COMP90038 Algorithms and Complexity

Lecture 2: Concepts

(with thanks to Hara ergaard)

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@tobycmurray

Approaching a problem

- Can we cover this board with 31 tiles of the following form?

- This is the **mutilated checkerboard problem**.

- There are only finitely many ways we can arrange the 31 tiles, so there is a brute-force (and very inefficient) way of solving the problem.

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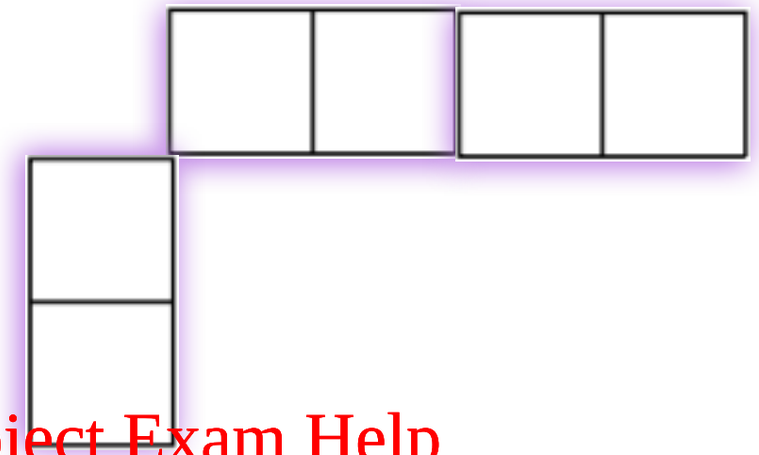

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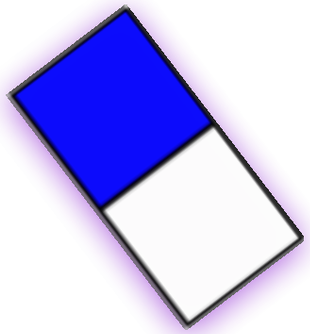

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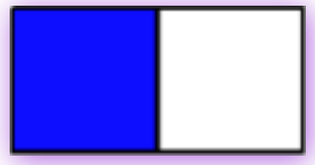

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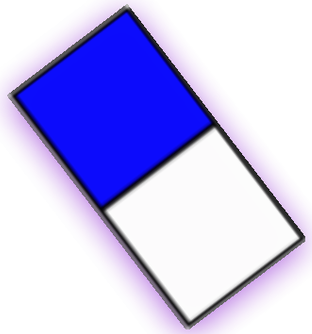

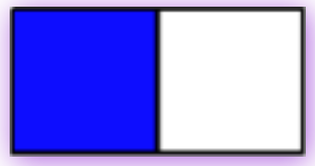

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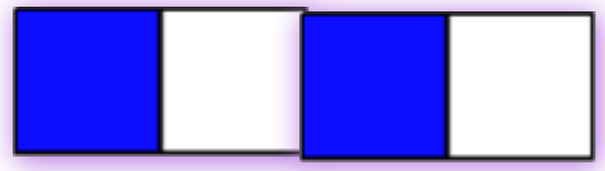

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Algorithms and Data Structures

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- **Algorithms**: for solving problems, transforming data.
- **Data structures**: for storing data, arranging data in a way that suits an algorithm.
  - **Linear** data structures: stacks and q
  - Trees and graphs
  - Dictionaries
- Which data structures are you familiar with?

Exercise

- Pick you favourite data structure and describe
  - How to insert and item
  - How to find an item
  - How to handle duplicate items

Primitive Data Structures: The Array

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- An array corresponds to a sequence of consecutive cells in memory.
- Depending on programming language: `A[0]` up to `A[n-1]`, or `A[1]` up to `A[n]`.
- Locating a cell, and storing or retrieving data at that cell is very fast.
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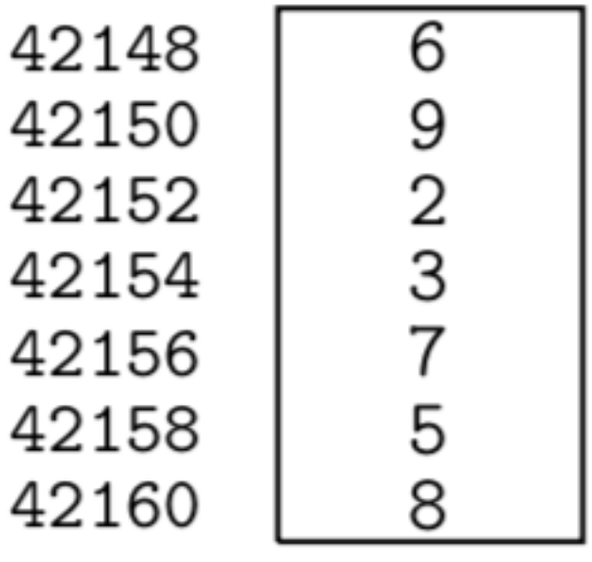

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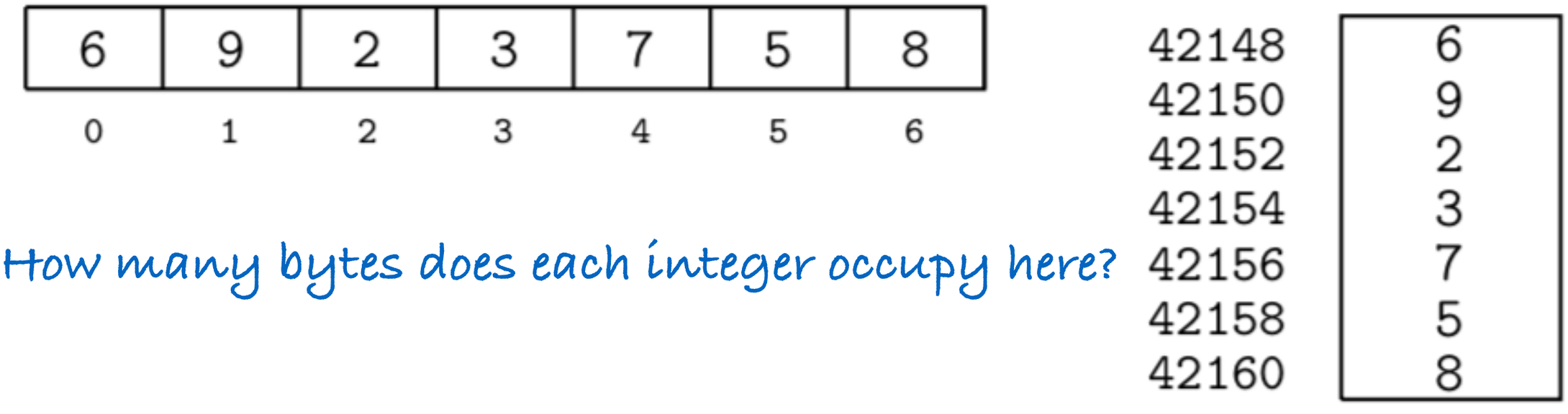

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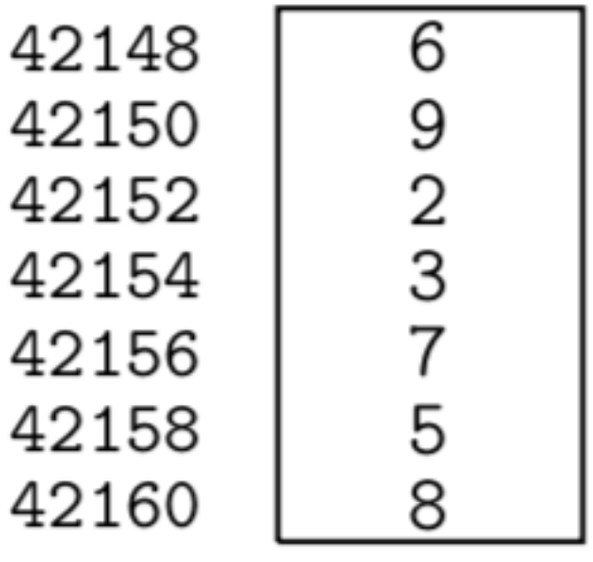
How many bytes does each integer occupy here?

Answer: 2 (16-bit integers)



Primitive Data Structures: The Linked List

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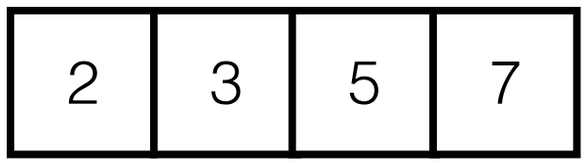
An array **x**:



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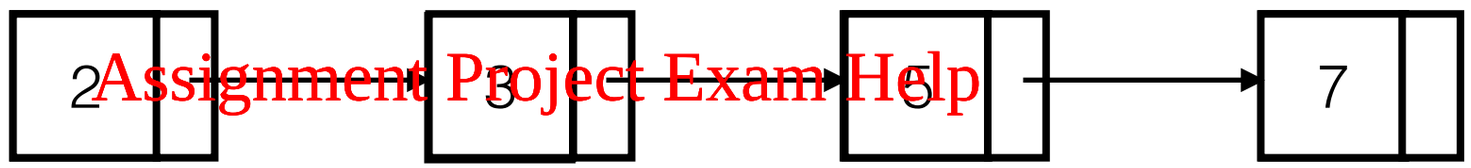

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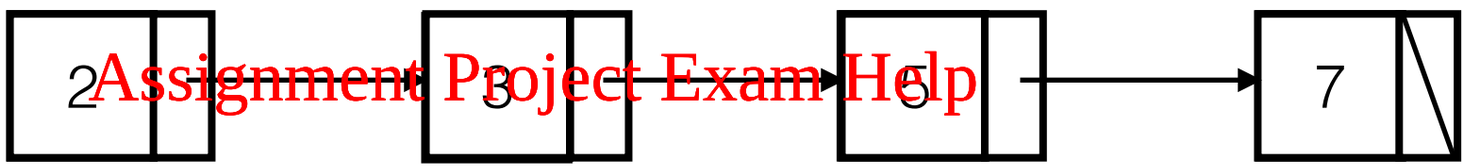

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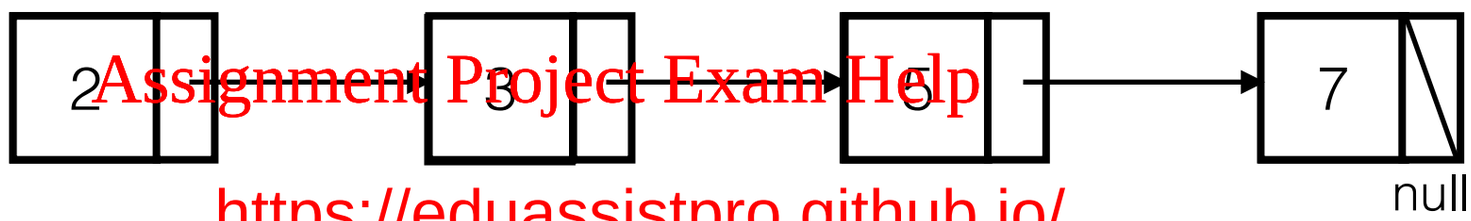

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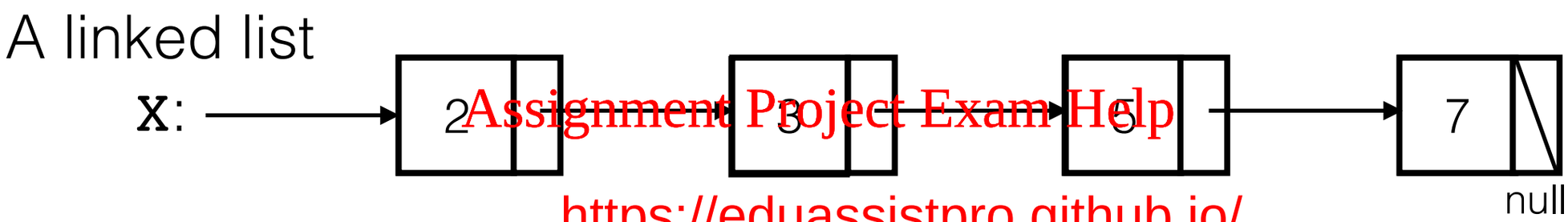

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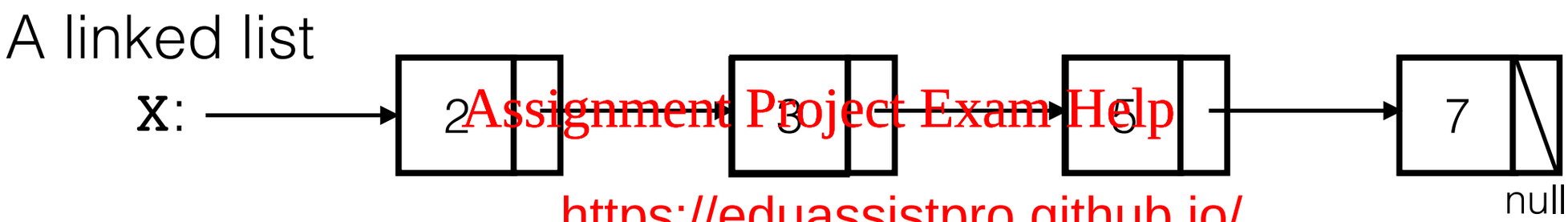

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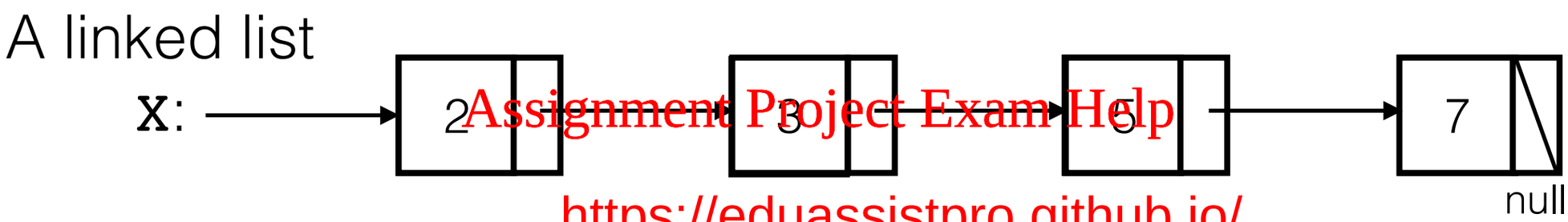

