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COMP90038

Algorithms and Complexity

Assignment Project Exam Help

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(with thanks to Hara versal ergaard)
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Toby Murray



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DMD 8.17 (Level 8, Doug McDonnell Bldg)



<http://people.eng.unimelb.edu.au/tobym>

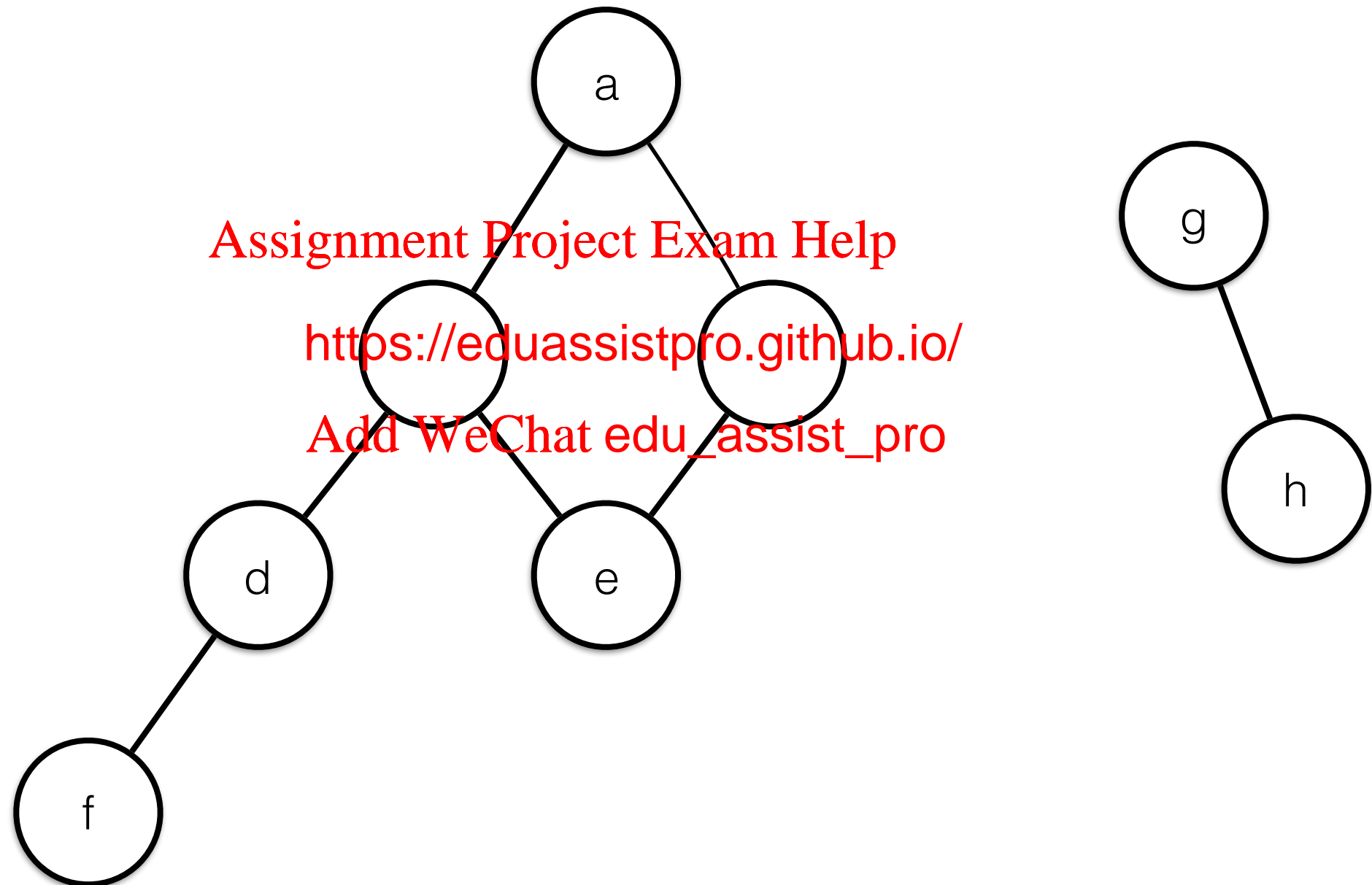


@tobycmurray

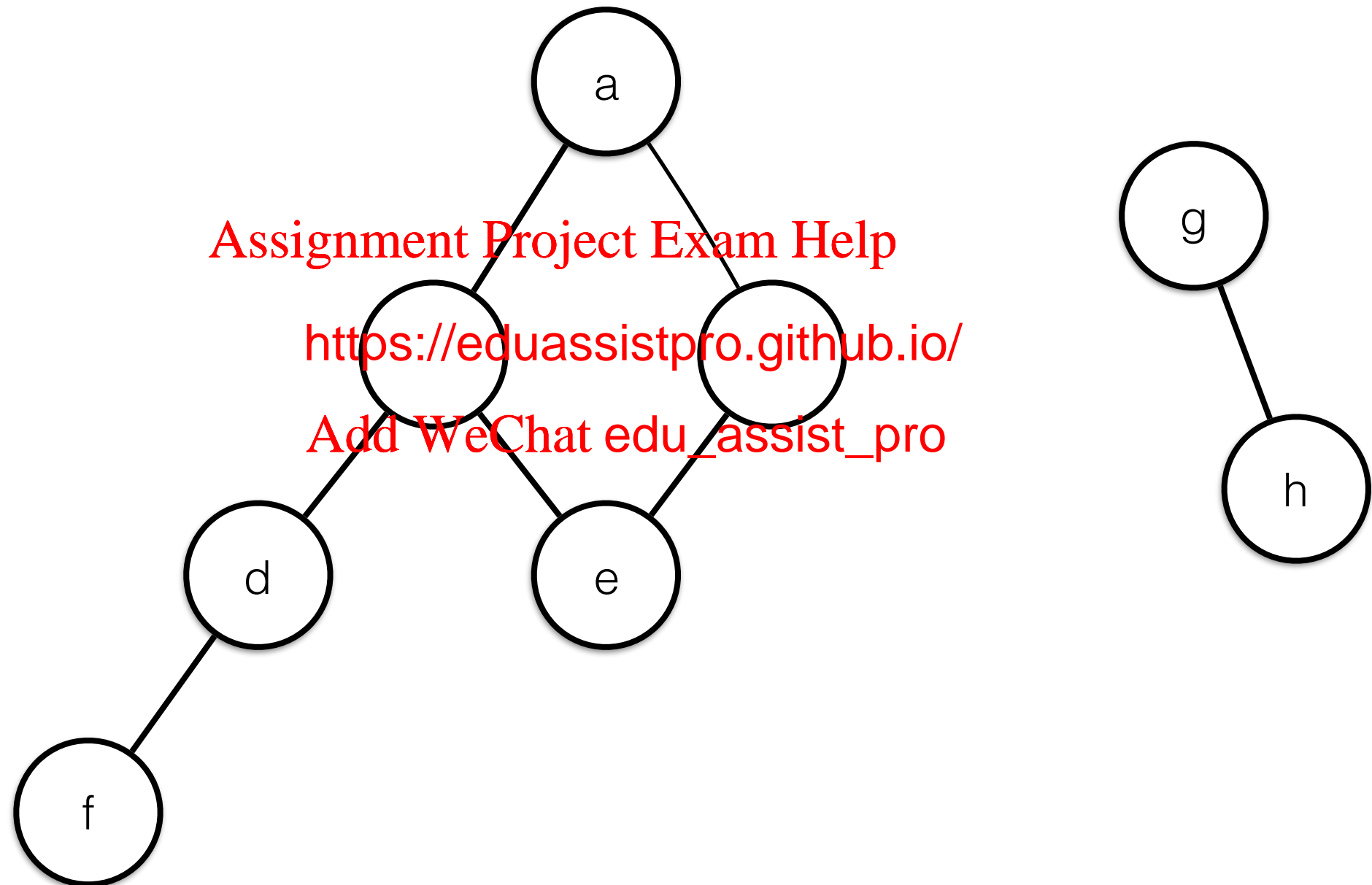
Breadth-First and Depth-First Traversal



- Used to **systematically** explore **all** nodes of a graph



Depth-First Traversal

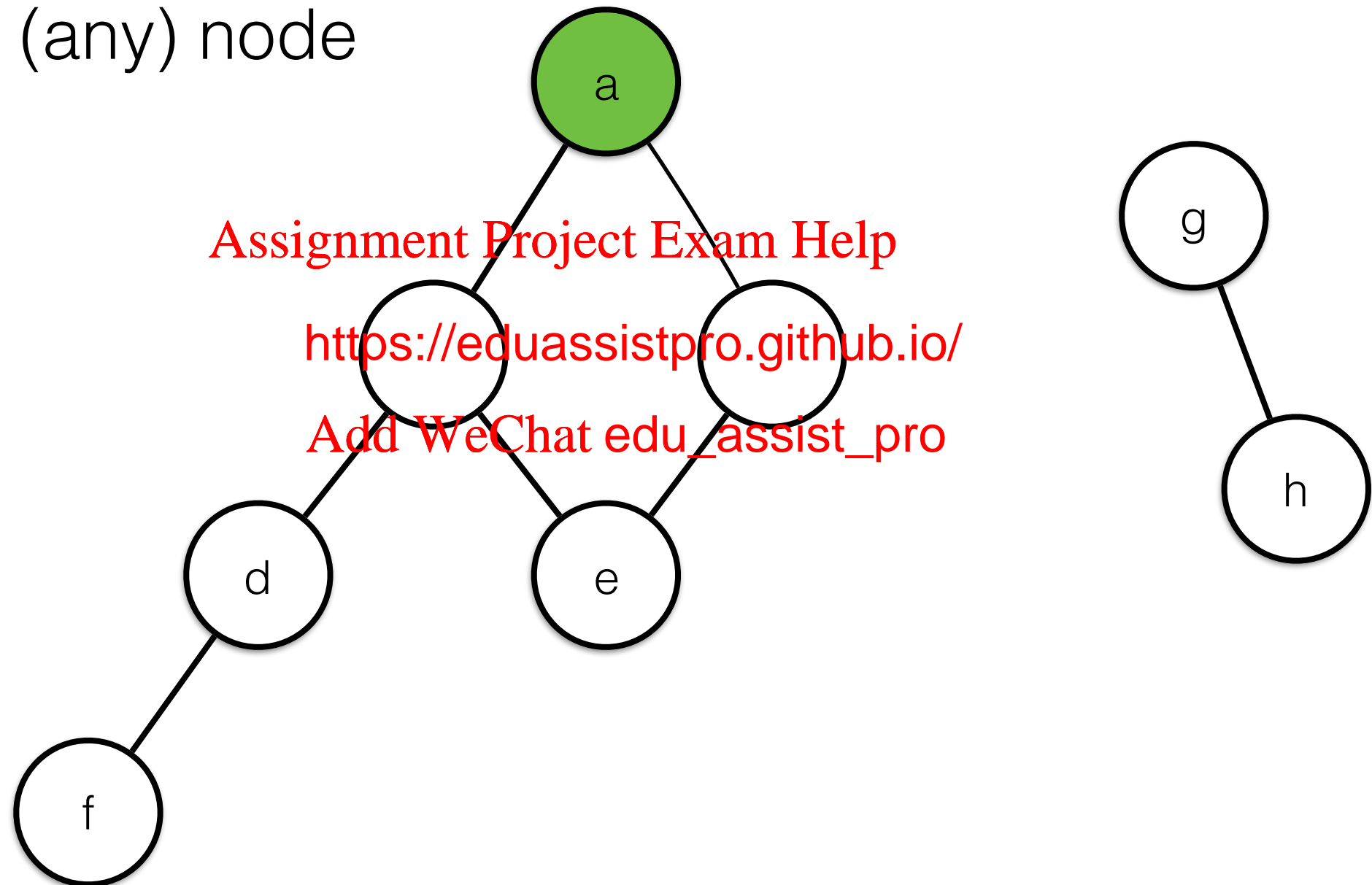


Depth-First Traversal



Nodes visited in this order: a

Start with (any) node

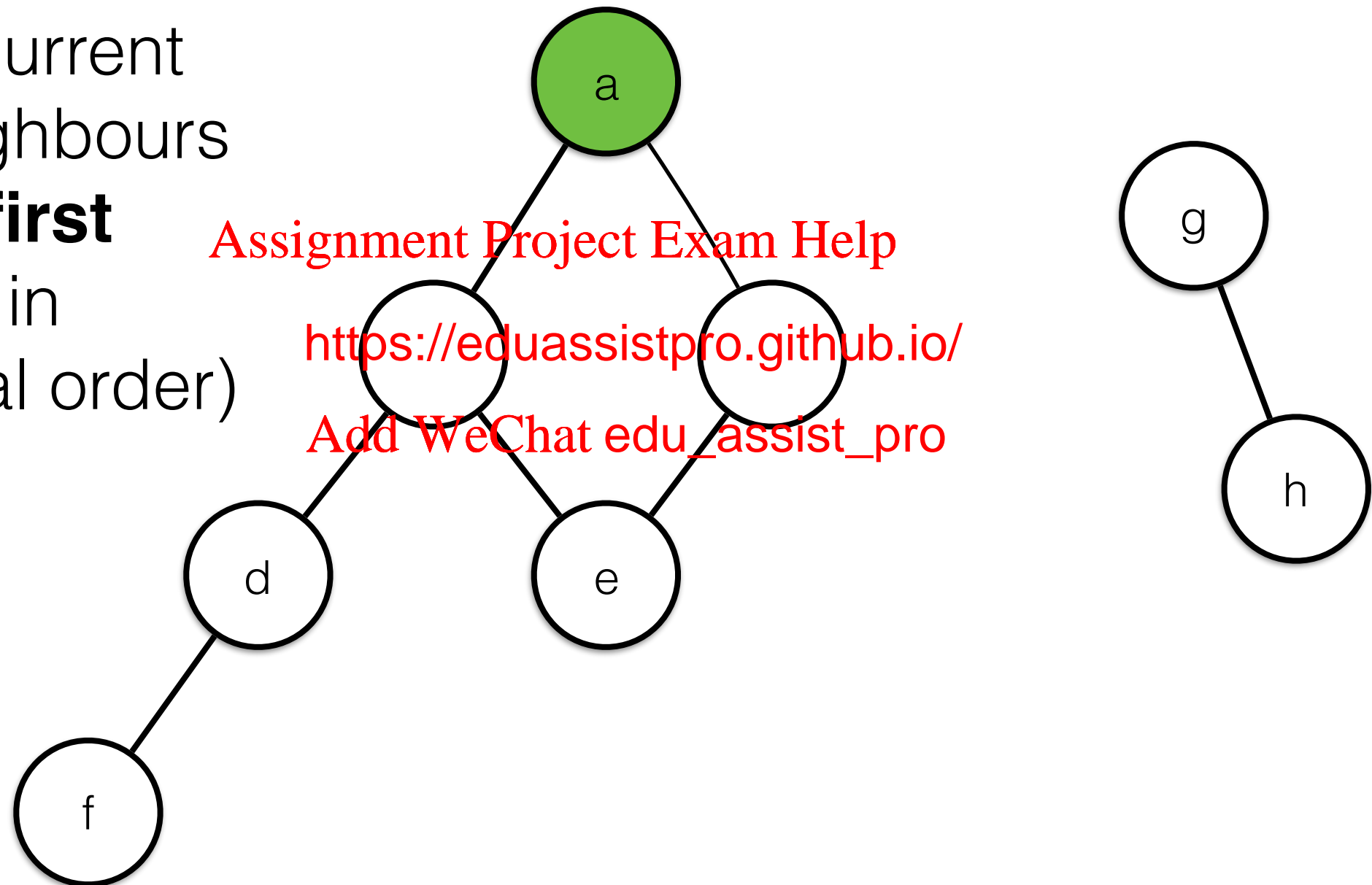


Depth-First Traversal



Nodes visited in this order: a

Visit the current
node's neighbours
depth first
(here in
alphabetical order)

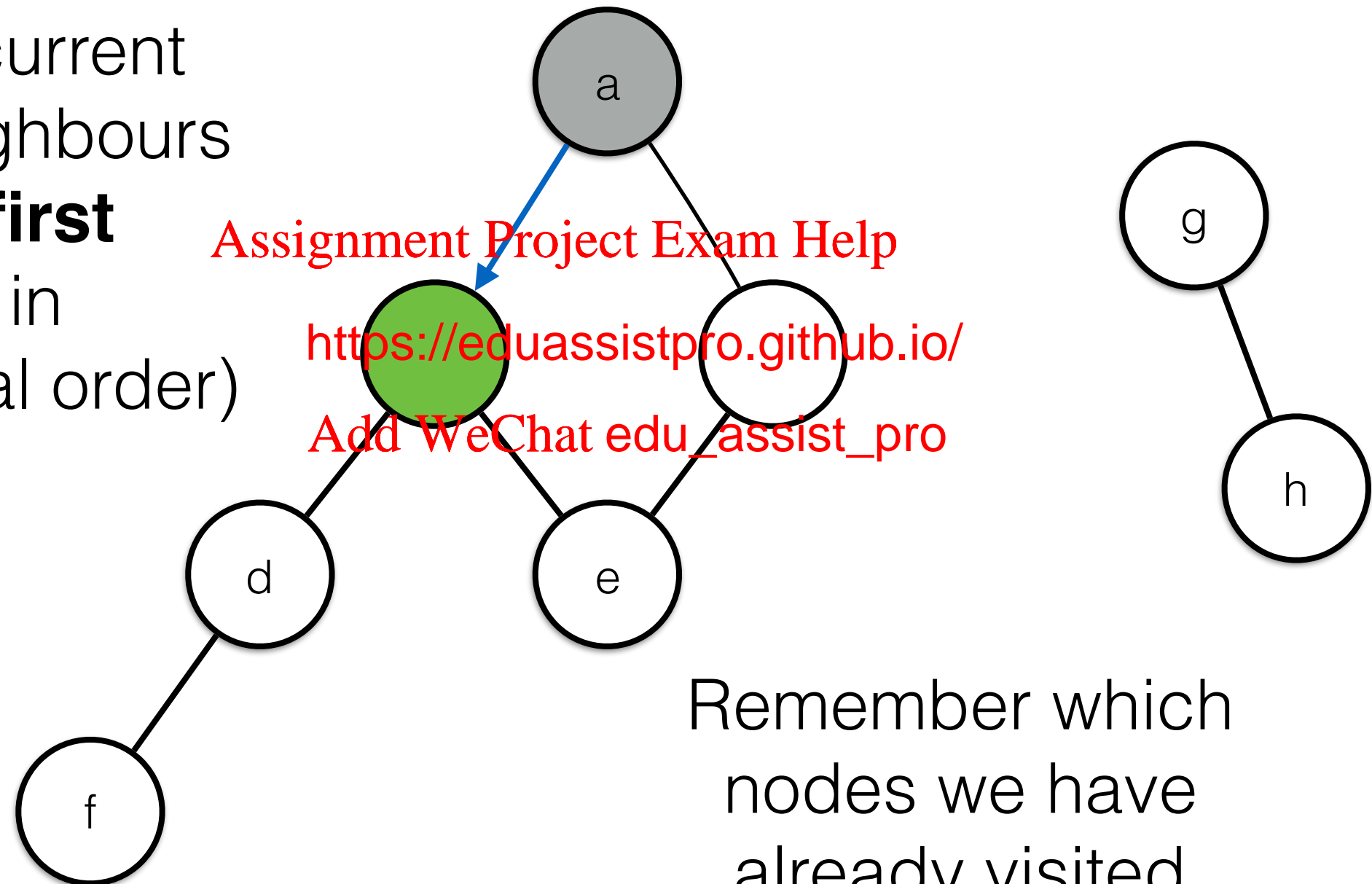


Depth-First Traversal



Nodes visited in this order: a b

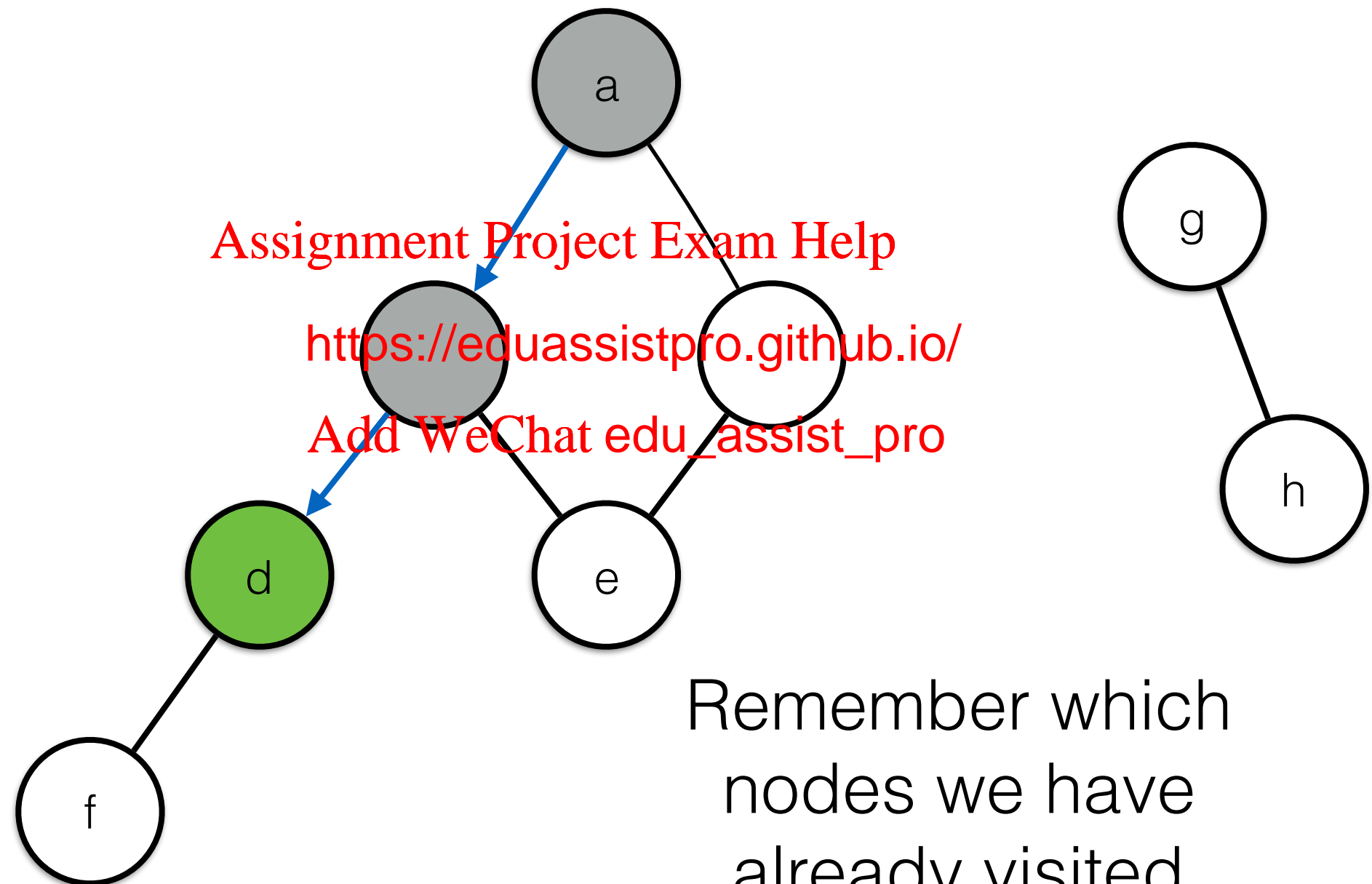
Visit the current
node's neighbours
depth first
(here in
alphabetical order)



