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COMP90038
Algorithms and Complexity

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Lecture 11: S

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and-Conquer

(with thanks to Hara ergaard)

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Divide and Conquer

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- We earlier studied recursion as a powerful problem solving technique.

- The **divide-and-conquer** strategy tries to make the most of this idea:

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1. Divide the given into smaller instances.

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2. Solve the smaller instances ely.

3. Combine the smaller solutions to solve the original instance.

- This works best when the smaller instances can be made to be of equal (or near-equal) size.

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Split-Solve-and-Join Approach

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Divide and Conquer Algorithms

- We will discuss:
  - The Master Theorem
  - Mergesort
  - Quicksort
  - Tree traversal
  - Closest Pair revisited

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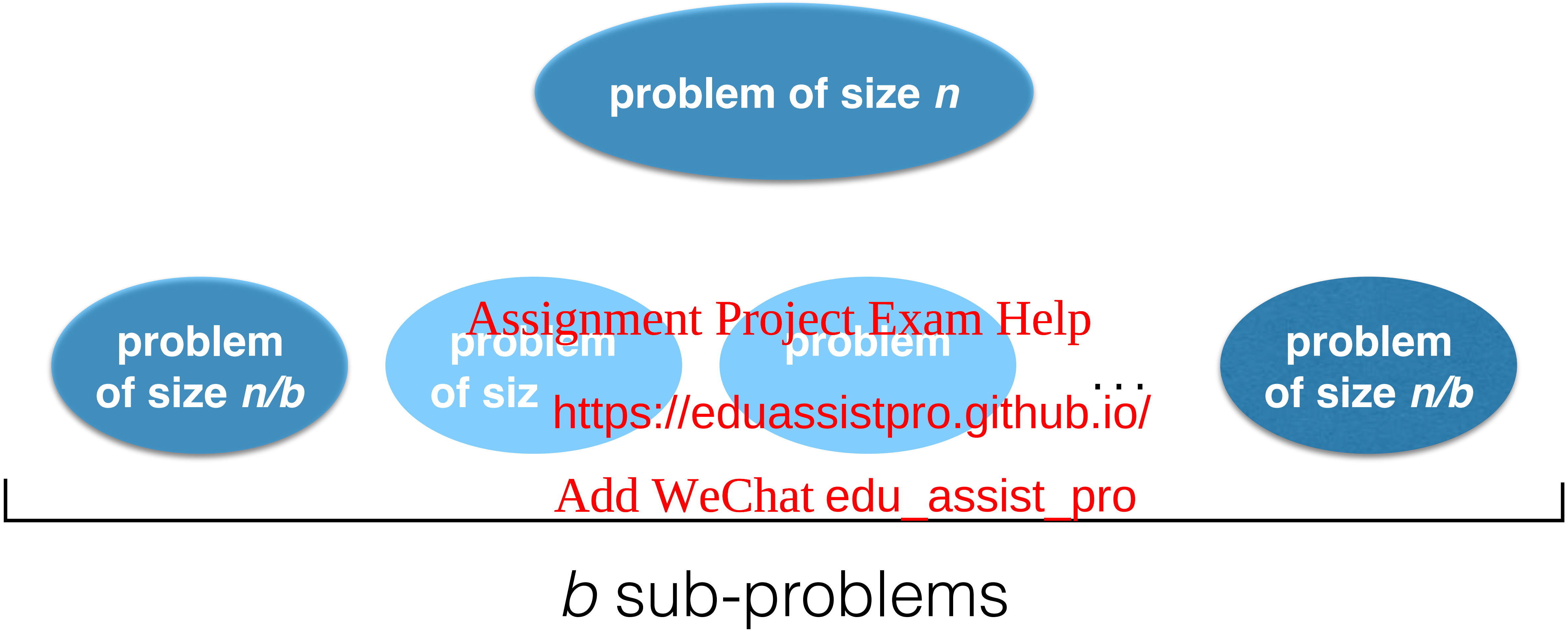
Divide-and-Conqer General Case



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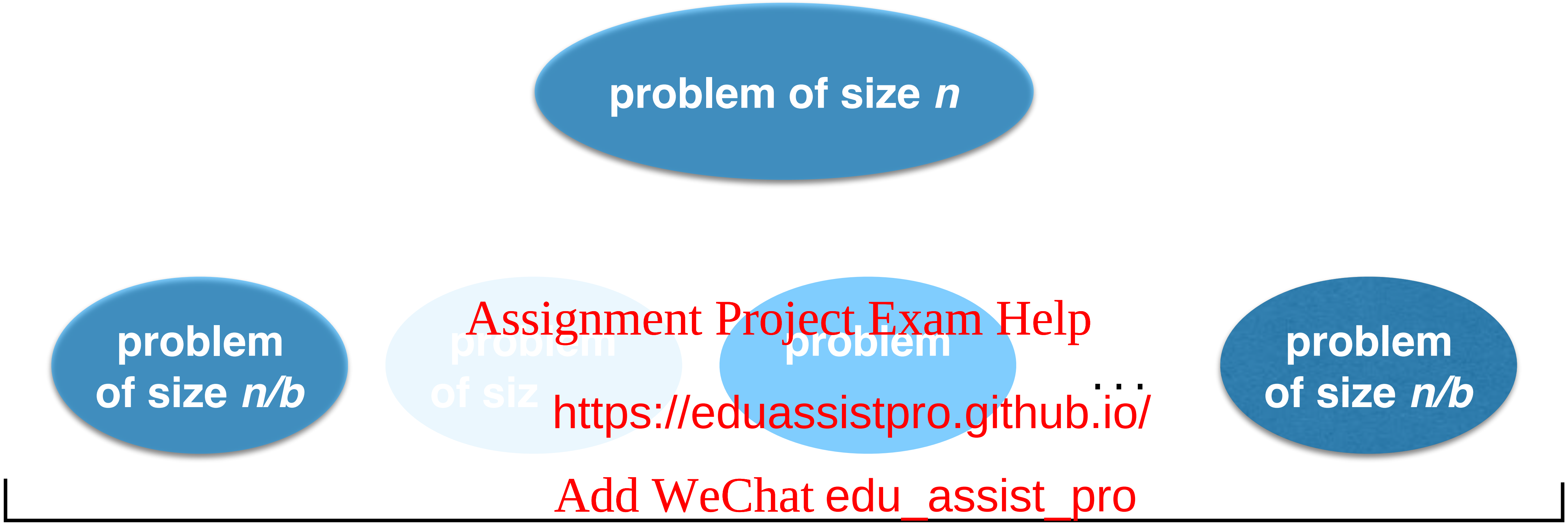
Divide-and-Conqer General Case



problem of size *n*

problem of size *n/b*

problem of siz

problem

. . .

problem of size *n/b*

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only *a* sub-problems need to be solved

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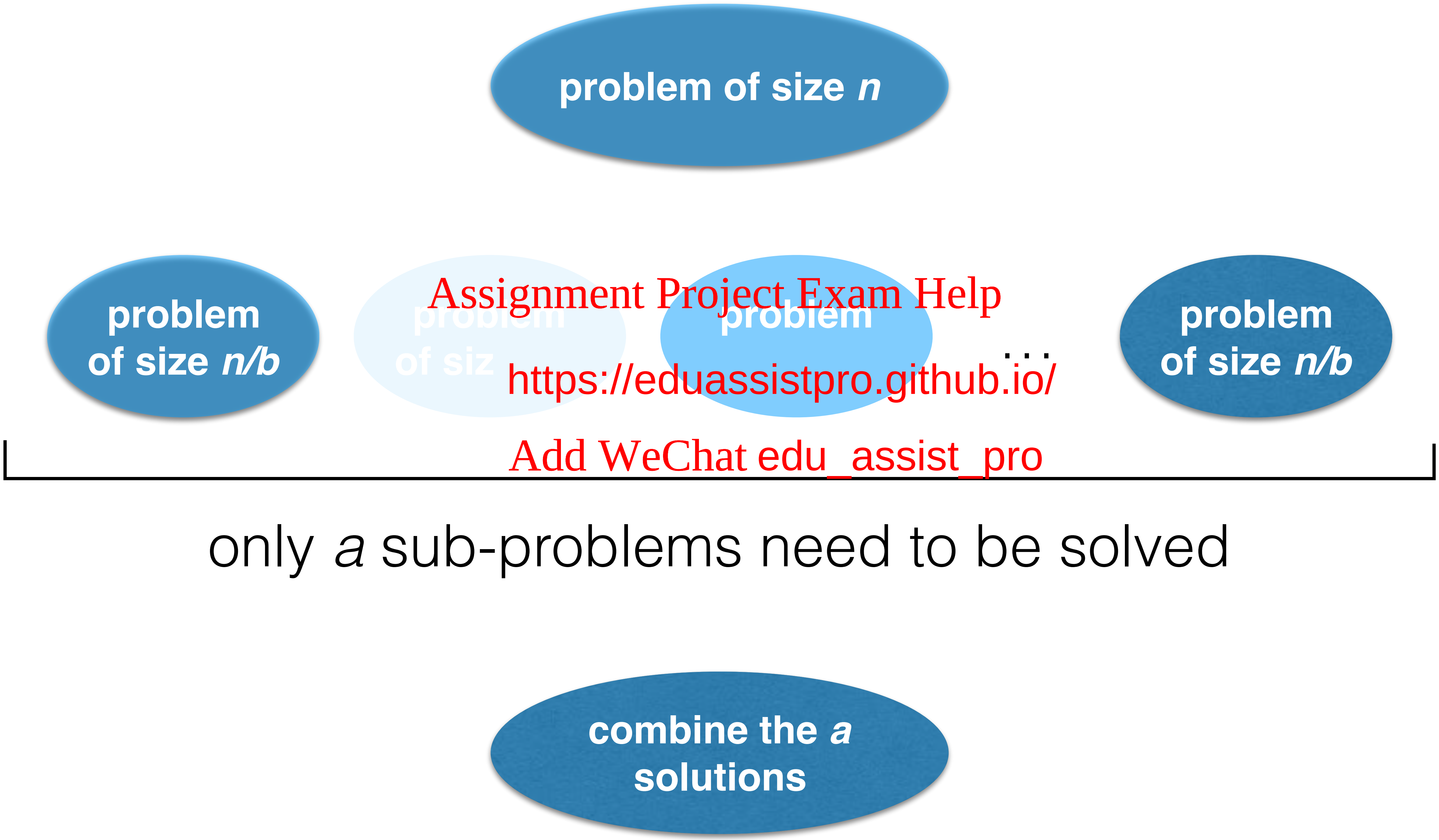
Divide-and-Conqer General Case



problem of size *n*

problem of size *n/b*

problem of siz

problem

. . .

problem of size *n/b*

only *a* sub-problems need to be solved

combine the *a* solutions

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Divide-and-Conquer Recurrences

- What is the time required to solve a problem of size *n* by divide-and-conquer?

- For the general case, assume we split the problem into *b* instances (each of size *n/b*), of which *a* need to be solved:

$$T(\qquad n)$$

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where *f*(*n*) expresses the time spent on dividing a problem into *b* sub-problems and combining the *a* results.

- (A very common case is $T(n) = 2T(n/2) + n$.)

- How to find closed forms for these recurrences?

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The Master Theorem

- (A proof is in Levitin's Appendix B.)

- For integer constants $a \geq 1$ and $b > 1$, and function f with $f(n) \in \Theta(n^d)$, $d \geq 0$, the recurrence

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$$T(\qquad n)$$

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(with T(1) = c) has solution

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$$T(n) = \begin{cases} \Theta(n^d) & \text{if } a < b^d \\ \Theta(n^d \log n) & \text{if } a = b^d \\ \Theta(n^{\log_b a}) & \text{if } a > b^d \end{cases}$$

- Note that we also allow *a* to be greater than *b*.

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Master Theorem: Example 1

$T(n) = 2T(n/2) + n$ a = 2, b = 2, d = 1

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Master Theorem: Example 1

$T(n) = 2T(n/2) + n$

$a = 2, b = 2, d = 1$

$a = b^d$

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So, by the Master Theorem, $T(n) \in \Theta(n \log n)$

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Master Theorem: Example 1

$T(n) = 2T(n/2) + n$ $a = 2, b = 2, d = 1$

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Master Theorem: Example 1

$T(n) = 2T(n/2) + n$ a = 2, b = 2, d = 1

$T(n) = 2(2T(n/4) + (n/2)) + n$

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Master Theorem: Example 1

$T(n) = 2T(n/2) + n$ a = 2, b = 2, d = 1

$T(n) = 4T(n/4) + 2(n/2) + n$

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Master Theorem: Example 1

$T(n) = 2T(n/2) + n$ a = 2, b = 2, d = 1

$T(n) = 4(2T(n/8) + n/4) + 2(n/2) + n$

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Master Theorem: Example 1

$T(n) = 2T(n/2) + n$ a = 2, b = 2, d = 1

$T(n) = 8T(n/8) + 4(n/4) + 2(n/2) + n$

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Master Theorem: Example 1

$T(n) = 2T(n/2) + n$ a = 2, b = 2, d = 1

$T(n) = 8(2T(n/16) + n/8) + 4(n/4) + 2(n/2) + n$

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Master Theorem: Example 1

$T(n) = 2T(n/2) + n$ a = 2, b = 2, d = 1

$T(n) = 16T(n/16) + 8(n/8) + 4(n/4) + 2(n/2) + n$

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Master Theorem: Example 1

$T(n) = 2T(n/2) + n$ a = 2, b = 2, d = 1

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Master Theorem: Example 1

$T(n) = 2T(n/2) + n$ 　　　　 a = 2, b = 2, d = 1

$T(n) \in \Theta(n \log n)$

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Master Theorem: Example 2

$T(n) = 4T(n/4) + n$

a = 4, b = 4, d = 1

$a = b^d$

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So, by the Master Theorem, $T(n) \in \Theta(n \log n)$

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Master Theorem: Example 2

$T(n) = 4T(n/4) + n$ $\qquad$ $a = 4, b = 4, d = 1$

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Master Theorem: Example 2

$T(n) = 4T(n/4) + n$ a = 4, b = 4, d = 1

$T(n) = 4(4T(n/16) + (n/4)) + n$

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Master Theorem: Example 2

$T(n) = 4T(n/4) + n$ a = 4, b = 4, d = 1

$T(n) = 16T(n/16) + 4(n/4) + n$

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Master Theorem: Example 2

$T(n) = 4T(n/4) + n$ a = 4, b = 4, d = 1

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Master Theorem: Example 2

$T(n) = 4T(n/4) + n$ a = 4, b = 4, d = 1

$T(n) = 16(4T(n/64) + n/16)\ 4(n/4) + n$

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Master Theorem: Example 2

$T(n) = 4T(n/4) + n$ a = 4, b = 4, d = 1

$T(n) = 64T(n/64) + 16(n/16) + 4(n/4) + n$

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Master Theorem: Example 2

$T(n) = 4T(n/4) + n$ a = 4, b = 4, d = 1

$T(n) = 64T(n/64) + 16(n/16) + 4(n/4) + n$

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$\vdots$

(log$_4$ n times)

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Master Theorem: Example 2

$T(n) = 4T(n/4) + n$ a = 4, b = 4, d = 1

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Master Theorem: Example 2

$T(n) = 4T(n/4) + n$ a = 4, b = 4, d = 1

$T(n) = 64T(n/64) + 16(n/16) + 4(n/4) + n$

$T(n) \in \Theta(n \log n)$

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Master Theorem: Example 3

$T(n) = T(n/2) + n$ $\qquad$ a = 1, b = 2, d = 1

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Master Theorem: Example 3

$T(n) = T(n/2) + n$

$a = 1,\ b = 2,\ d = 1$

$a < b^d$

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So, by the Master Theorem, $T(n) \in \Theta(n)$

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Master Theorem: Example 3

$T(n) = T(n/2) + n$ $\qquad$ $a = 1, b = 2, d = 1$

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Master Theorem: Example 3

$T(n) = T(n/2) + n$ a = 1, b = 2, d = 1

$T(n) = T(n/4) + n/2 + n$

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Master Theorem: Example 3

$T(n) = T(n/2) + n$ a = 1, b = 2, d = 1

$T(n) = T(n/8) + n/4 + n/2 + n$

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Master Theorem: Example 3

$T(n) = T(n/2) + n$ a = 1, b = 2, d = 1

$T(n) = T(n/8) + n/4 + n/2 + n$

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Master Theorem: Example 3

$T(n) = T(n/2) + n$ a = 1, b = 2, d = 1

$T(n) = T(n/8) + n/4 + n/2 + n$

$T(n) \in \Theta(n)$

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Master Theorem: Example 4

$T(n) = 2T(n/2) + n^2$ a = 2, b = 2, d = 2

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Master Theorem: Example 4

$T(n) = 2T(n/2) + n^2$

$a = 2, b = 2, d = 2$

$a < b^d$

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So, by the Master Theorem, $T(n) \in \Theta(n^2)$

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Master Theorem: Example 4

$T(n) = 2T(n/2) + n^2$ a = 2, b = 2, d = 2

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Master Theorem: Example 4

$T(n) = 2T(n/2) + n^2$ a = 2, b = 2, d = 2

$T(n) = 2(2T(n/4) + (n/2)^2) + n^2$

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Master Theorem: Example 4

$T(n) = 2T(n/2) + n^2$ a = 2, b = 2, d = 2

$T(n) = 4T(n/4) + 2(n/2)^2 + n^2$

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Master Theorem: Example 4

$T(n) = 2T(n/2) + n^2$ a = 2, b = 2, d = 2

$T(n) = 4(2T(n/8) + (n/4)^2) + 2(n/2)^2 + n^2$

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Master Theorem: Example 4

$T(n) = 2T(n/2) + n^2$ $\qquad$ $a = 2,\ b = 2,\ d = 2$

$T(n) = 8T(n/8) + 4(n/4)^2 + 2(n/2)^2 + n^2$

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Master Theorem: Example 4

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$T(n) = 2T(n/2) + n^2$ a = 2, b = 2, d = 2

$T(n) = 8T(n/8) + 4(n/4)^2 + 2(n/2)^2 + n^2$

$T(n) \in \Theta(n^2)$

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Mergesort

- Perhaps the most obvious application of divide-and-conquer:
- To sort an array (or a list), cut it into two halves, sort each half, and merge the two results.