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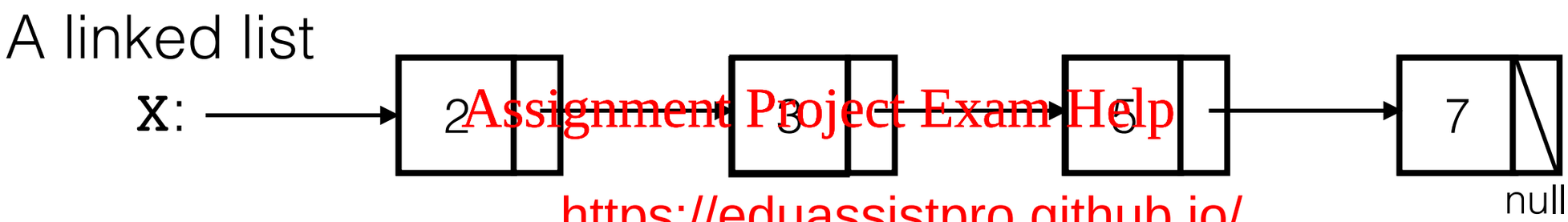

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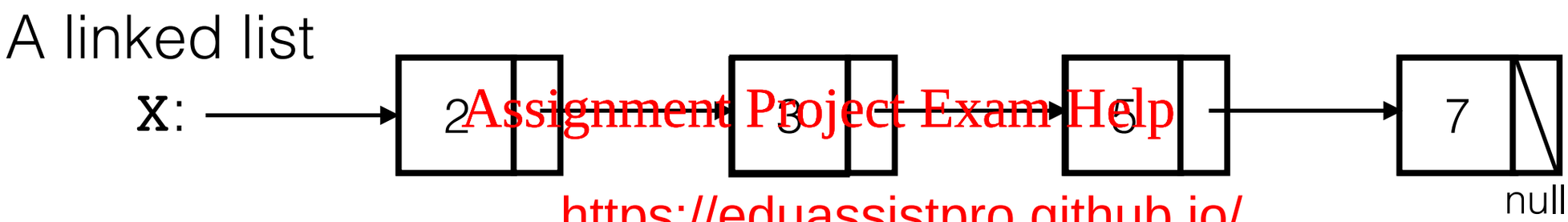

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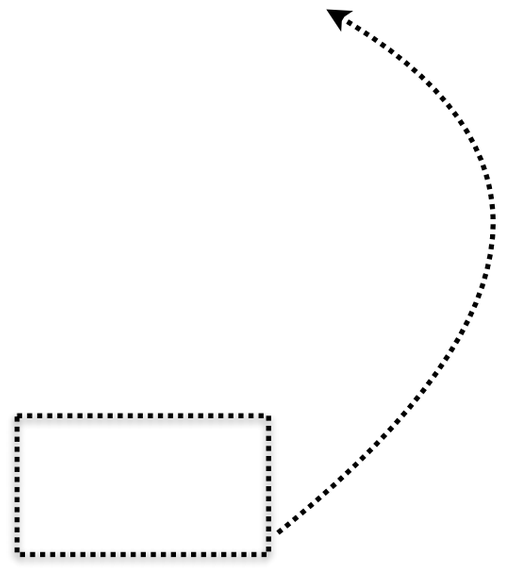

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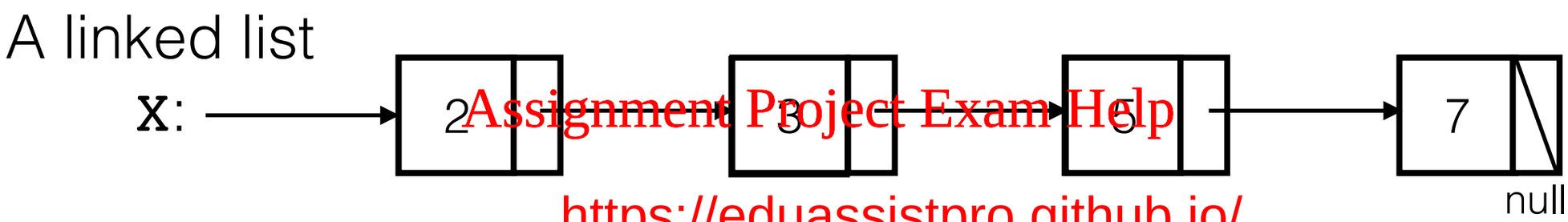

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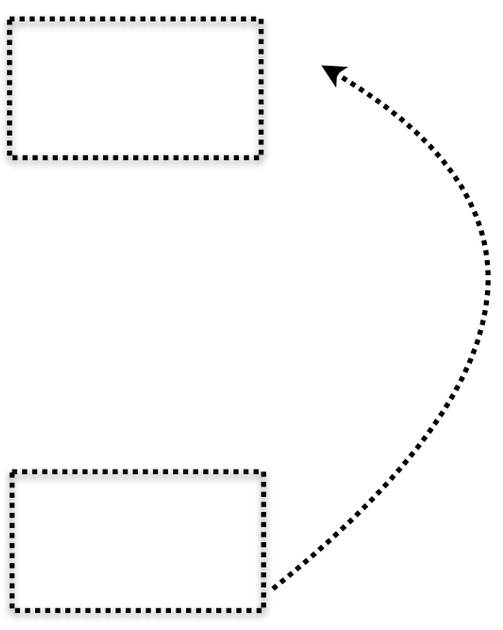

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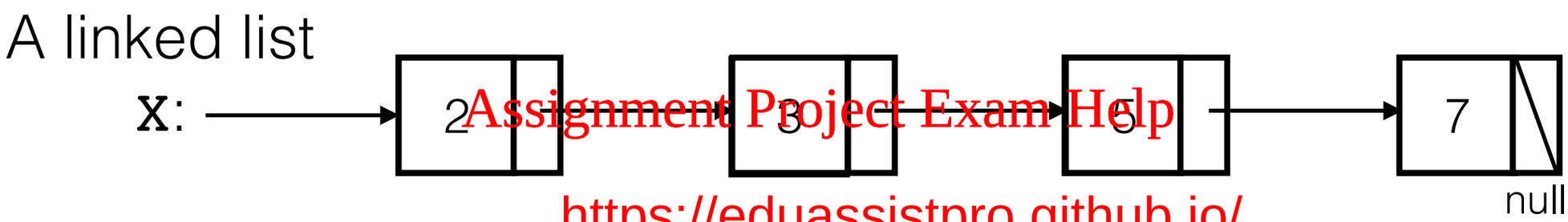

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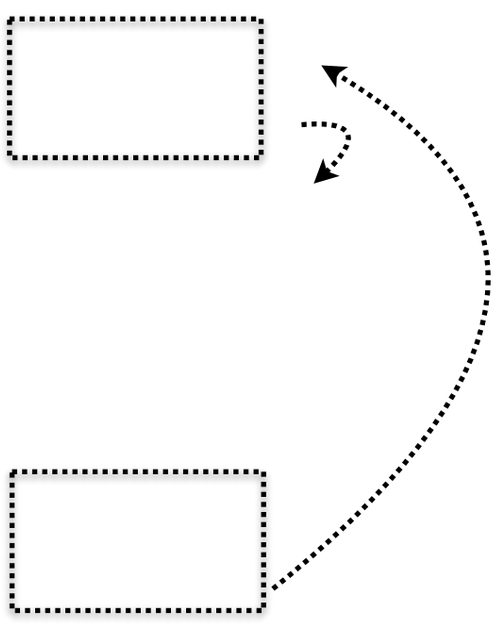

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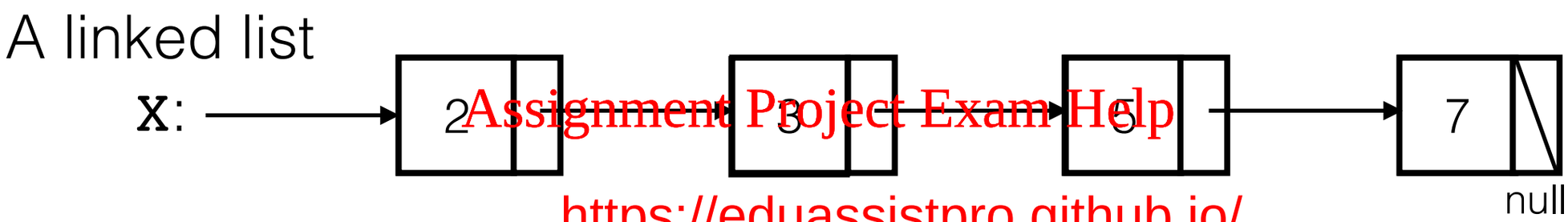

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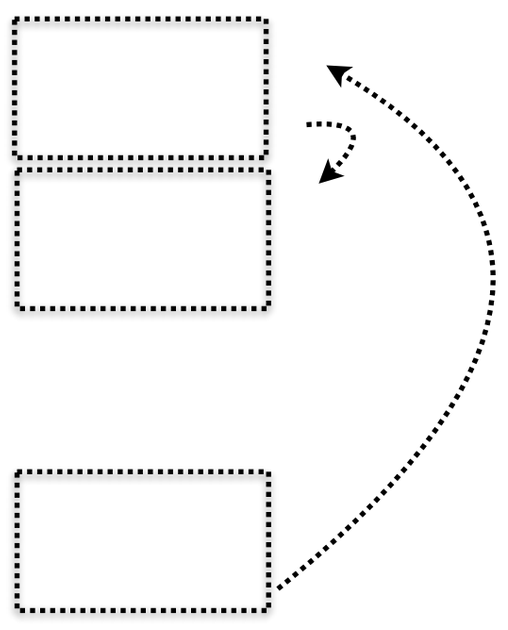

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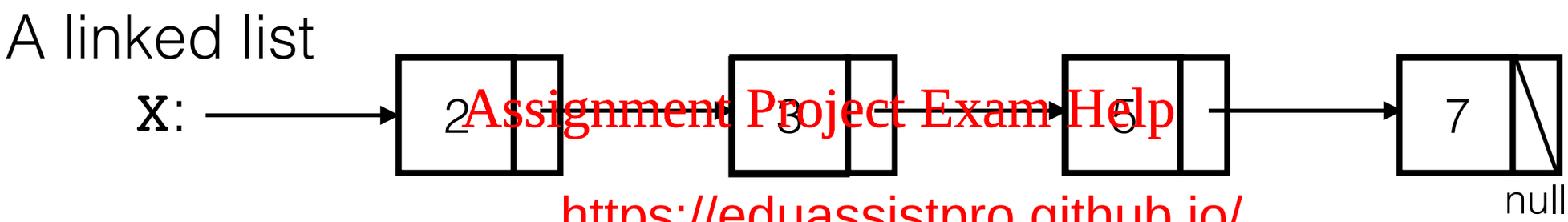

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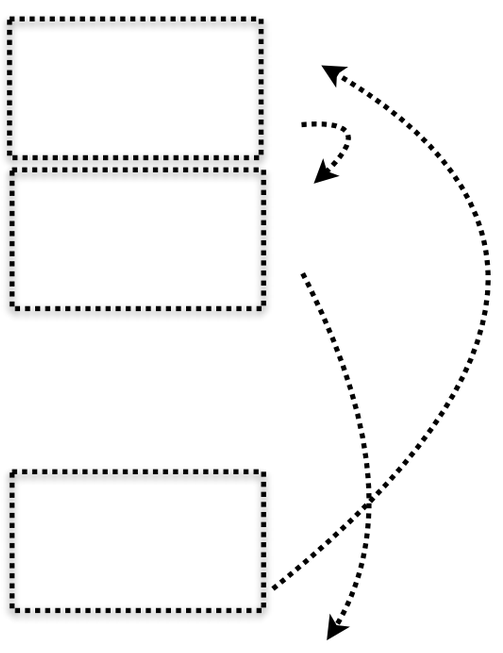

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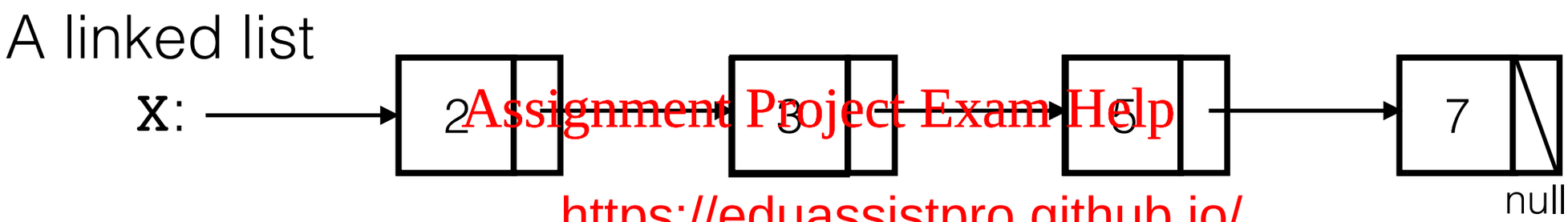

