School of Computing and Information Systems
COMP90038 Algorithms and Complexity Tutorial Week 9

Sample Answers

The exercises

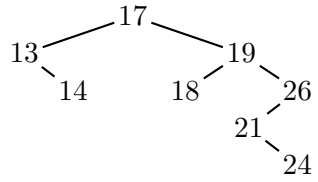
(Solutions to 56–59 were provided earlier.)

60. Given a list of n distinct integers and a sequence of n boxes with fixed inequality signs between them, design an algorithm to place the numbers into the boxes while satisfying each inequality. For example, given 4, 6, 3, 1, 8 and the boxes $\square < \square > \square < \square < \square$, we can place the numbers as follows: $\boxed{1} < \boxed{8} > \boxed{3} < \boxed{4} < \boxed{6}$.

    Answer: At first it is not clear that this is always possible. However, here is an algorithm. First sort the sequence. Now fill in the boxes from left to right, as follows. If there is a $<$ after the box, remove the smallest element from the list and place it in the box. If there is a $>$ after the box, remove the largest element from the list and place it in the box. Finally place the only remaining element in the last box. Note that some problem instances will have many solutions, but this algorithm is guaranteed to produce one solution.
    To see why the algorithm works, think recursively. We want to be able to produce a solution for n boxes, assu . We can always do that if we have reserved t e is a $<$ after the box, c

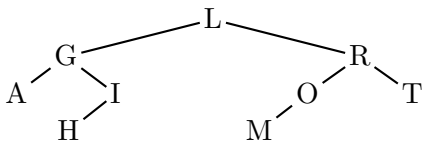
61. Construct a binary search tree (BST) by starting from an empty t keys, in the given order: 17, 19, 13, 26, 14, 18, 21, 24.

    Answer:



62. Construct an AVL tree from the empty tree by inserting the following keys in the given order: A, L, G, O, R, I, T, H, M.

    Answer:

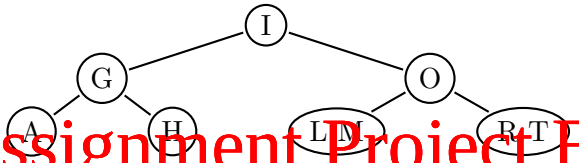


63. Consider the set of five keys (let us say they are positive integers) $\{k_1, k_2, k_3, k_4, k_5\}$, satisfying $k_1 < k_2 < k_3 < k_4 < k_5$. There are 120 different permutations of these five keys. For exactly two of the 120 permutations, the following happens, when the keys are inserted one by one, in the order given by the permutation, into an initially empty AVL tree: First an LR-rotation takes place, then an RL-rotation takes place. Which two permutations generate that behaviour?

Answer: Without loss of generality, assume the set of keys is $\{1, 2, 3, 4, 5\}$. After the first three keys are inserted to make an AVL tree, that tree must be perfectly balanced. Hence the first three keys inserted must cause the $\langle$ zigzag path that requires an LR-rotation. After that, insertion of the two remaining keys must cause the $\rangle$ zigzag path that requires an RL-rotation. The only permutations that will achieve this are 3, 1, 2, 5, 4 and 5, 1, 4, 3, 2.

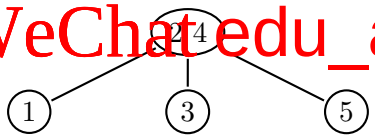
64. Construct a 2–3 tree from the empty tree by inserting the following keys in the given order: A, L, G, O, R, I, T, H, M.

Answer:

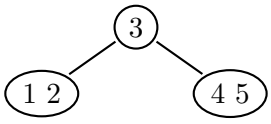


65. Construct a 2–3 tree by inserting keys 1, 2, 3, 4, 5, in that order. Repeat the exercise, but this time insert the ke
possible orderings produced?

Answer: Insert



Inserting 2, 3, 4, 5, 1, in that order, we produce this 2–3 tree:



These are the only possible 2–3 trees with the five given keys.