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Mergesort

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Mergesort

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Mergesort

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Mergesort

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Mergesort: Merging Arrays

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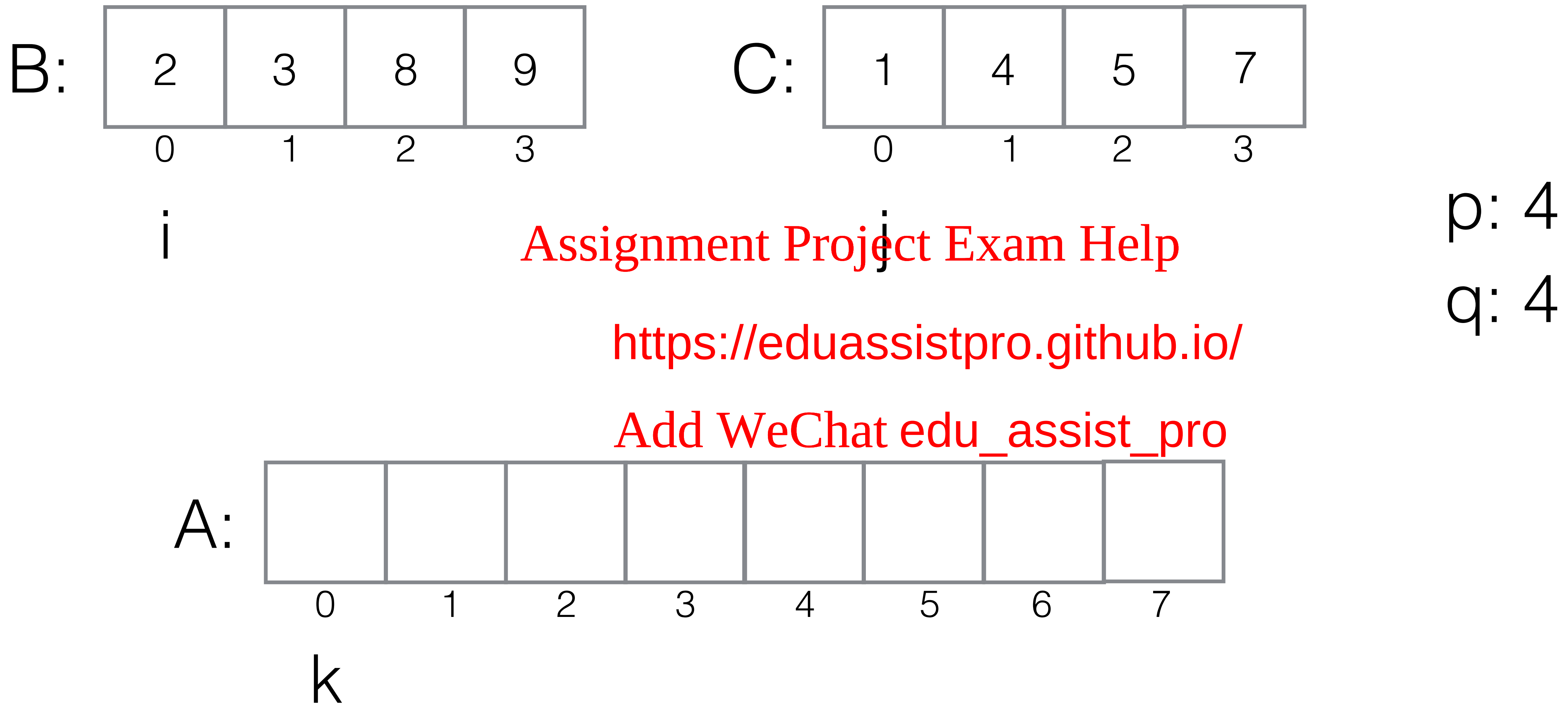
Mergesort: Merging Arrays



B:

2	3	8	9
0	1	2	3

C:

1	4	5	7
0	1	2	3

p: 4

q: 4

A:

0	1	2	3	4	5	6	7

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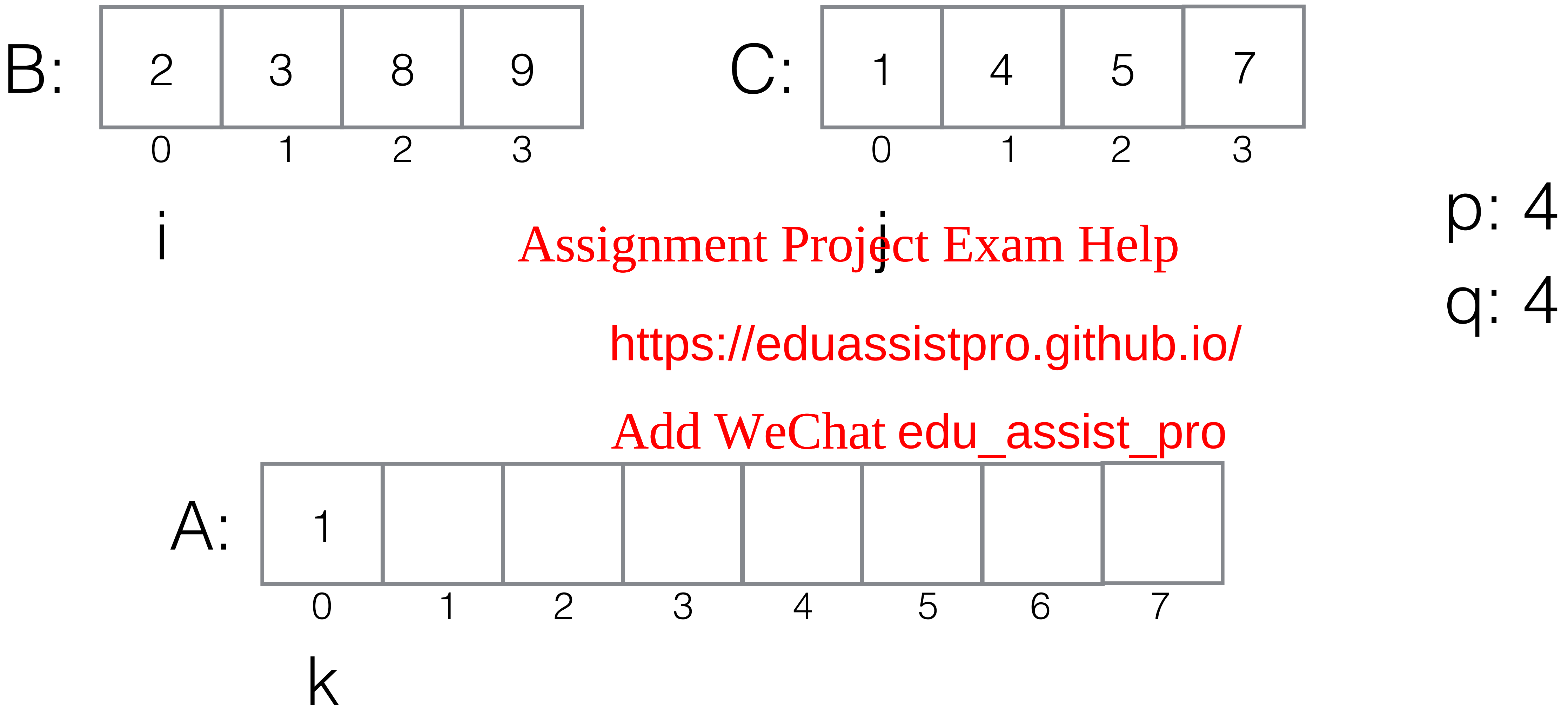
Mergesort: Merging Arrays



B:

2	3	8	9
0	1	2	3

i

C:

1	4	5	7
0	1	2	3

j

p: 4

q: 4

A:

1							
0	1	2	3	4	5	6	7

k

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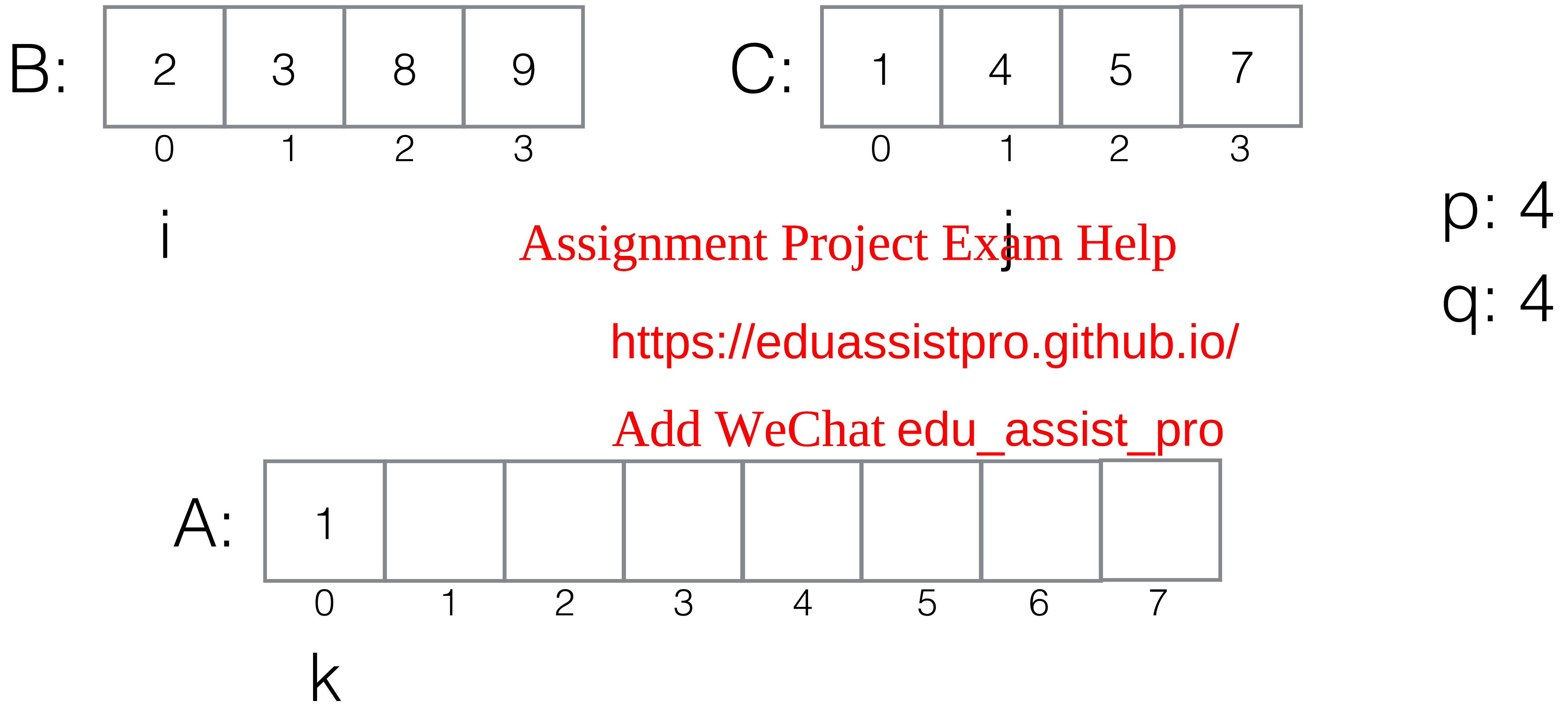
Mergesort: Merging Arrays



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1							

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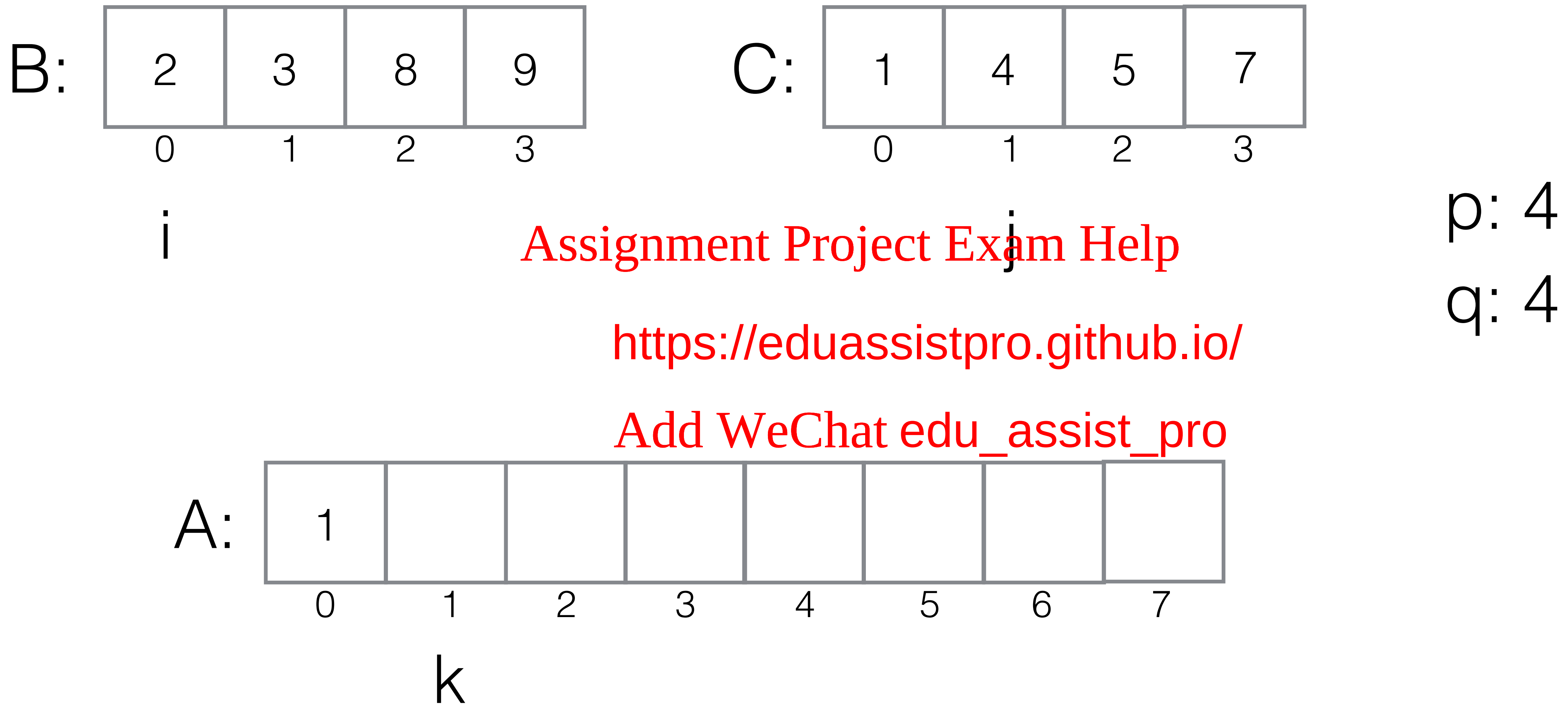
Mergesort: Merging Arrays



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1							

k

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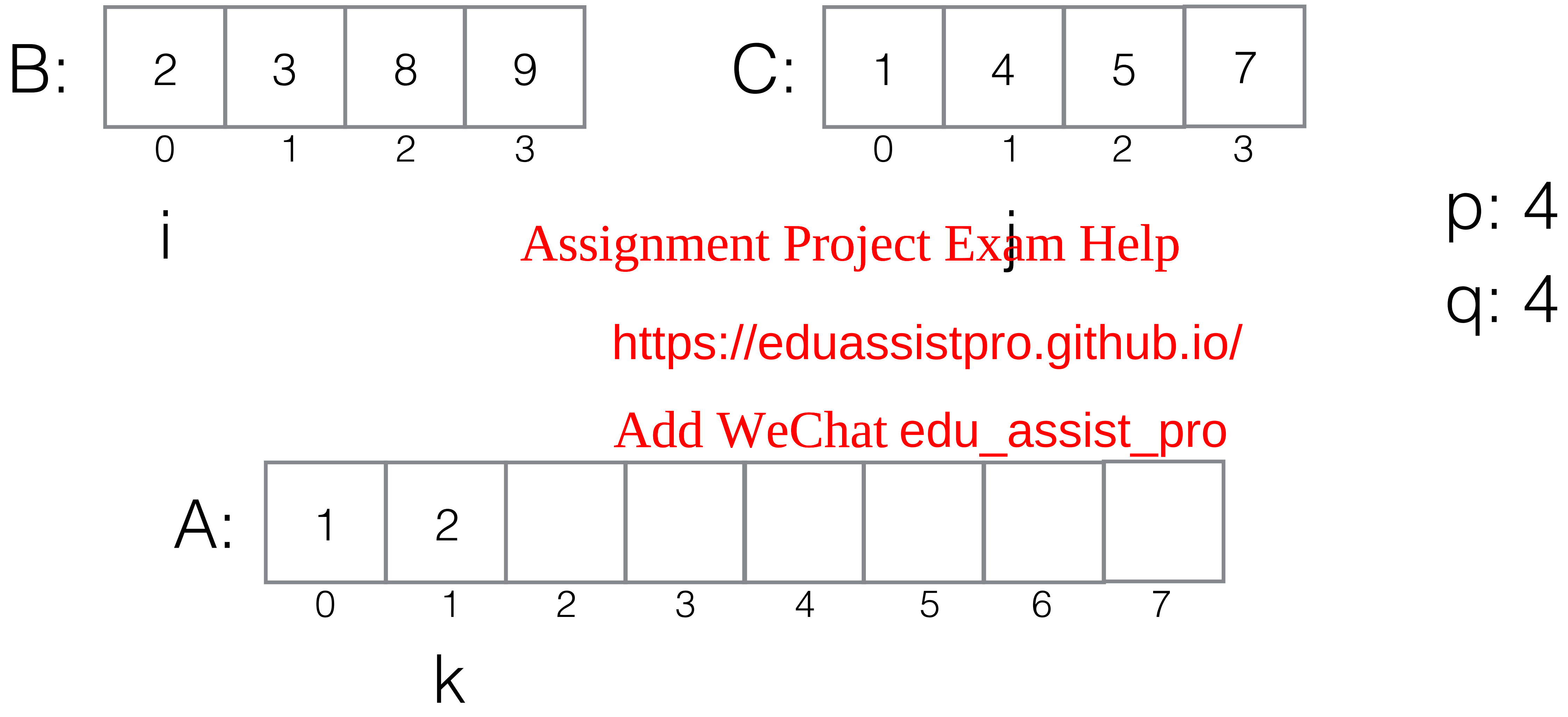
Mergesort: Merging Arrays