Remember which
nodes we have
already visited
to avoid revisiting them

Depth-First Traversal

Nodes visited in this order: a b d



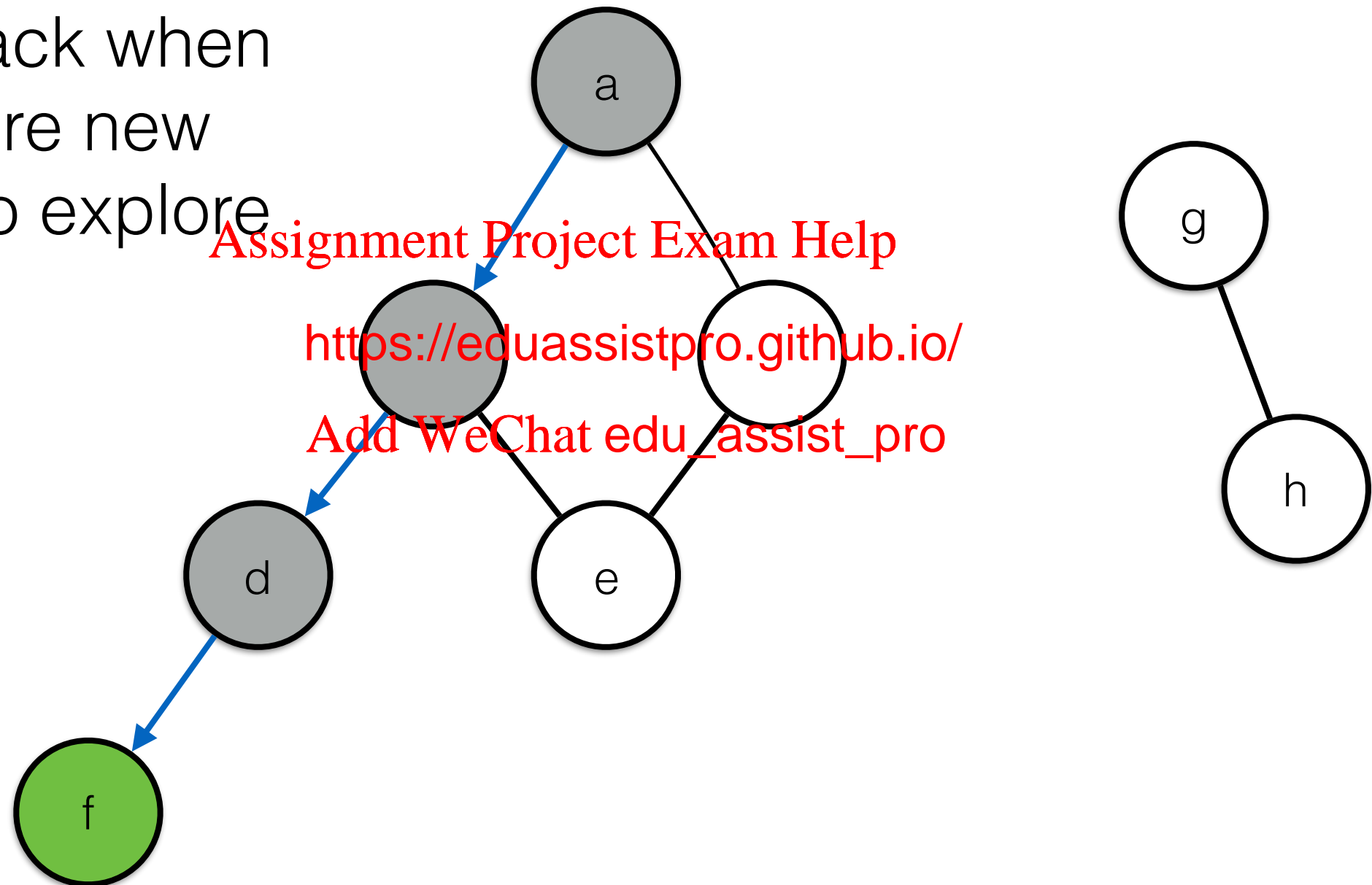
Remember which
nodes we have
already visited
to avoid revisiting them

Depth-First Traversal



Nodes visited in this order: a b d f

Back-track when
no more new
nodes to explore

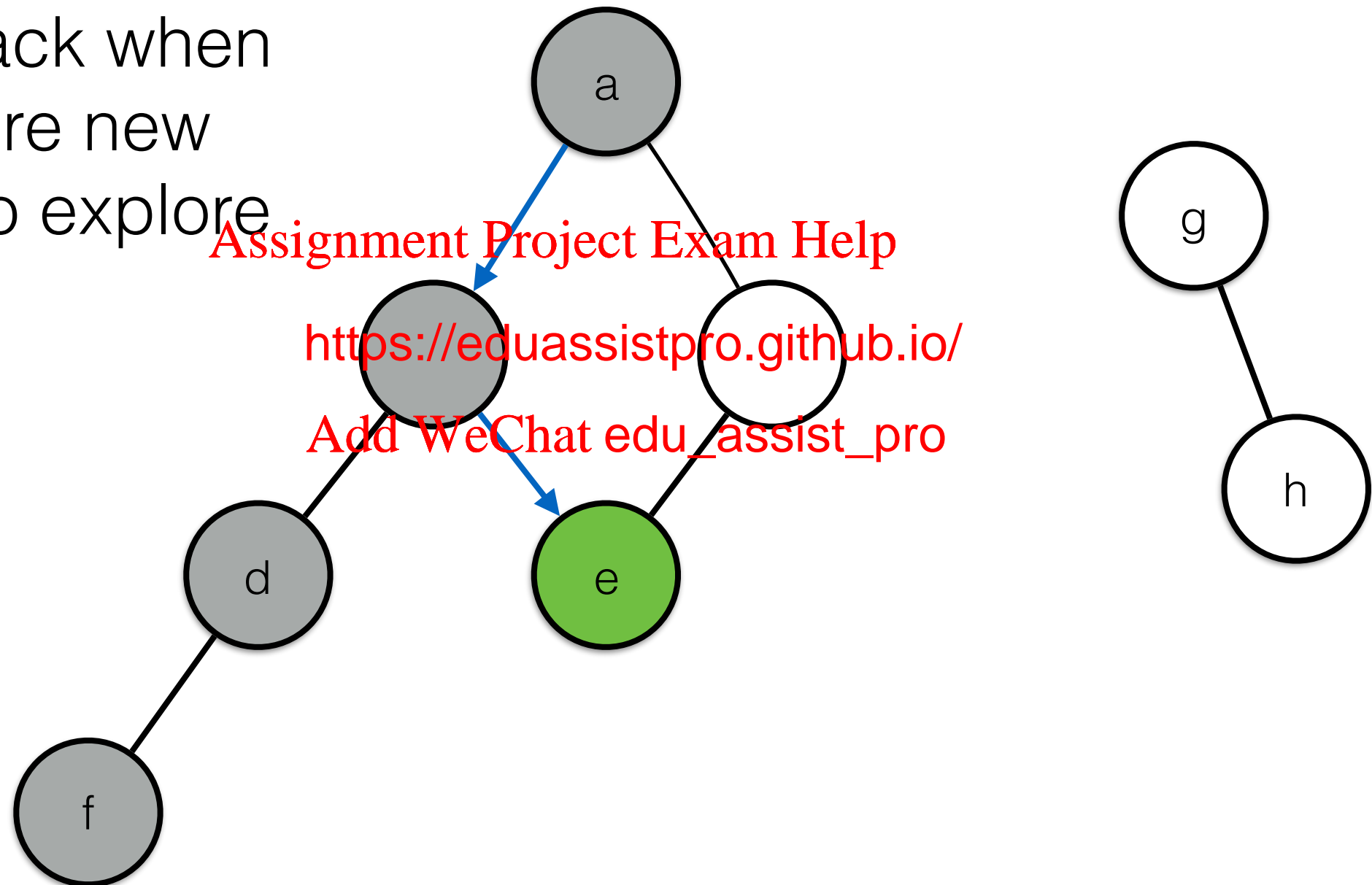


Depth-First Traversal



Nodes visited in this order: a b d f e

Back-track when
no more new
nodes to explore

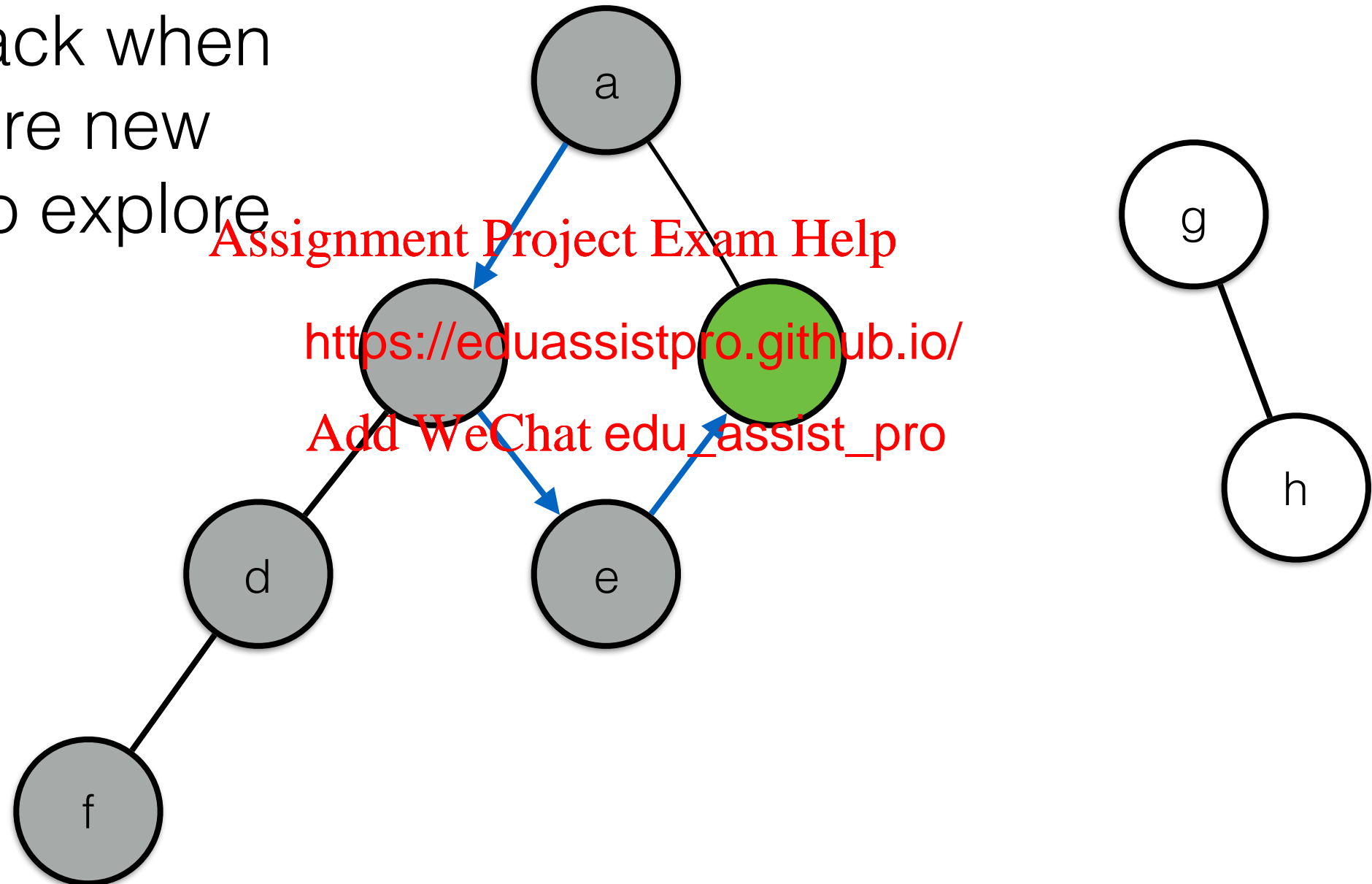


Depth-First Traversal



Nodes visited in this order: a b d f e c

Back-track when
no more new
nodes to explore

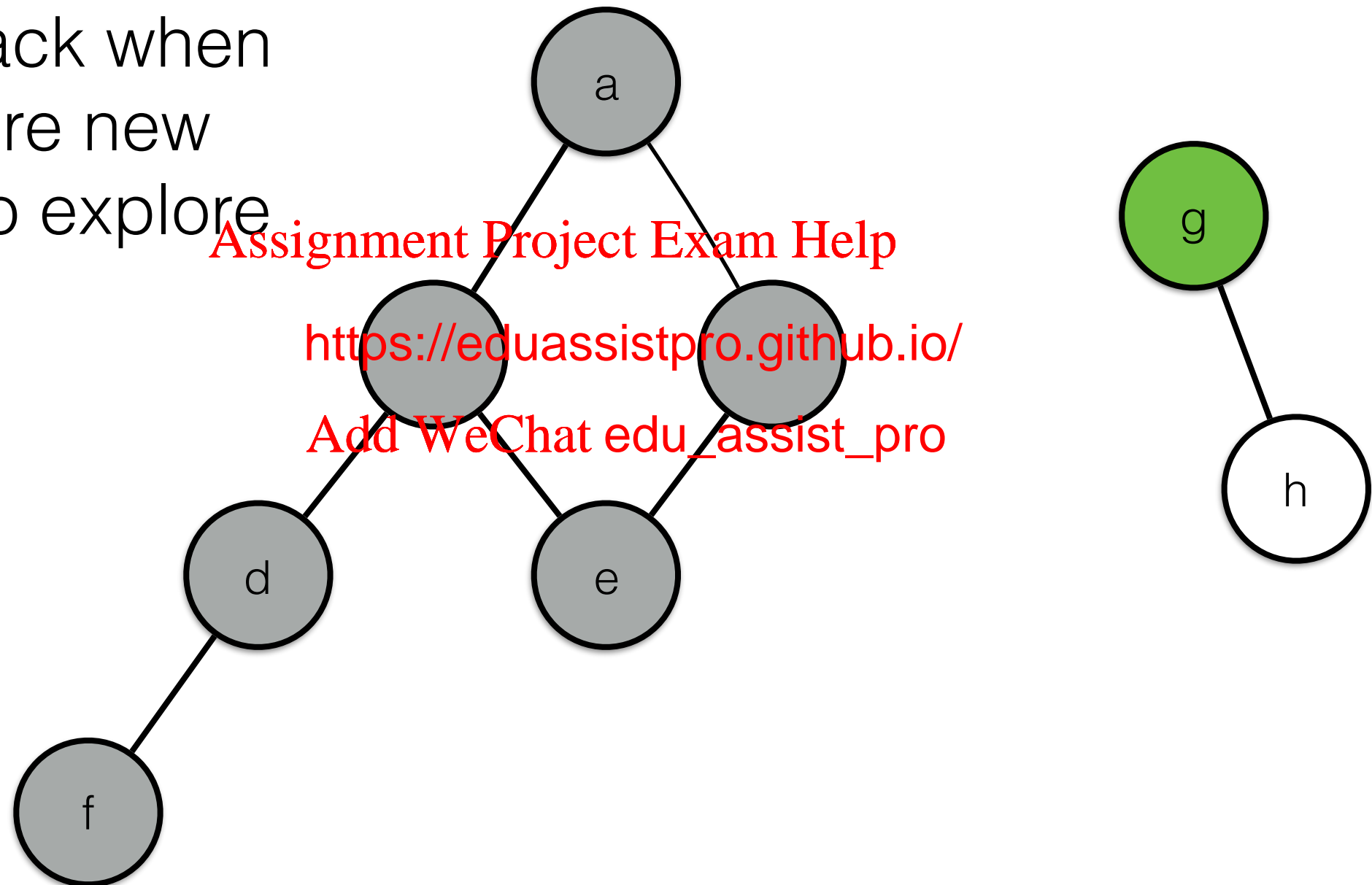


Depth-First Traversal



Nodes visited in this order: a b d f e c g

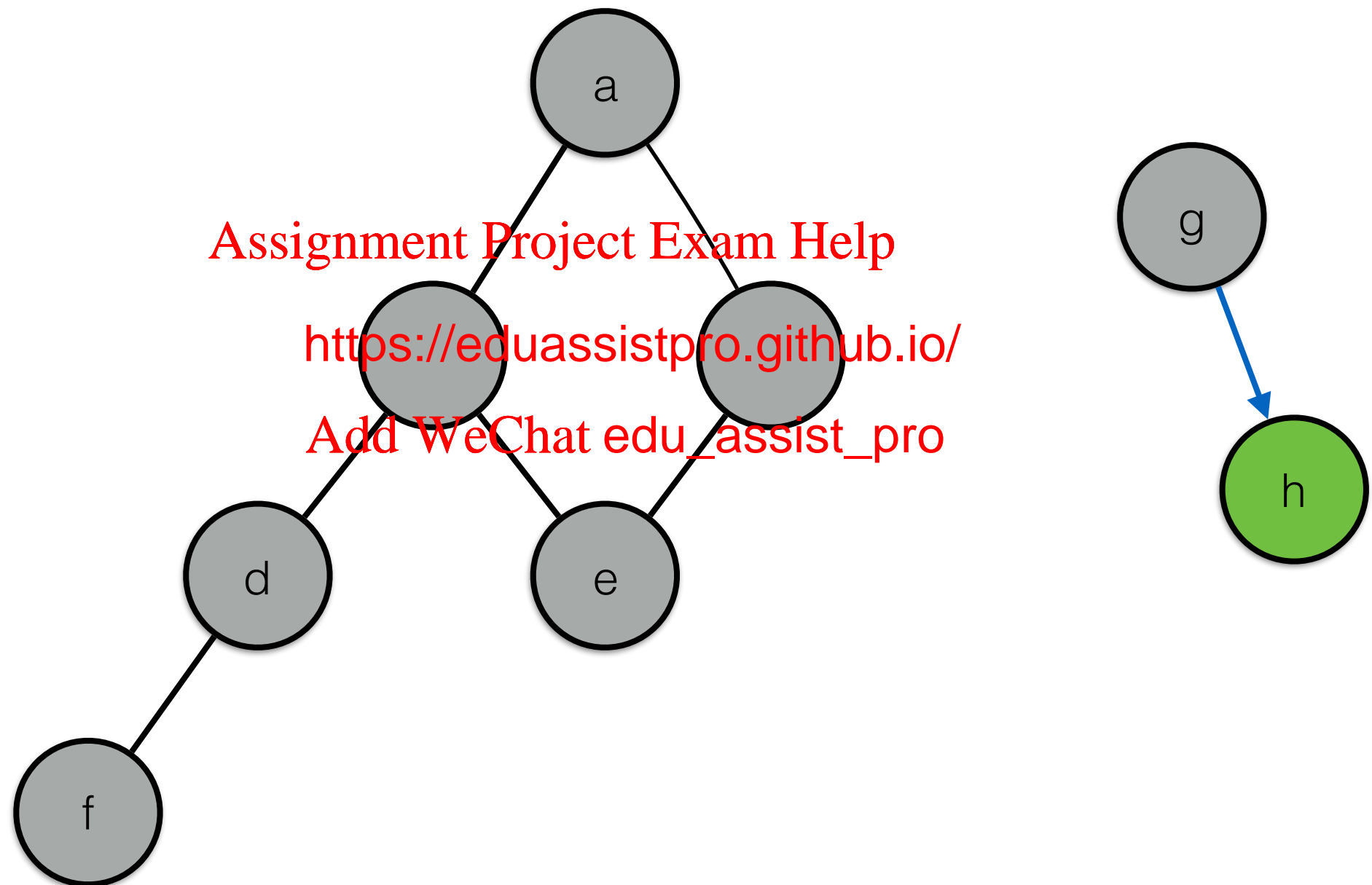
Back-track when
no more new
nodes to explore



Depth-First Traversal



Nodes visited in this order: a b d f e c g h



Depth-First Traversal: Stack Discipline



When back-tracking, we go back to the most recently-visited node that still has unvisited neighbours

This is simulated by pushing each node onto a stack
as it is visited.

