process** the pattern so as to find out when the brute-force method's index *i* can be incremented by more than 1.
- Unlike Horspool's method, KMP works by comparing from left to right in the pattern.

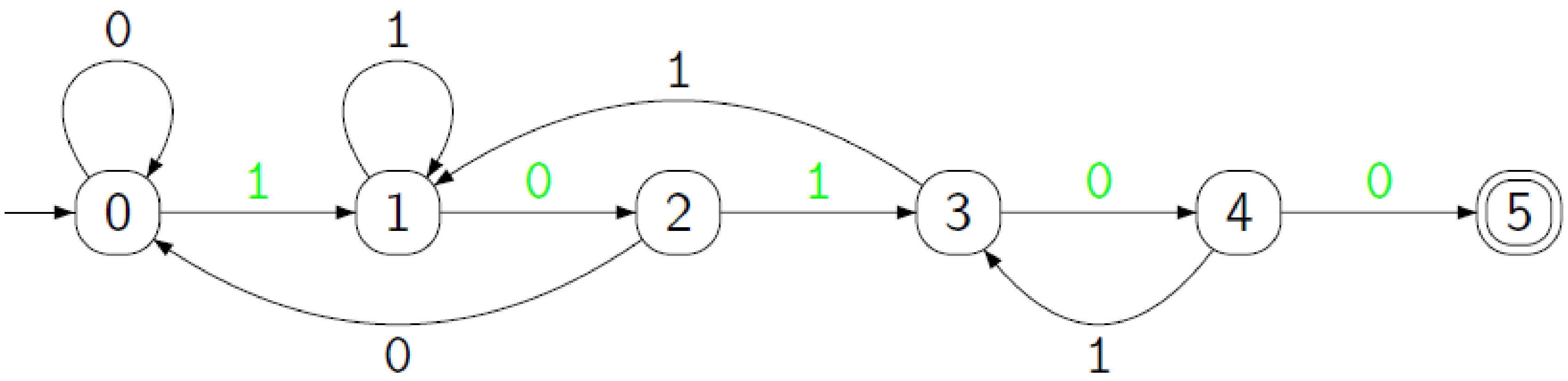
Knuth-Morris-Pratt as Running an FSA

- Given the pattern [1 0 1 0 0] we want to construct the following **finite-state automaton**:

- We can capture the behaviour of this automaton in a table.

Knuth-Morris-Pratt Automaton

- We can represent the finite-state automaton as a 2-dimensional “transition” array *T*, where *T*[c][j] is the state to go to up the character *c* in stat



Constructing the Automaton

- The automaton (or the table *T*) can be constructed step-by-step:

- Somewhat tricky but fas

- *x* is a "backtrack point".
- For next state *j*:

  - First *x*'s transitions are copied (in red).
  - Then the success arc is updated, determined by *P*[*j*] (in green).

- Finally *x* is updated based on *P*[*j*].

Constructing the Automaton

Constructing the Automaton

Constructing the Automaton

$T['0'][0] \leftarrow 0$
$T['1'][0] \leftarrow 0$
$T[P[0]][0] \leftarrow 1$
$x \leftarrow 0$
$j \leftarrow 1$
while $j < m$ **do**
    $T['0'][j] \leftarrow T['0'][x]$
    $T['1'][j] \leftarrow T['1'][x]$
    $T[P[j]][j] \leftarrow j + 1$
    $x \leftarrow T[P[j]][x]$
    $j \leftarrow j + 1$

Pattern Compilation: Hard-Wiring the Pattern

- Even better, we can directly produce code that is specialised to find the given pattern. As a C program, for the example `p = 1 0 1 0 0`:

```
int kmp(char *s) {
        int i = -1;
        s0: i++; if (
        s1: i++; if (s[i] == '1')
        s2: i++; if (s[i] == '0')
        s3: i++; if (s[i] == '1')
        s4: i++; if (s[i] == '1') goto s3;
        s5: return i-4;
}
```


- Again, this assumes that we have posted a sentinel, that is, appended `p` to the end of `s` before running `kmp(s)`.

Next week

- We look at the hugely f **hashing**, a standard way of implementing
- Hashing is arguably the best example of how to gain speed by using additional space to great effect.