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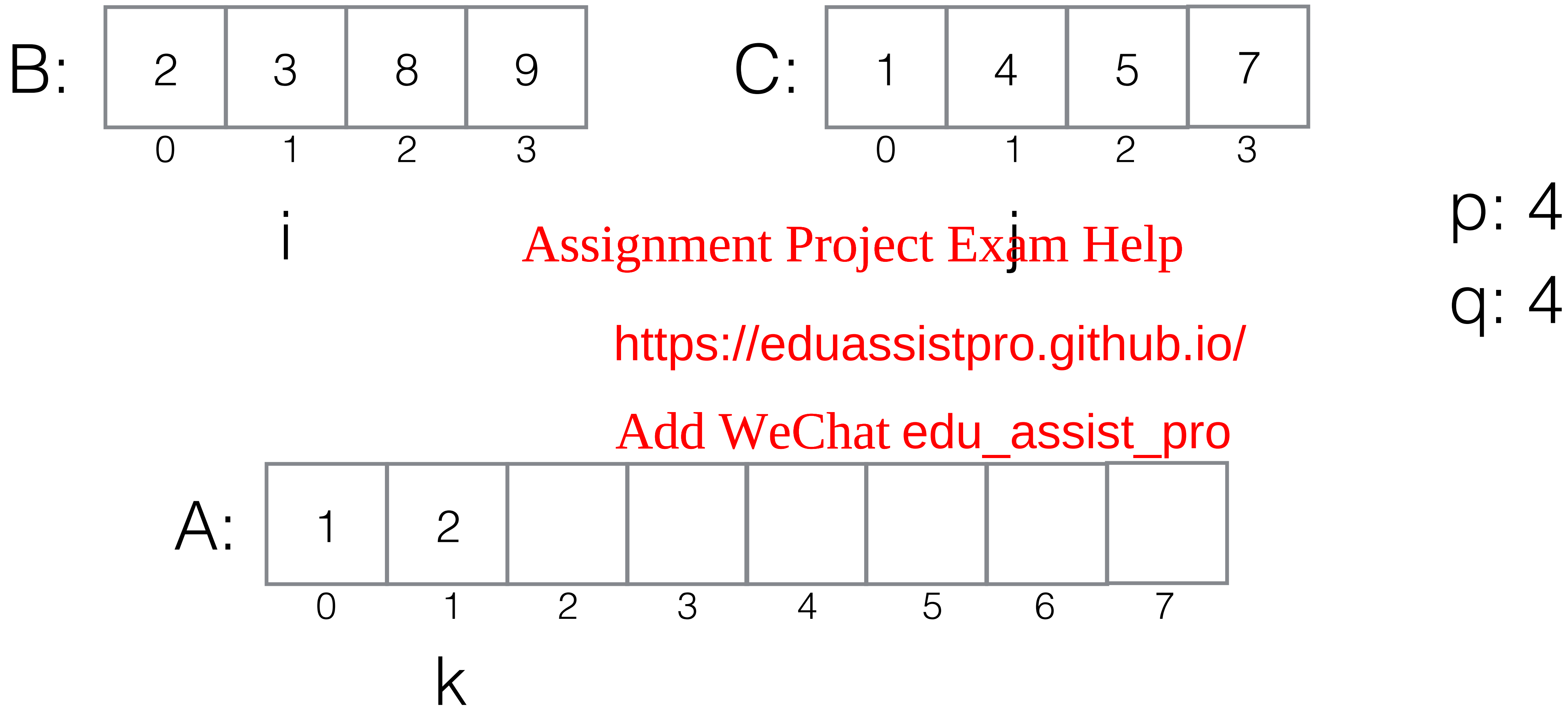
Mergesort: Merging Arrays



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0	1	2	3	4	5	6	7
	k						

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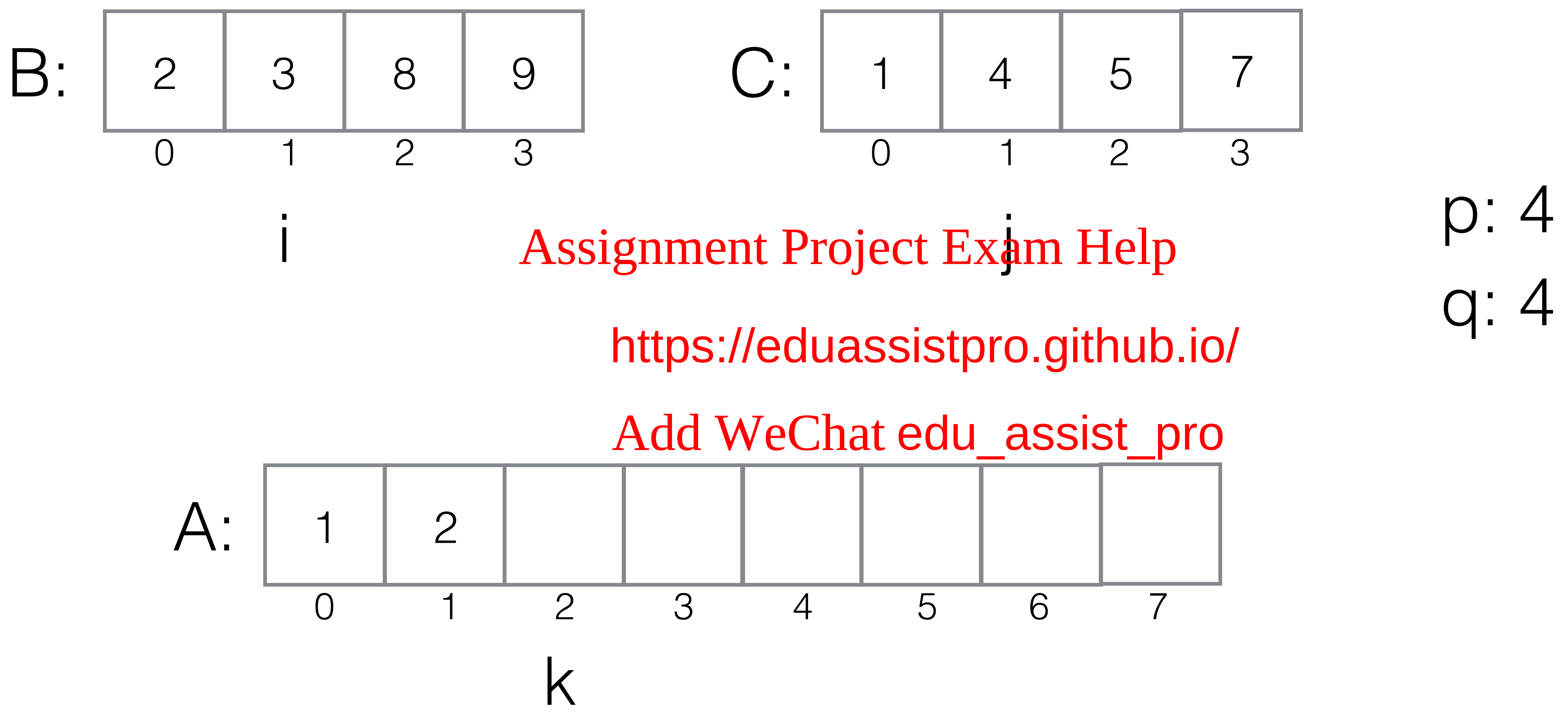
Mergesort: Merging Arrays



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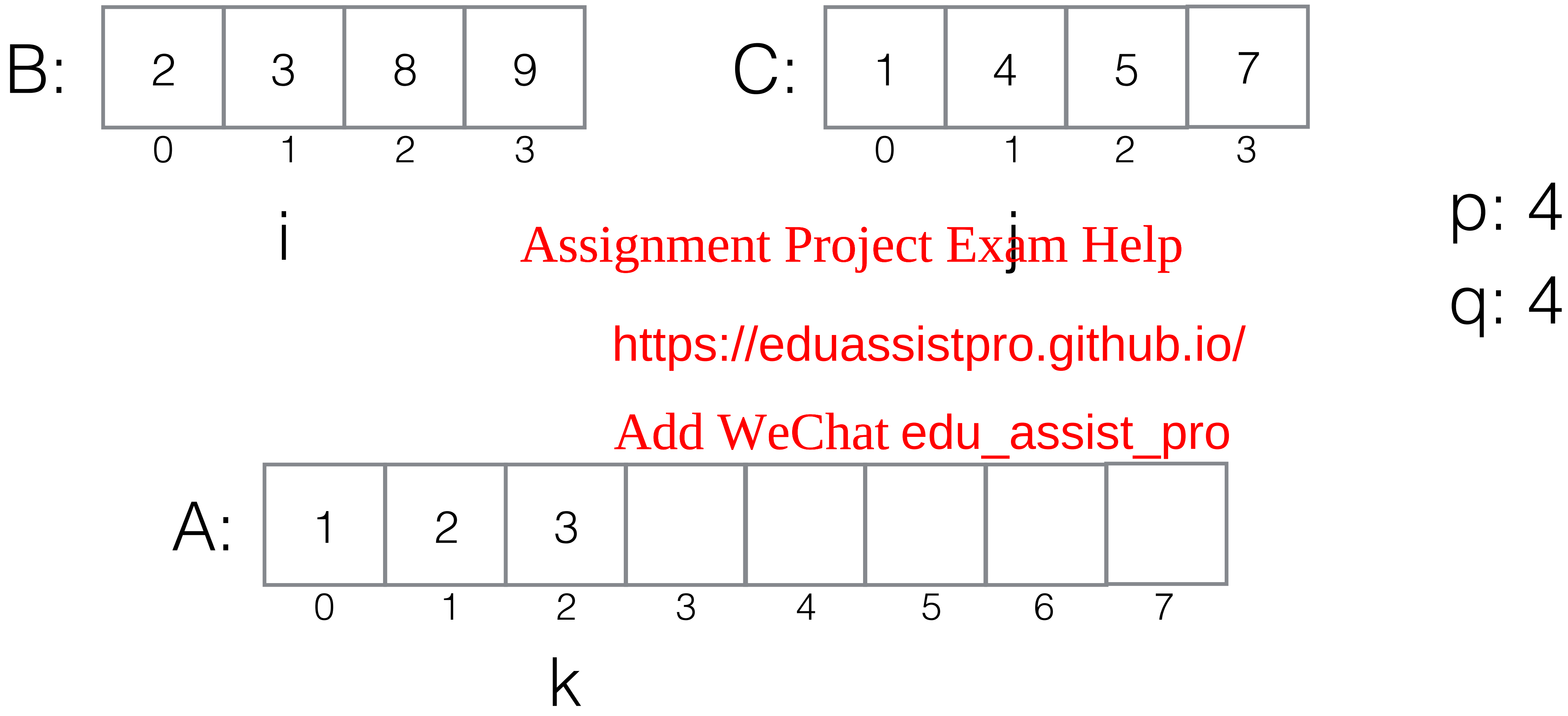
Mergesort: Merging Arrays



B:

2	3	8	9
0	1	2	3

i

C:

1	4	5	7
0	1	2	3

j

p: 4

q: 4

A:

1	2	3					
0	1	2	3	4	5	6	7

k

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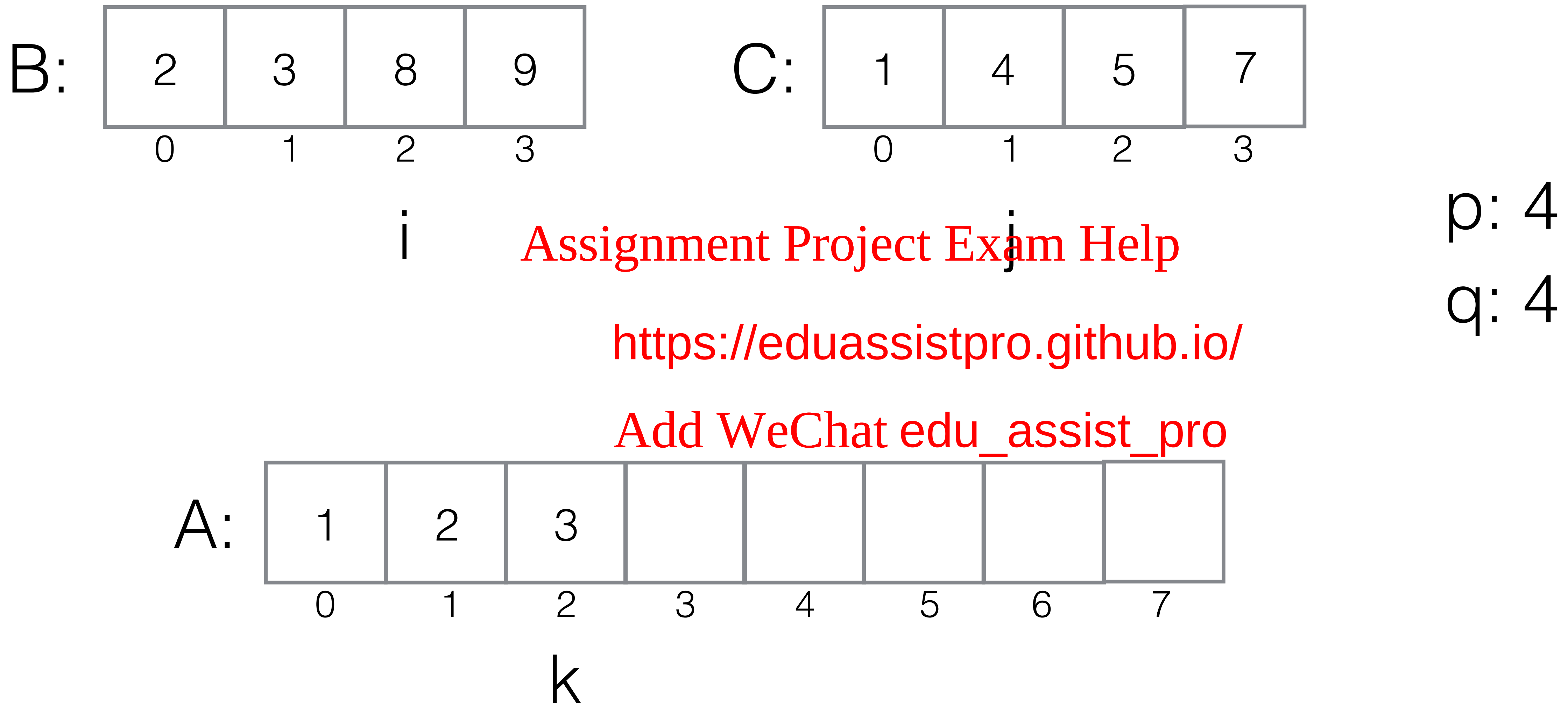

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0	1	2	3

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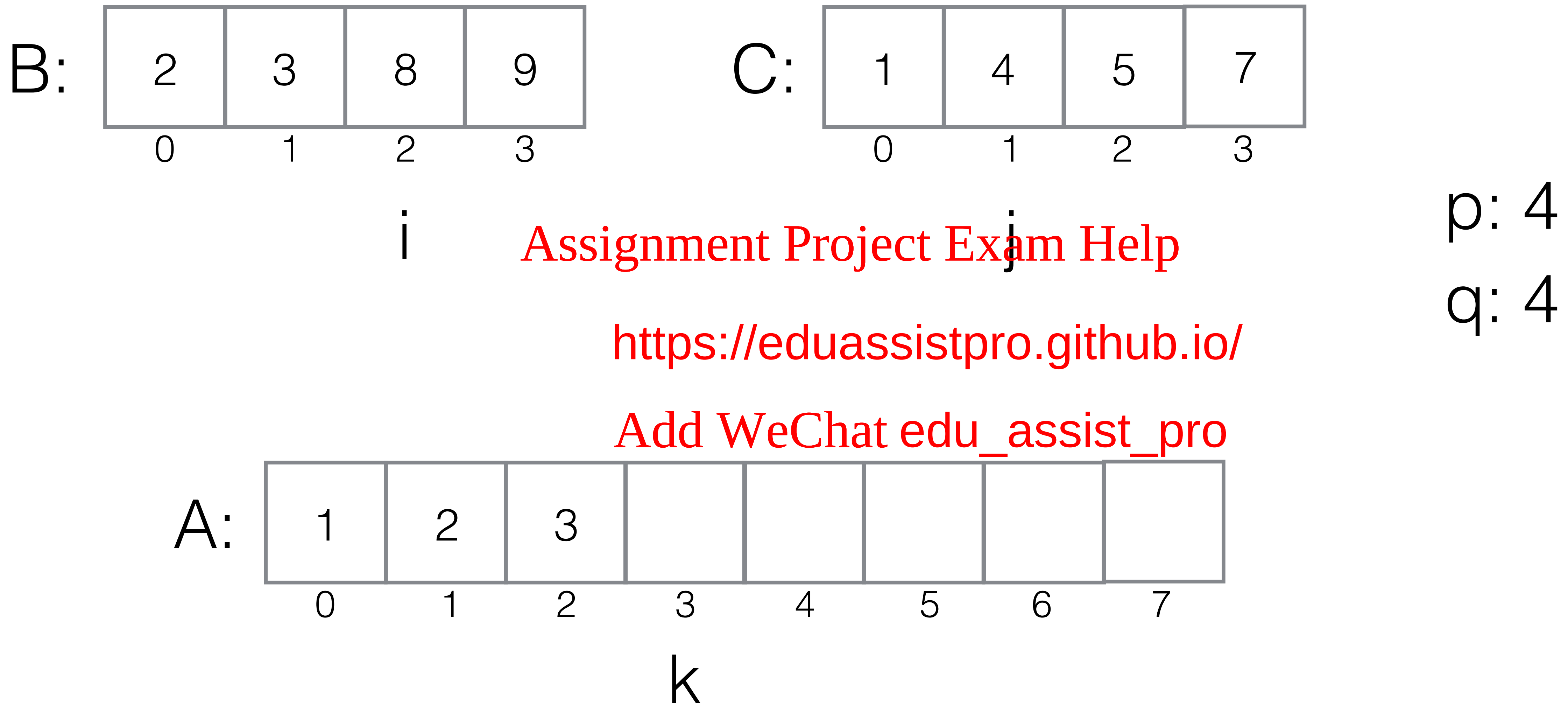

B:

0	1	2	3
2	3	8	9

C:

0	1	2	3
1	4	5	7

p: 4

q: 4

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0	1	2	3	4	5	6	7
1	2	3					

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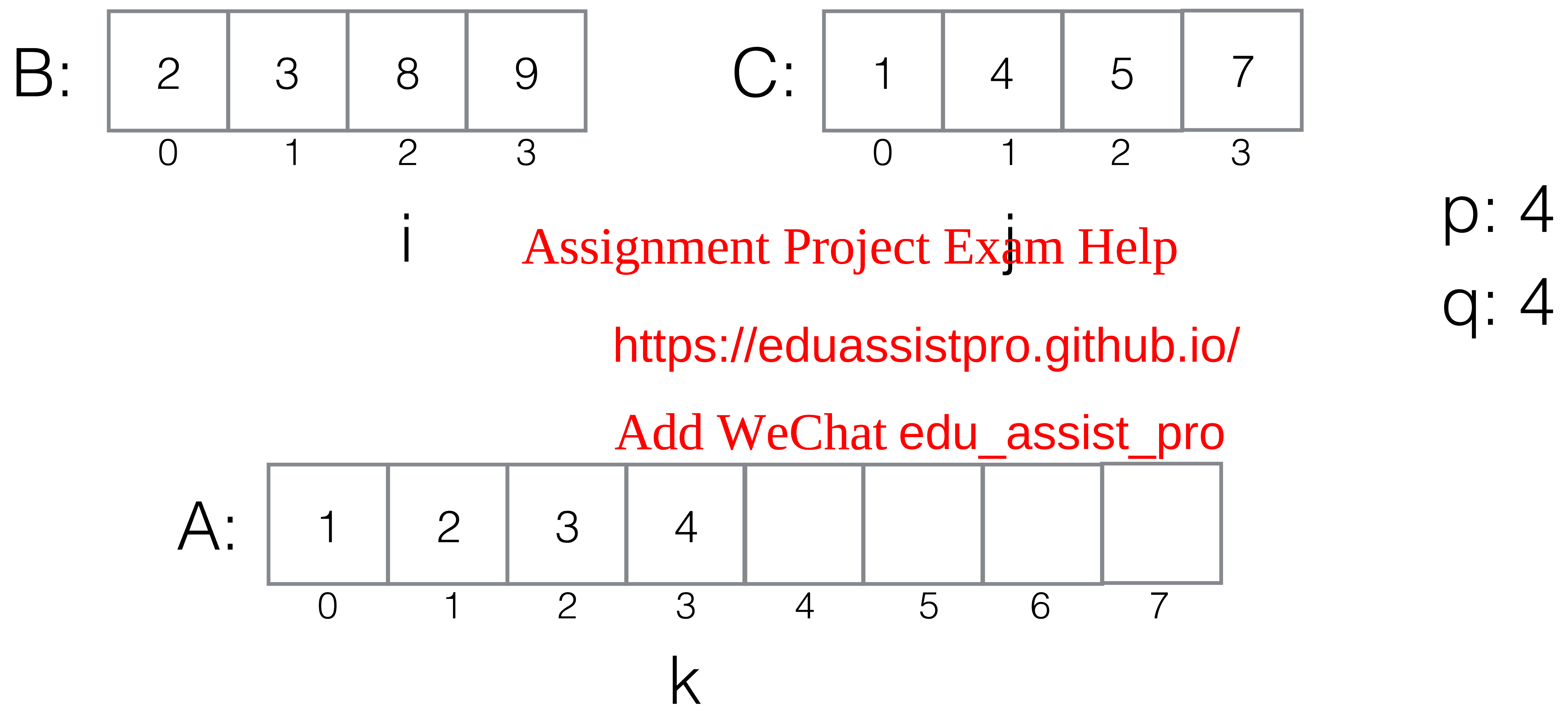

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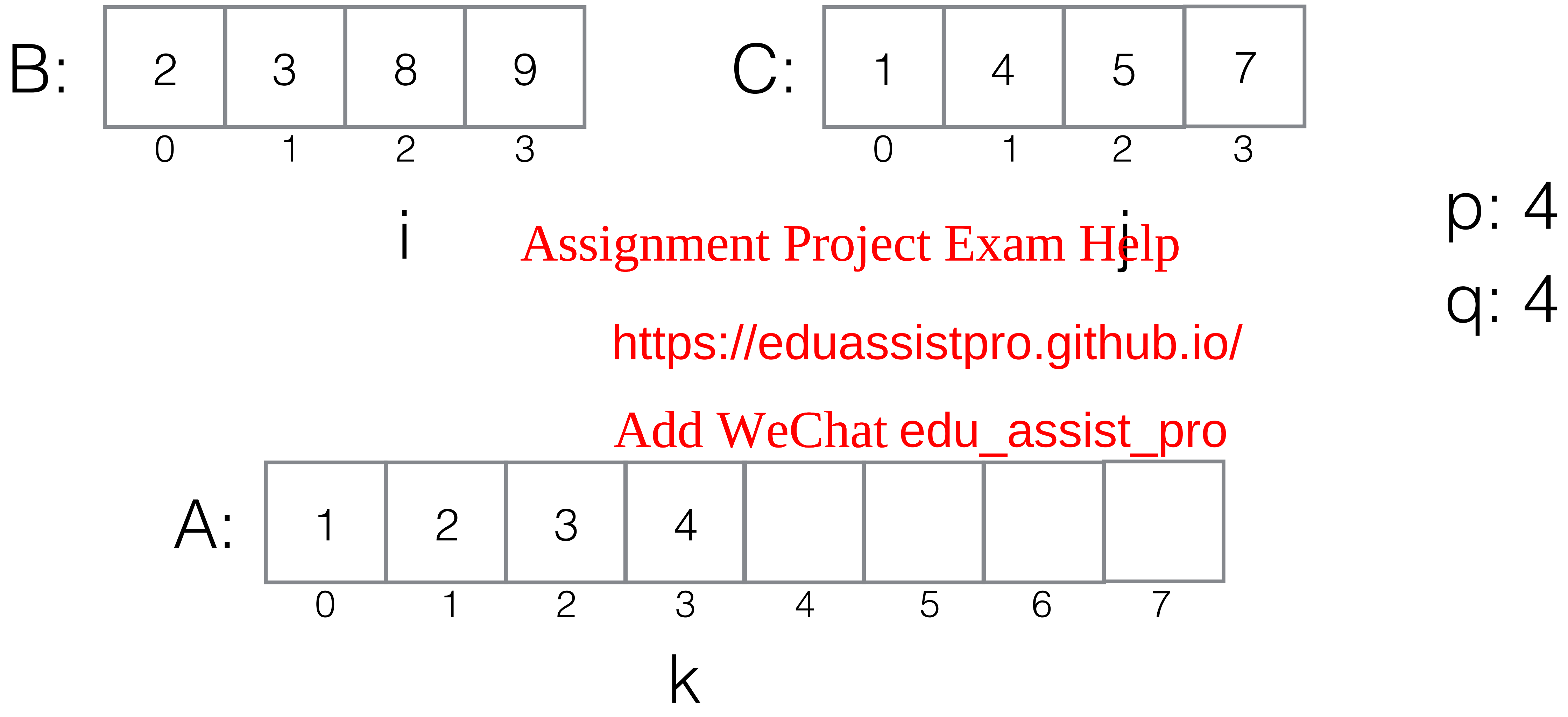

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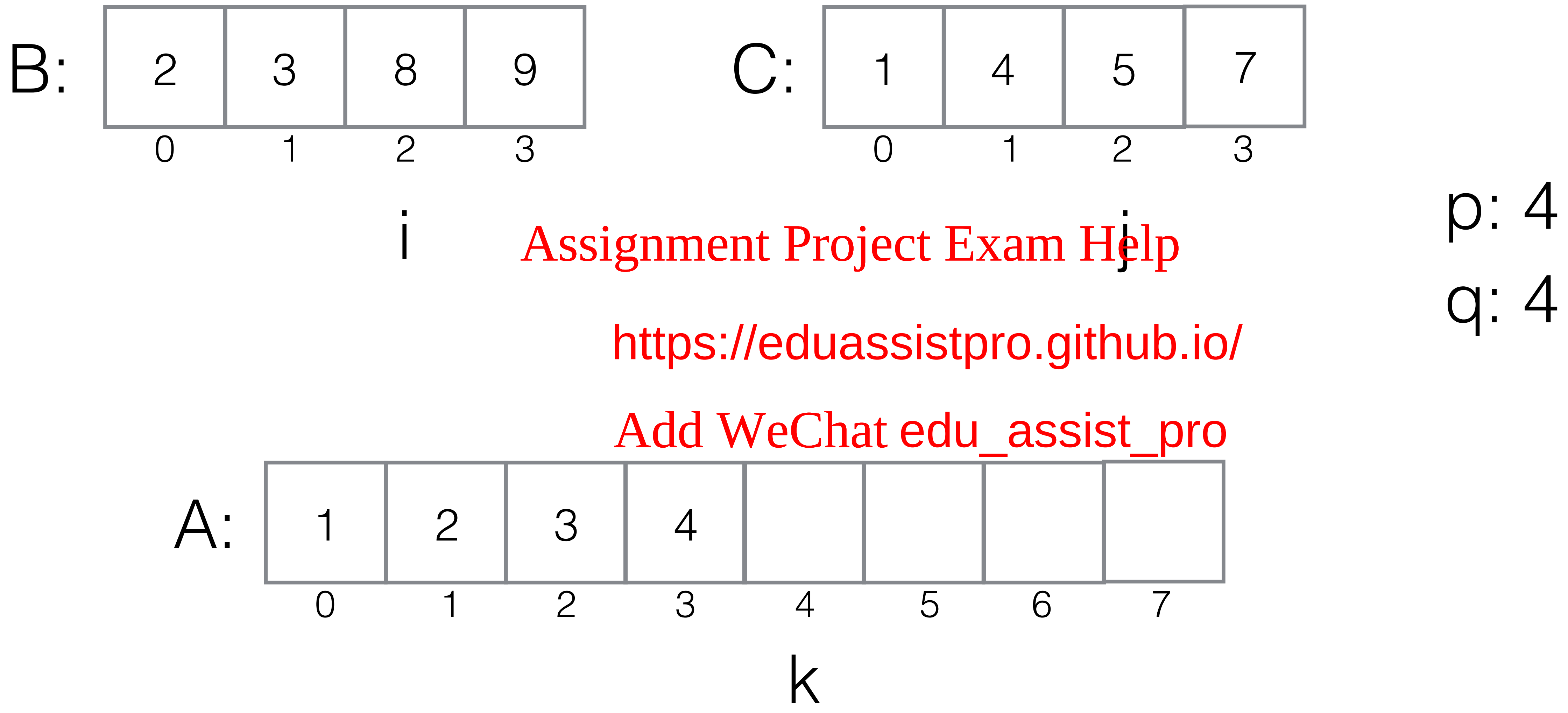

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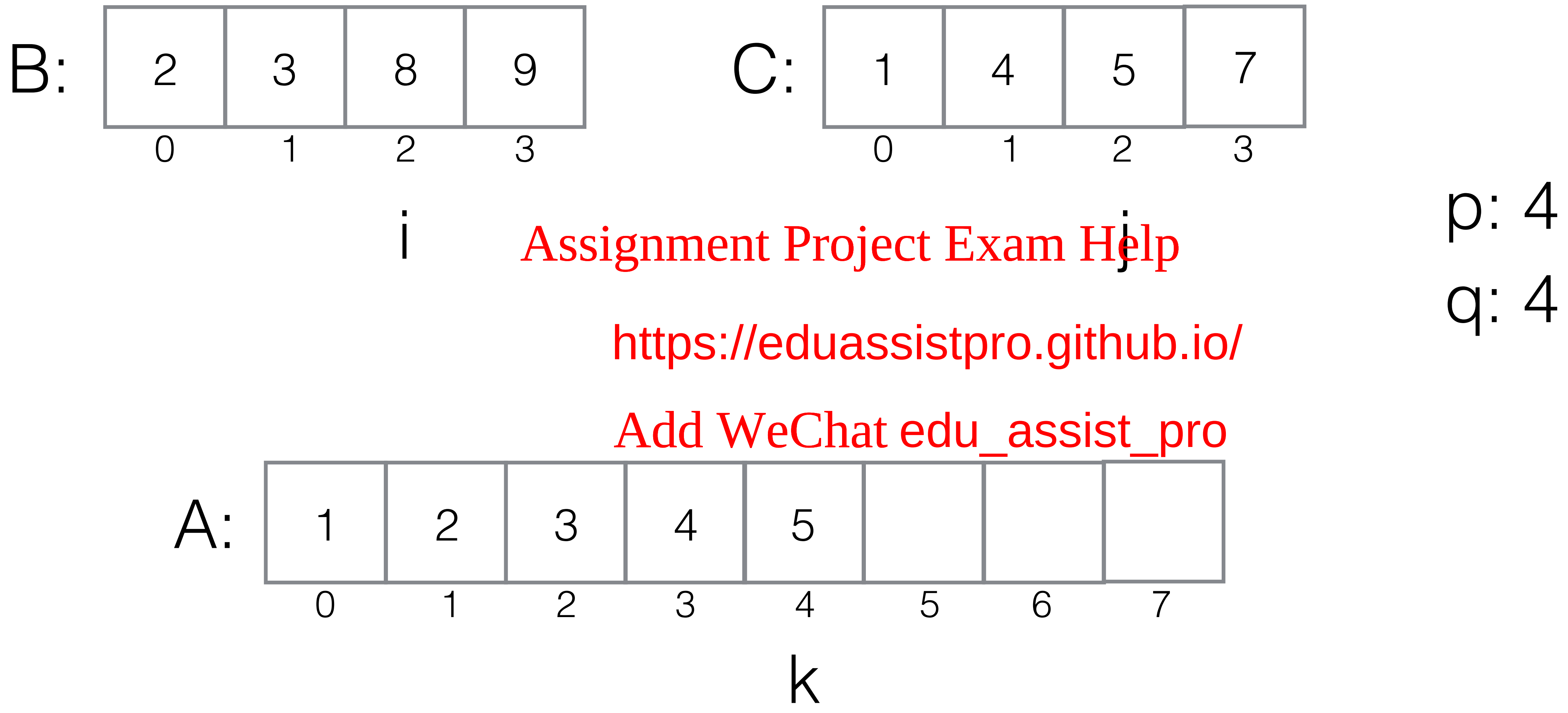

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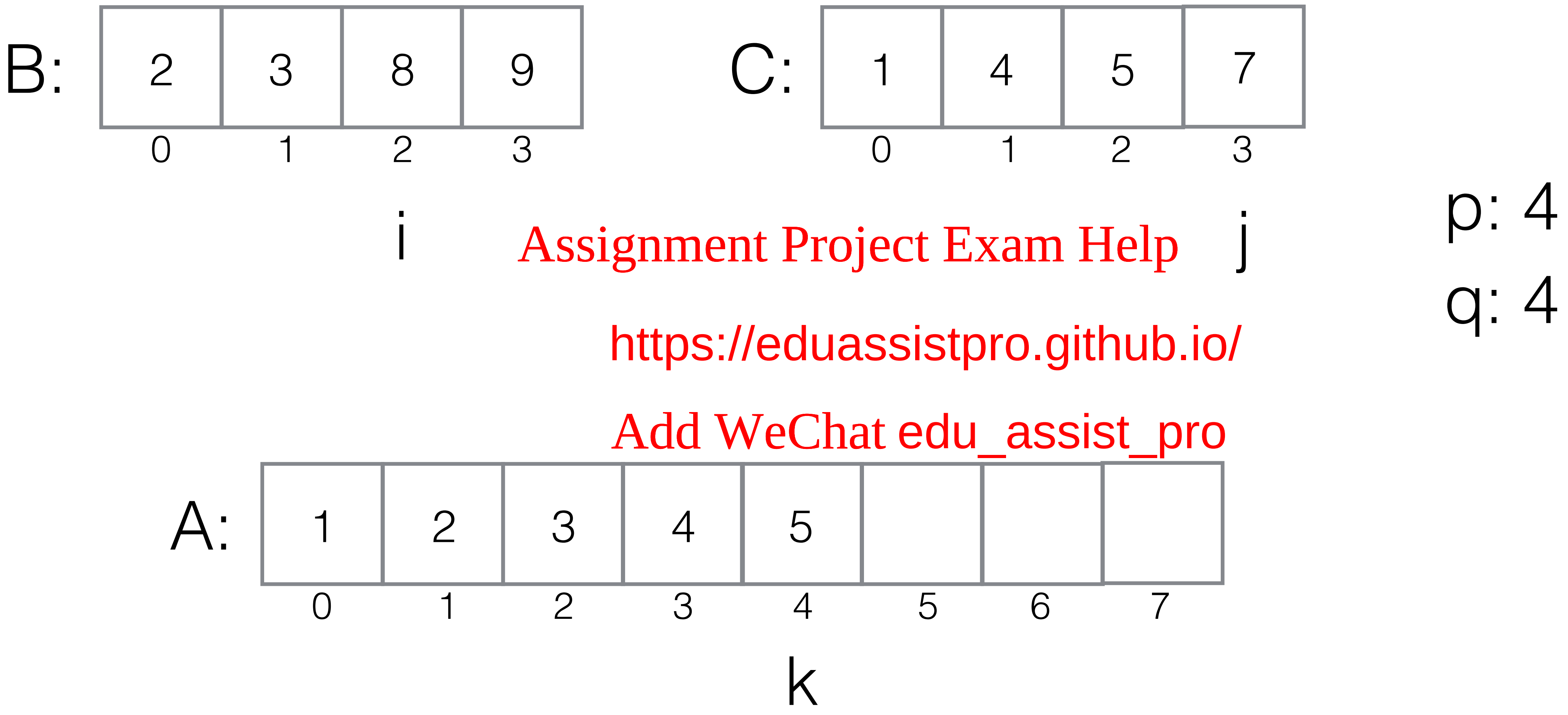