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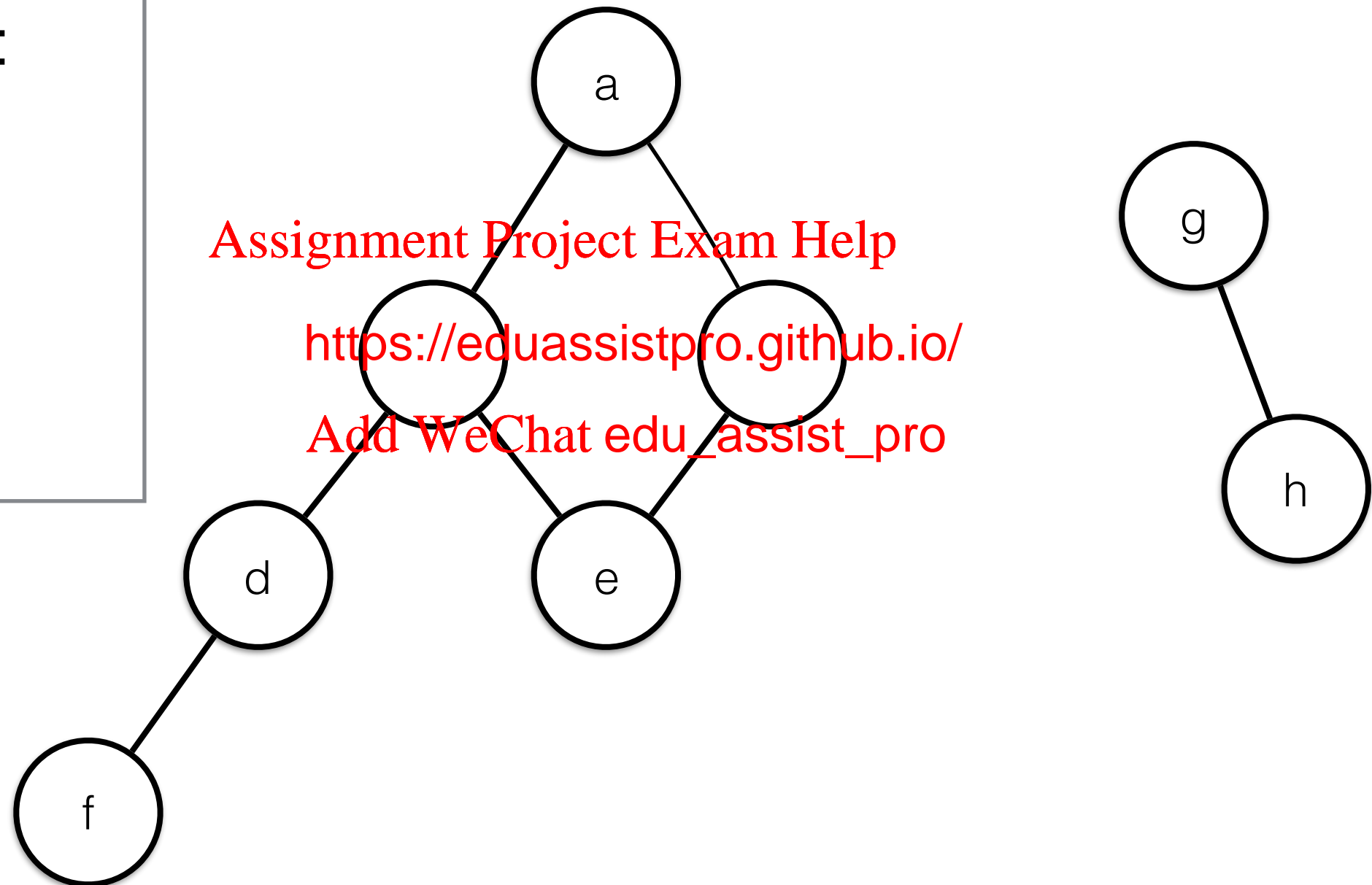
Back-tracking then corresponds to popping the stack.

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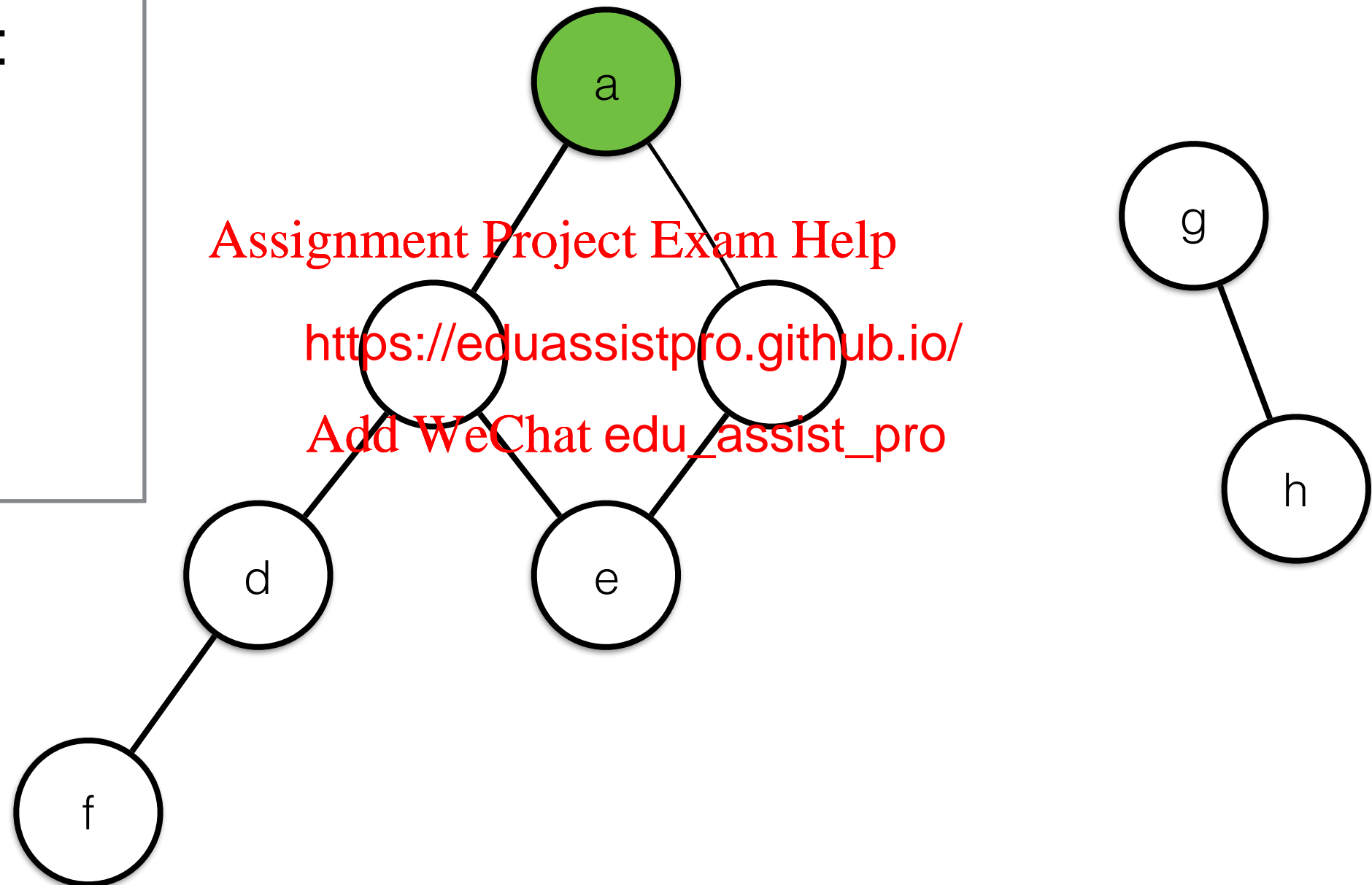
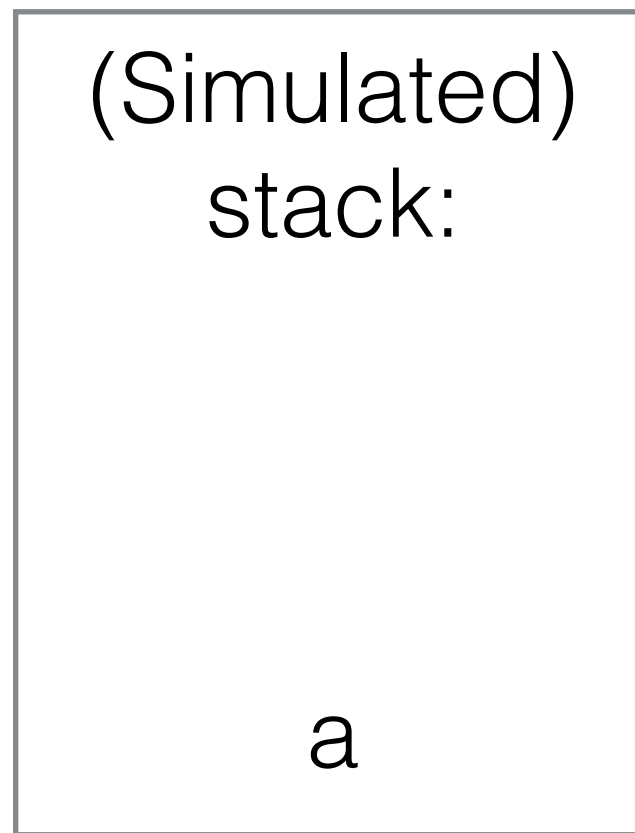
Depth-First Traversal: Stack Discipline



(Simulated)
stack:



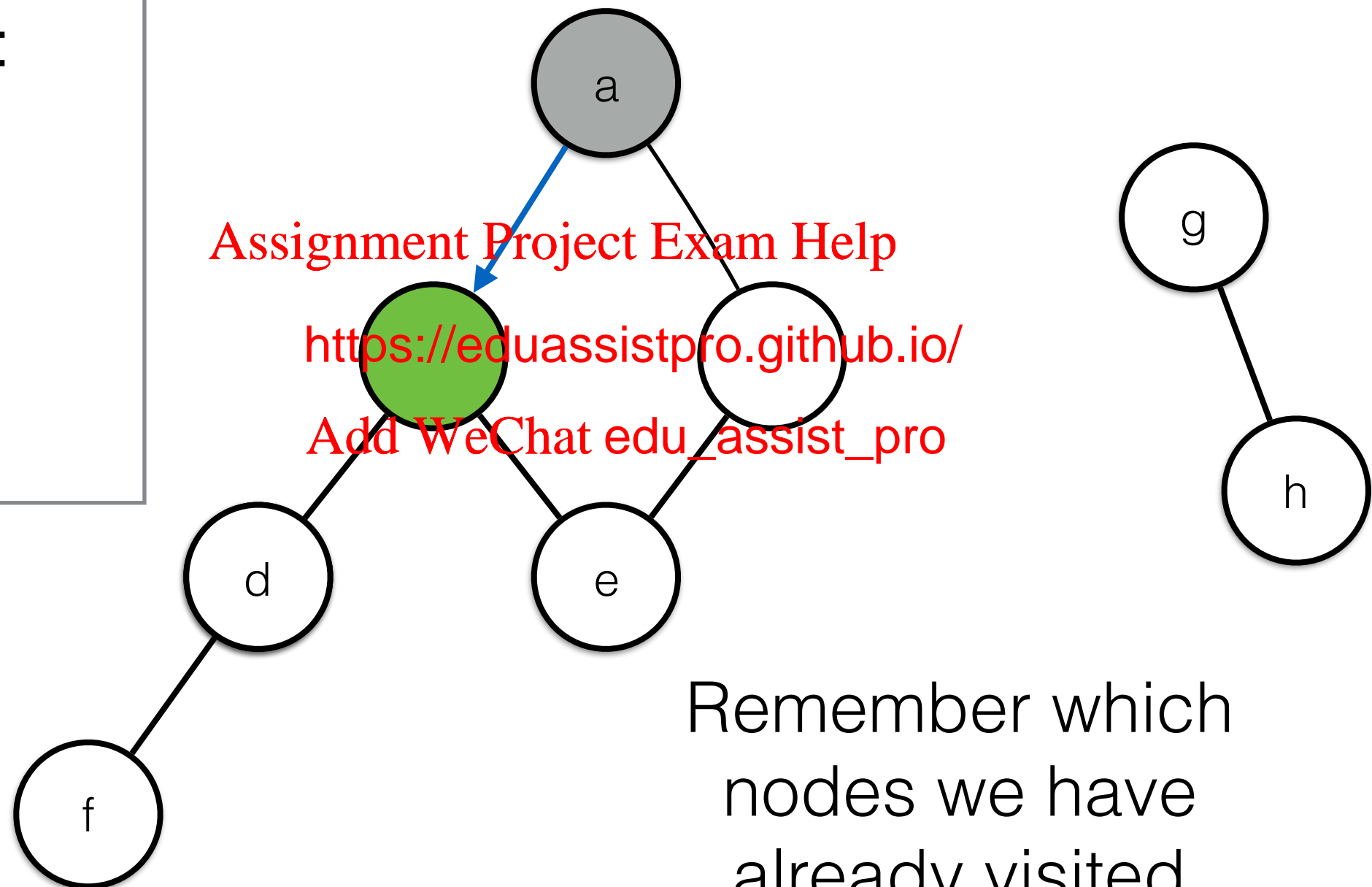
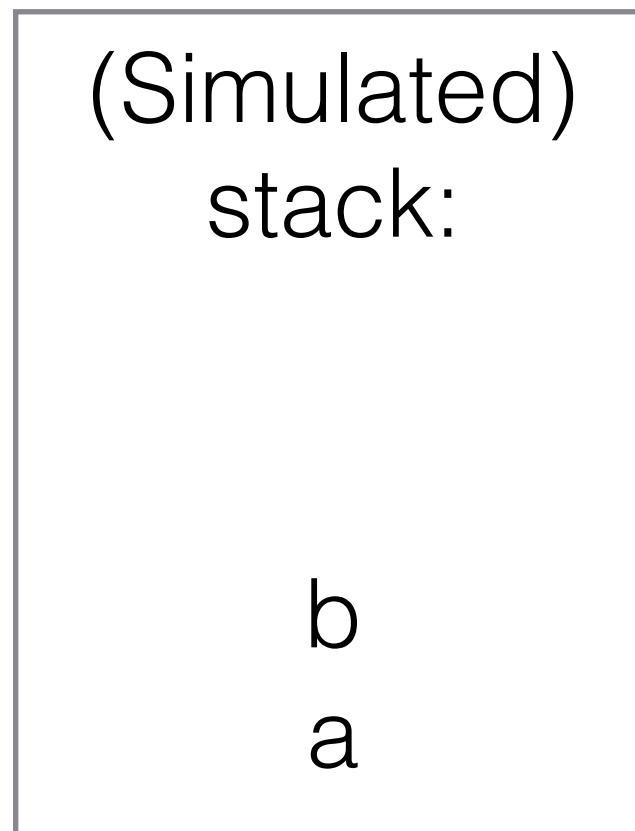
Depth-First Traversal: Stack Discipline



Depth-First Traversal: Stack Discipline



a b



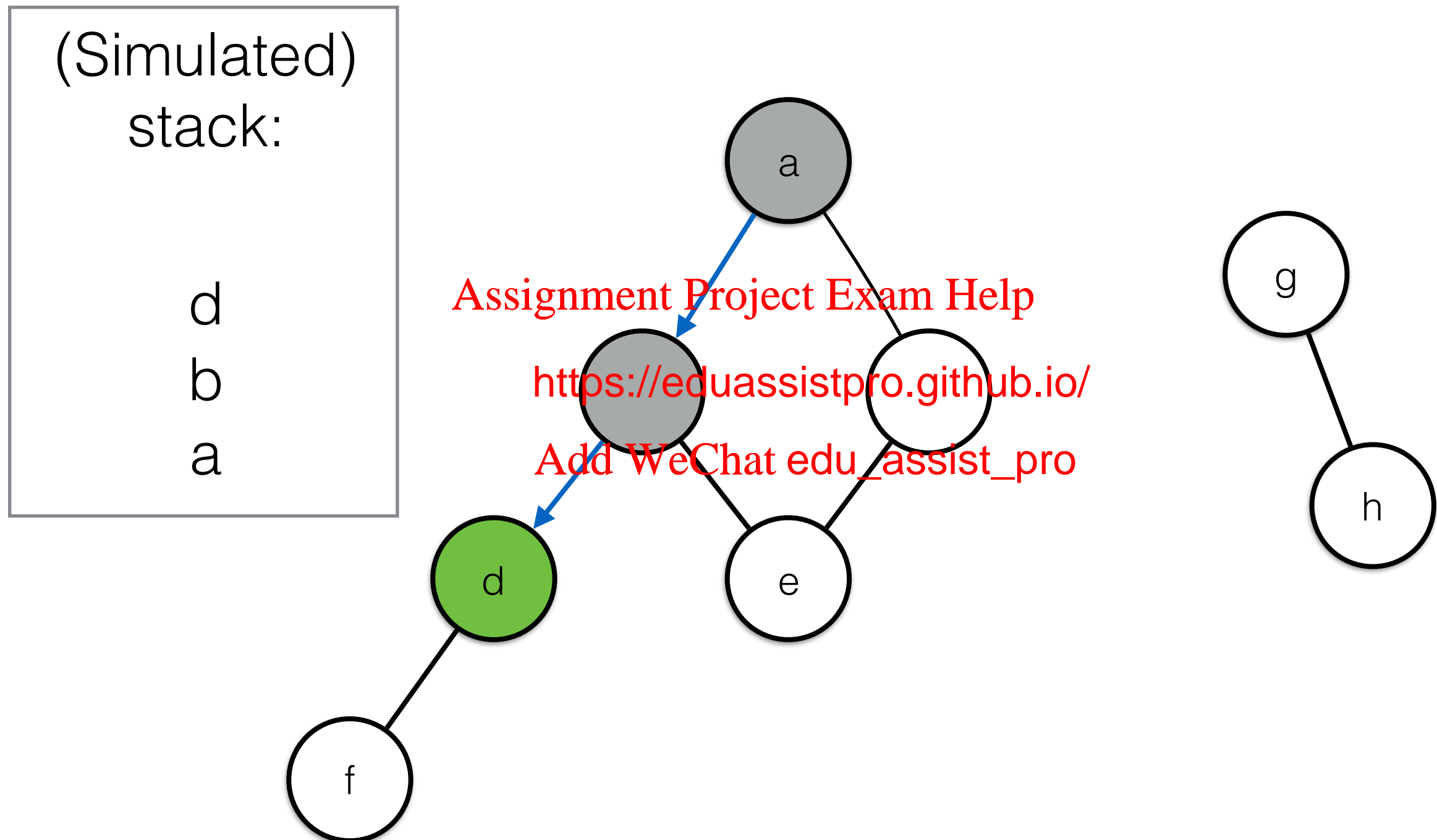
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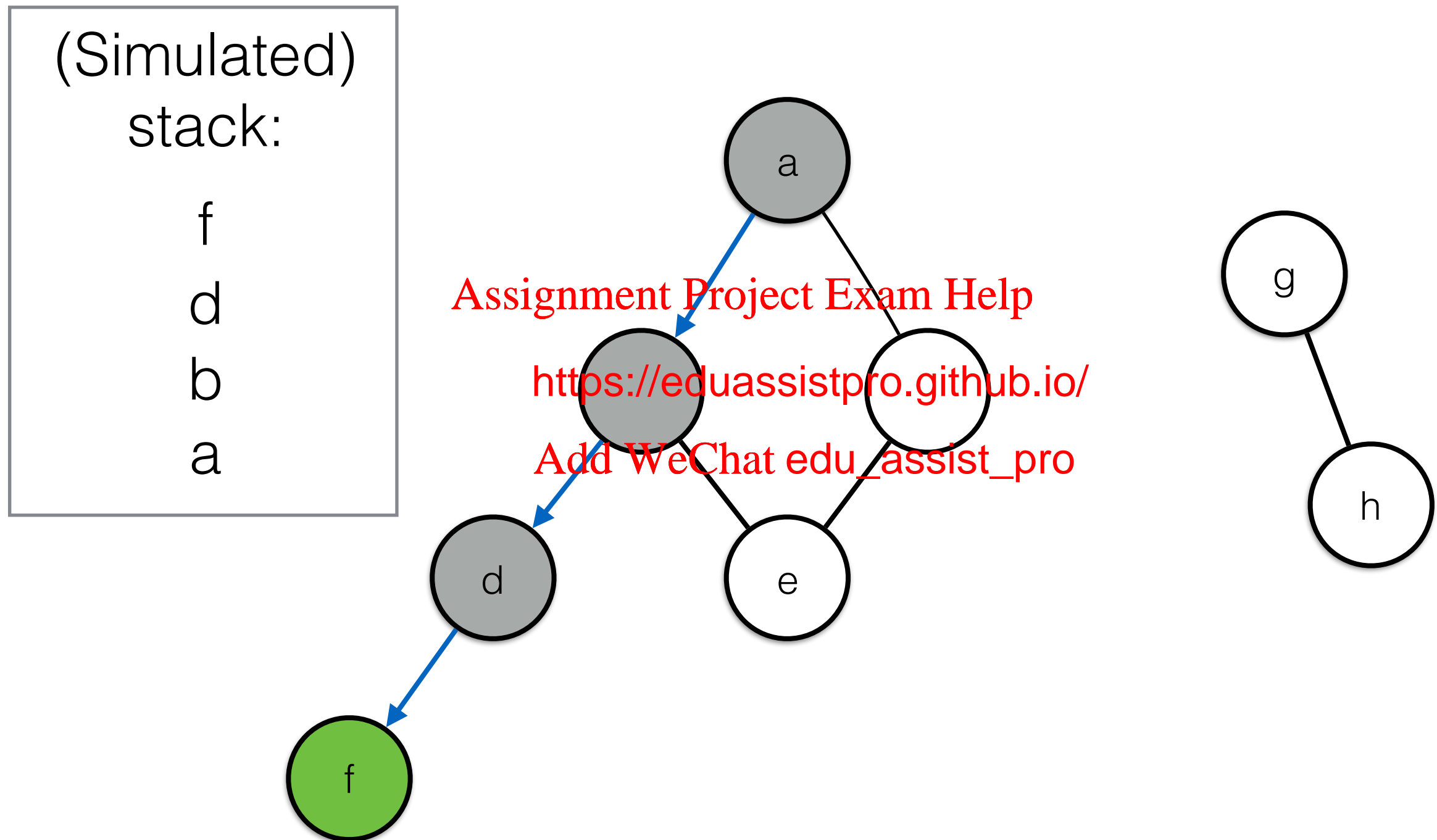
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Remember which
nodes we have
already visited
to avoid revisiting them