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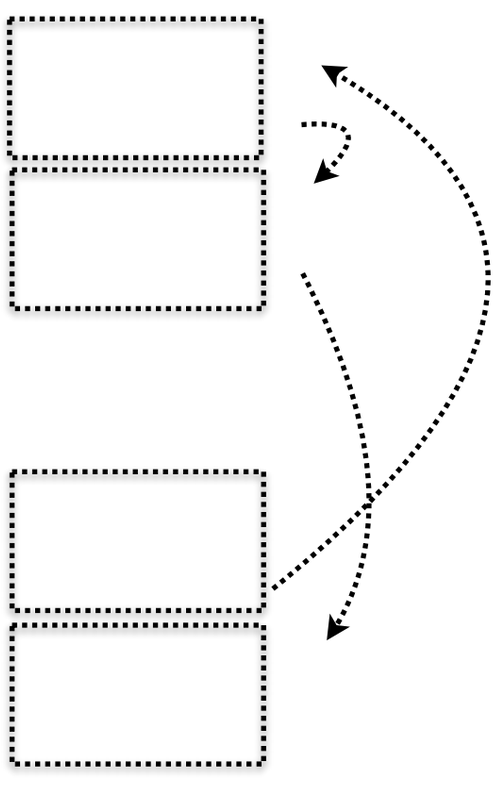

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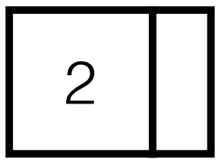

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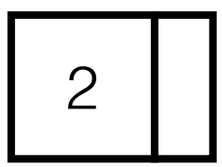

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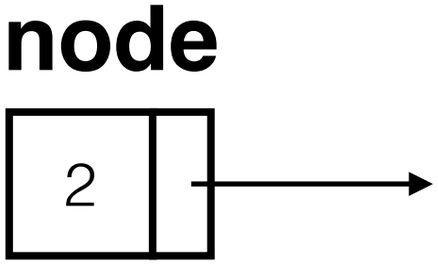

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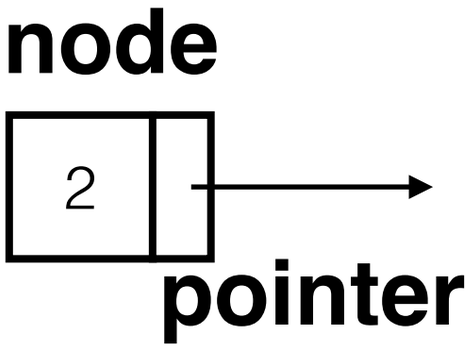

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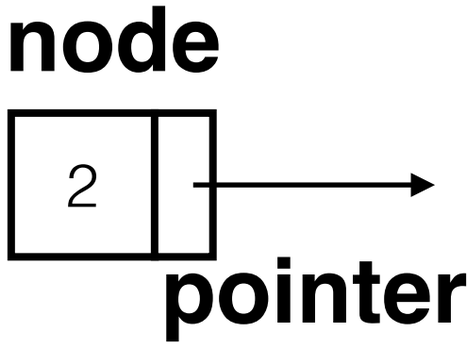

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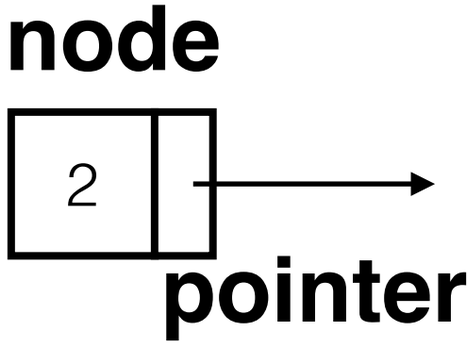

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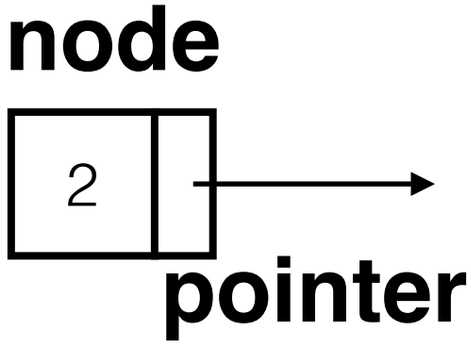

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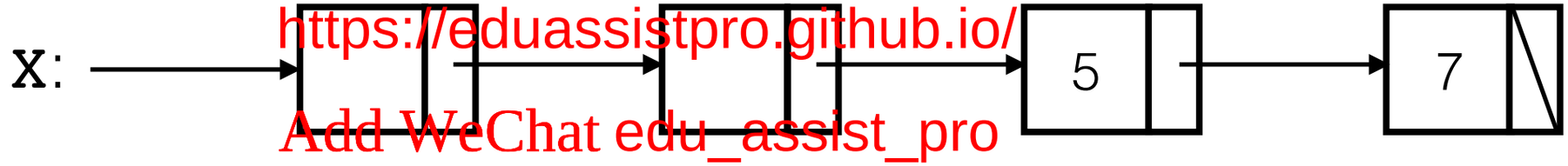

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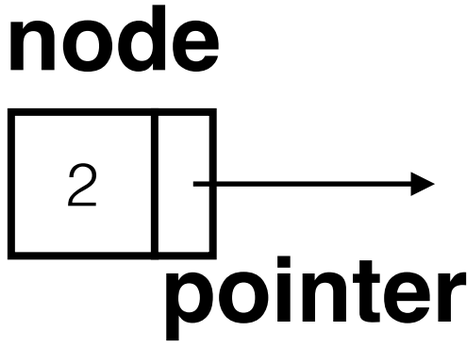

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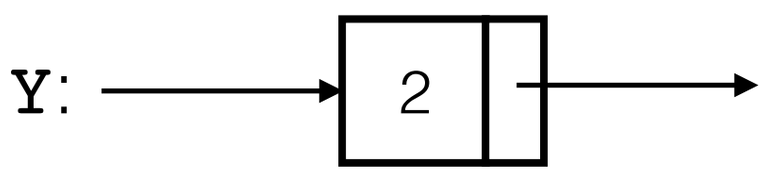

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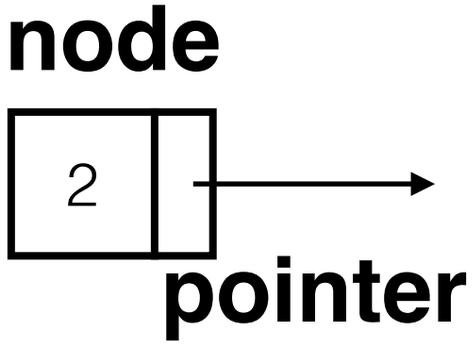

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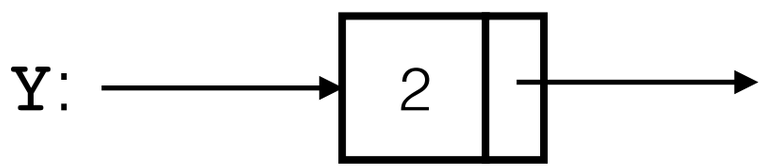

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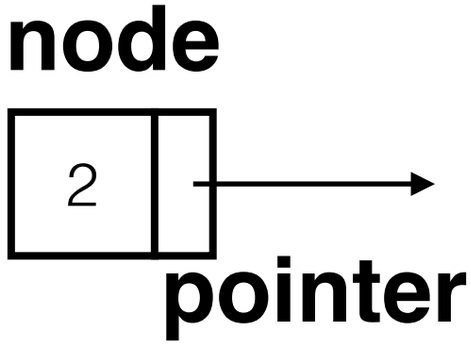

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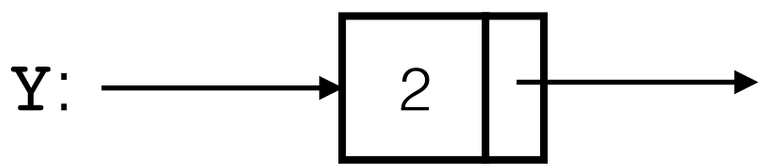

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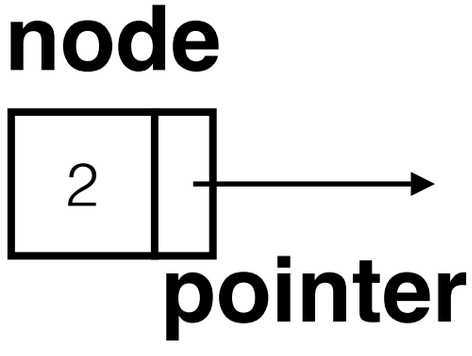

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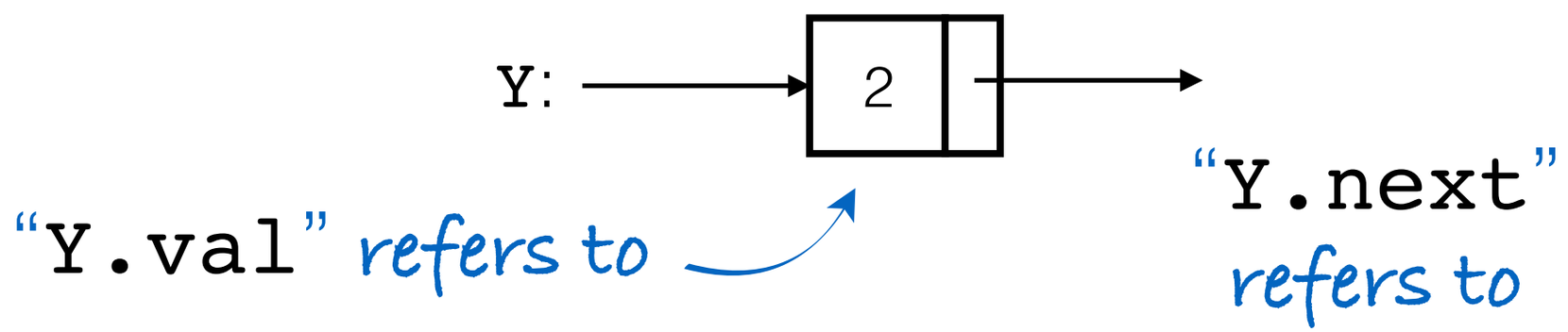

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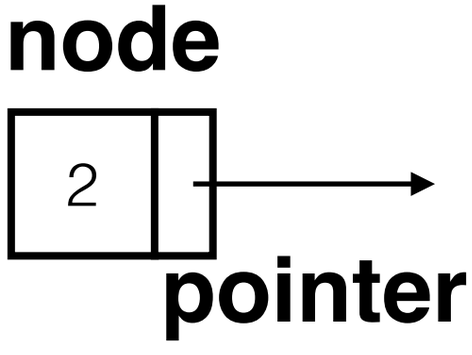

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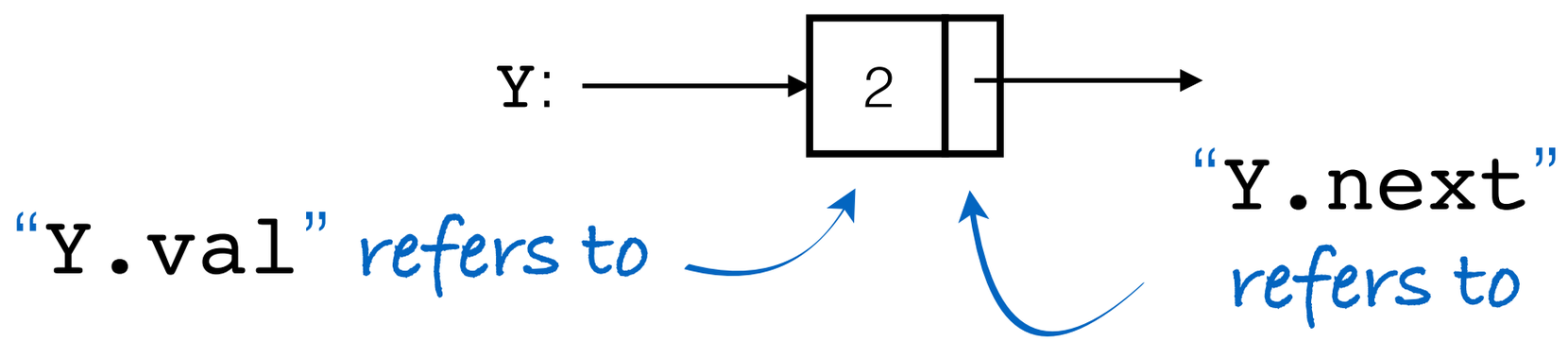

Linked List

- Often we use a dummy head node that points to the first object, or to a special `null` object that represents an empty list. This makes it easier to write functions that insert
- Inserting and deleting elements is very f ve a few links around.
- Finding the *i*th element can be time-consuming.

Iterative Processing: Array

THE UNIVERSITY OF MELBOURNE

- Walk through the array (of length n)
- For example, to locate item **x**.

```
function find(A,x,n)
    j ← 0
    while j < n
        if A[j] = x
            return j
        j ← j+1
    return -1
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        j ← j+1
    return -1
```


Y:

6	9	2	3	7	5	8
0	1	2	3	4	5	6

Iterative Processing: Array

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        j ← j+1
    return -1
```


Y:

6	9	2	3	7	5	8
0	1	2	3	4	5	6

Let's trace the execution of find(Y,7,6)

Iterative Processing: Array

- Walk through the array (of length n)

- For example, to locate item **x**.

A: Y

```
function find(A,x,n)
    j ← 0
    while j < n
        if A[j] = x
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        j ← j+1
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```


Y:

6	9	2	3	7	5	8
0	1	2	3	4	5	6

Let's trace the execution of find(Y,7,6)

Iterative Processing: Array

- Walk through the array (of length n)

- For example, to locate item **x**.

A: Y x: 7

```
function find(A,x,n)
    j ← 0
    while j < n
        if A[j] = x
            return j
        j ← j+1
    return -1
```


Y:

6	9	2	3	7	5	8
0	1	2	3	4	5	6

Let's trace the execution of find(Y,7,6)

Iterative Processing: Array

THE UNIVERSITY OF MELBOURNE

- Walk through the array (of length n)
- For example, to locate item **x**.

A: Y x: 7 n: 6

```
function find(A,x,n)
    j ← 0
    while j < n
        if A[j] = x
            return j
        j ← j+1
    return -1
```


Y:

6	9	2	3	7	5	8
0	1	2	3	4	5	6

Let's trace the execution of find(Y,7,6)

Iterative Processing: Array

- Walk through the array (of length n)

- For example, to locate item **x**.