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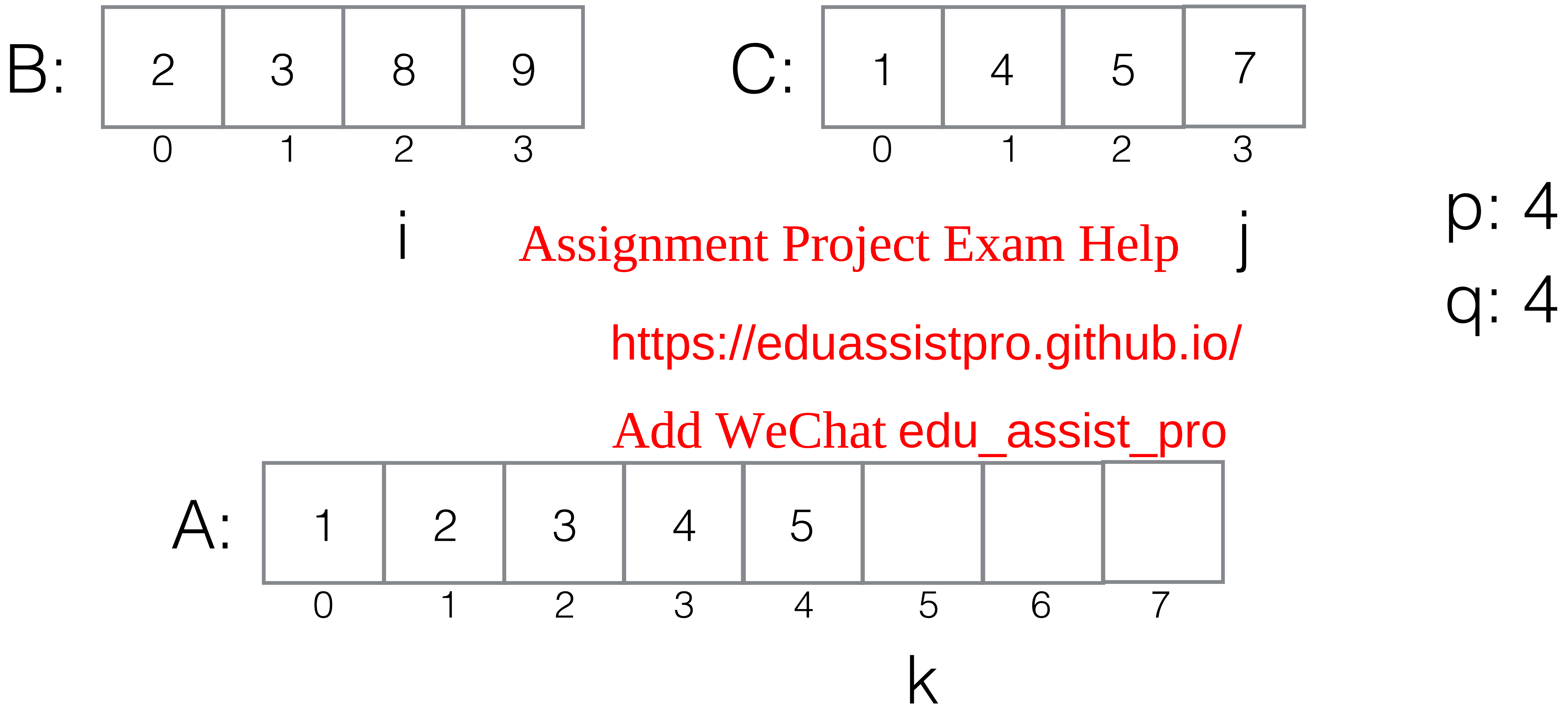

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1	2	3	4	5			

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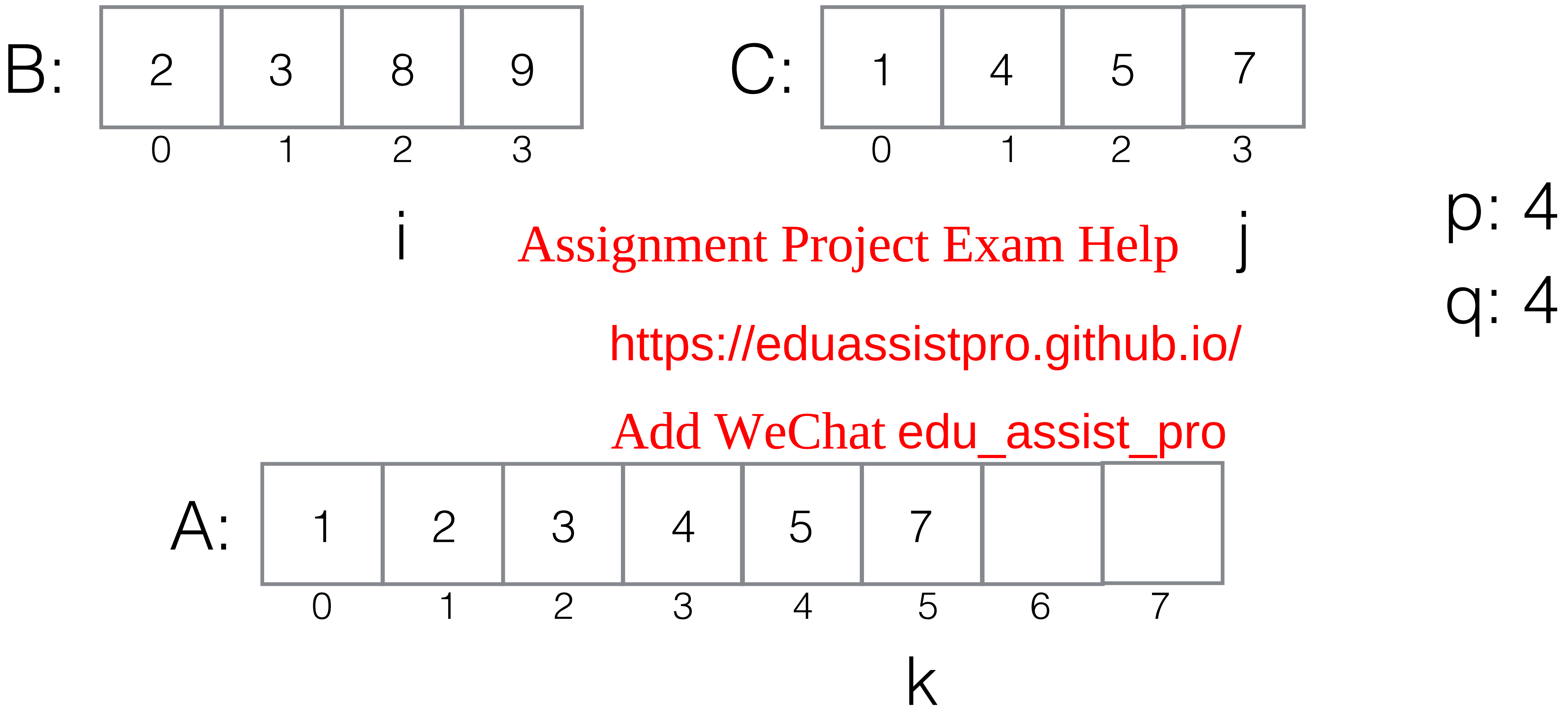
Mergesort: Merging Arrays



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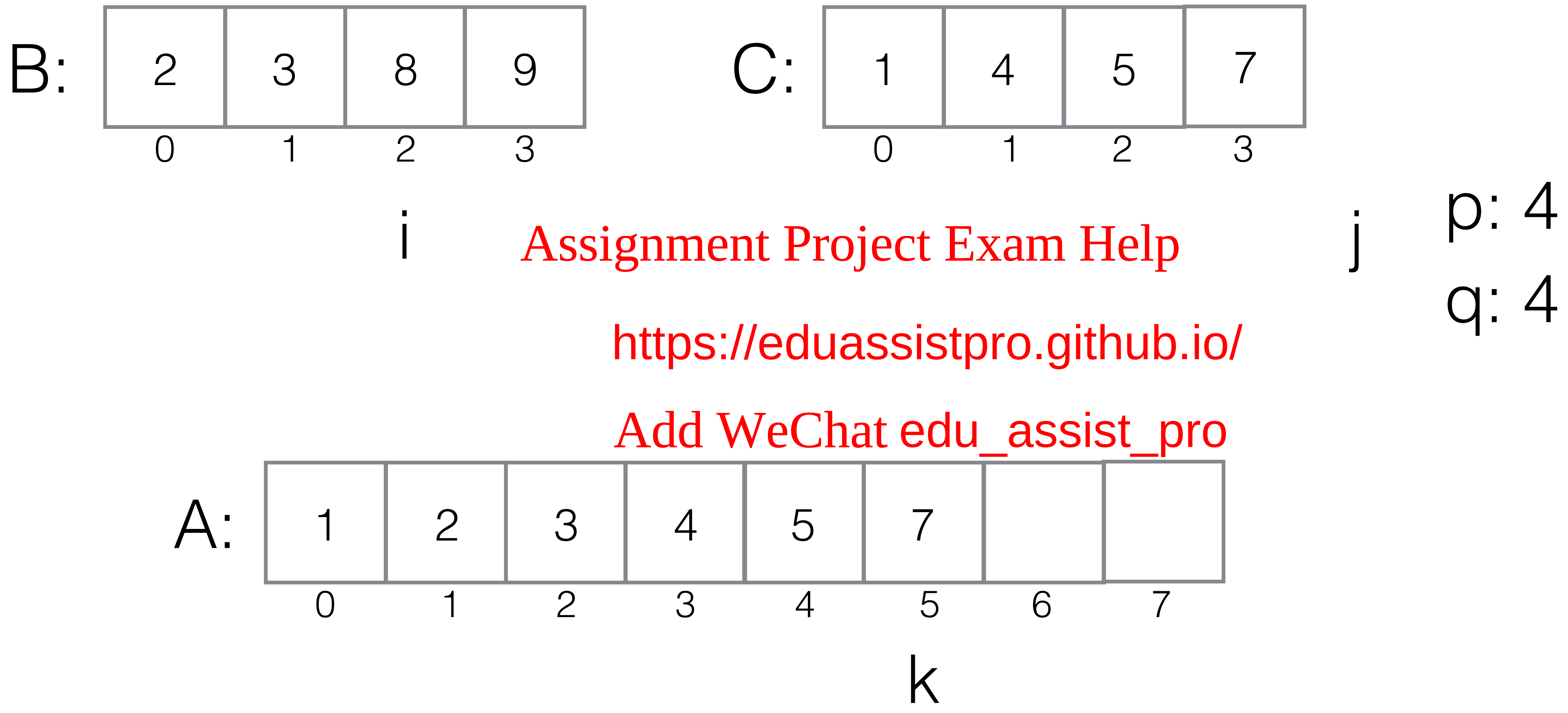

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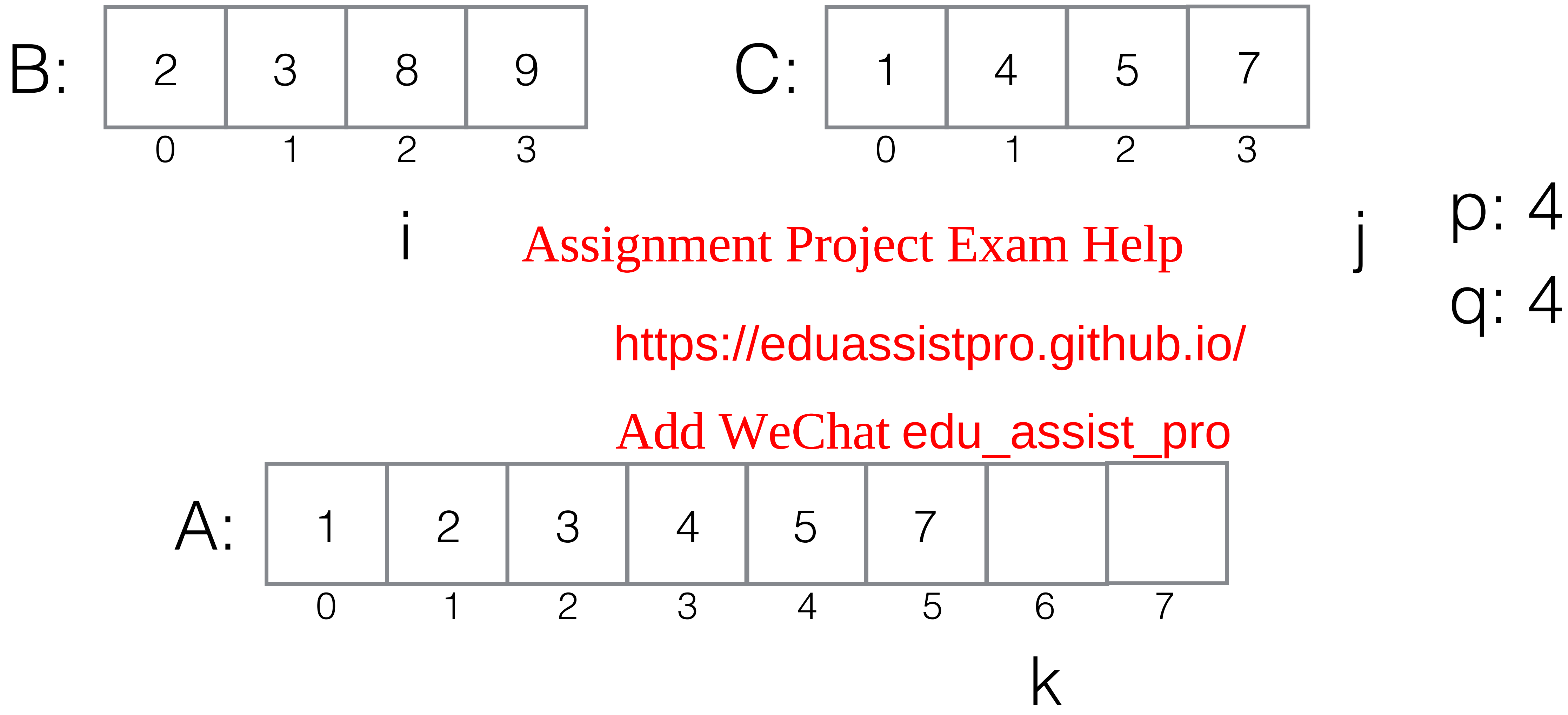

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Mergesort: Merging Arrays

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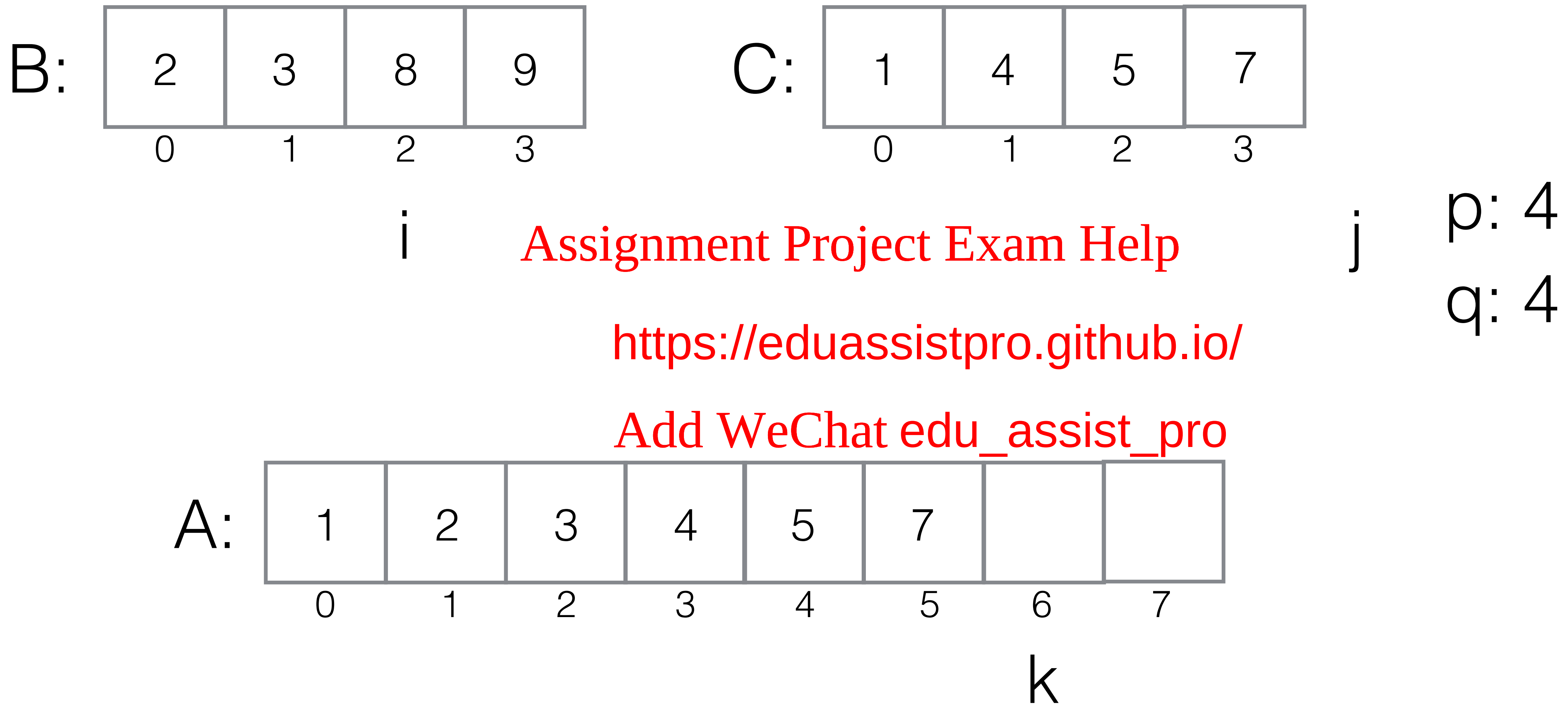