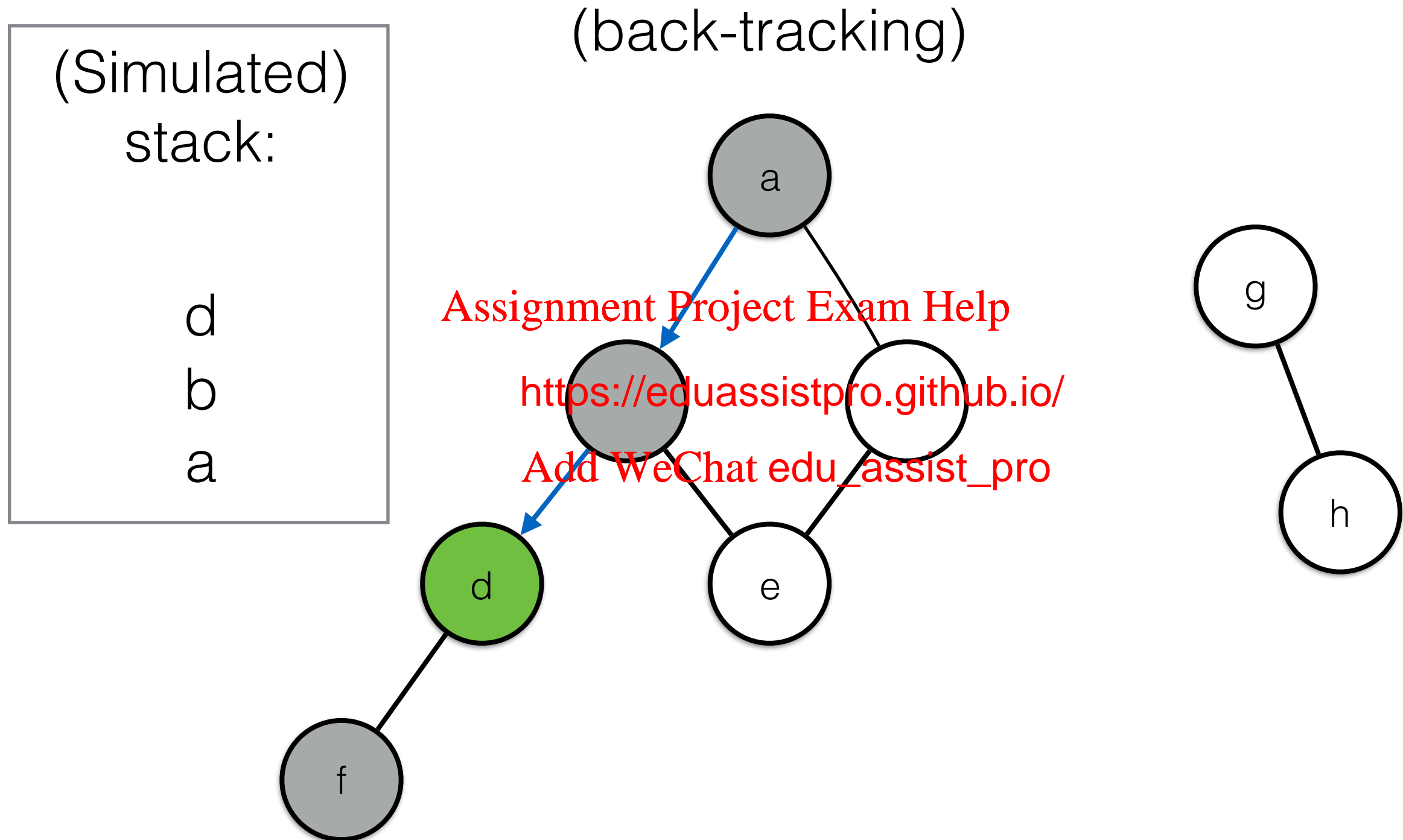
Depth-First Traversal: Stack Discipline



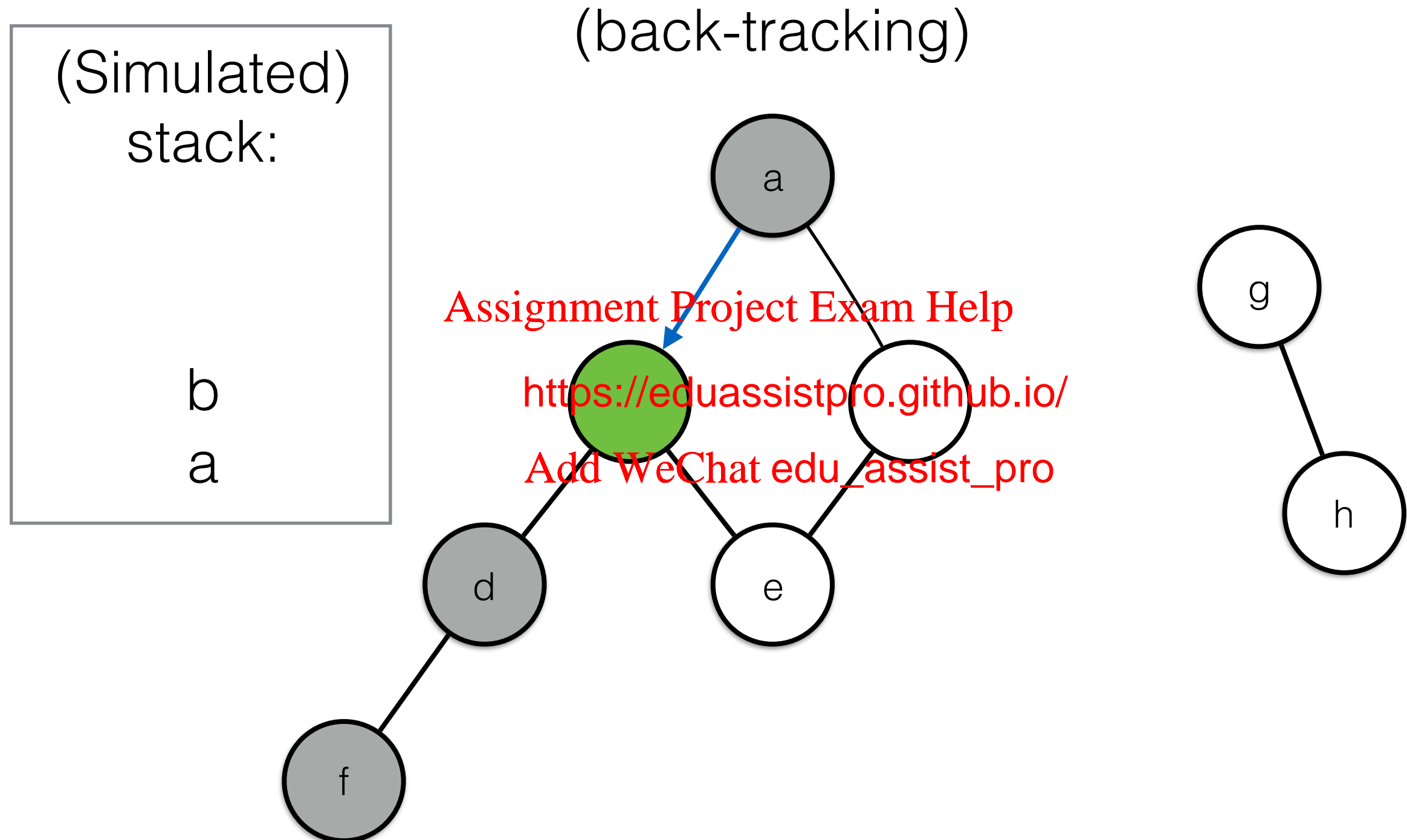
Depth-First Traversal: Stack Discipline



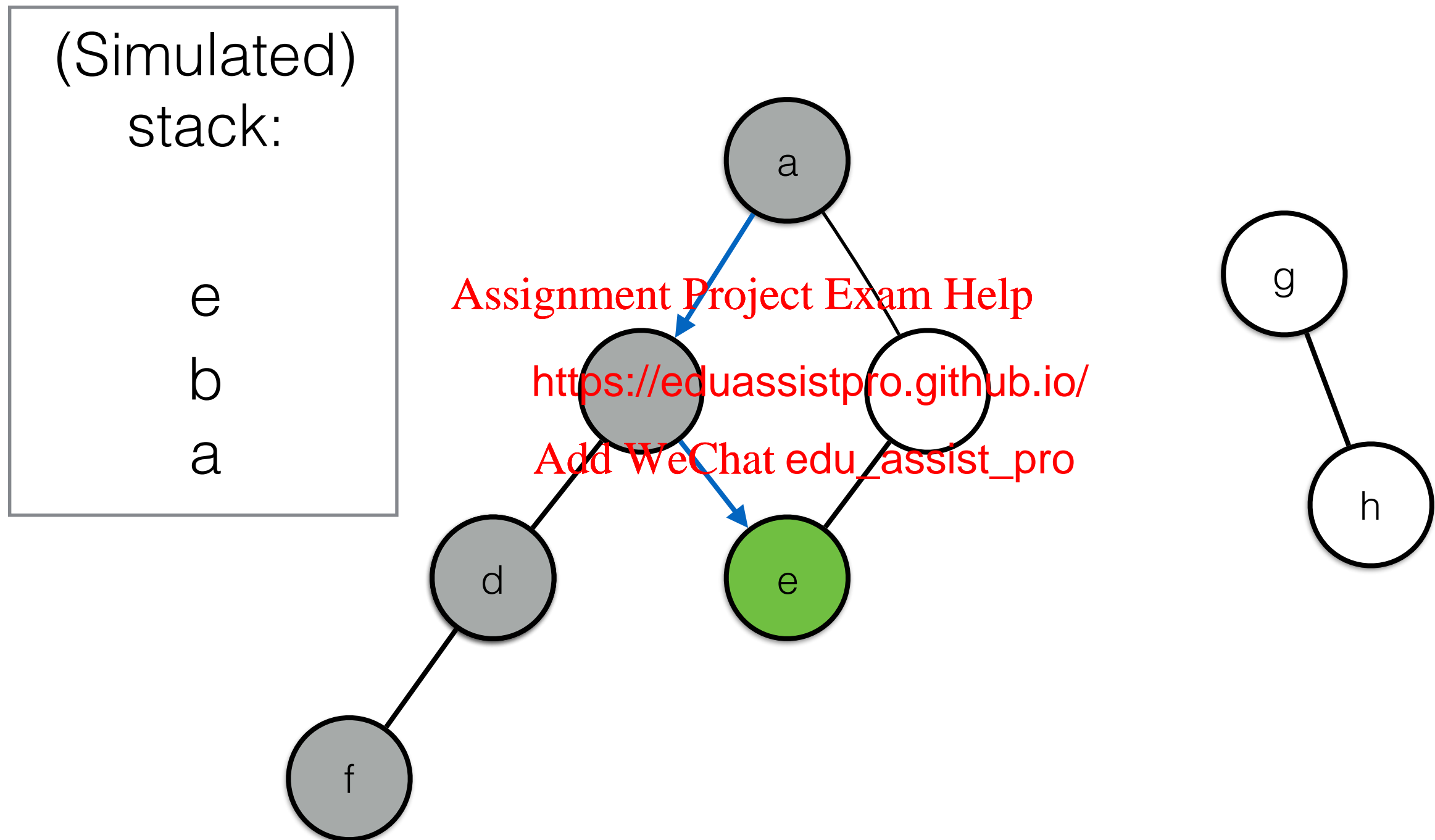
Depth-First Traversal: Stack Discipline



Depth-First Traversal: Stack Discipline



Depth-First Traversal: Stack Discipline

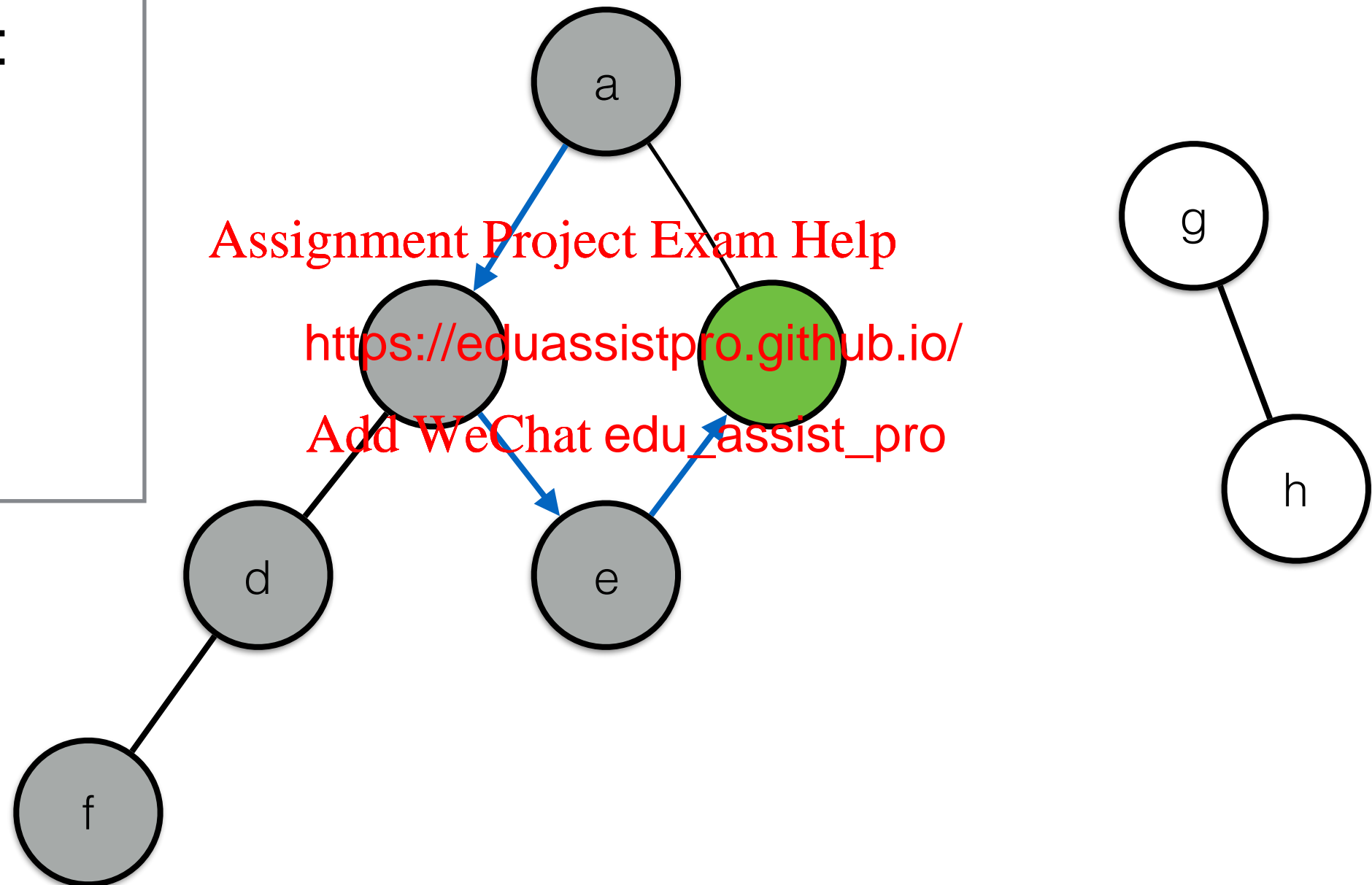


Depth-First Traversal: Stack Discipline



(Simulated)
stack:

c
e
b
a



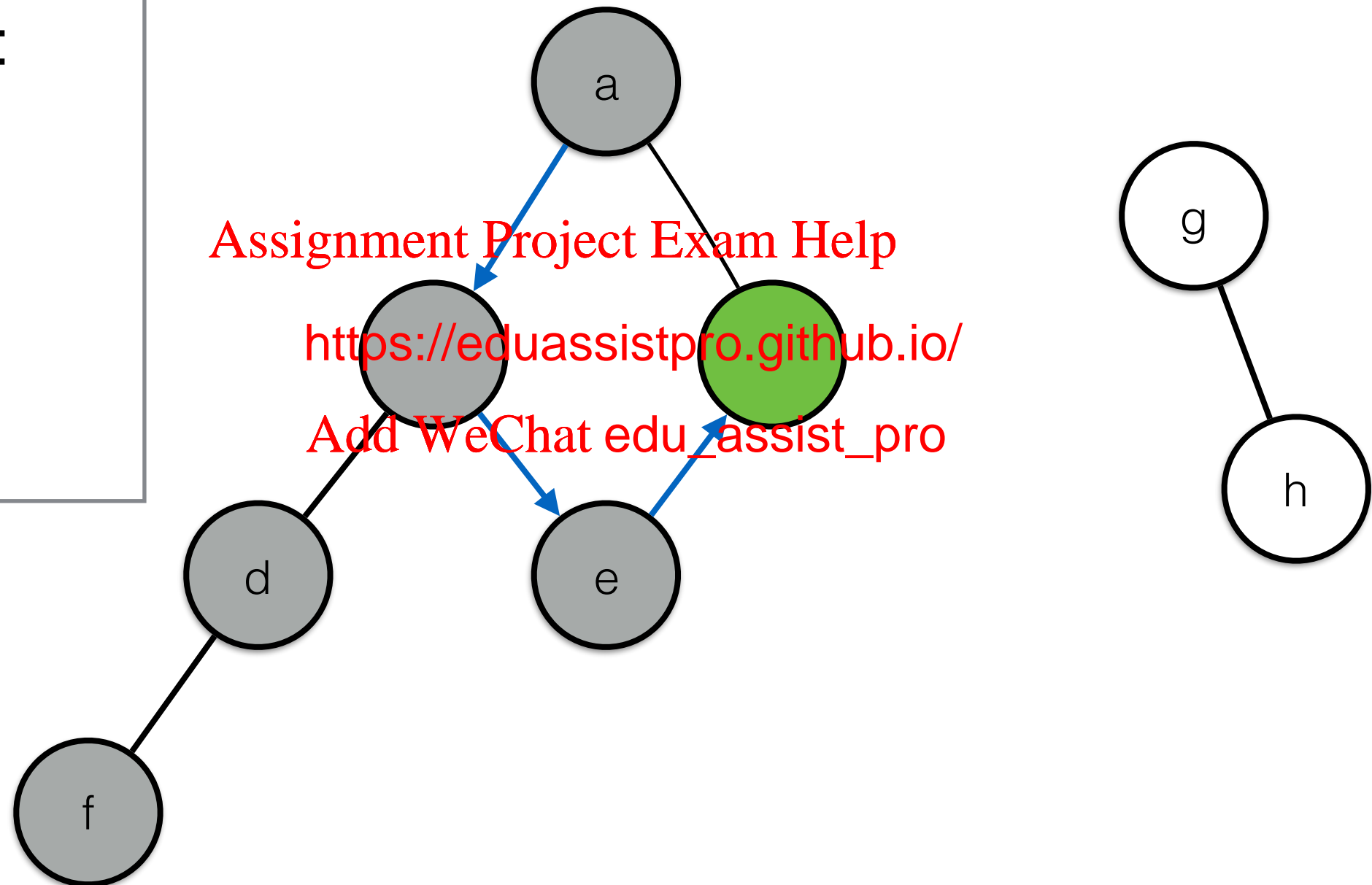
Depth-First Traversal: Stack Discipline



back-tracking ...
details omitted ...