A: Y x: 7 n: 6 j: 0

```
function find(A,x,n)
    j ← 0
    while j < n
        if A[j] = x
            return j
        j ← j+1
    return -1
```


Y:

6	9	2	3	7	5	8
0	1	2	3	4	5	6

Let's trace the execution of find(Y,7,6)

Iterative Processing: Array

- Walk through the array (of length n)

- For example, to locate item **x**.

A: Y x: 7 n: 6 j: 0

```
function find(A,x,n)
    j ← 0
    while j < n
        if A[j] = x
            return j
        j ← j+1
    return -1
```


Y:

Let's trace the execution of find(Y,7,6)

Iterative Processing: Array

- Walk through the array (of length n)

- For example, to locate item **x**.

A: Y x: 7 n: 6 j: 1

```
function find(A,x,n)
    j ← 0
    while j < n
        if A[j] = x
            return j
        j ← j+1
    return -1
```


Y:

6	9	2	3	7	5	8
0	1	2	3	4	5	6

Let's trace the execution of find(Y,7,6)

Iterative Processing: Array

THE UNIVERSITY OF MELBOURNE

- Walk through the array (of length n)
- For example, to locate item **x**.

A: Y x: 7 n: 6 j: 1

```
function find(A,x,n)
    j ← 0
    while j < n
        if A[j] = x
            return j
        j ← j+1
    return -1
```


Y:

6	9	2	3	7	5	8
0	1	2	3	4	5	6

Let's trace the execution of find(Y,7,6)

Iterative Processing: Array

- Walk through the array (of length n)

- For example, to locate item **x**.

A: Y x: 7 n: 6 j: 2

```
function find(A,x,n)
    j ← 0
    while j < n
        if A[j] = x
            return j
        j ← j+1
    return -1
```


Y:

Let's trace the execution of find(Y,7,6)

Iterative Processing: Array

THE UNIVERSITY OF MELBOURNE

- Walk through the array (of length n)
- For example, to locate item **x**.

A: Y x: 7 n: 6 j: 3

```
function find(A,x,n)
    j ← 0
    while j < n
        if A[j] = x
            return j
        j ← j+1
    return -1
```


A[j]

Y:

6	9	2	3	7	5	8
0	1	2	3	4	5	6

Let's trace the execution of find(Y,7,6)

Iterative Processing: Array

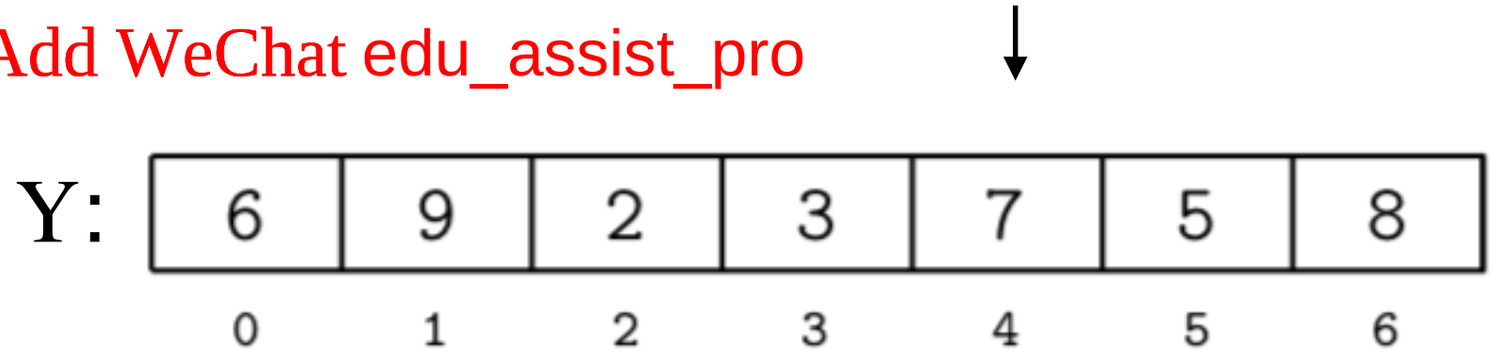
- Walk through the array (of length n)

- For example, to locate item **x**.

A: Y x: 7 n: 6 j: 4

```
function find(A,x,n)
    j ← 0
    while j < n
        if A[j] = x
            return j
        j ← j+1
    return -1
```


A[j]



Let's trace the execution of find(Y,7,6)

Iterative Processing: Array

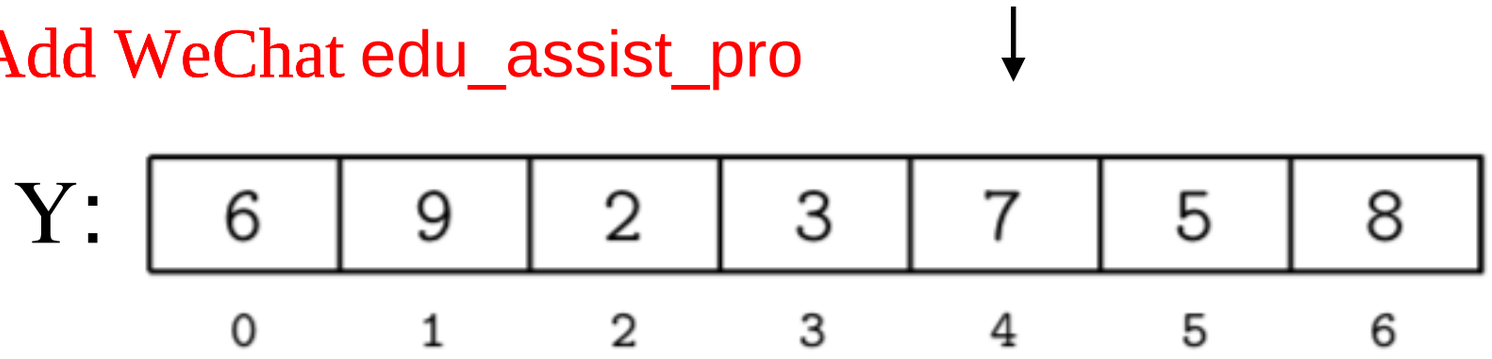
THE UNIVERSITY OF MELBOURNE

- Walk through the array (of length n)
- For example, to locate item **x**.

A: Y x: 7 n: 6 j: 4

```
function find(A,x,n)
    j ← 0
    while j < n
        if A[j] = x
            return j
        j ← j+1
    return -1
```


A[j]



Let's trace the execution of find(Y,7,6)

(returns 4)

Iterative Processing: List

- Walk through a linked list.

- For example, to locate item `x`.

```
function find(head,x)
    p ← head
    while p ≠ null
        if p.val = x
            return p
        p ← p.next
    return null
```

Iterative Processing: List

- Walk through a linked list.

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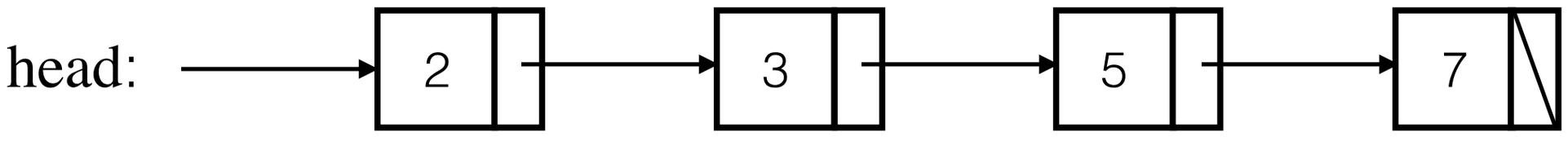


Iterative Processing: List

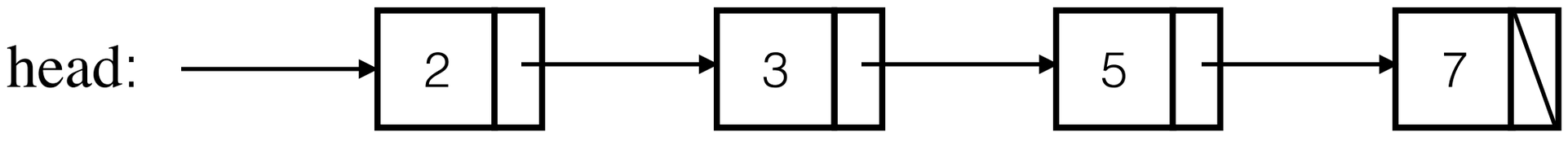
- Walk through a linked list.

- For example, to locate item **x**.

(note similarity to array version)

```
function find(head,x)
    p ← head
  while p ≠ null
    if p.val = x
      return p
    p ← p.next
  return null
```

```
function find(A,x,n)
                  n
               = x
          return j
       j ← j+1
    return -1
```

Iterative Processing: List

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- For example, to locate item **x**.

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```
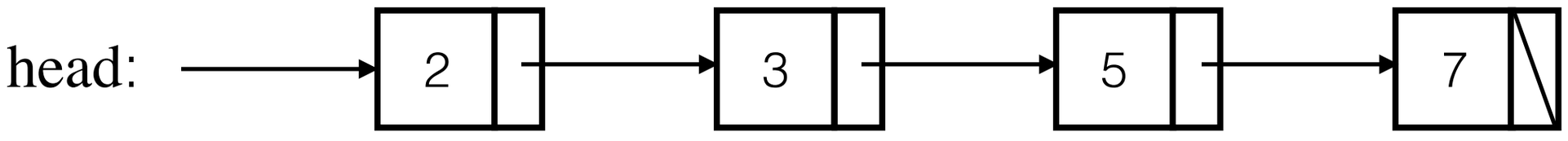

```
function find(A,x,n)
    j ← 1
    while j ≤ n
        if A[j] = x
            return j
        j ← j+1
    return -1
```