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	0	1	2	3

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q: 4

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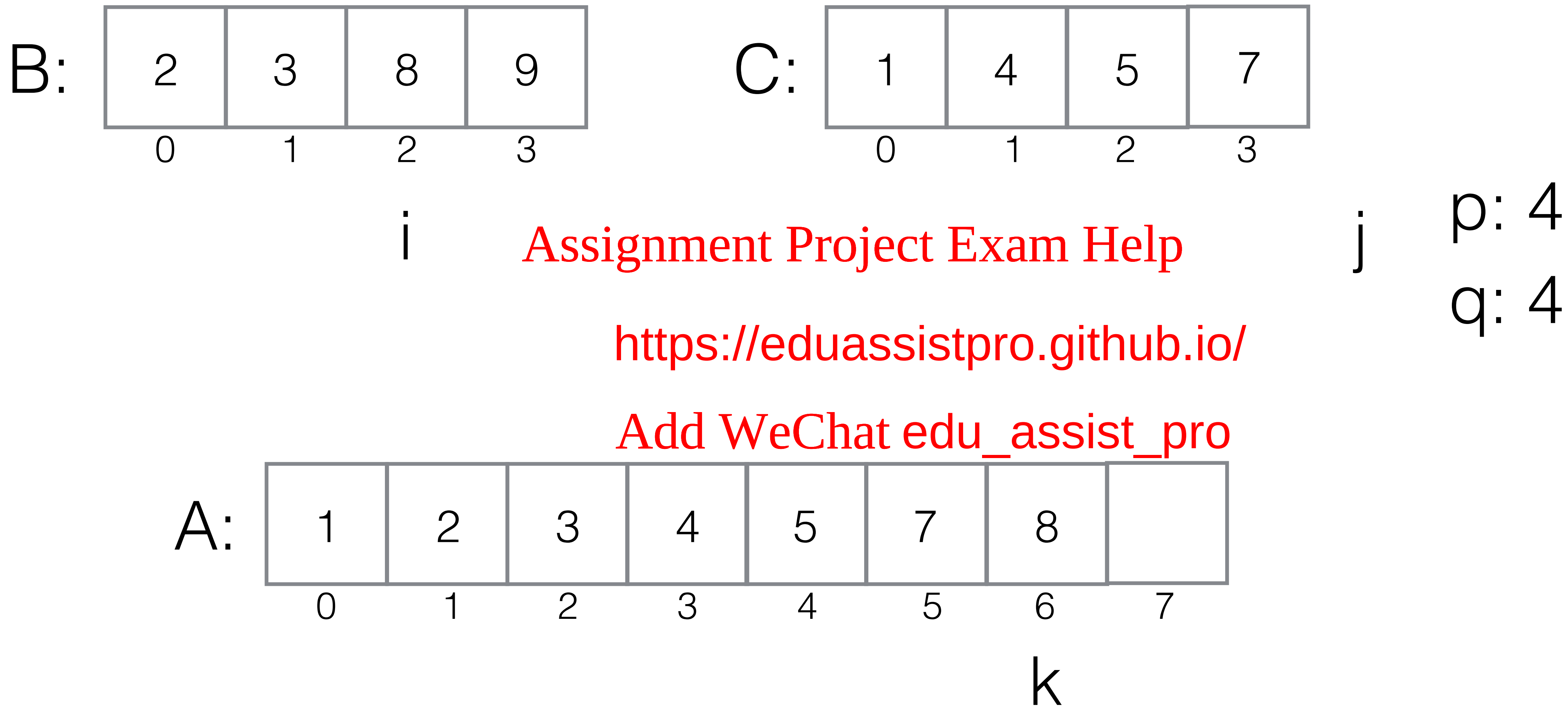

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C:

1	4	5	7
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Mergesort: Merging Arrays

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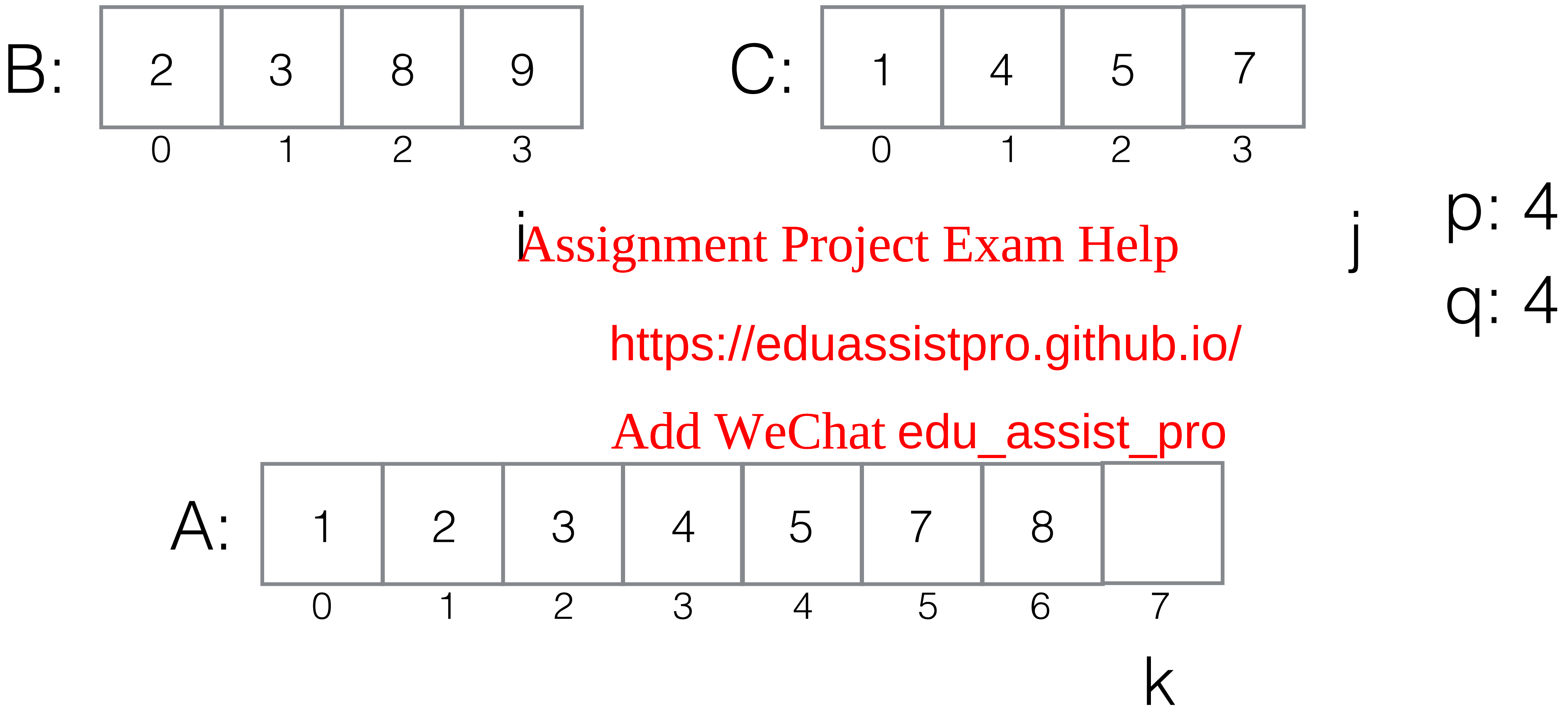
Mergesort: Merging Arrays



B:

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2	3	8	9

C:

0	1	2	3
1	4	5	7

A:

0	1	2	3	4	5	6	7
1	2	3	4	5	7	8	

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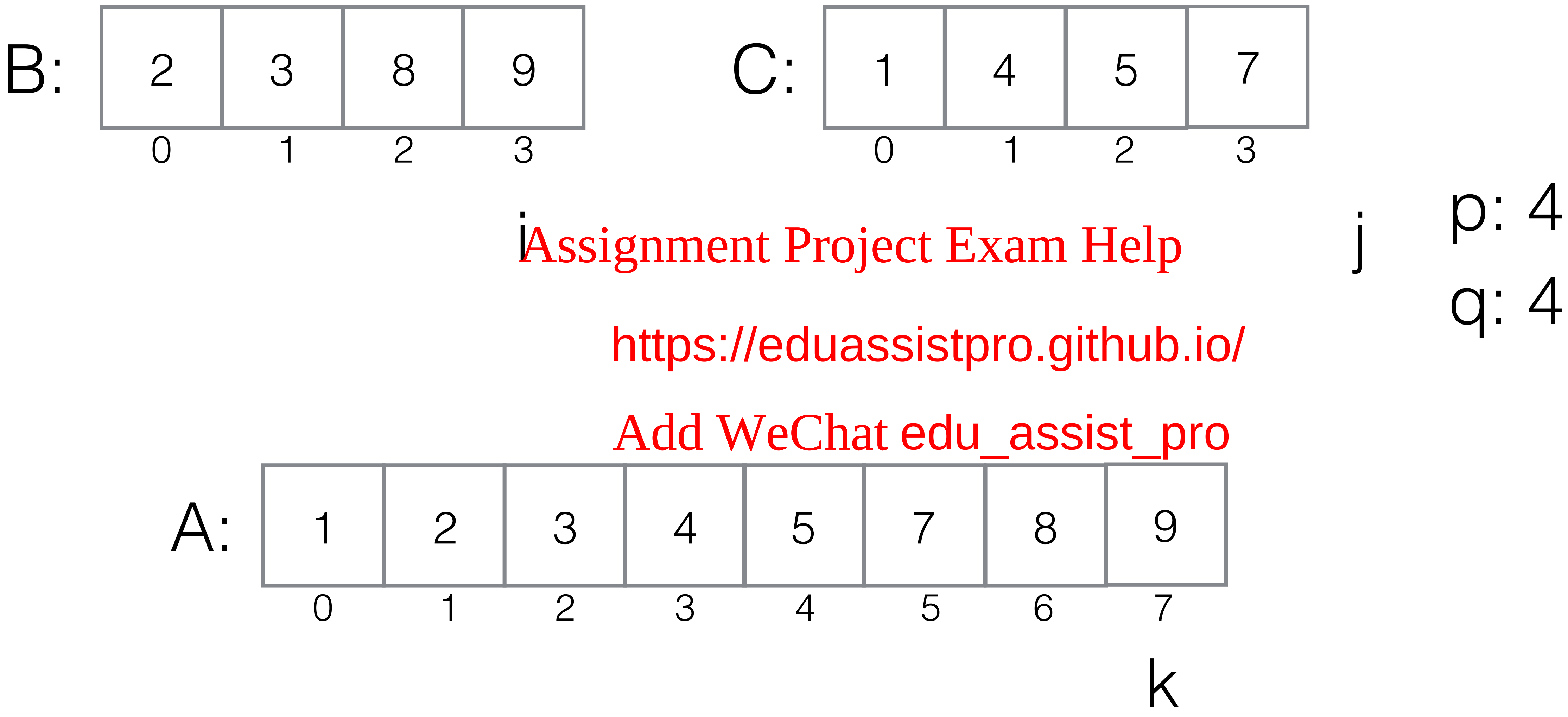
Mergesort: Merging Arrays



B:

2	3	8	9
0	1	2	3

C:

1	4	5	7
0	1	2	3

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A:

1	2	3	4	5	7	8	9
0	1	2	3	4	5	6	7

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Mergesort: Analysis

- How many comparisons will MERGE need to make in the worst case, when given arrays of size ⌊n/2⌋ and ⌈n/2⌉ ?

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- If the largest and elements are in different arrays, then n − 1 risons. Hence the cost equation for Mergesor

$$C(n) = \begin{cases} 0 & \text{if } n < 2 \\ 2C(n/2) + n - 1 & \text{otherwise} \end{cases}$$

- By the Master Theorem, C(n) ∈ Θ(n log n).

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Mergesort: Properties

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- For large n, the number of comparisons made tends to be around 75% of the worst-case scenario.
- Is mergesort stable? ?

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- Is mergesort in-pl

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- If comparisons are fast, me ranks between quicksort and heapsort (covered next week) for time, assuming random data.
- Mergesort is the method of choice for linked lists and for very large collections of data.

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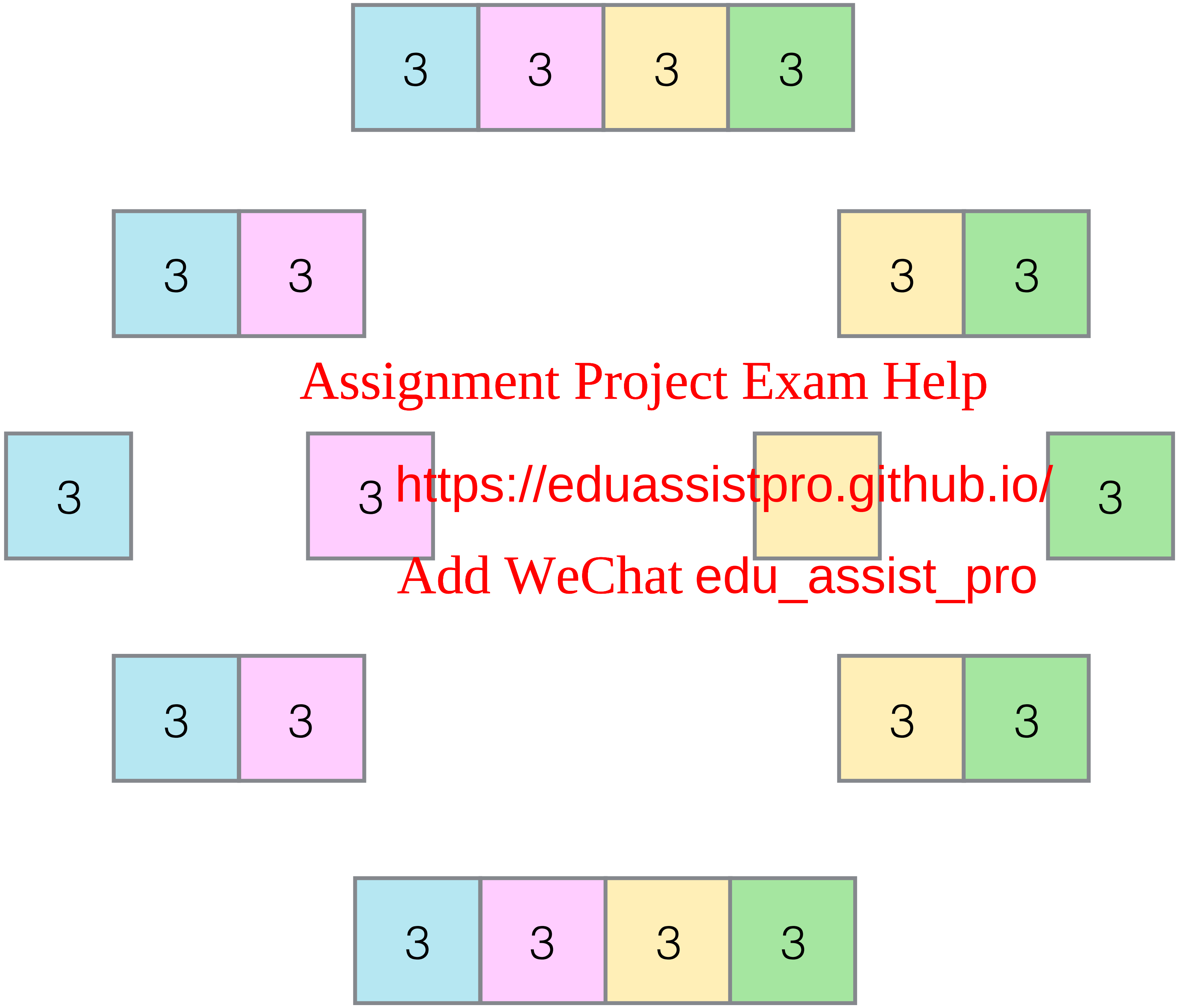
Mergesort: Stability



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Mergesort: Properties

- For large n, the number of comparisons made tends to be around 75% of the worst-case scenario.

- Is mergesort stable? yes

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- Is mergesort in-pl

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- If comparisons are fast, me ranks between quicksort and heapsort (covered next week) for time, assuming random data.

- Mergesort is the method of choice for linked lists and for very large collections of data.

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Bottom-Up Mergesort

- An alternative way of doing mergesort:

- Generate **runs** of length 2, then of length 4, and so on:

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Quicksort

- Quicksort takes a divide-and-conquer approach that is different to mergesort's.

- It uses the **partitioning** idea from QuickSelect, picking a pivot element, and partitioning the array around that, so as to obtain this situation:

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- The element A[s] will be in its final position (it is the (s + 1)th smallest element).

- All that then needs to be done is to sort the segment to the left, recursively, as well as the segment to the right.

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Quicksort

- Very short and elegant:

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- Initial call: Quicksort(A, 0, n – 1).