(Simulated)
stack:

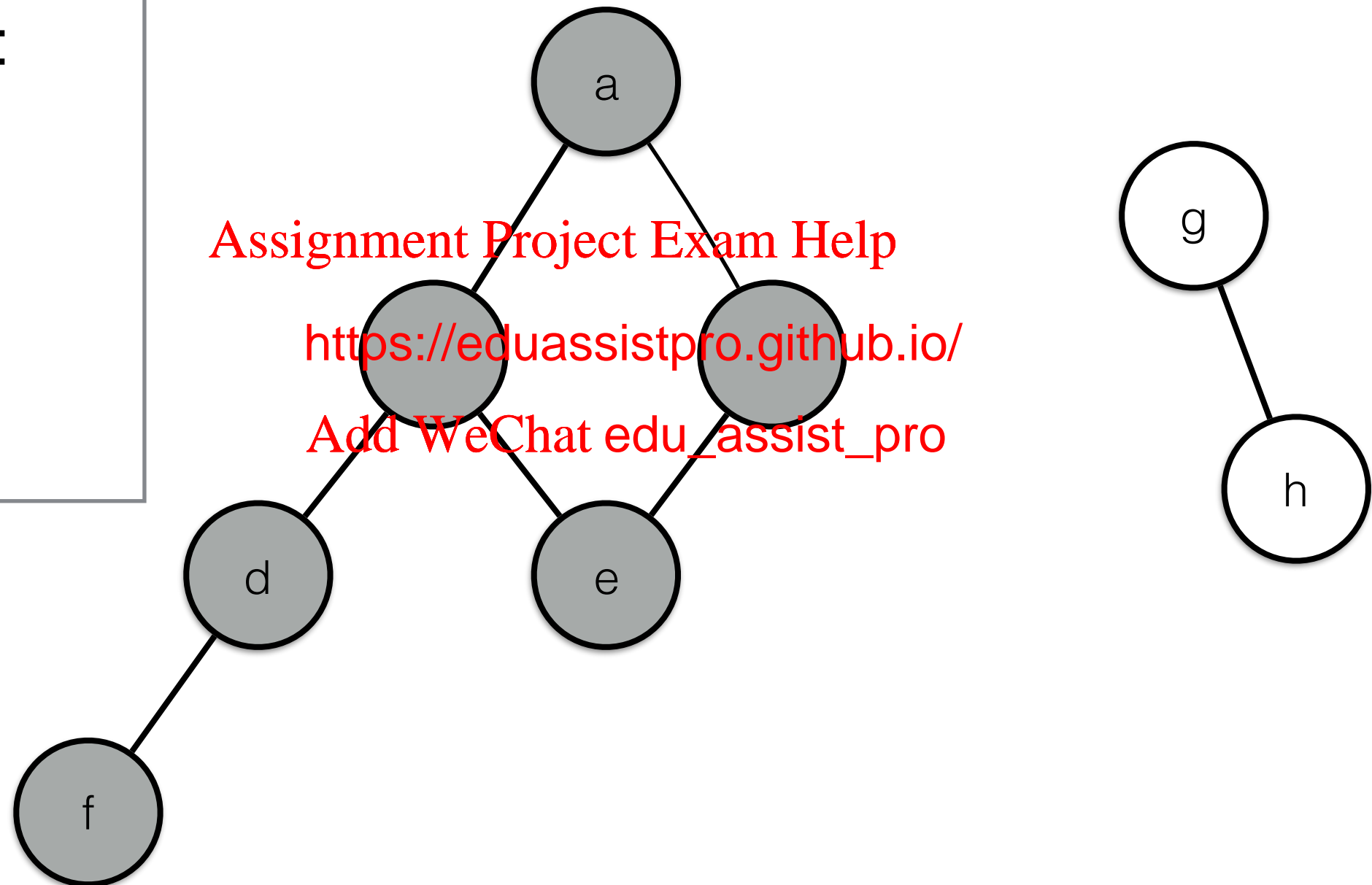
c
e
b
a



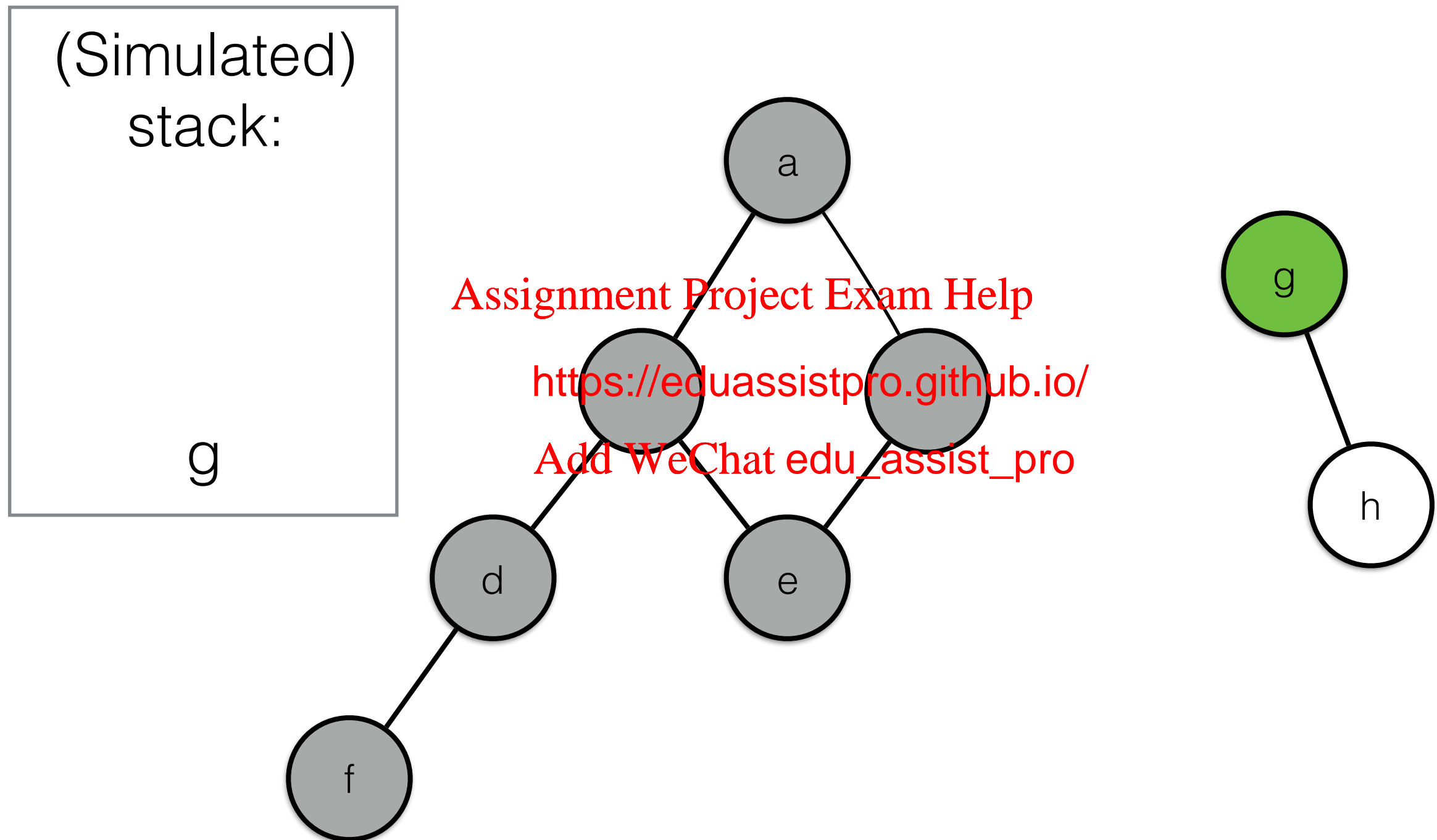
Depth-First Traversal: Stack Discipline



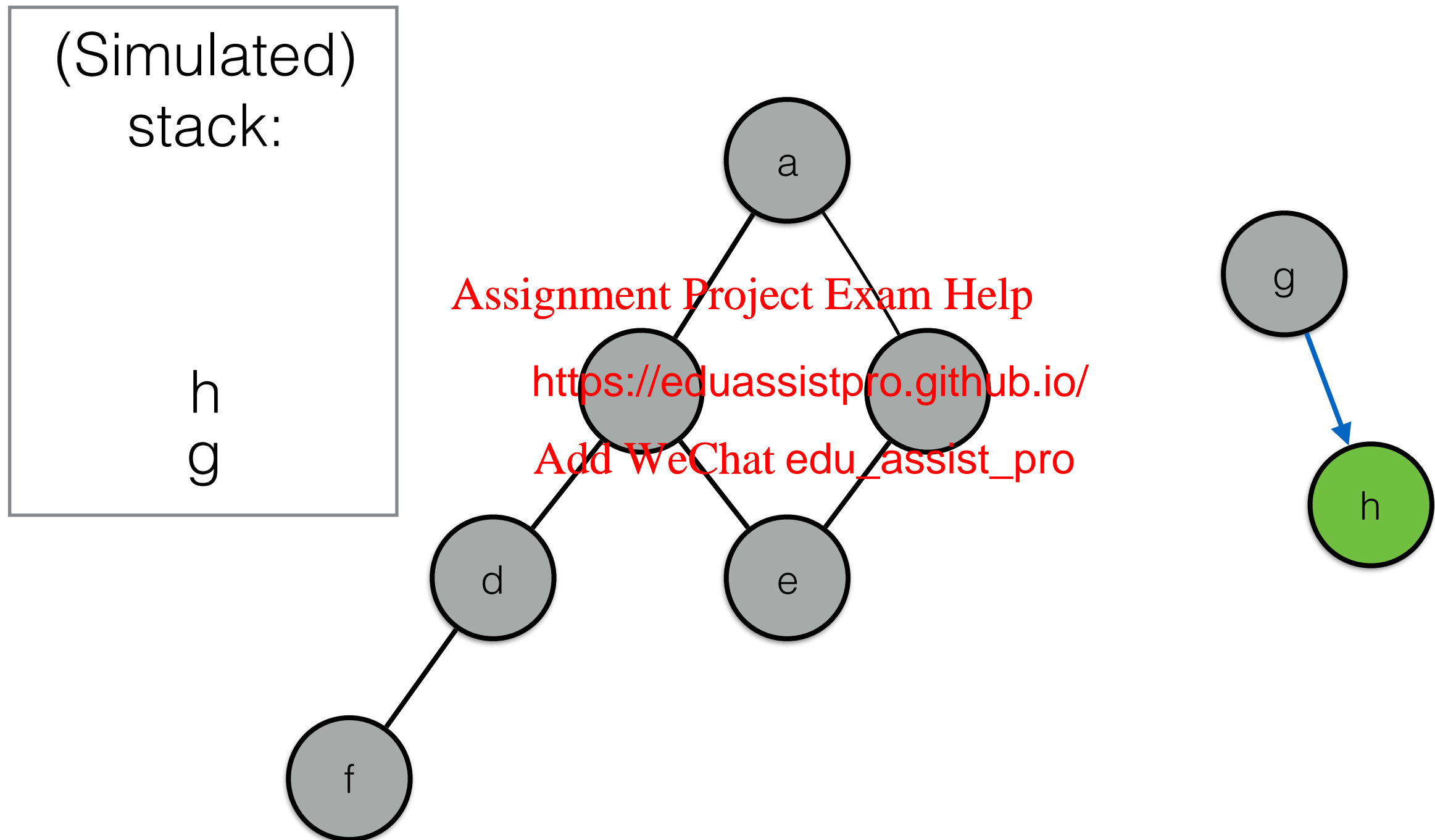
(Simulated)
stack:



Depth-First Traversal: Stack Discipline



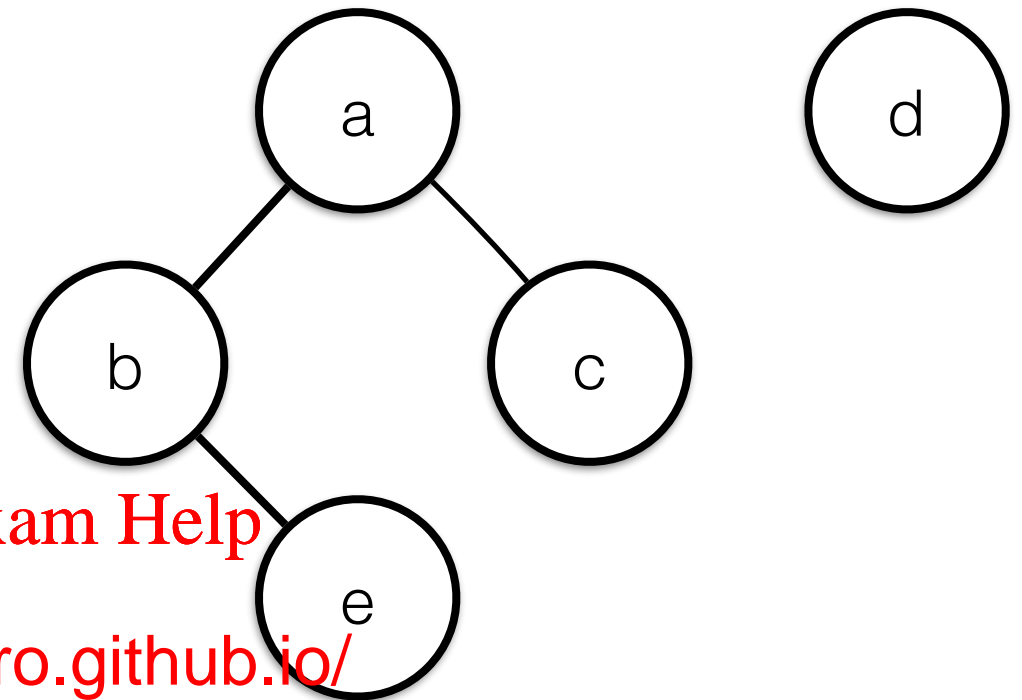
Depth-First Traversal: Stack Discipline



Depth-First Traversal: Recursive Implementation



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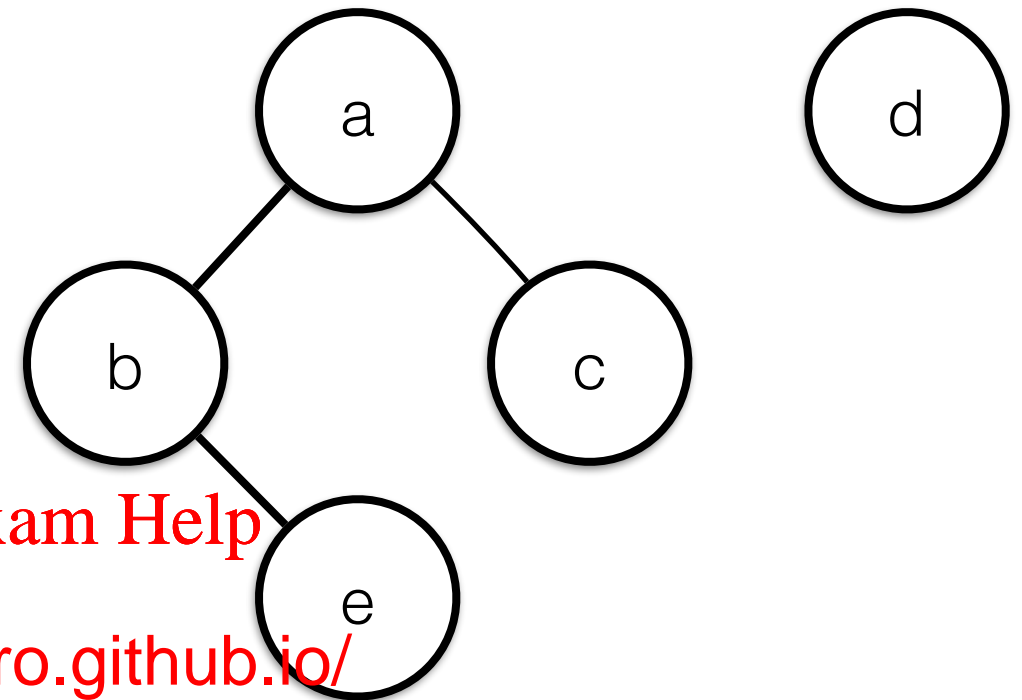
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Call Stack

Depth-First Traversal: Recursive Implementation



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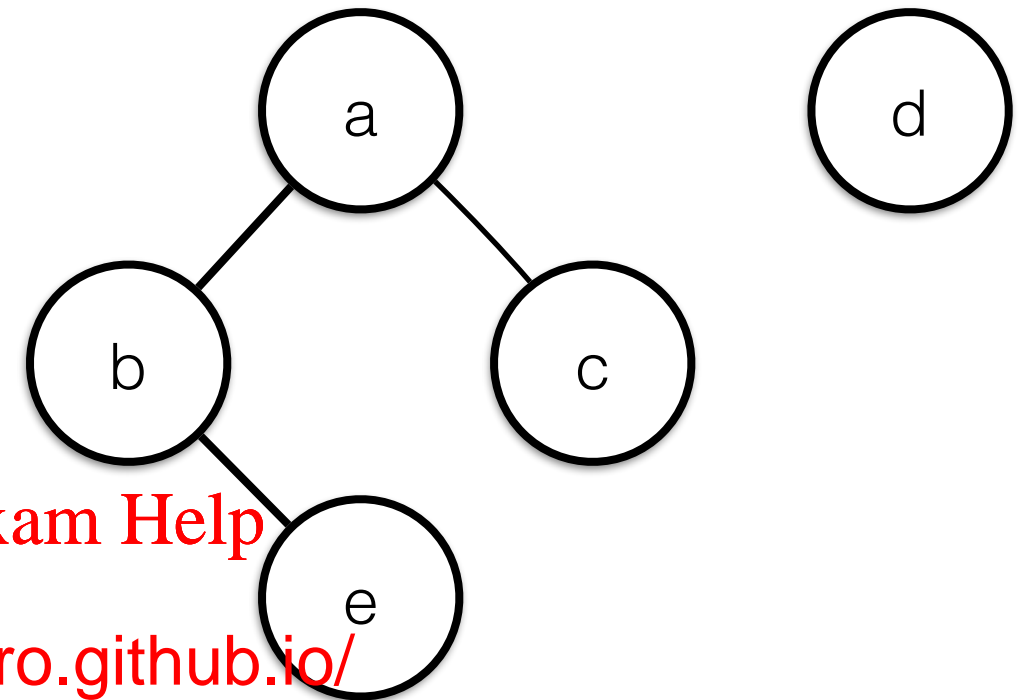
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DFS($\langle V, E \rangle$)
Call Stack