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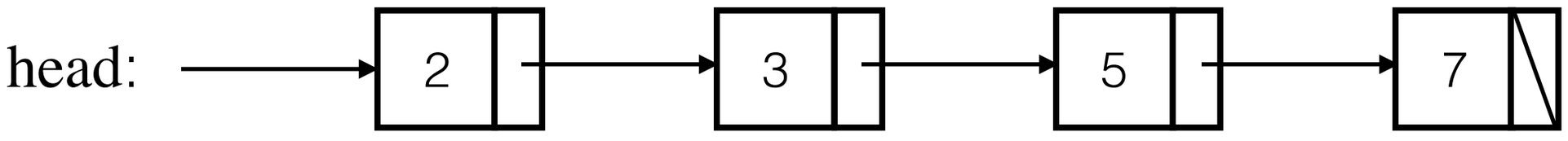
        j ← j+1
    return -1
```

p:



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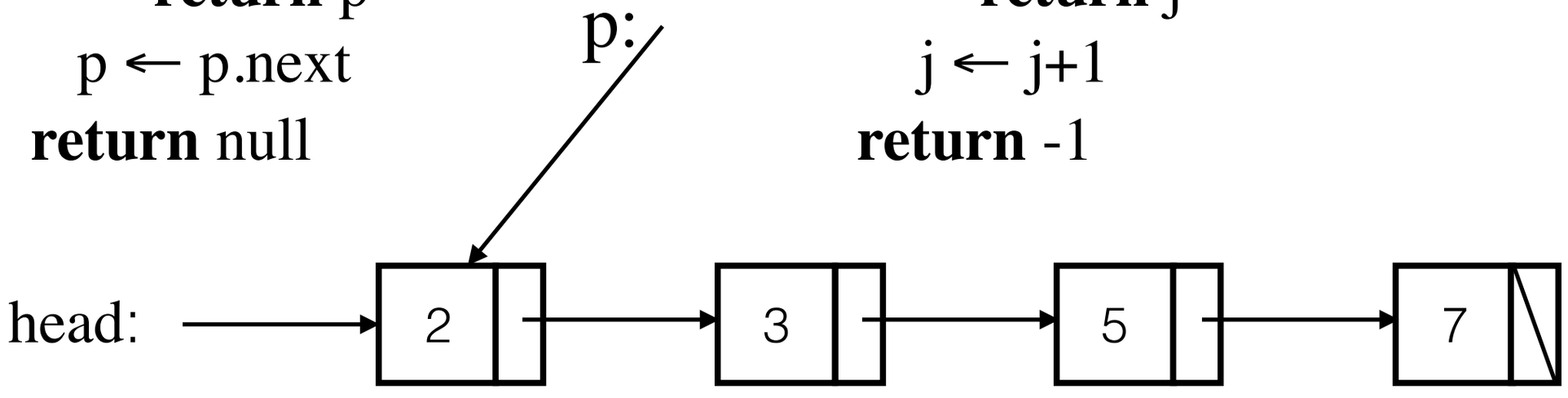

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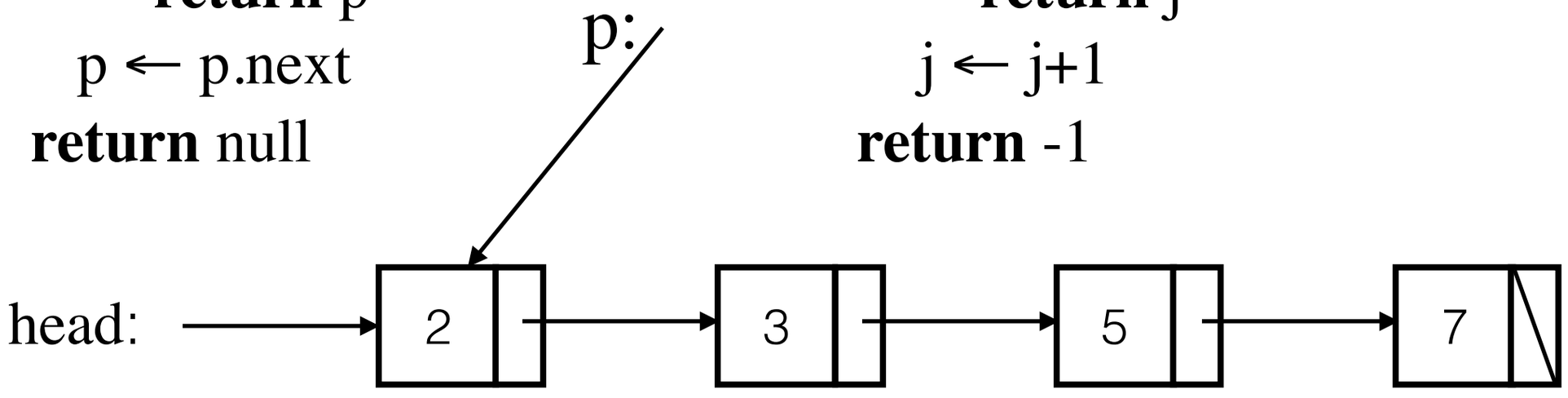

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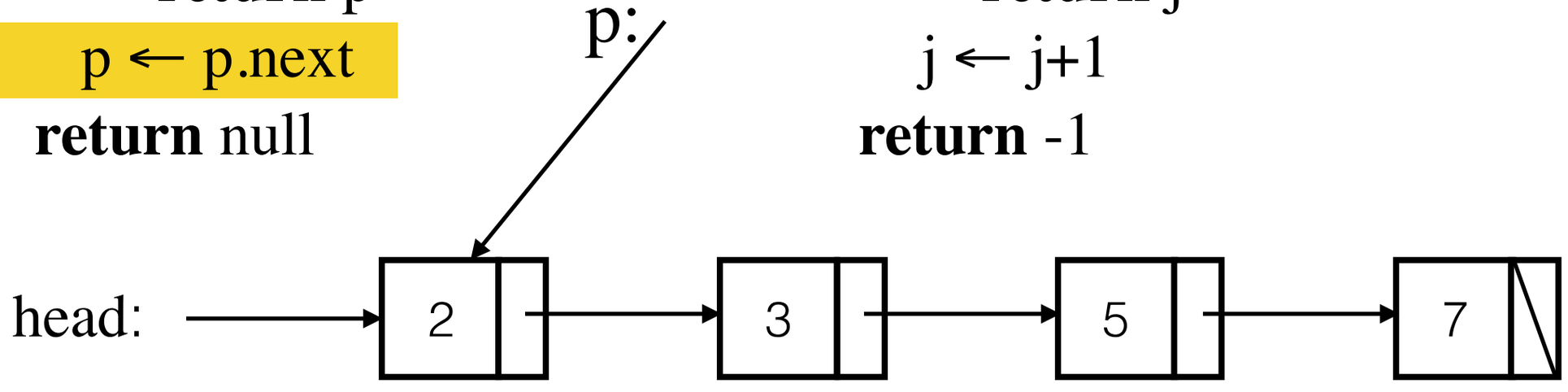

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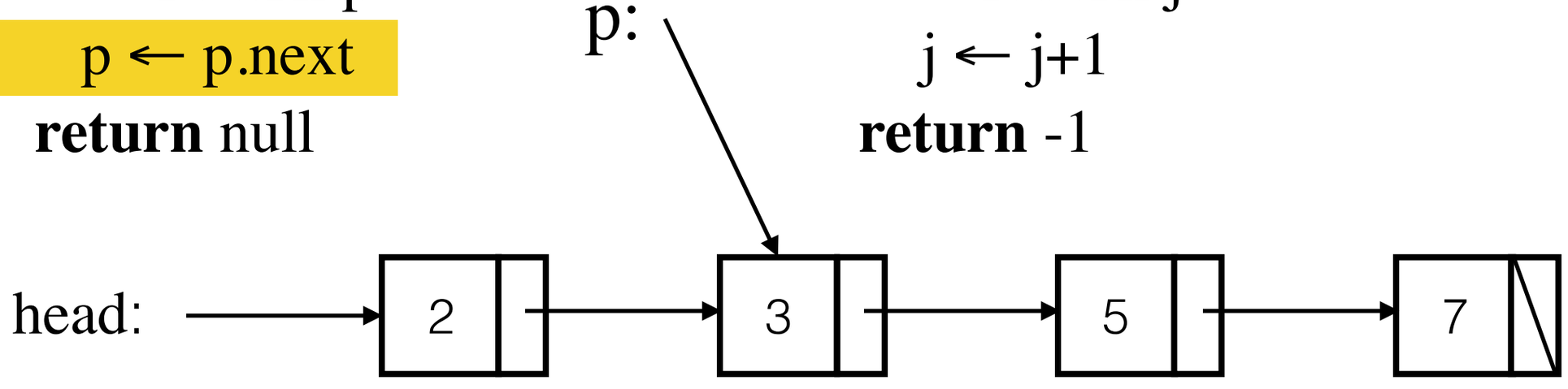

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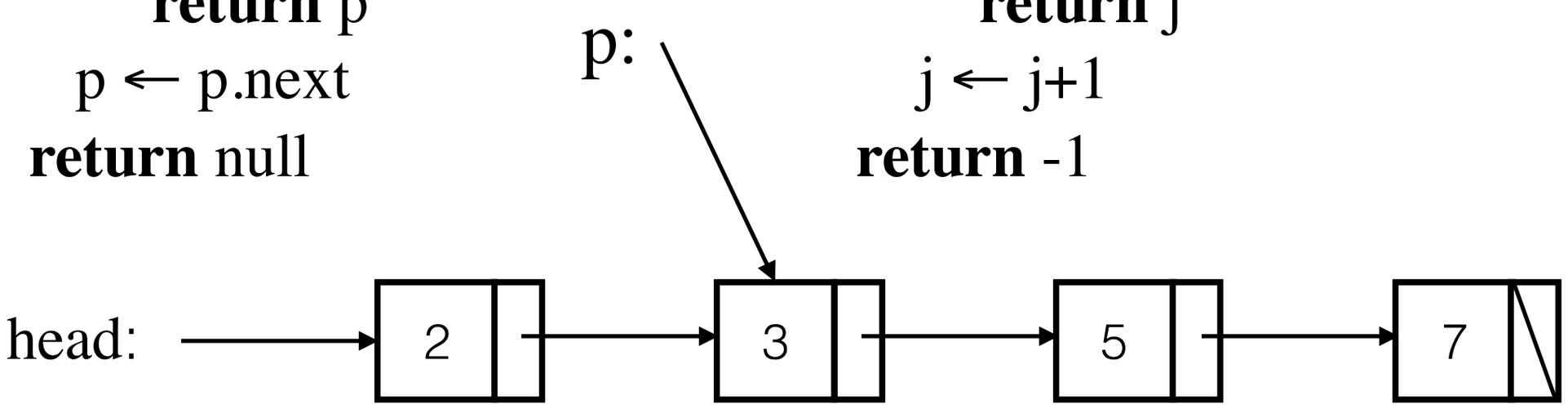

Recursive Processing: Array

THE UNIVERSITY OF MELBOURNE

- Solve the problem for a sub-instance and use the solution to solve the full instance

- For example, to locate item **x**.

```
function find(A,x,lo,hi)
    if lo > hi
        return -1
    else if A[lo] = x
        return lo
    else
        return find(A,x,lo+1,hi)
```

Recursive Processing: Array

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```

Initial call: find(A,x,0,n-1)

Recursive Processing: Array

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```


Y:

6	9	2	3	7	5	8
0	1	2	3	4	5	6

Initial call: find(A,x,0,n-1)

Recursive Processing: Array

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Initial call: find(A,x,0,n-1)

Let's trace the execution of find(Y,7,0,6)

Recursive Processing: Array

THE UNIVERSITY OF MELBOURNE

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- For example, to locate item **x**.

A: Y


```
function find(A,x,lo,hi)
    if lo > hi
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Y:

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0	1	2	3	4	5	6

Initial call: find(A,x,0,n-1)

Let's trace the execution of find(Y,7,0,6)

Recursive Processing: Array

- Solve the problem for a sub-instance and use the solution to solve the full instance

- For example, to locate item **x**.

A: Y x: 7


```
function find(A,x,lo,hi)
    if lo > hi
        return -1
    else if A[lo] = x
        return lo
    else
        return find(A,x,lo+1,hi)
```

Y:

6	9	2	3	7	5	8
0	1	2	3	4	5	6

Initial call: find(A,x,0,n-1)

Let's trace the execution of find(Y,7,0,6)

Recursive Processing: Array

THE UNIVERSITY OF MELBOURNE

- Solve the problem for a sub-instance and use the solution to solve the full instance

- For example, to locate item **x**.

A: Y x: 7 lo: 0

```
function find(A,x,lo,hi)
    if lo > hi
        return -1
    else if A[lo] = x
        return lo
    else
        return find(A,x,lo+1,hi)
```


Y:

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Initial call: find(A,x,0,n-1)

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Recursive Processing: Array

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- Solve the problem for a sub-instance and use the solution to solve the full instance

- For example, to locate item **x**.

A: Y x: 7 lo: 0 hi: 6

```
function find(A,x,lo,hi)
    if lo > hi
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```
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    if lo > hi
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    else if A[lo] = x
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        return find(A,x,lo+1,hi)
```


A[hi]

Y:

6	9	2	3	7	5	8
0	1	2	3	4	5	6

Initial call: find(A,x,0,n-1)

Let's trace the execution of find(Y,7,0,6)

Recursive Processing: Array

THE UNIVERSITY OF MELBOURNE

- Solve the problem for a sub-instance and use the solution to solve the full instance

- For example, to locate item **x**.

A: Y x: 7 lo: 1 hi: 6

```
function find(A,x,lo,hi)
    if lo > hi
        return -1
    else if A[lo] = x
        return lo
    else
        return find(A,x,lo+1,hi)
```


A[hi]

Y:

6	9	2	3	7	5	8
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Recursive Processing: Array

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```


A[hi]

Y:

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0	1	2	3	4	5	6

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Recursive Processing: Array

THE UNIVERSITY OF MELBOURNE

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Initial call: find(A,x,0,n-1)

Let's trace the execution of find(Y,7,0,6)

Recursive Processing: Array

THE UNIVERSITY OF MELBOURNE

- Solve the problem for a sub-instance and use the solution to solve the full instance

- For example, to locate item **x**.

A: Y x: 7 lo: 2 hi: 6

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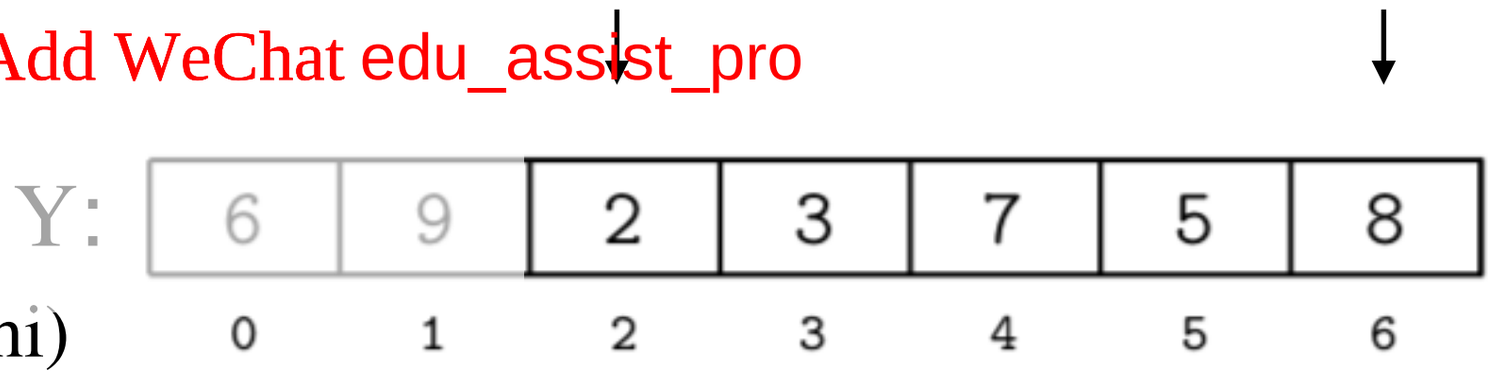

Initial call: find(A,x,0,n-1)

Let's trace the execution of find(Y,7,0,6)

Recursive Processing: Array

THE UNIVERSITY OF MELBOURNE

- Solve the problem for a sub-instance and use the solution to solve the full instance

- For example, to locate item **x**.

A: Y x: 7 lo: 3 hi: 6

```
function find(A,x,lo,hi)
    if lo > hi
        return -1
    else if A[lo] = x
        return lo
    else
        return find(A,x,lo+1,hi)
```


A[lo] A[hi]

Initial call: find(A,x,0,n-1)

Let's trace the execution of find(Y,7,0,6)

Recursive Processing: Array

THE UNIVERSITY OF MELBOURNE

- Solve the problem for a sub-instance and use the solution to solve the full instance

- For example, to locate item **x**.

A: Y x: 7 lo: 4 hi: 6

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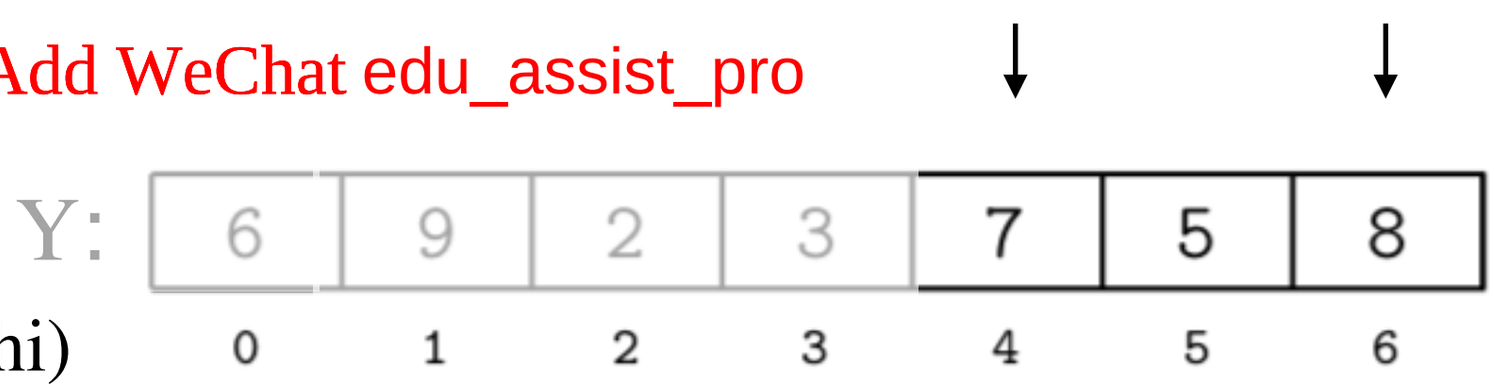

Initial call: find(A,x,0,n-1)

Let's trace the execution of find(Y,7,0,6)

Recursive Processing: Array

THE UNIVERSITY OF MELBOURNE

- Solve the problem for a sub-instance and use the solution to solve the full instance

- For example, to locate item **x**.