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Quicksort: Example

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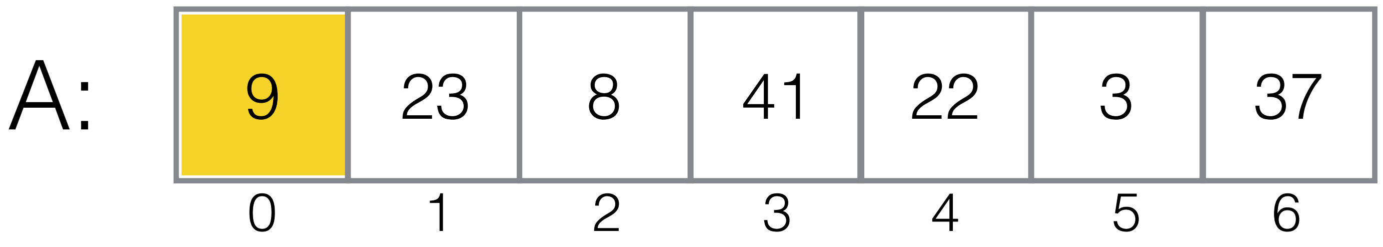

A:	0	1	2	3	4	5	6
	9	23	8	41	22	3	37

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Quicksort: Example

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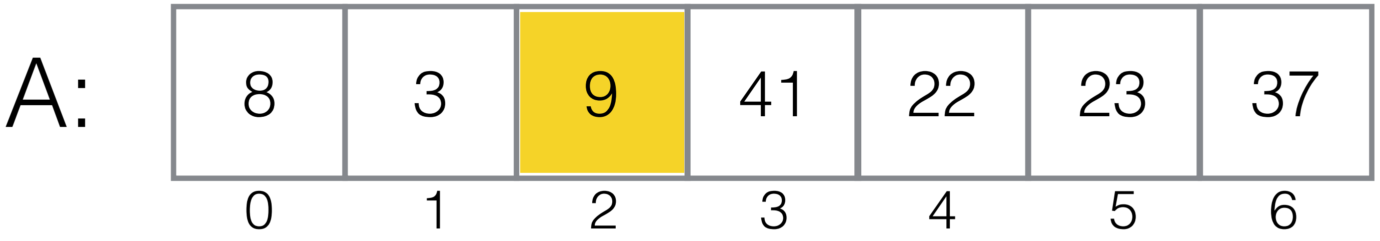

A:						
8	3	9	41	22	23	37
0	1	2	3	4	5	6

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Quicksort: Example

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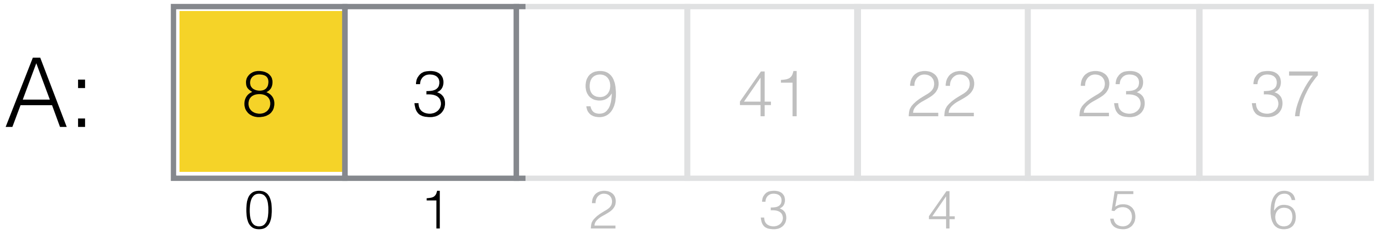

A:	8	3	9	41	22	23	37
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Quicksort: Example

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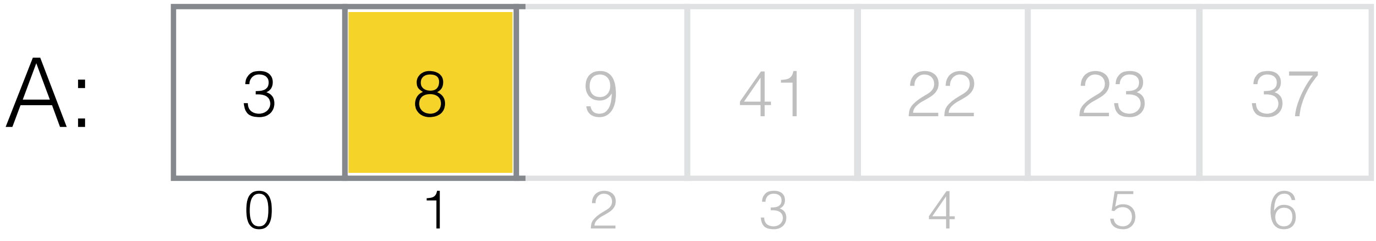

A:

3	8	9	41	22	23	37
0	1	2	3	4	5	6

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Quicksort: Example

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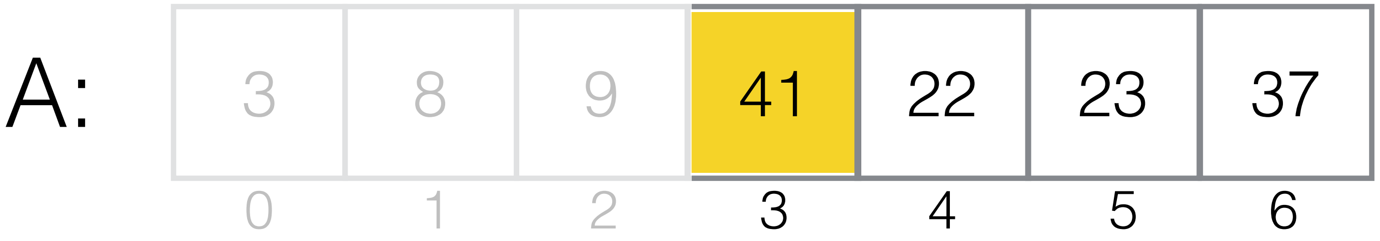

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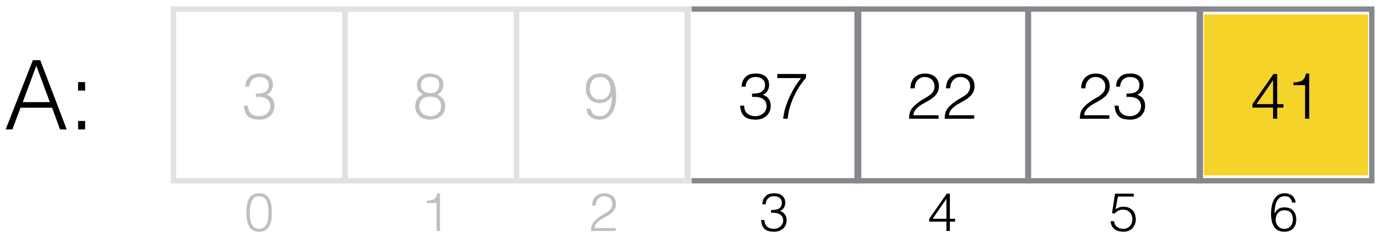

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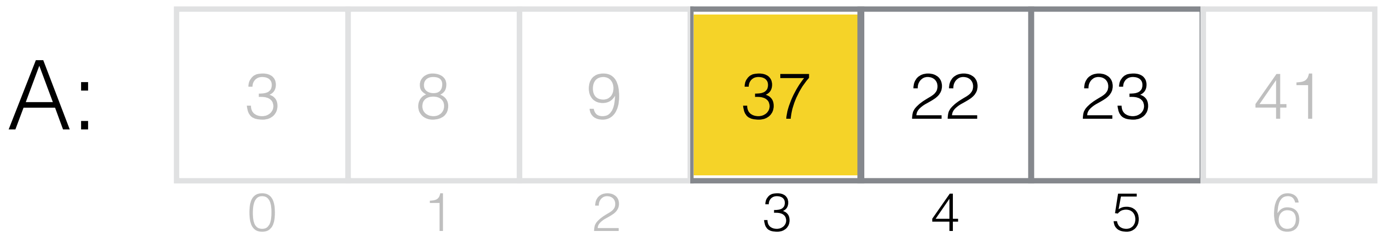

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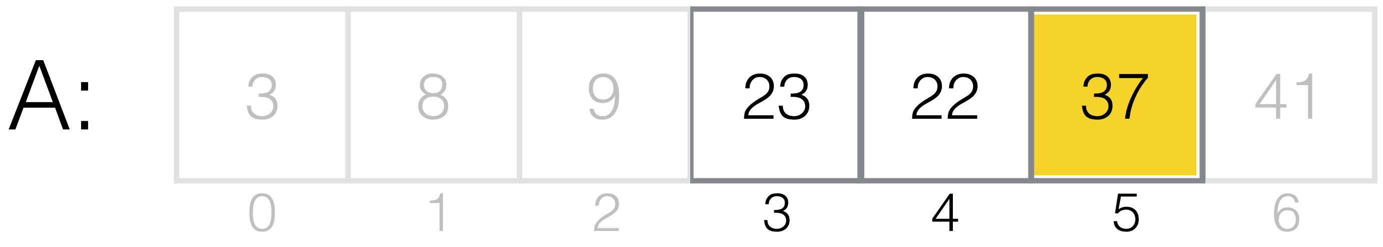

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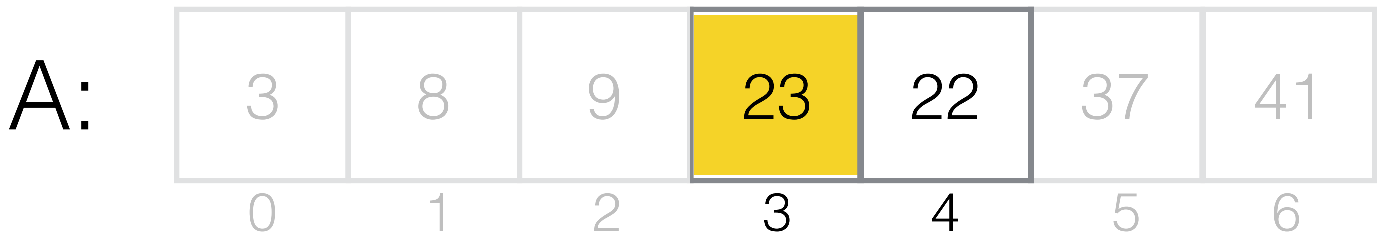

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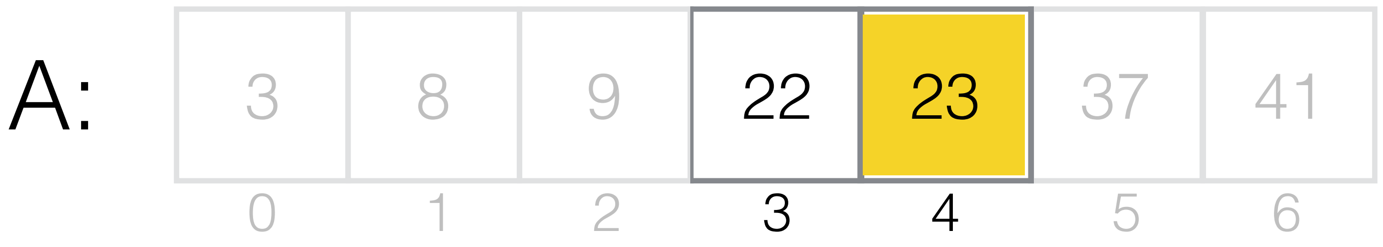

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Hoare Partitioning

- The standard way of doing partitioning in Quicksort

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Hoare Partitioning

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Hoare Partitioning

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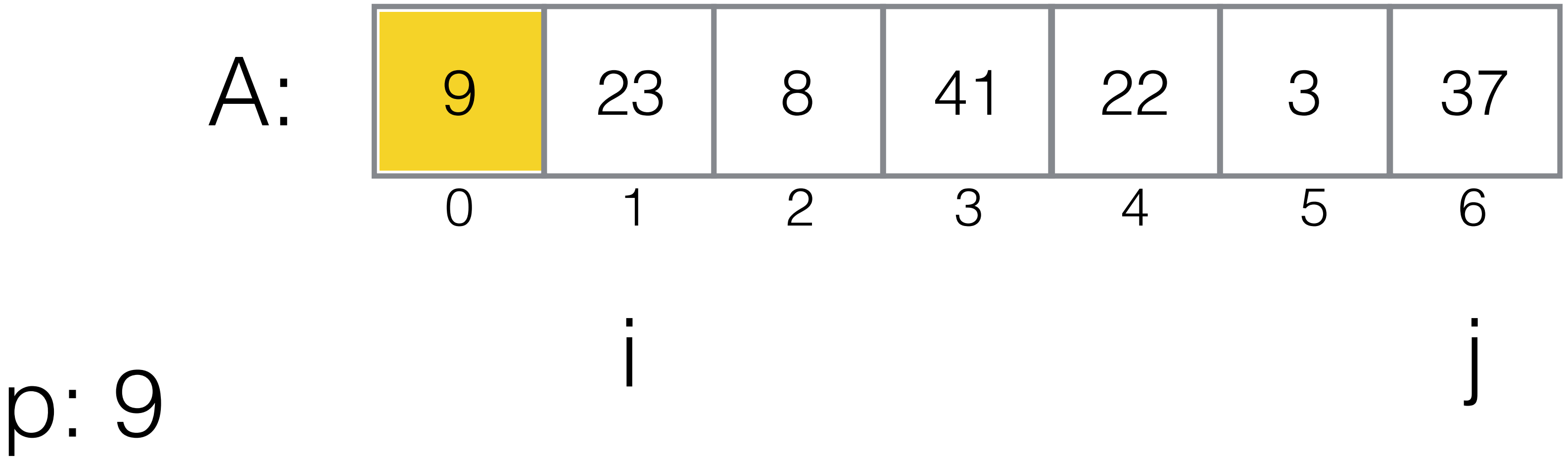

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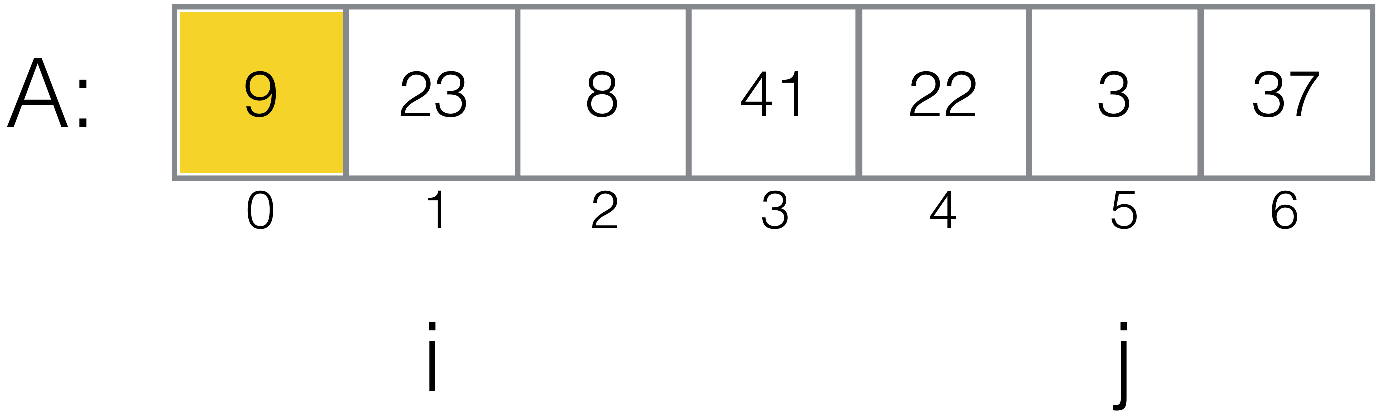

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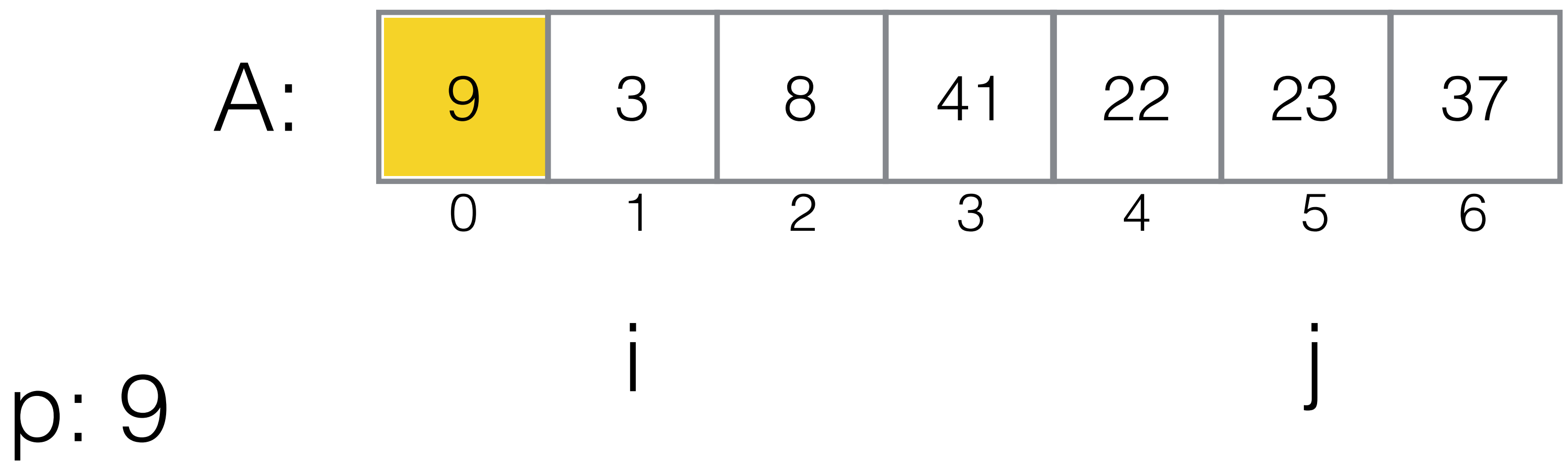

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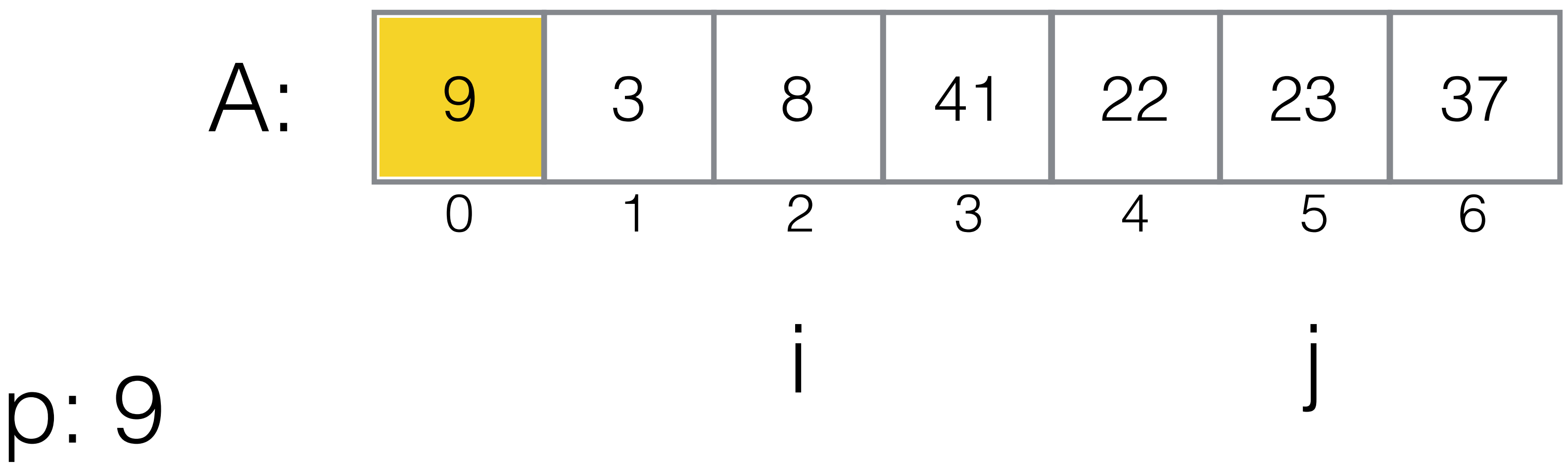