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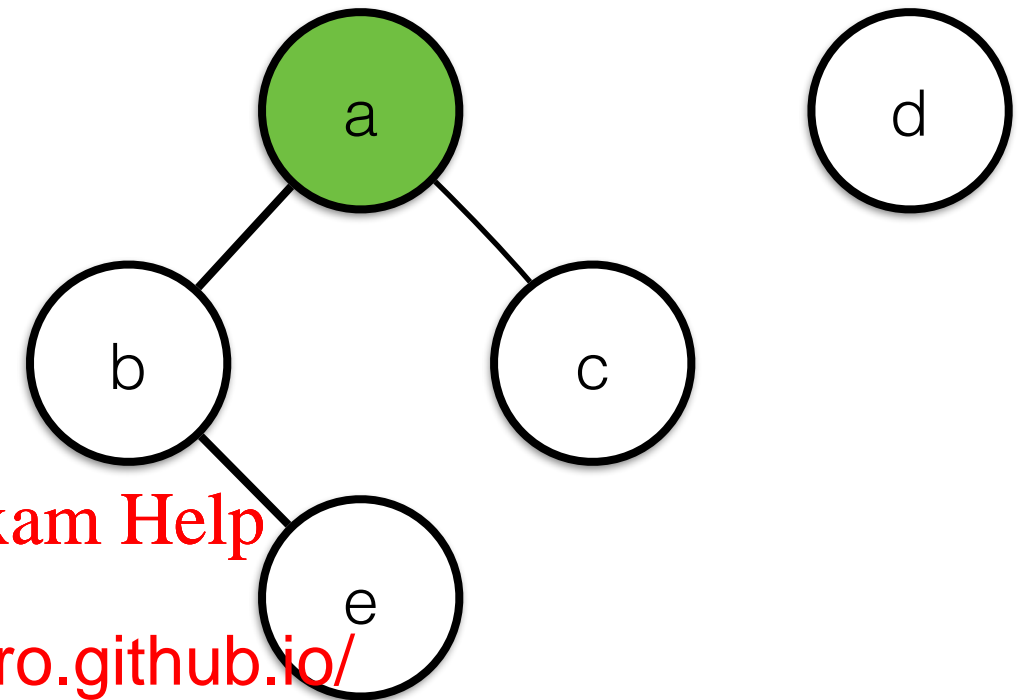
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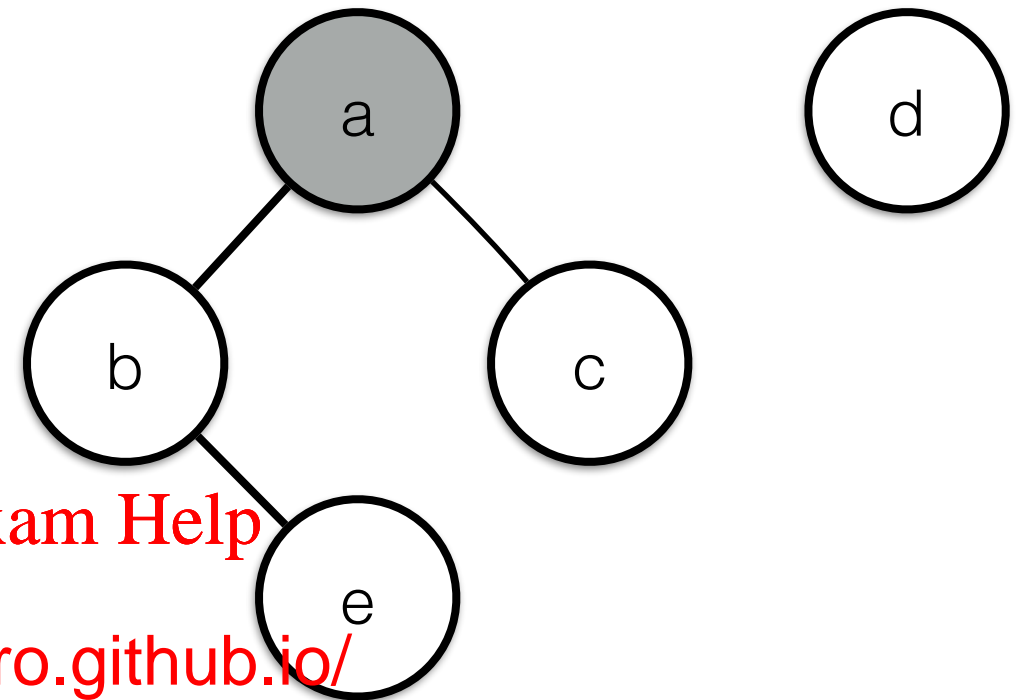
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DFSEXPLORE(a)
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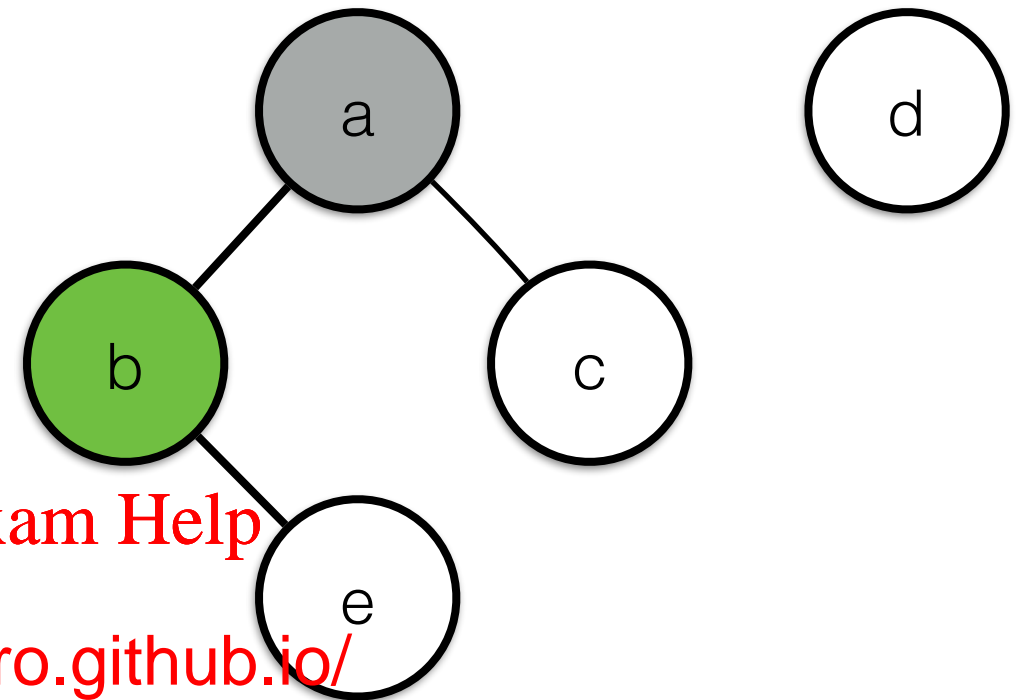
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DFSEXPLORE(a)
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count:
1

DFSEXPLORE(b)

DFSEXPLORE(a)

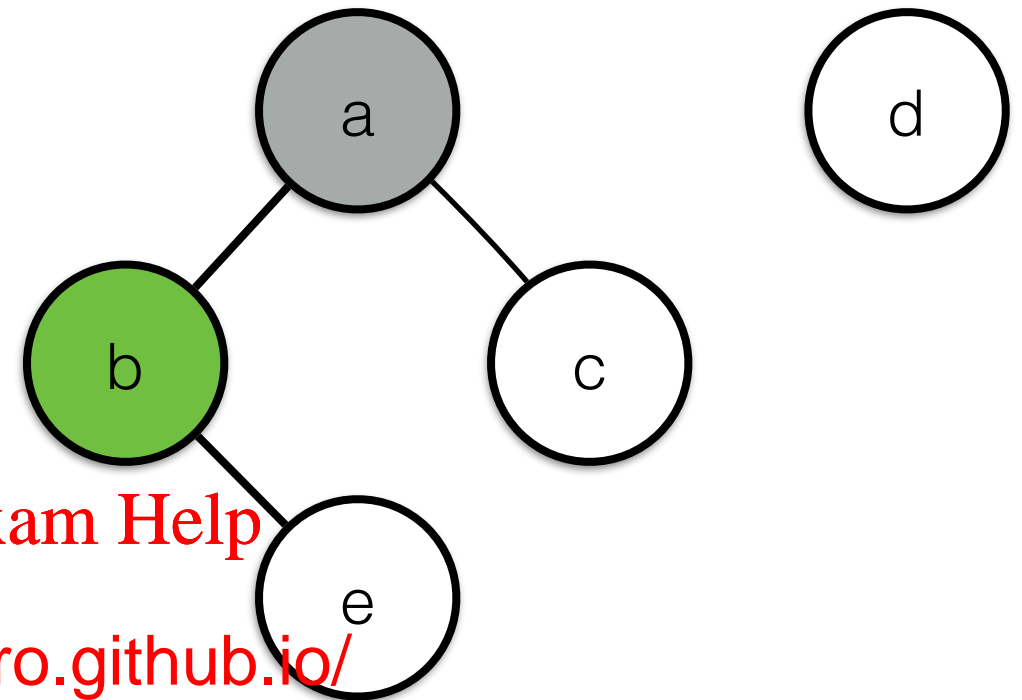
DFS($\langle V, E \rangle$)

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count:
2

DFSEXPLORE(b)

DFSEXPLORE(a)

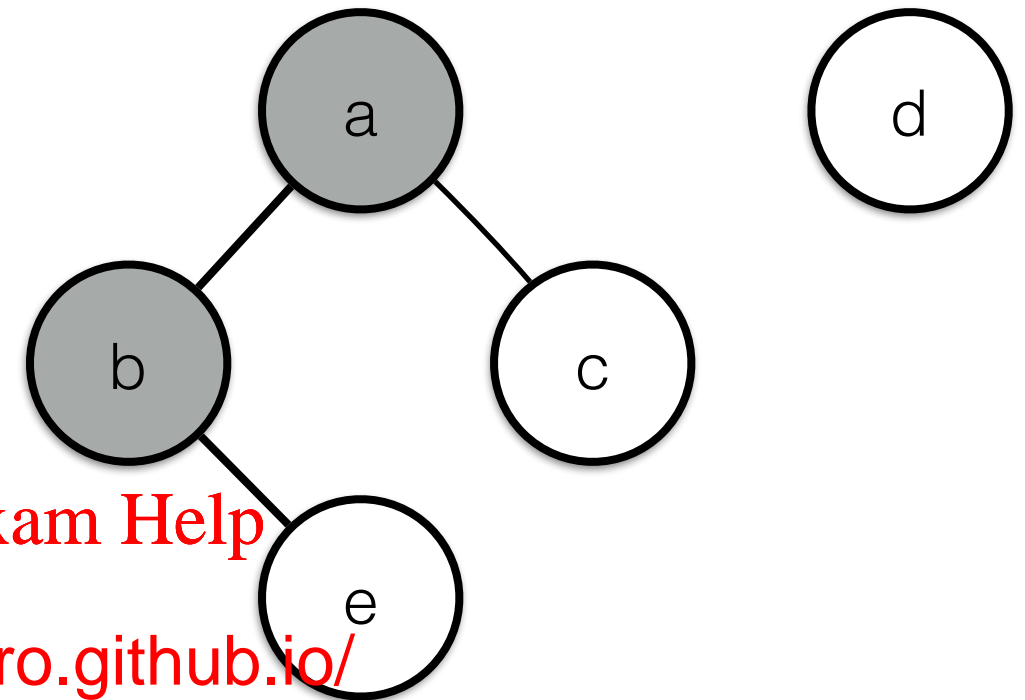
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count:
2

DFSEXPLORE(b)

DFSEXPLORE(a)

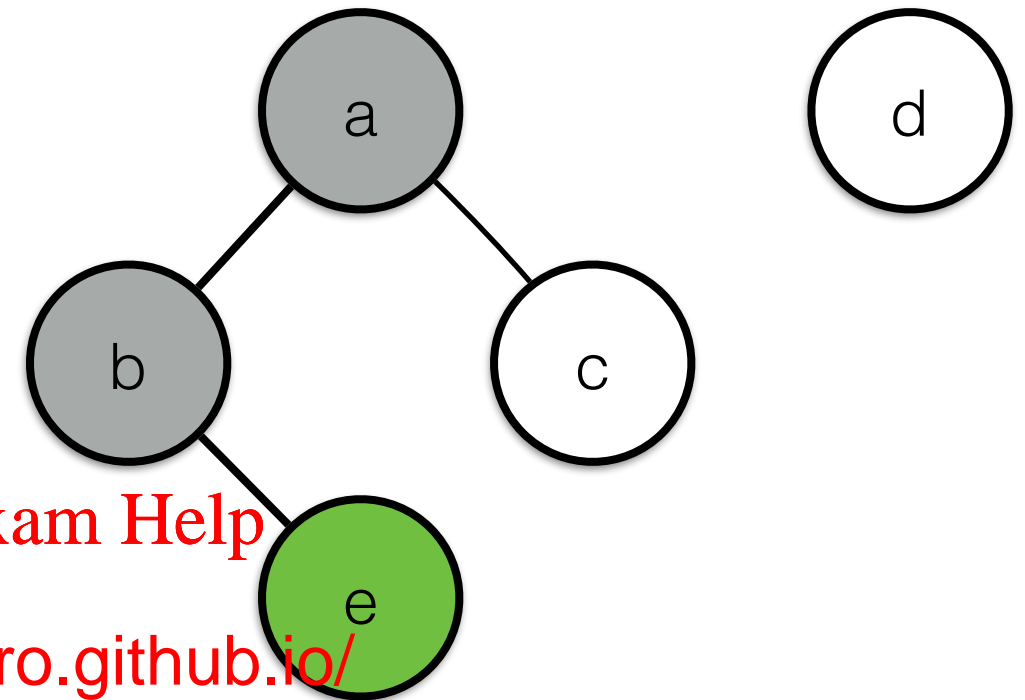
DFS($\langle V, E \rangle$)

Call Stack

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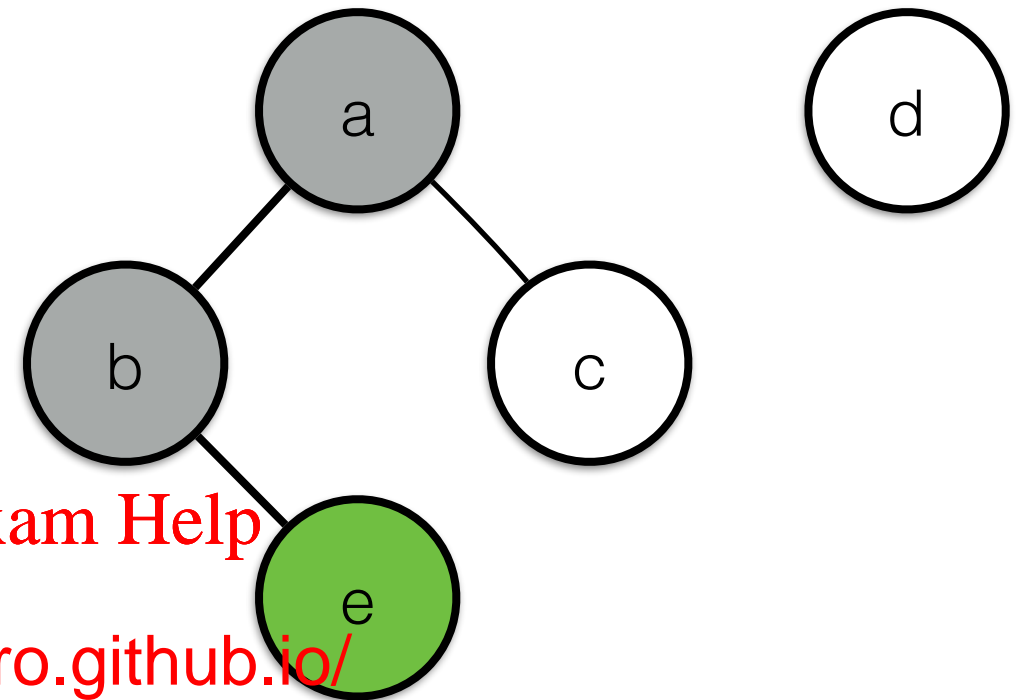
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DFSEXPLORE(e)
DFSEXPLORE(b)
DFSEXPLORE(a)
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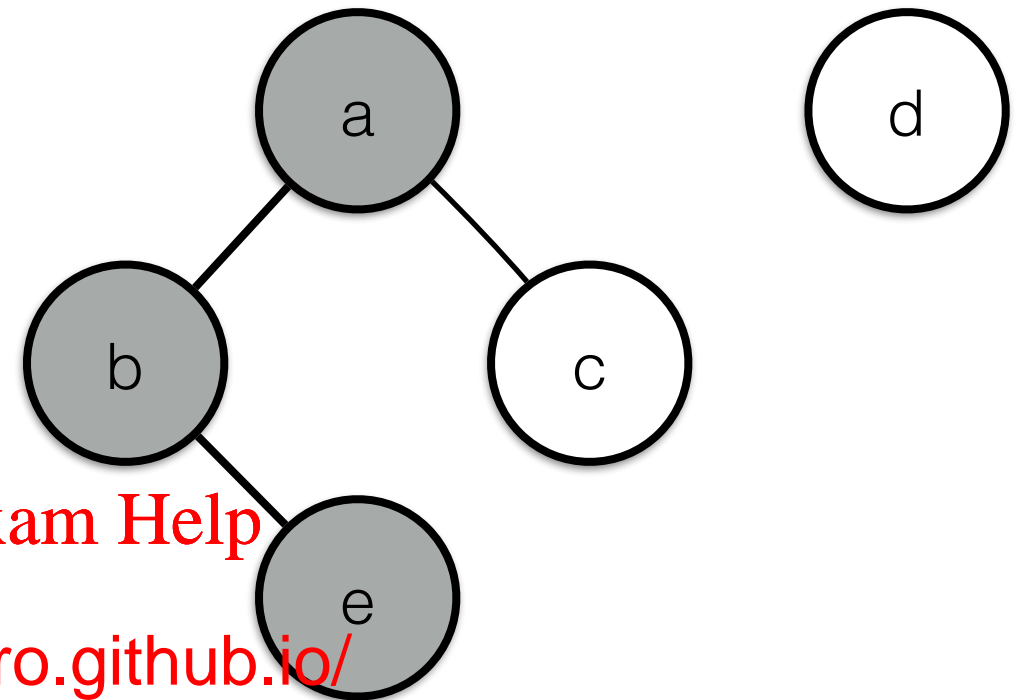
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DFSEXPLORE(e)
DFSEXPLORE(b)
DFSEXPLORE(a)
DFS($\langle V, E \rangle$)
Call Stack

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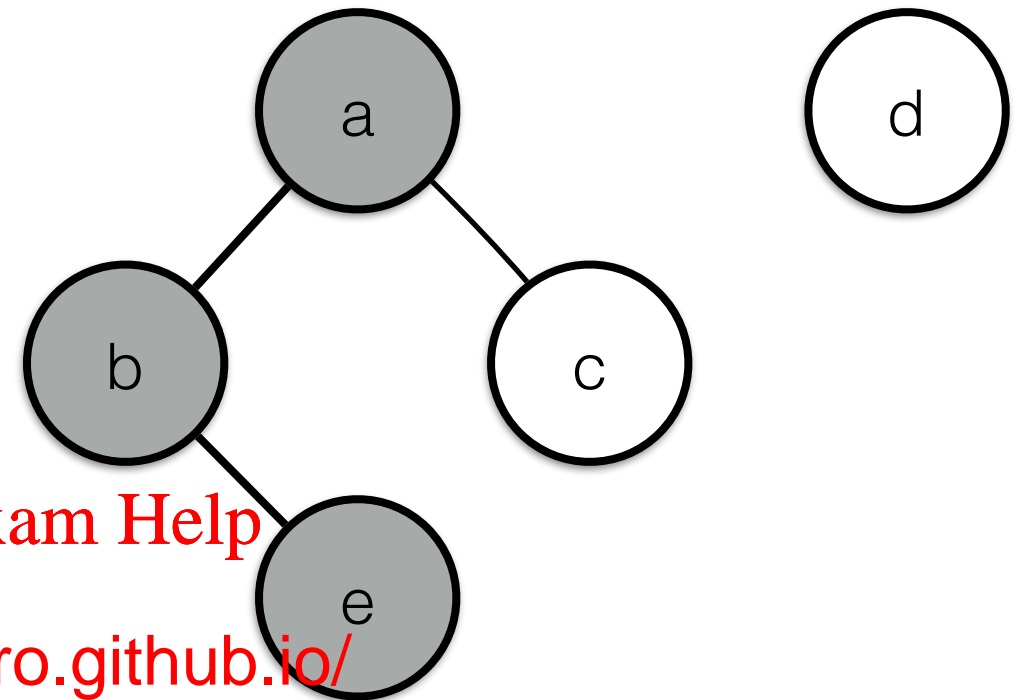
count:
3