A: Y x: 7 lo: 4 hi: 6

```
function find(A,x,lo,hi)
    if lo > hi
        return -1
    else if A[lo] = x
        return lo
    else
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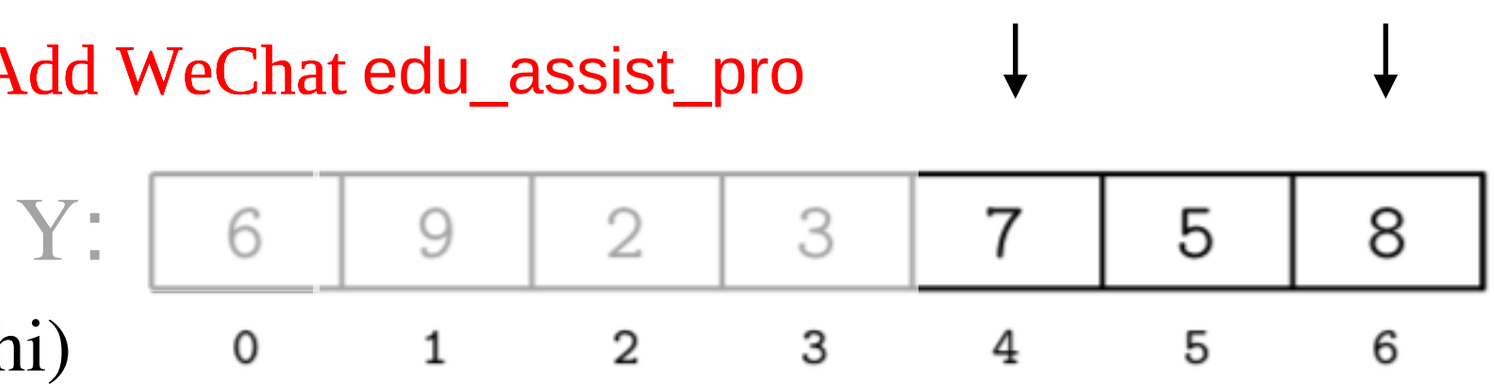

Initial call: find(A,x,0,n-1)

Let's trace the execution of find(Y,7,0,6)

(returns 4)

Recursive Processing: List

THE UNIVERSITY OF MELBOURNE

- Solve the problem for a sub-instance and use the solution to solve the full instance

```
function find(p,x)
    if p = null
        return p
    else if p.val = x
        return p
    else
        return find(p.next,x)
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Recursive Processing: List

THE UNIVERSITY OF MELBOURNE

- Solve the problem for a sub-instance and use the solution to solve the full instance

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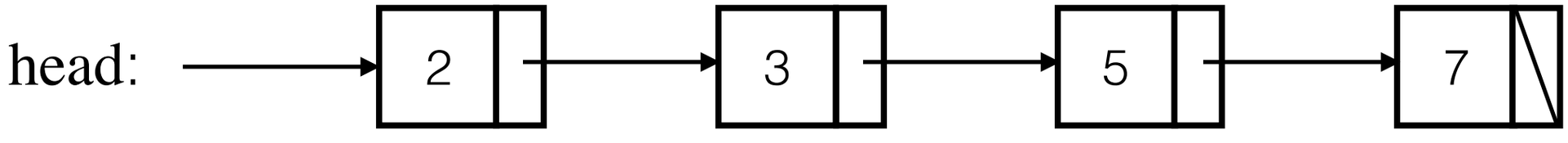