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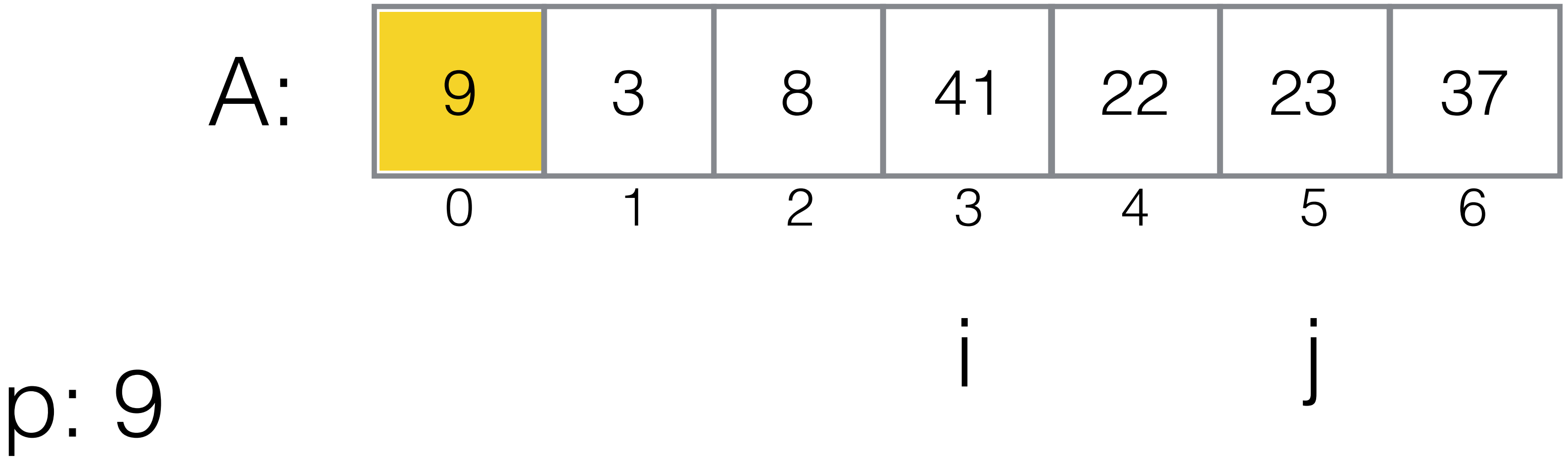

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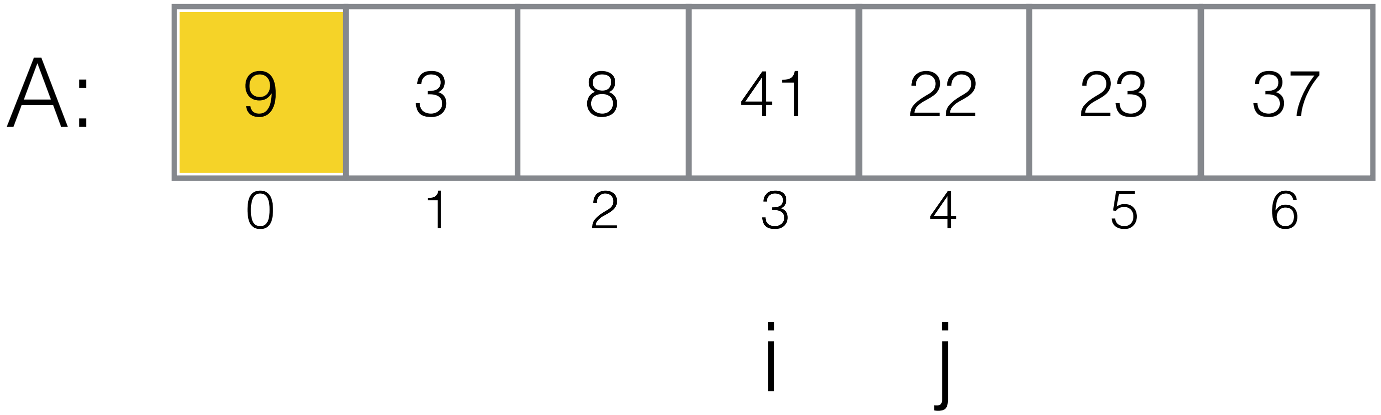

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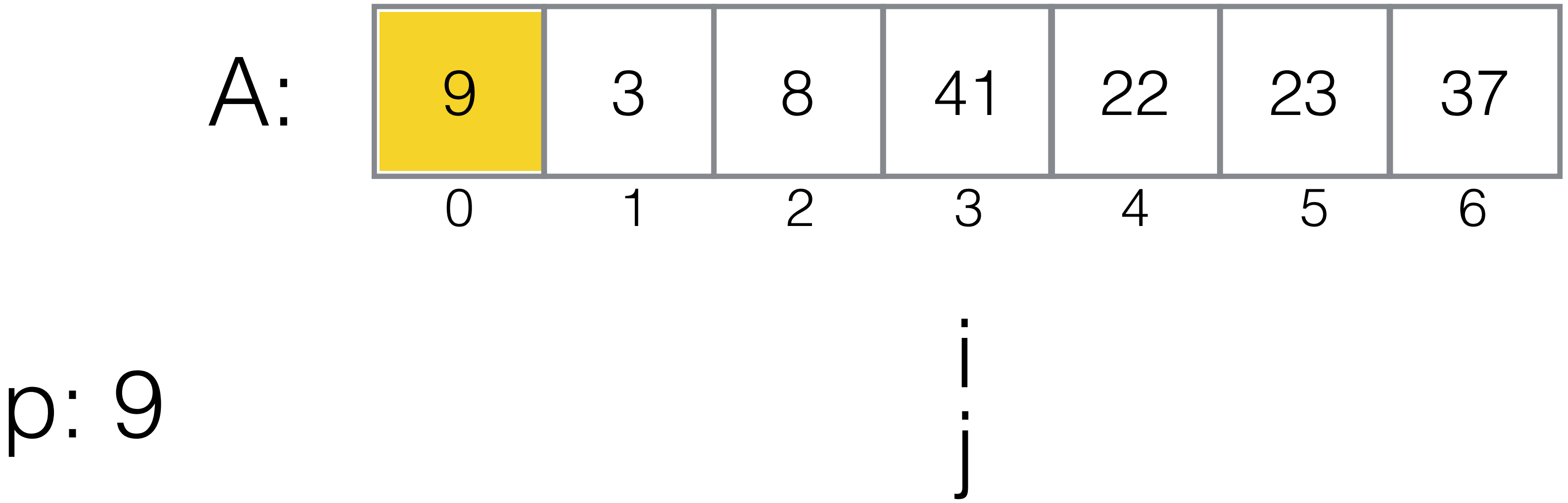

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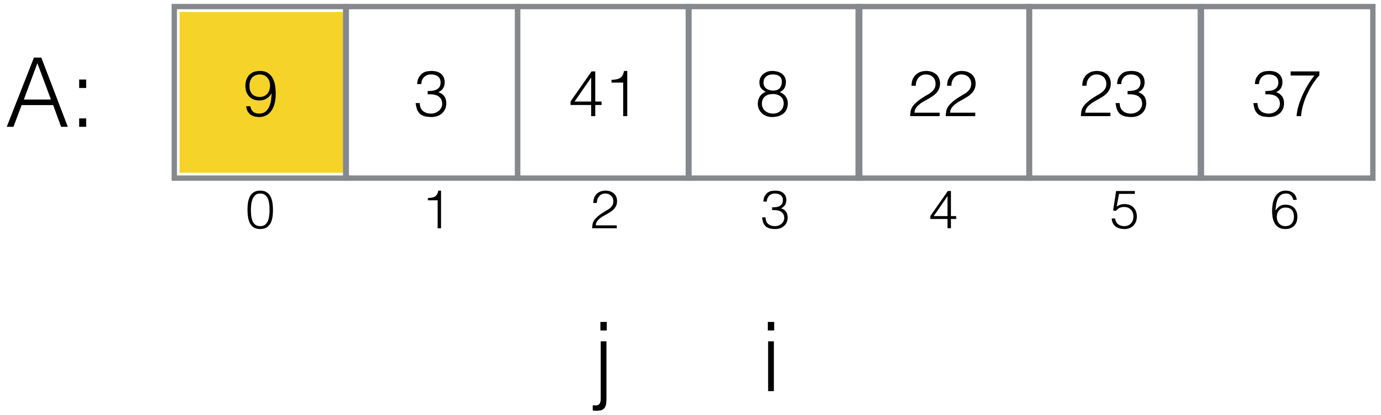

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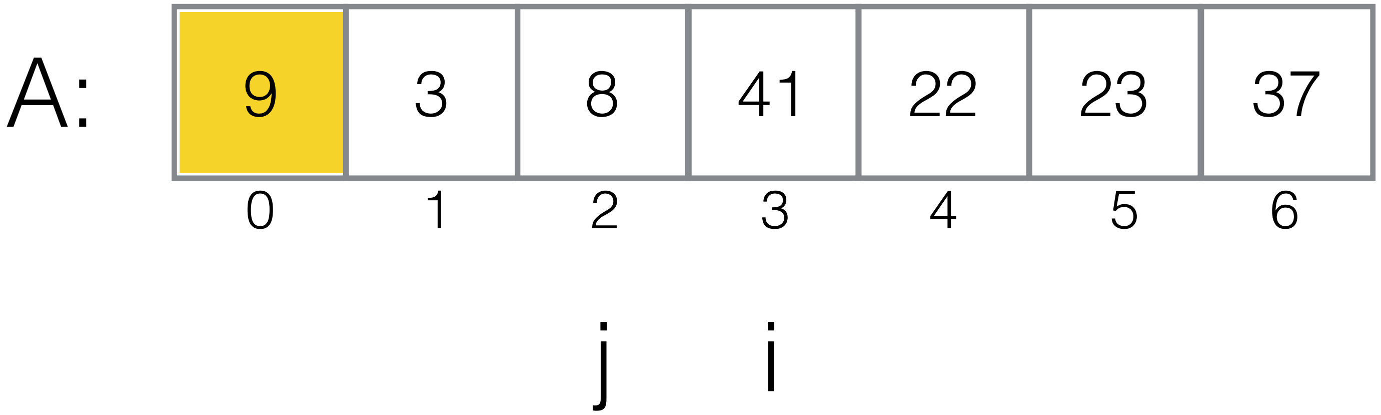

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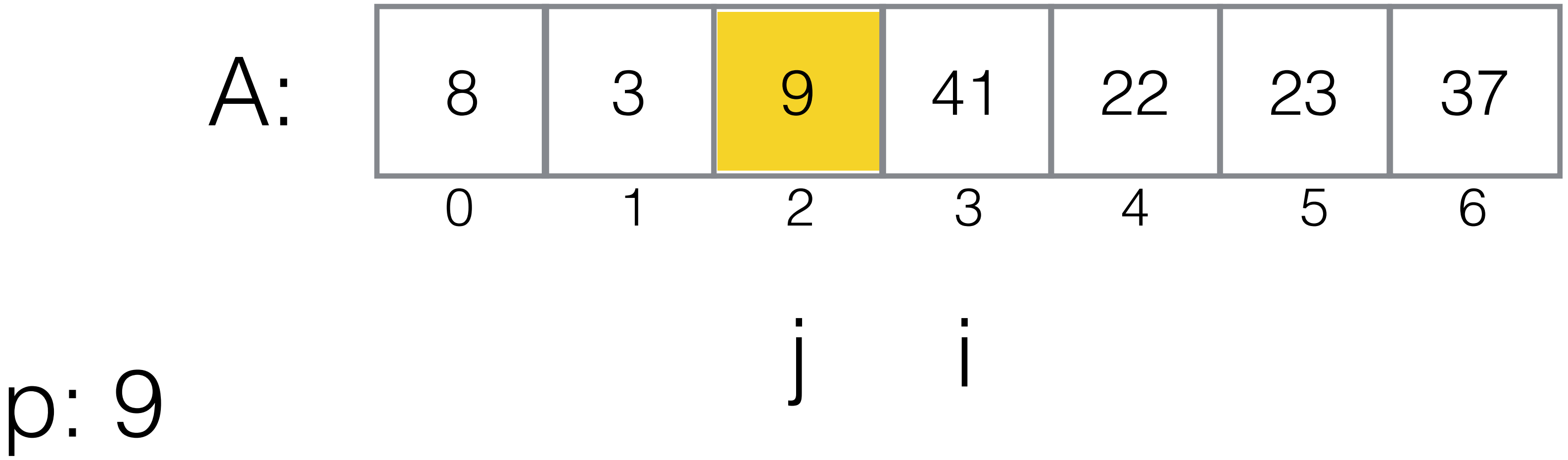

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Quicksort Analysis: Best Case Analysis

- The best case happens when the pivot is the median; that results in two sub-tasks of equal size.

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The ‘n’ is for the https://eduassistpro.github.io/ ns performed by Partition. Add WeChat edu_assist_pro

- By the Master Theorem, $C_{best}(n) \in \Theta(n \log n)$, just as for mergesort, so quicksort’s best case is (asymptotically) no better than mergesort’s worst case.

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Quicksort Worst Case

A:

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Quicksort Analysis: Worst Case Analysis

- The worst case happens if the array is already sorted.

- In that case, we don't really have divide-and-conquer, becaus e call deals with a problem size that decremented by 1:

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- That is, $C_{worst}(n) = n + (n - 1) + \cdot \cdot \cdot + 3 + 2 \in \Theta(n^2)$.

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Quicksort Improvements: Median-of-Three

- It would be better if the pivot was chosen randomly.

- A cheap and useful approximation to this is to take the median of three candidates, A[lo], A[hi], and A[$\lfloor(lo + hi)/2\rfloor$].

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- Reorganise the three elements so that p_1 is the median, and p_3 is the largest of the three.

- Now run quicksort as before.

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Quicksort Improvements: Median-of-Three

- In fact, with median-of-three, we can have a much faster version than before, simplifying tests in the innermost loops:

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Quicksort Improvements: Early Cut-Off

- A second useful improvement is to stop quicksort early and switch to insertion sort. This is easily implemented:

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Quicksort Properties

- With these (and other) improvements, quicksort is considered the best available sorting method for arrays of random data.

- A major reason for its speed is the very tight inner loop in PARTITION.

- Although mergesort has a better performance guarantee, quicksort is faster on average.

- In the best case, we get ⌈log2 n⌉ re vels. It can be shown that on random data, the expected number is 2 $\log_e n \approx 1.38 \log_2 n$. So quicksort’s average behaviour is very close to the best-case behaviour.

- Is quicksort stable? ?

- Is it in-place? yes

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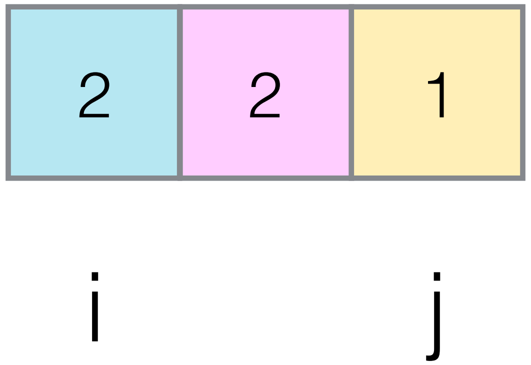
Quicksort Stability



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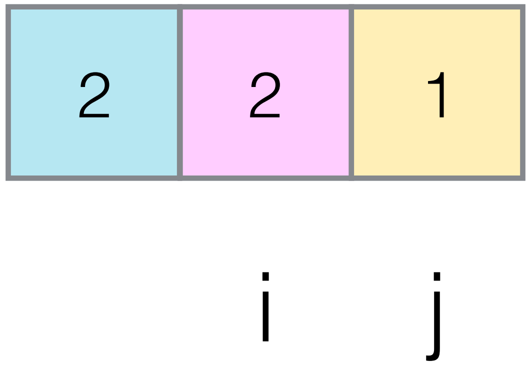
Quicksort Stability



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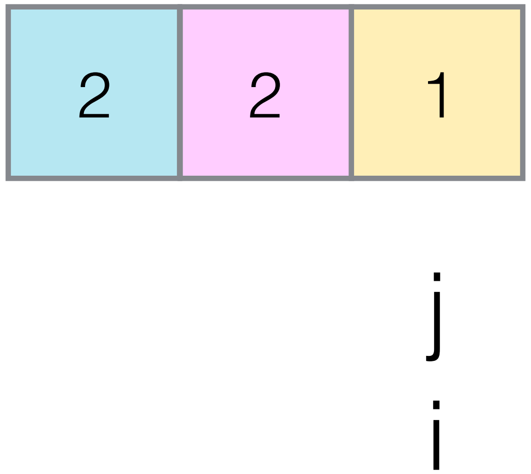
Quicksort Stability



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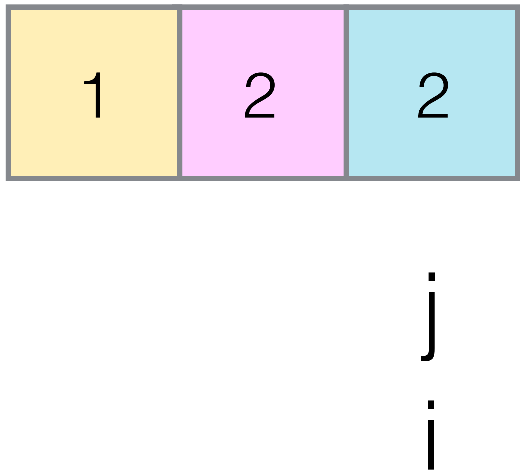
Quicksort Stability



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Quicksort Stability

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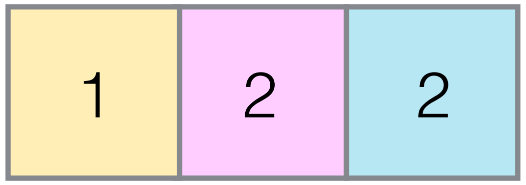
Quicksort Stability



This is where we finished

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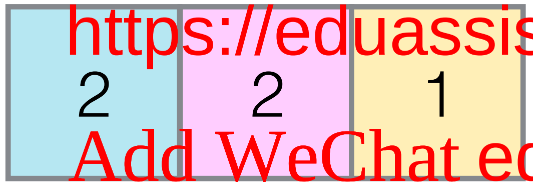

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This is where we started

Not stable

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Quicksort Properties

- With these (and other) improvements, quicksort is considered the best available sorting method for arrays of random data.

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- Although mergesort has a better performance guarantee, quicksort is faster on average.

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- In the best case, we get $\lceil \log_2 n \rceil$ recursion levels. It can be shown that on random data, the expected number is $2 \log_e n \approx 1.38 \log_2 n$. So quicksort's average behaviour is very close to the best-case behaviour.

- Is quicksort stable? no

- Is it in-place? yes

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Next up

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- Tree traversal m apply the divide-and-conquer technique to est-pair problem.

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