DFSEXPLORE(e)
DFSEXPLORE(b)
DFSEXPLORE(a)
DFS($\langle V, E \rangle$)
Call Stack

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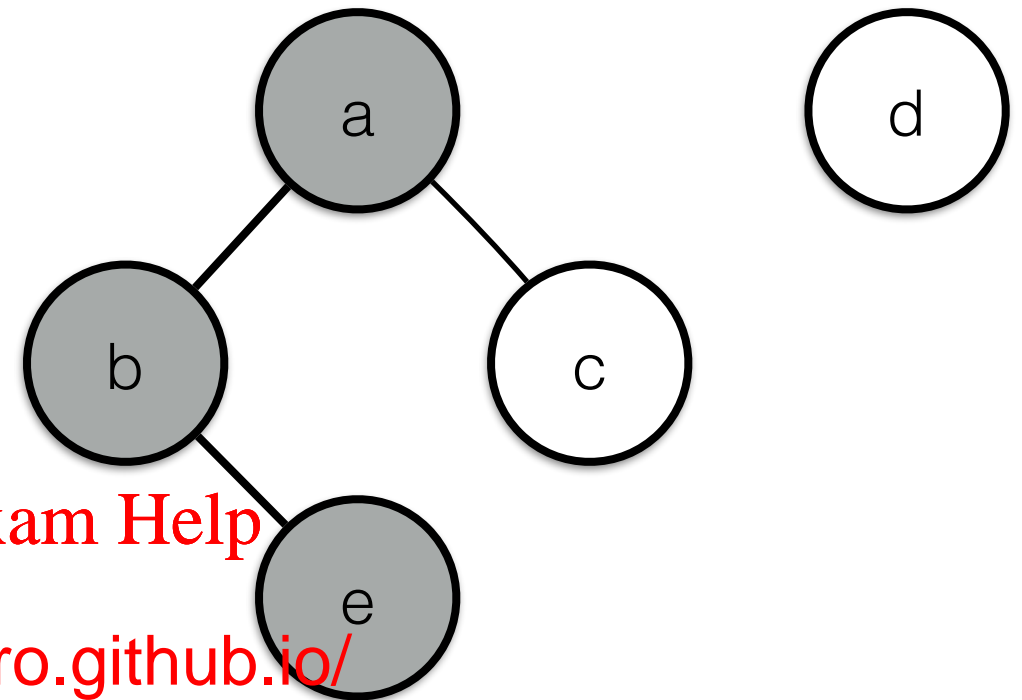
count:
3

DFSEXPLORE(b)
DFSEXPLORE(a)
DFS($\langle V, E \rangle$)
Call Stack

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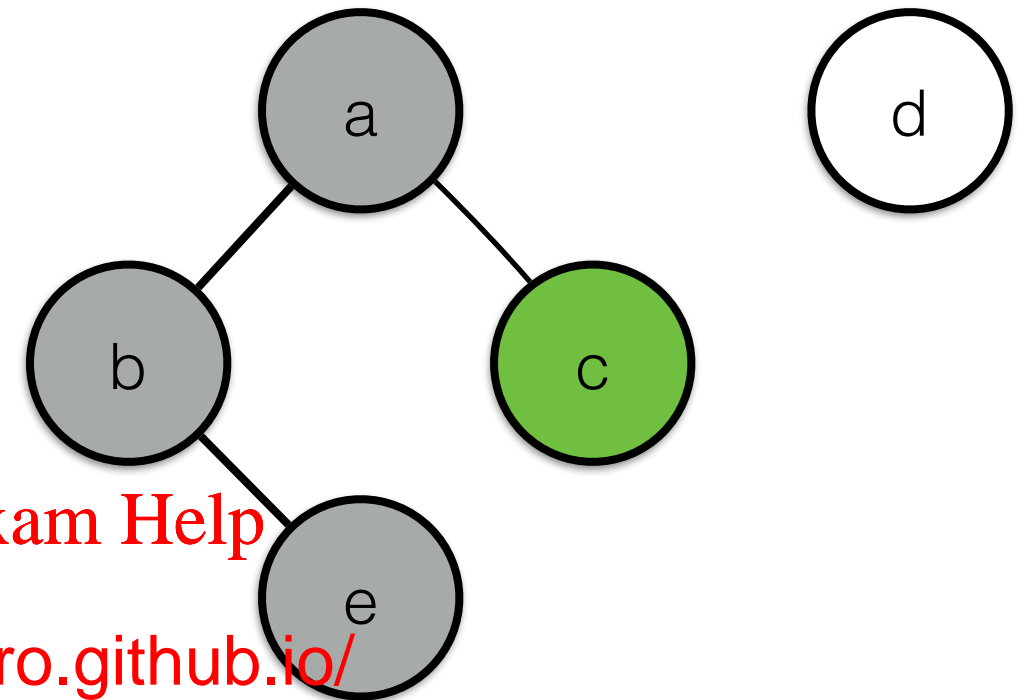
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DFSEXPLORE(a)
DFS($\langle V, E \rangle$)
Call Stack

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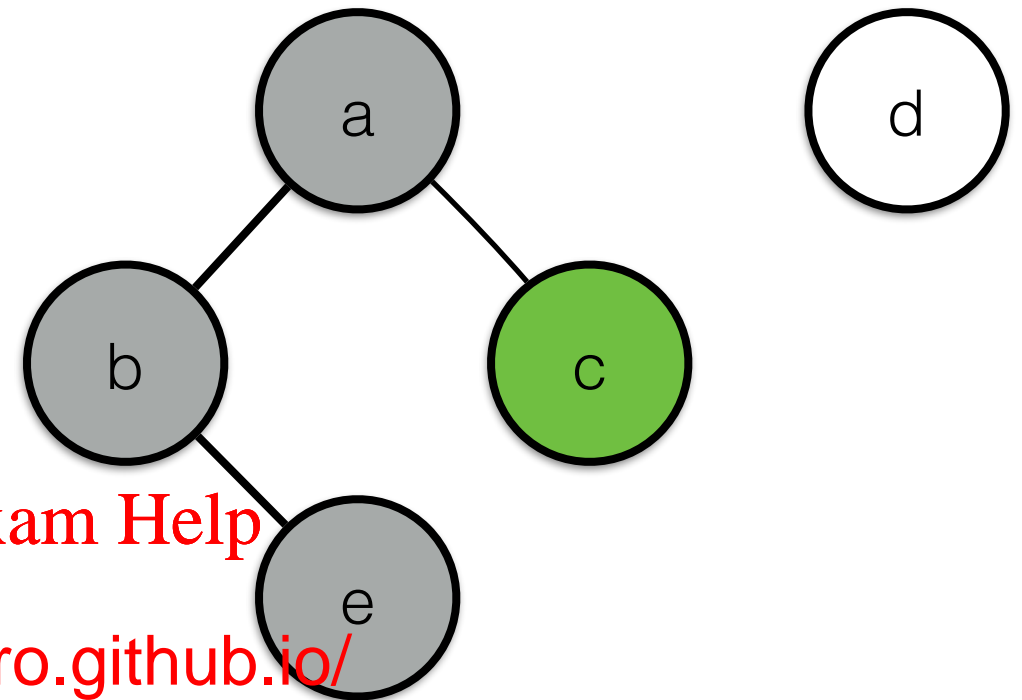
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DFSEXPLORE(c)
DFSEXPLORE(a)
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Call Stack

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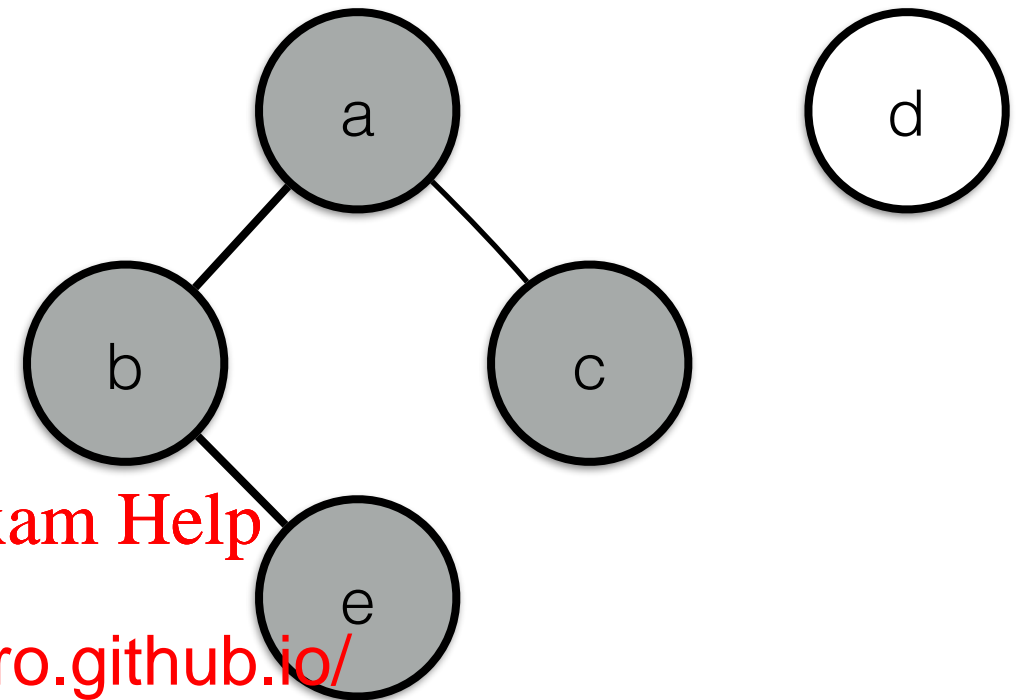
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DFSEXPLORE(c)
DFSEXPLORE(a)
DFS($\langle V, E \rangle$)
Call Stack

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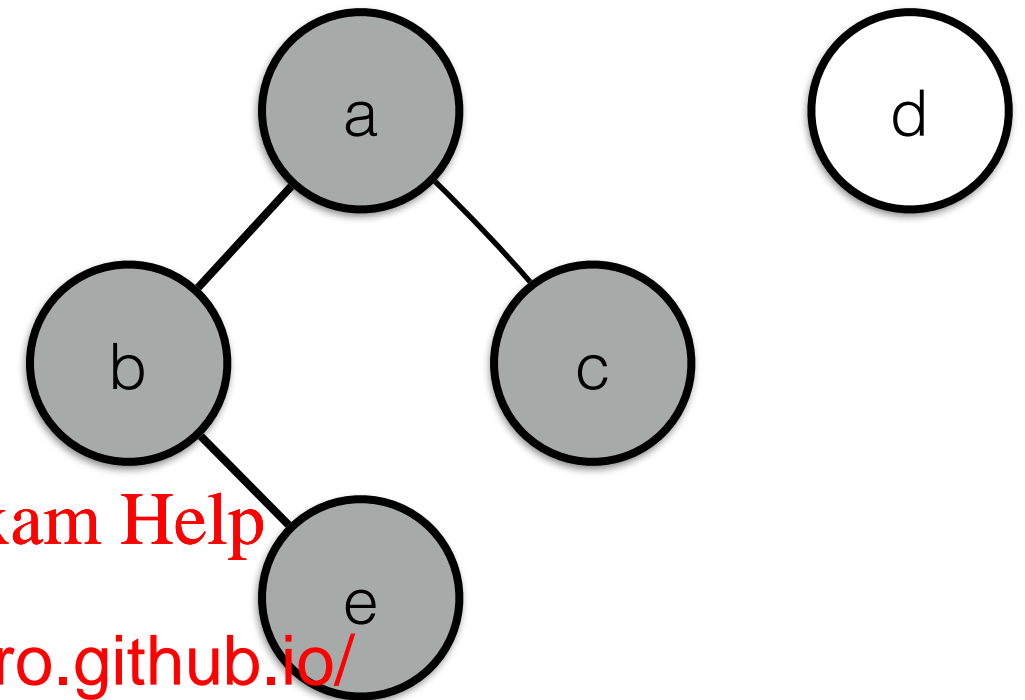
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DFSEXPLORE(c)
DFSEXPLORE(a)
DFS($\langle V, E \rangle$)
Call Stack

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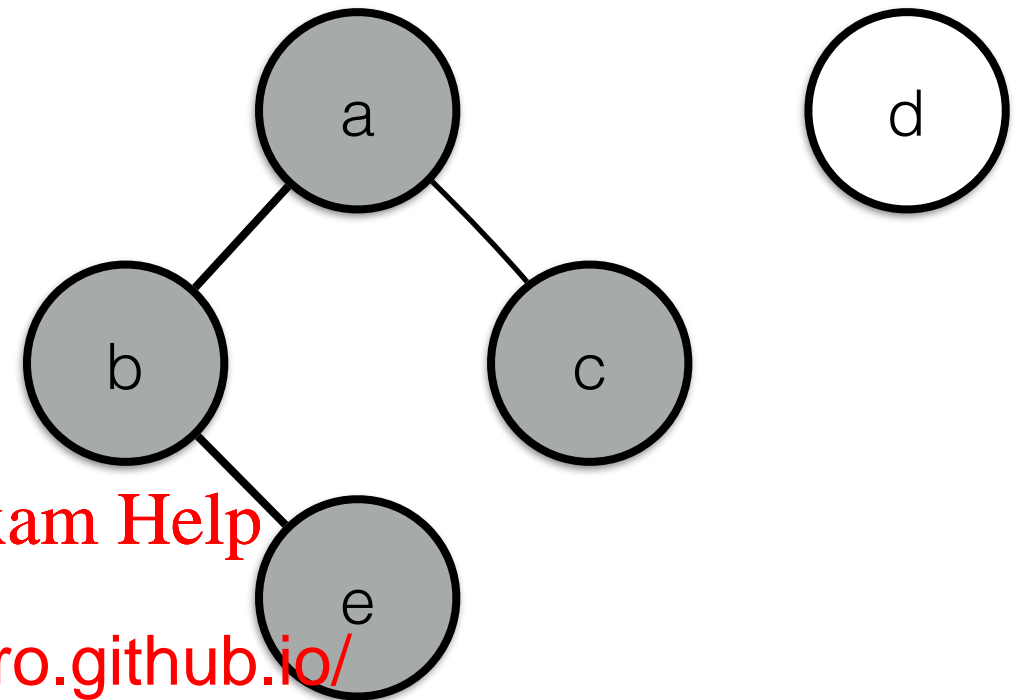
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DFSEXPLORE(a)
DFS($\langle V, E \rangle$)
Call Stack

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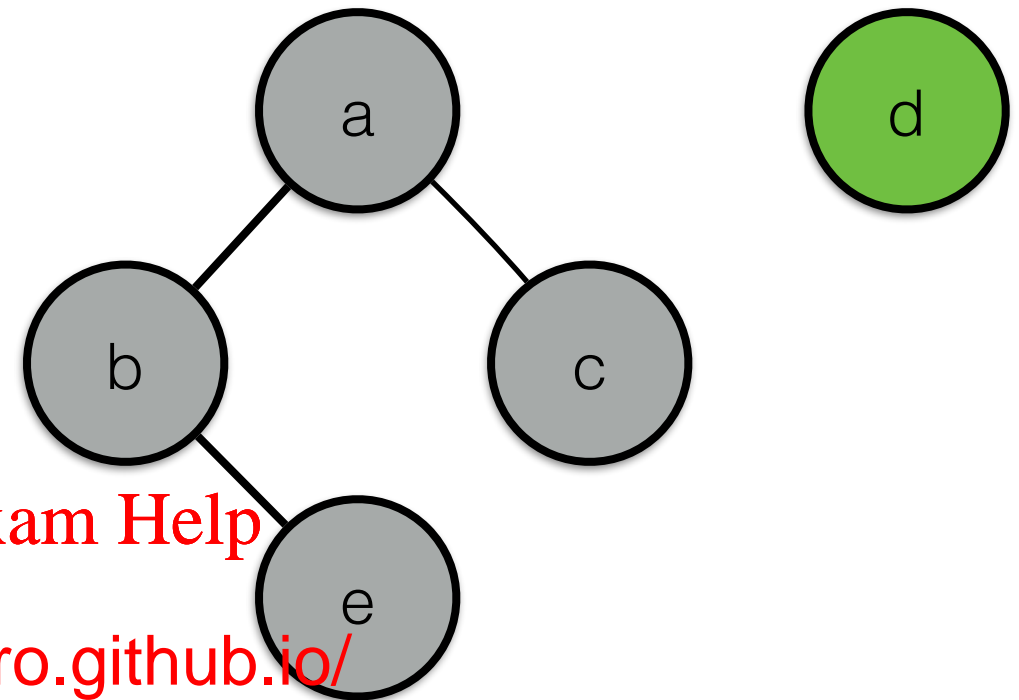
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DFS($\langle V, E \rangle$)
Call Stack

Depth-First Traversal: Recursive Implementation



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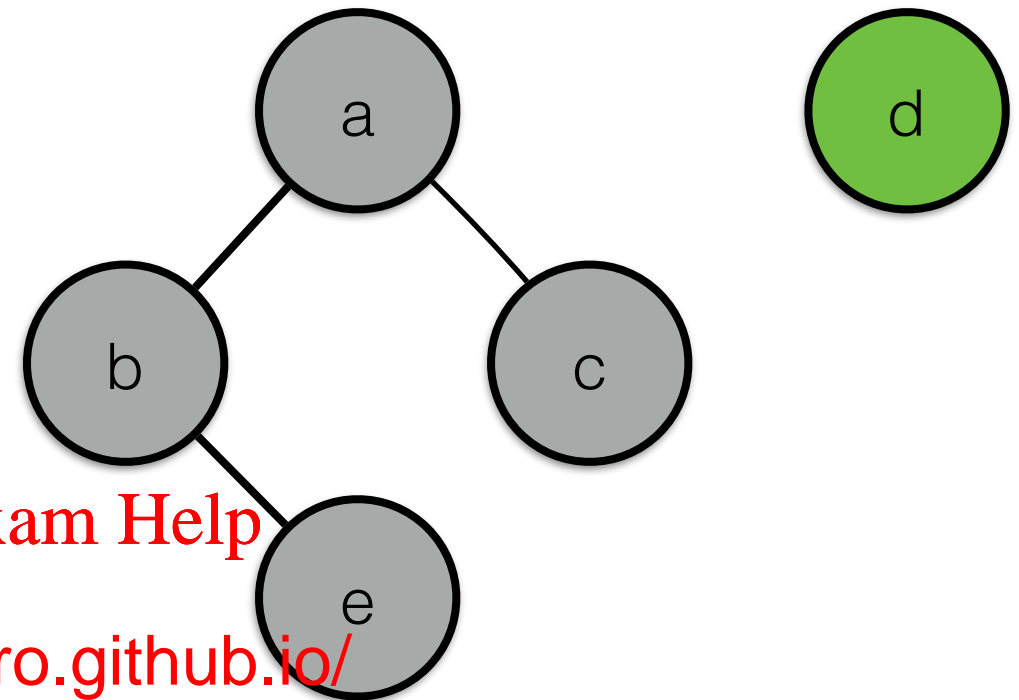
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DFSEXPLORE(d)
DFS($\langle V, E \rangle$)
Call Stack

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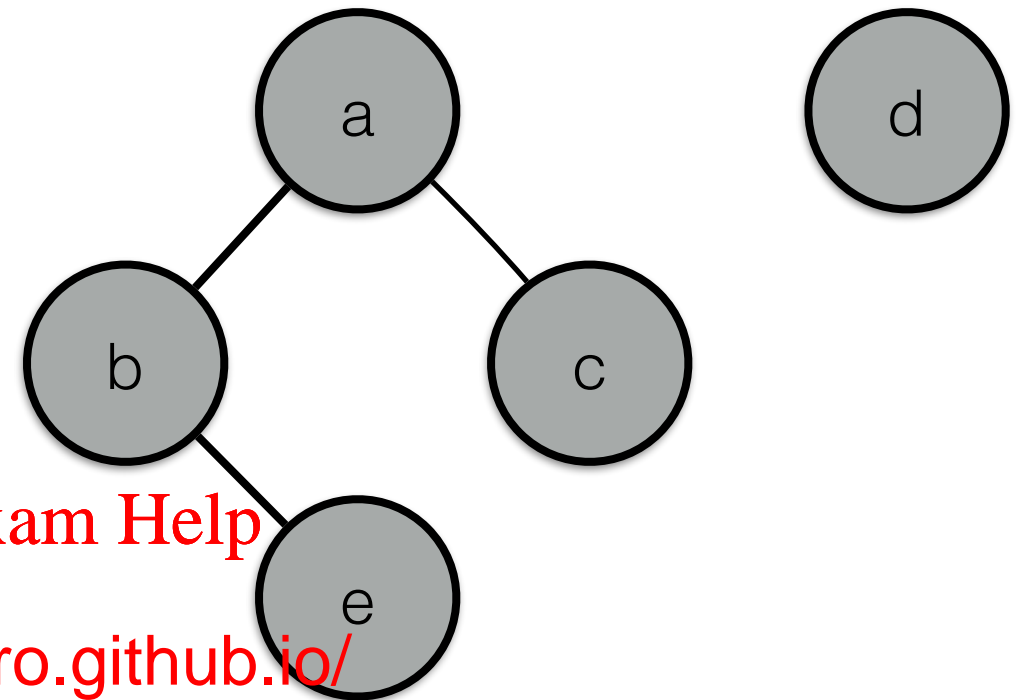
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DFSEXPLORE(d)
DFS($\langle V, E \rangle$)
Call Stack