Recursive Processing: List

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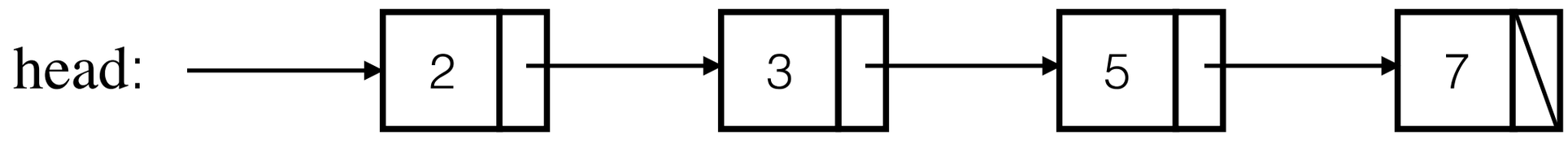
```


Initial call: find(head,x)



Recursive Processing: List

- Solve the problem for a sub-instance and use the solution to solve the full instance

(note similarity to array version)

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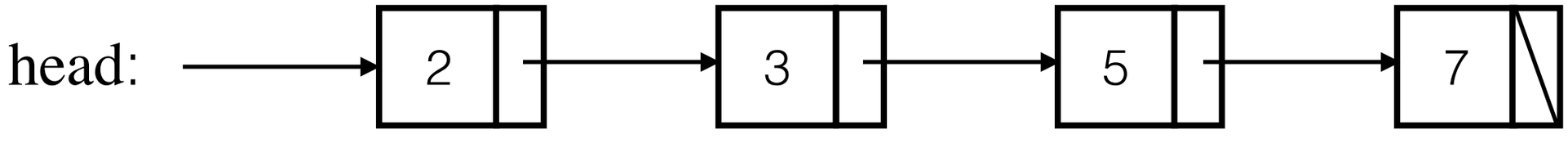

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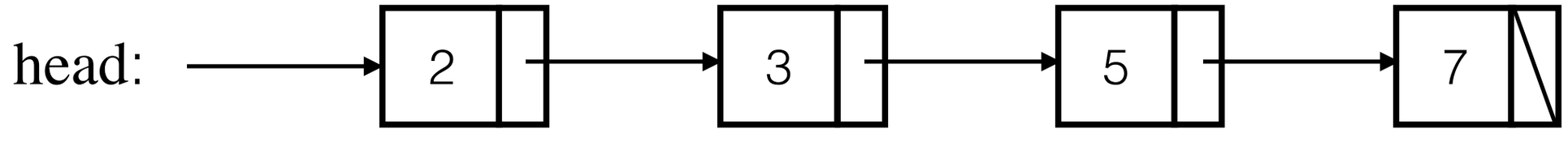

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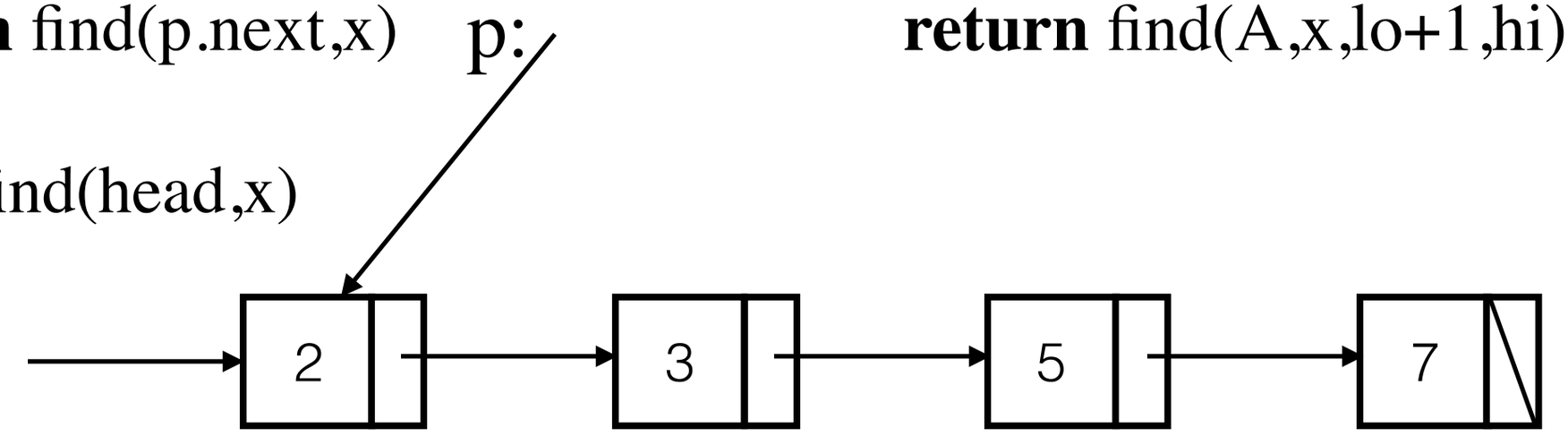

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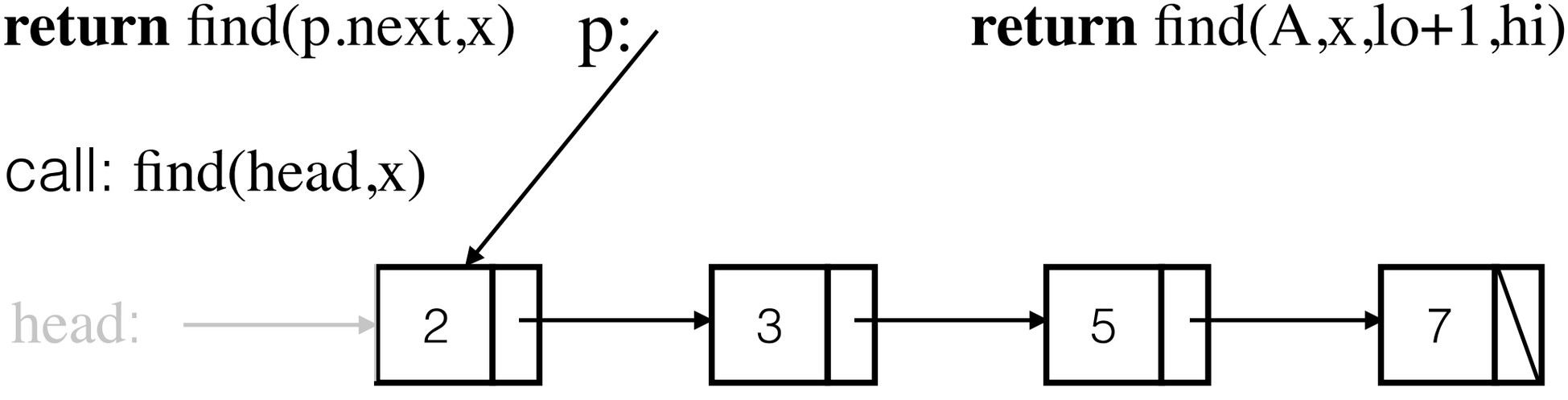

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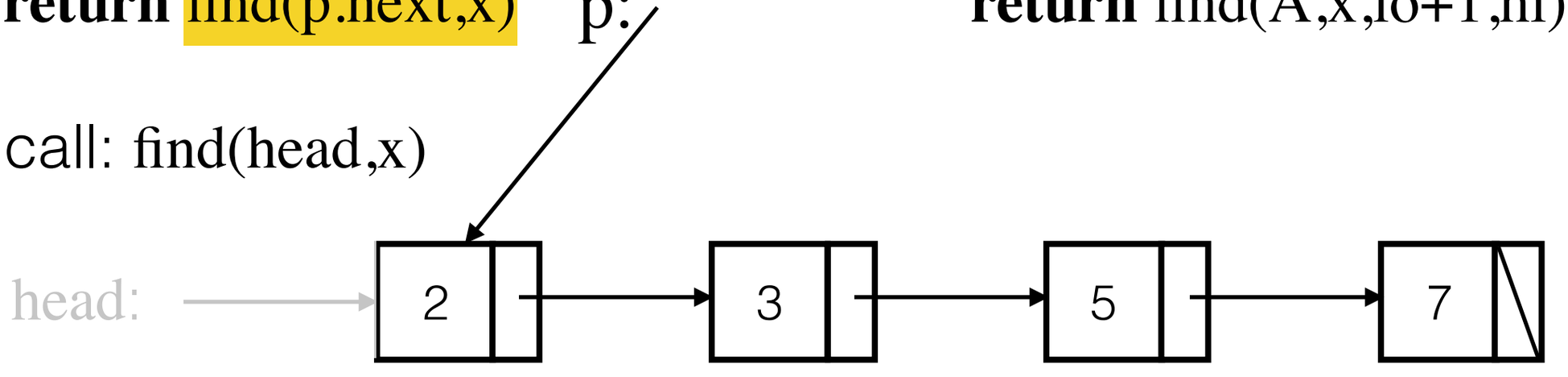

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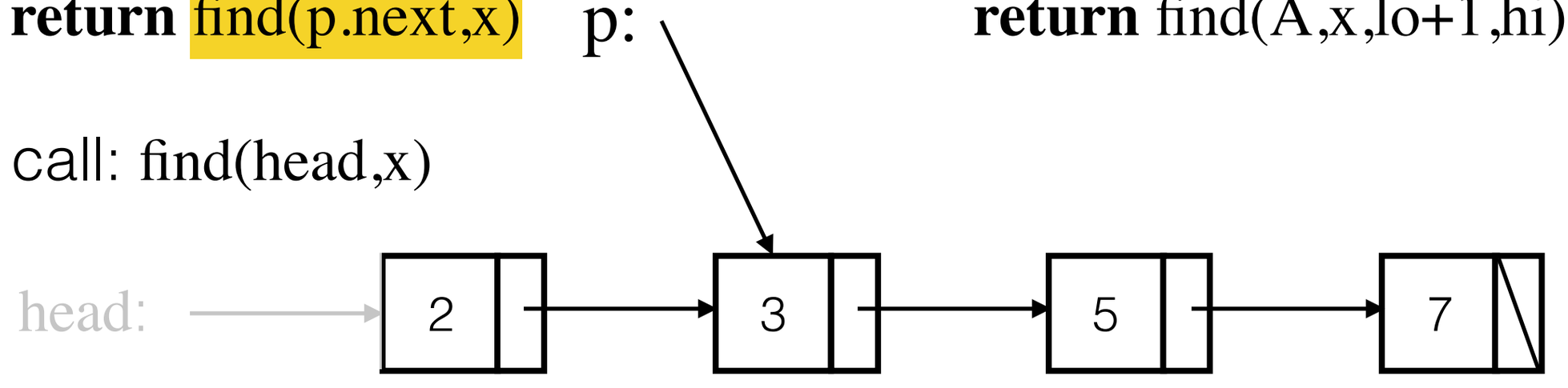

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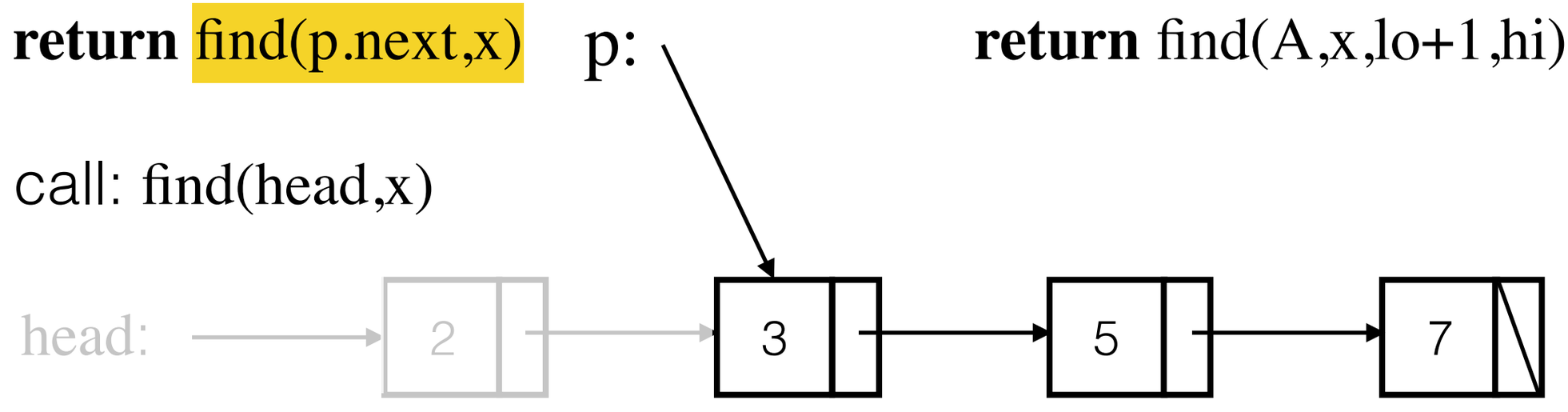

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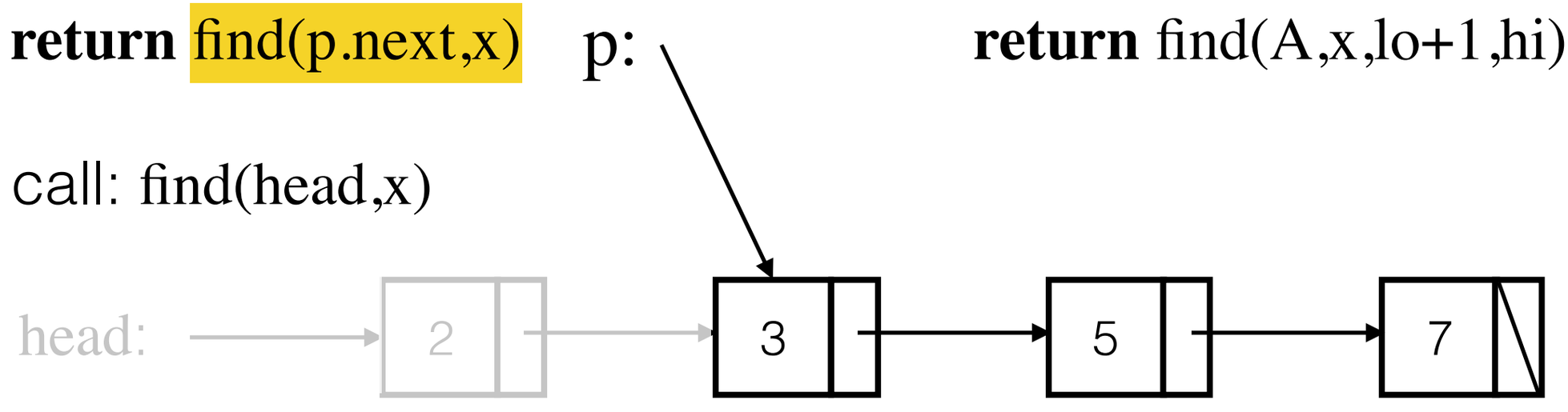

Abstract DataTypes

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- A collection of data items, and a family of operations that operate on that data
- Think of an ADT as a set of contracts, an **interface**
- We must still **implement** t an advantage to separate the implementation of the ADT from the “ i.e. the interface it provides)
- Good programming practice is to support this separation
  - Nothing outside of the definition of the ADT should refer to anything inside, except through function calls and basic operations

Fundamental Data Structure: The Stack

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- Last-In-First-Out (LIFO)
- Operations:
  - CreateStack
  - Push
  - Pop
  - Top
  - EmptyStack?
  - …
- Usually implemented as an ADT

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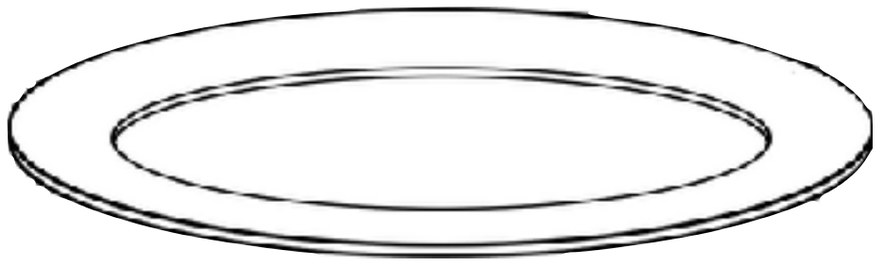

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Stack Implementation: Array

Stack Implementation: Array

Stack Implementation: Array

top: 5

Stack Implementation: Array

top: 5

Push(5)

Stack Implementation: Array

top: 5

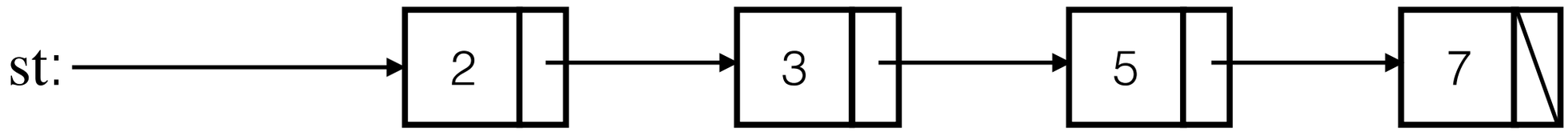
Push(5)

Stack Implementation: Array

top: 6

Push(5)

Stack Implementation: Linked List



function push(st,x)

  ew ← node

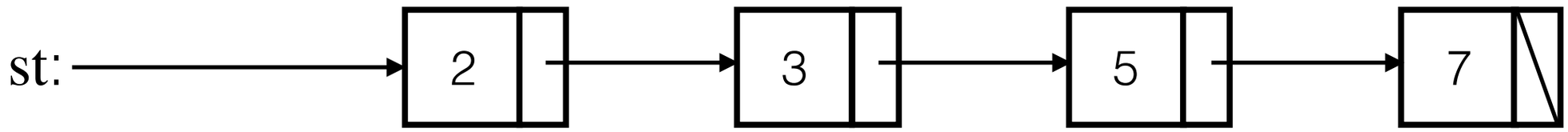
  x

  st

  st ← elt

  return st

Stack Implementation: Linked List



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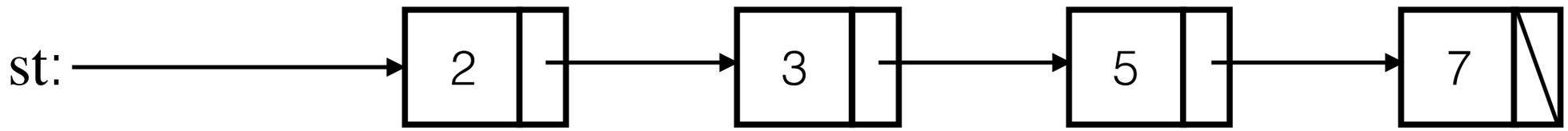


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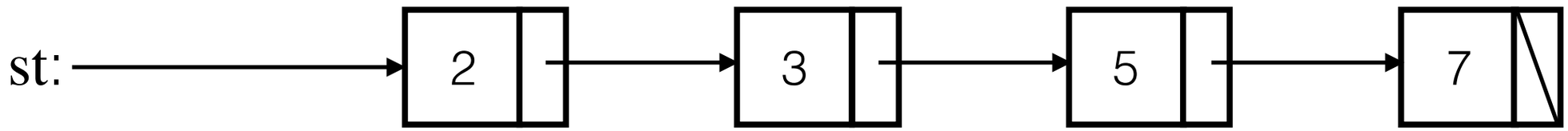
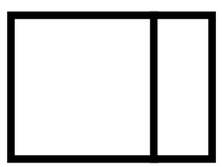
Push(5)

Stack Implementation: Linked List



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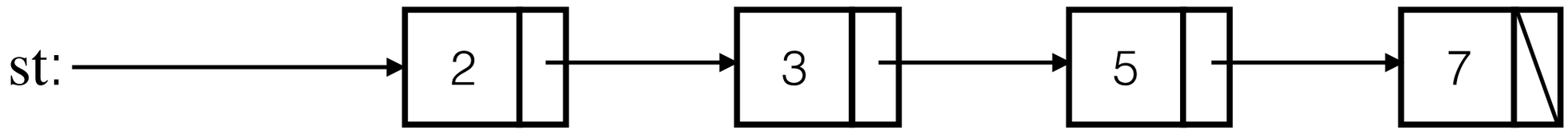

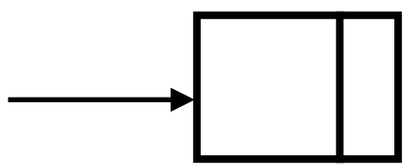


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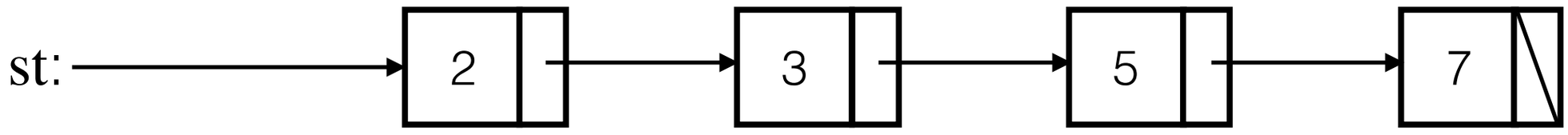

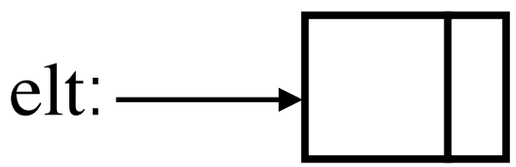

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Push(5)

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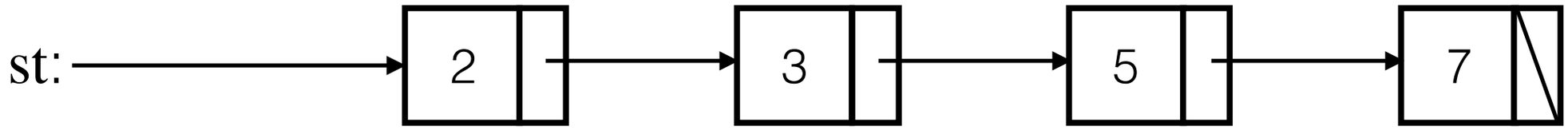

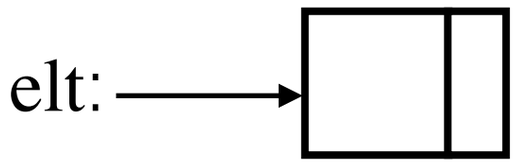


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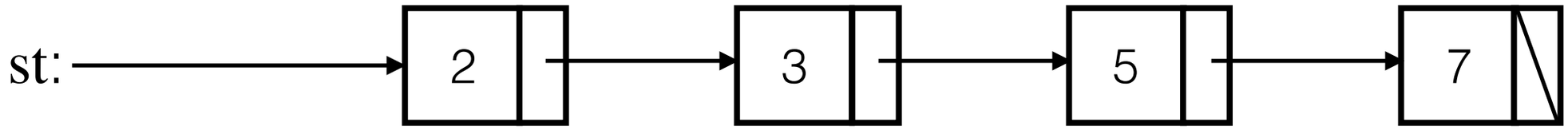

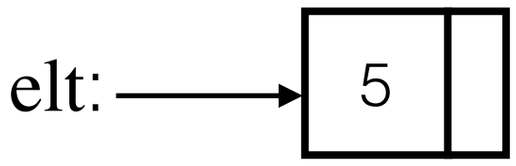


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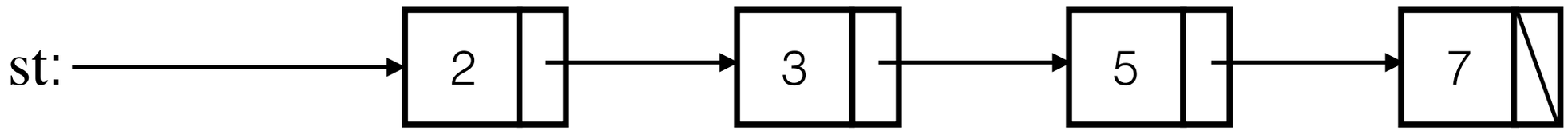

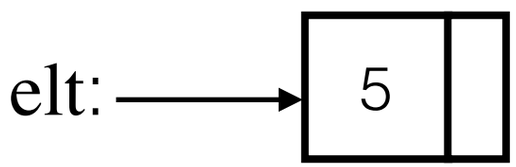


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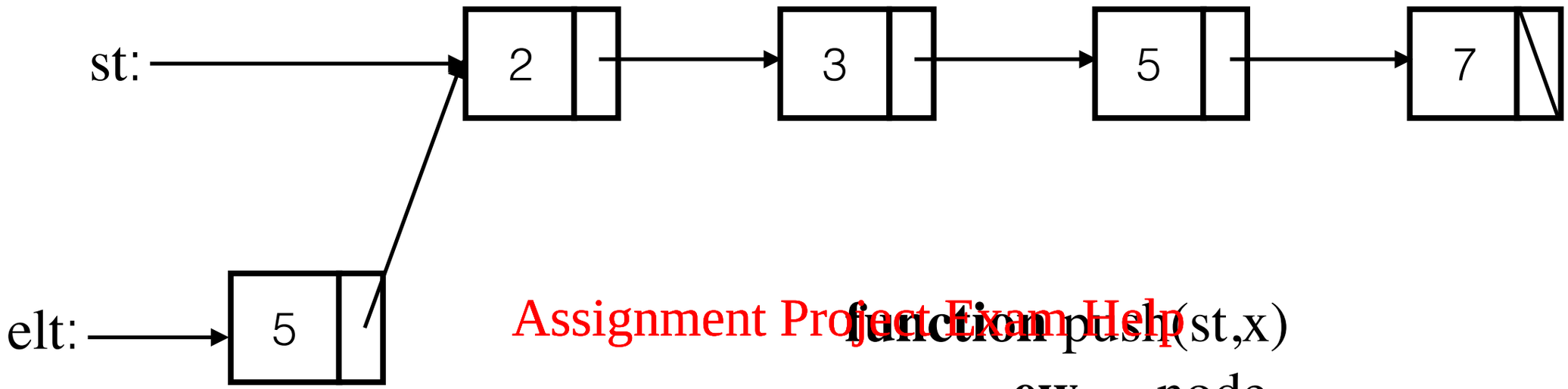


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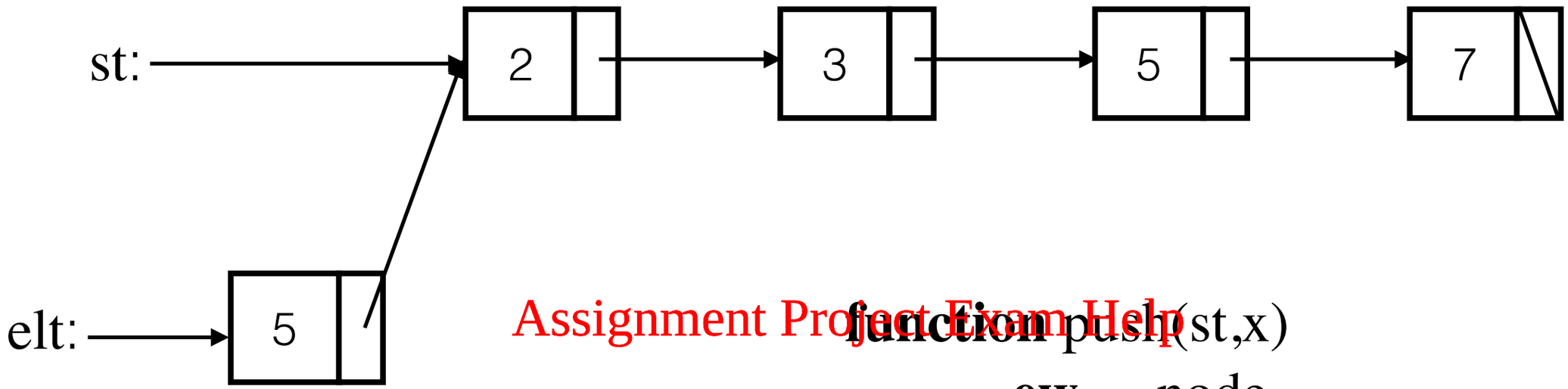


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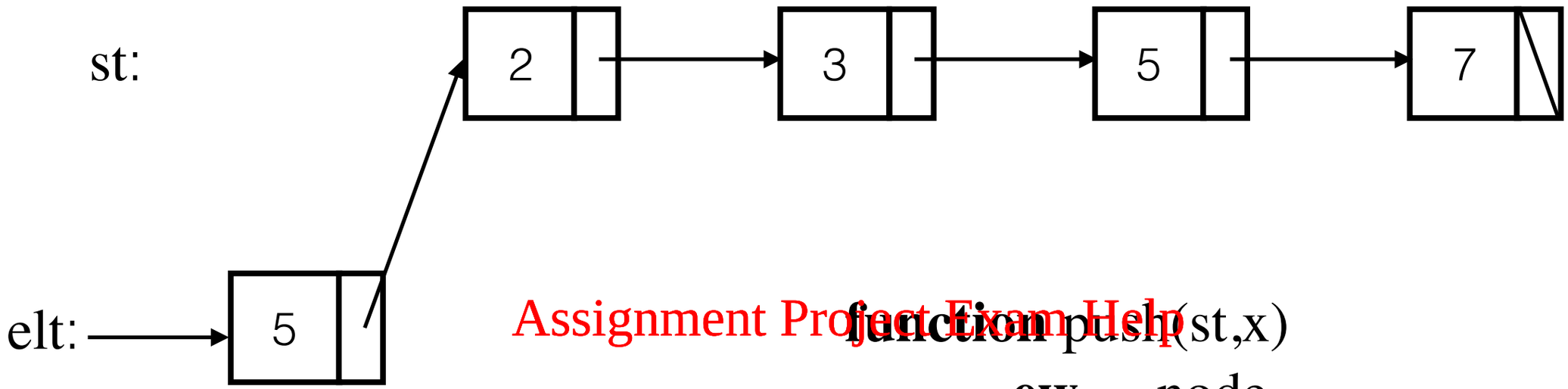


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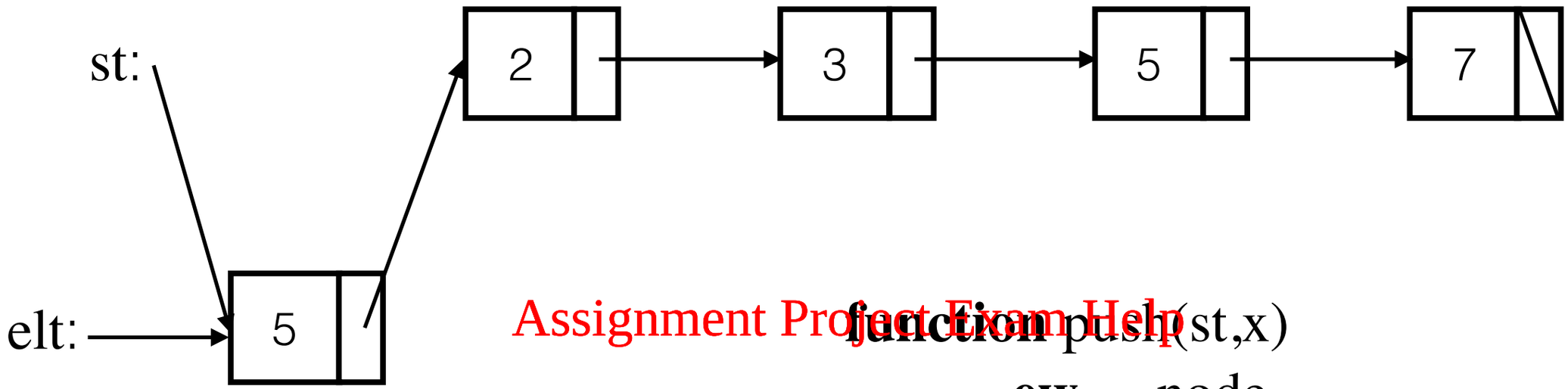

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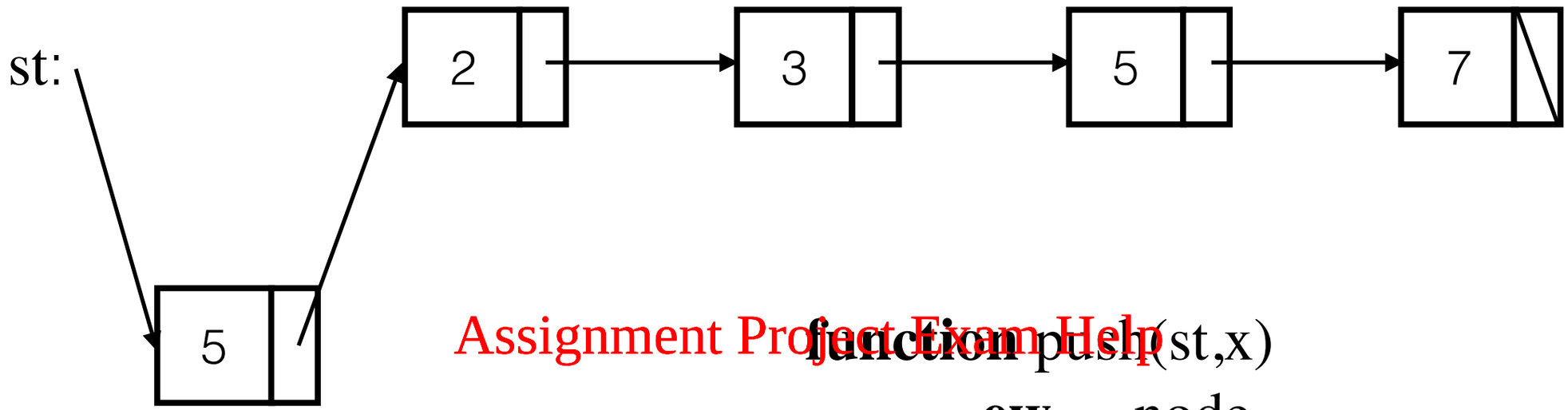

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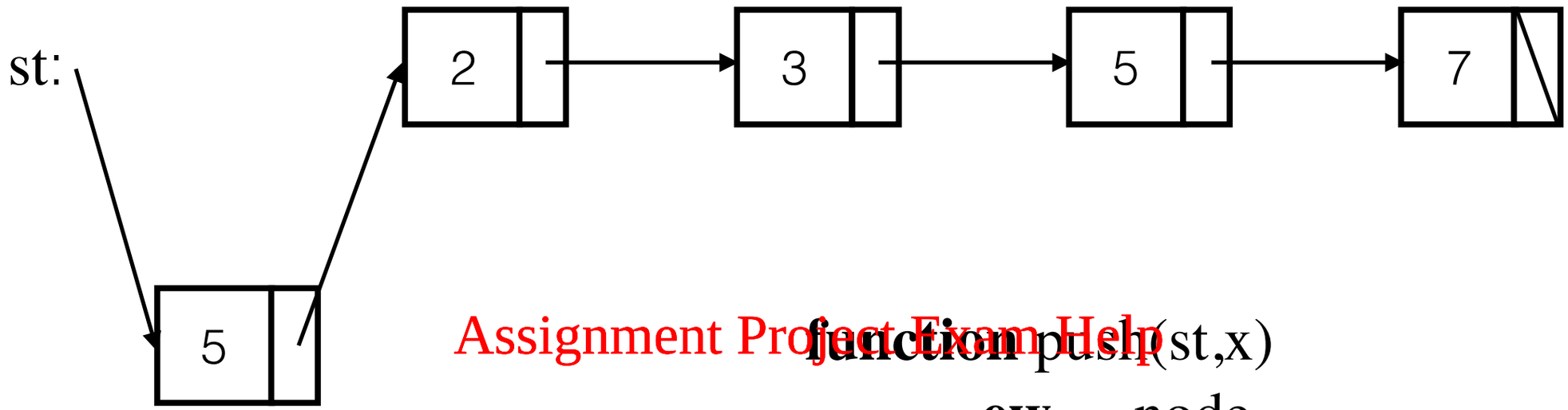


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    return st
```


Push(5)

See
https://www.cs.usfca.edu/~galles/visualization/Algorithms.html
for more visualisations

Pseudo Code

- On the previous slide, we assumed that a "node" has two attributes: a "val" which is its value, and a "next" which points to the rest of the list.

- There is no standard for pseudo-code. mples in Levitin as a guide. Cormen et al. pages 20–22 (in Re urces) has a list of standard conventions used with pseudo-code which are good to follow, except we use ← as the assignment operator.

Fundamental Data Structure: Queues

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- First-In-First-Out (FIFO)
- Operations:
  - CreateQueue
  - Enqueue
  - Dequeue
  - Head
  - EmptyQueue?
  - …

Fundamental Data Structure: Queues

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  - …

Fundamental Data Structure: Queues

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Other Data Structures

- We will meet many other (abstract) data structures, e.g.
  - The priority queue
  - Various types of “tree”
  - Various types of “graph”

- If you check out algorithm animation tools or advanced algorithm books, you will meet exotic data structures such as splay trees and skip lists.

Next Week

- Algorithm analysis—how t rithm's resource consumption.