Depth-First Traversal: Recursive Implementation



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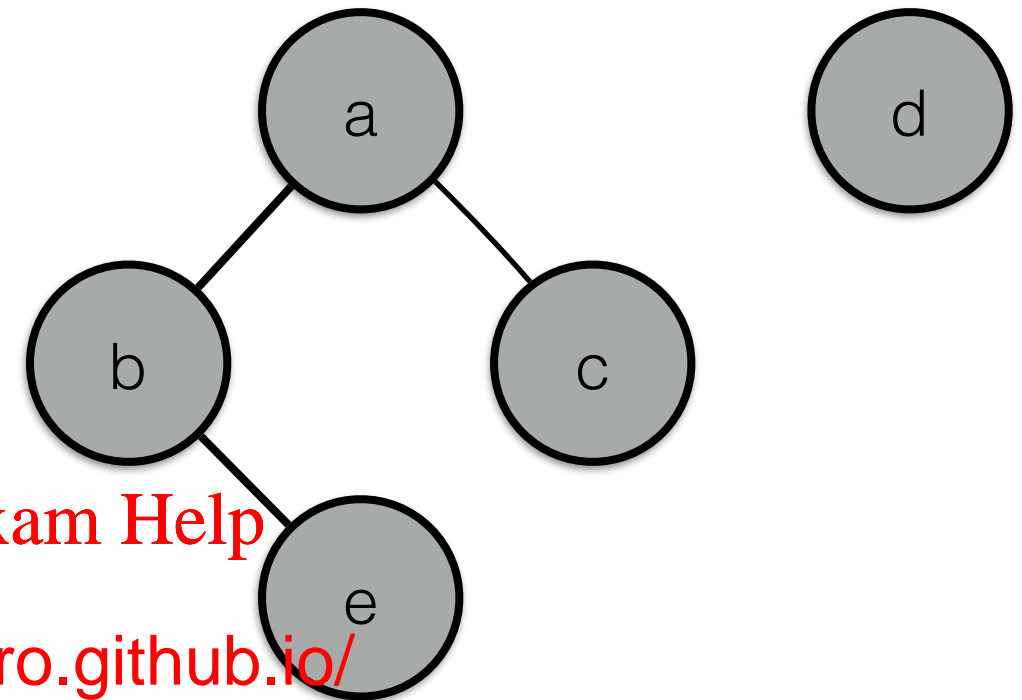
count:
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DFSEXPLORE(d)
DFS($\langle V, E \rangle$)
Call Stack

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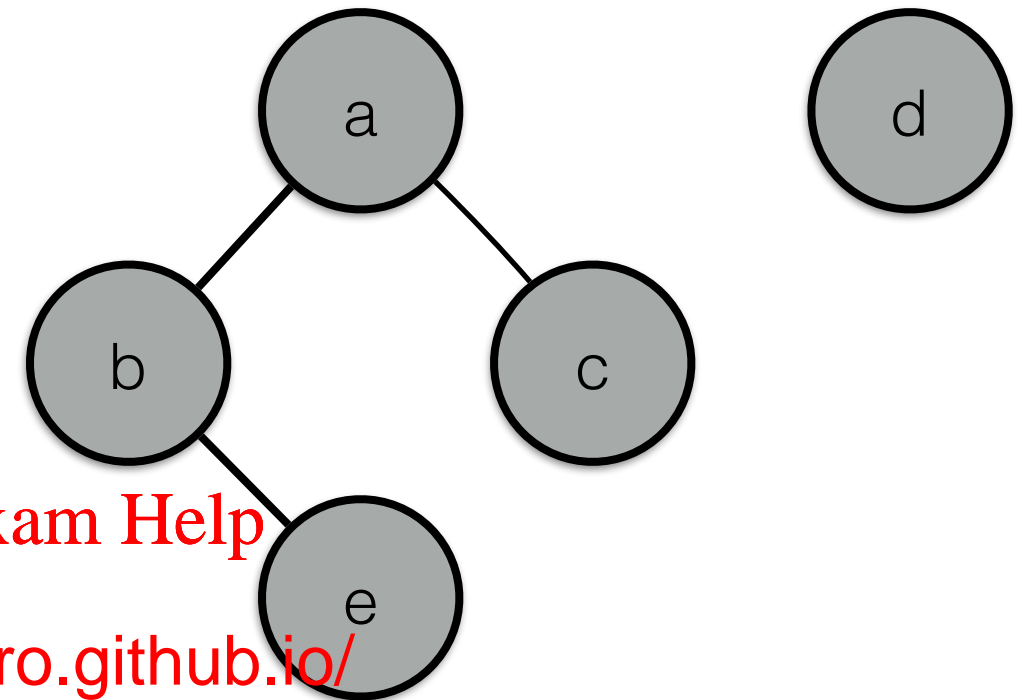
count:
5

DFS($\langle V, E \rangle$)
Call Stack

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count:
5

Call Stack

Depth-First Search: Recursive Algorithm Notes



- Works both for directed and undirected graphs.
- The “marking” of nodes is usually done by maintaining a separate array, **mark**, indexed by V .
- For example, when we wrote “mark v with count”, that would be implemented as “**mark**”
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- How to find the nodes adjacent to v on the graph representation used.
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- Using an adjacency **matrix** **adj**, we need to consider **adj**[v , w] for each w in V . Here the complexity of graph traversal is $\Theta(|V|^2)$.
- Using adjacency **lists**, for each v , we traverse the list **adj**[v]. In this case, the complexity of traversal is $\Theta(|V| + |E|)$.

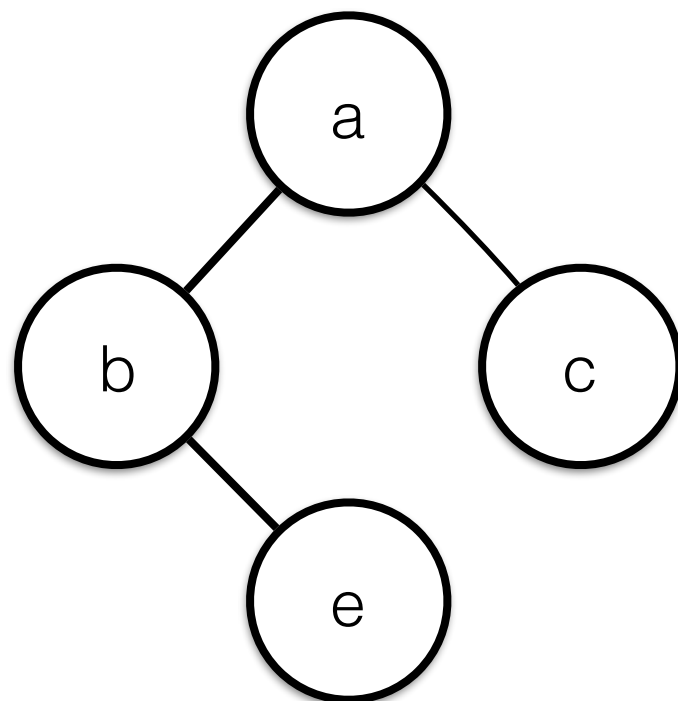
Applications of Depth-First Search (DFS)

- Easy to adapt DFS to decide if a graph is connected.
- How?

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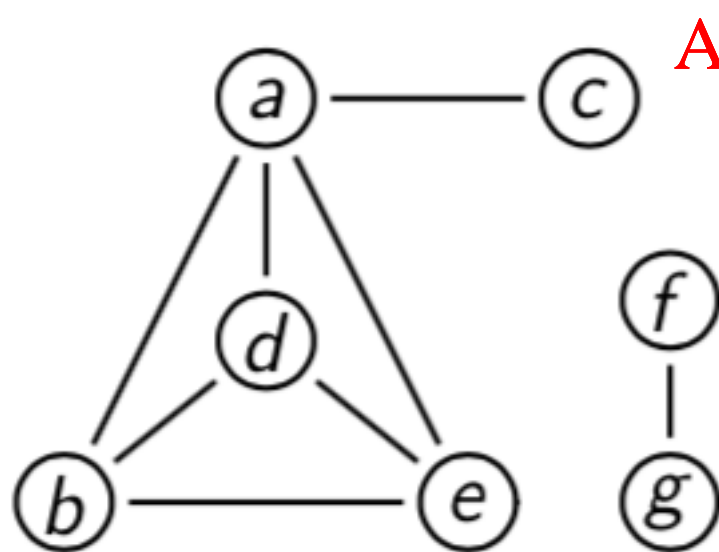
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Depth-First Search: Node Orderings

- We can order nodes **either** by the order in which they get **pushed onto** the stack, or by the order in which they are **popped from** the stack



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Levitin's compact stack notation

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The first subscripts give the order in which nodes are pushed, the second the order in which they are popped off the stack.

Depth-First Search Forest



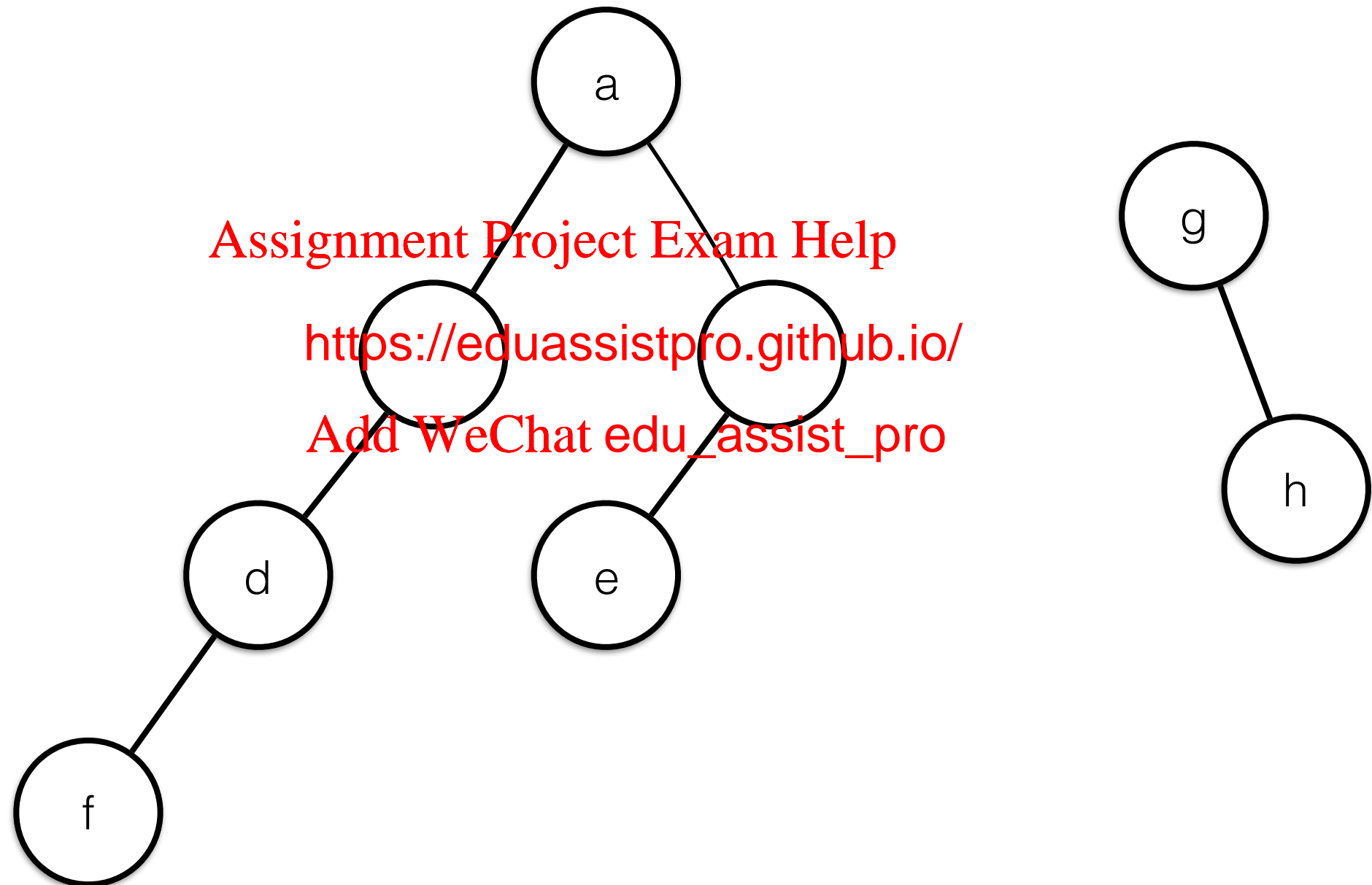
DFS can be depicted by the
resulting **DFS Forest**
(DFS **Tree** for a connected graph)

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Breadth-First Traversal

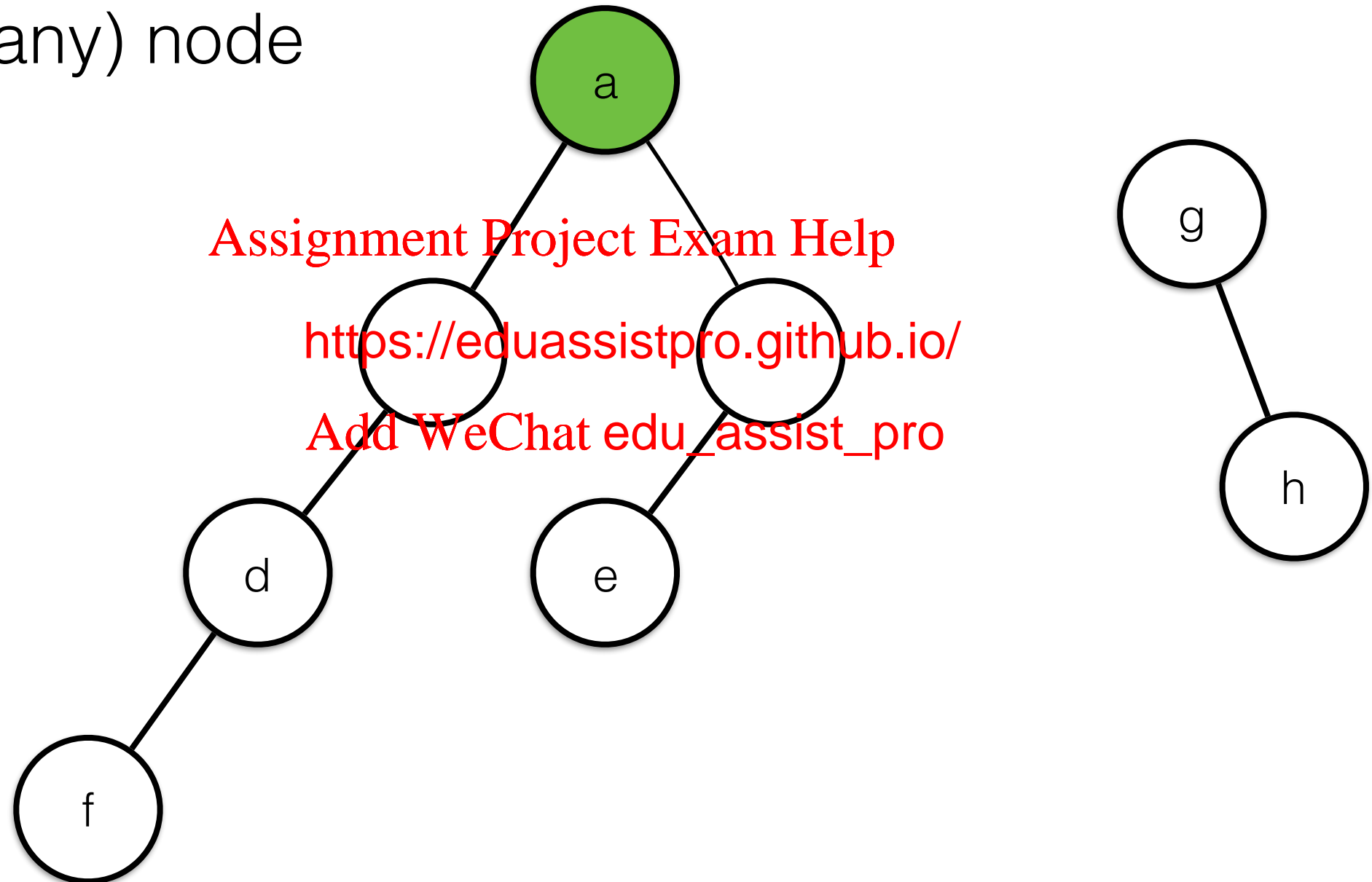


Breadth-First Traversal



Nodes visited in this order: a

Start with (any) node

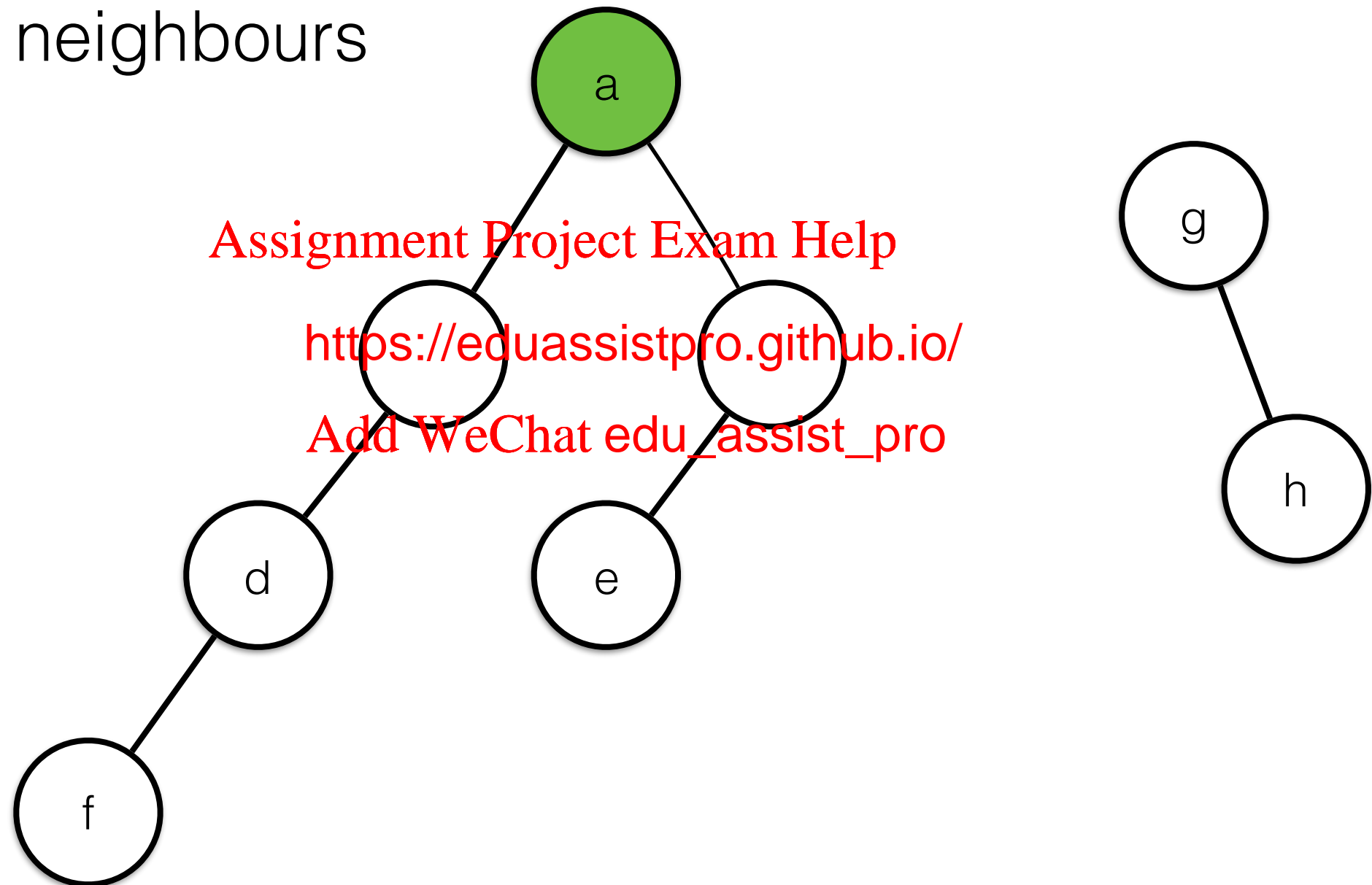


Breadth-First Traversal



Nodes visited in this order: a

Visit all of its neighbours

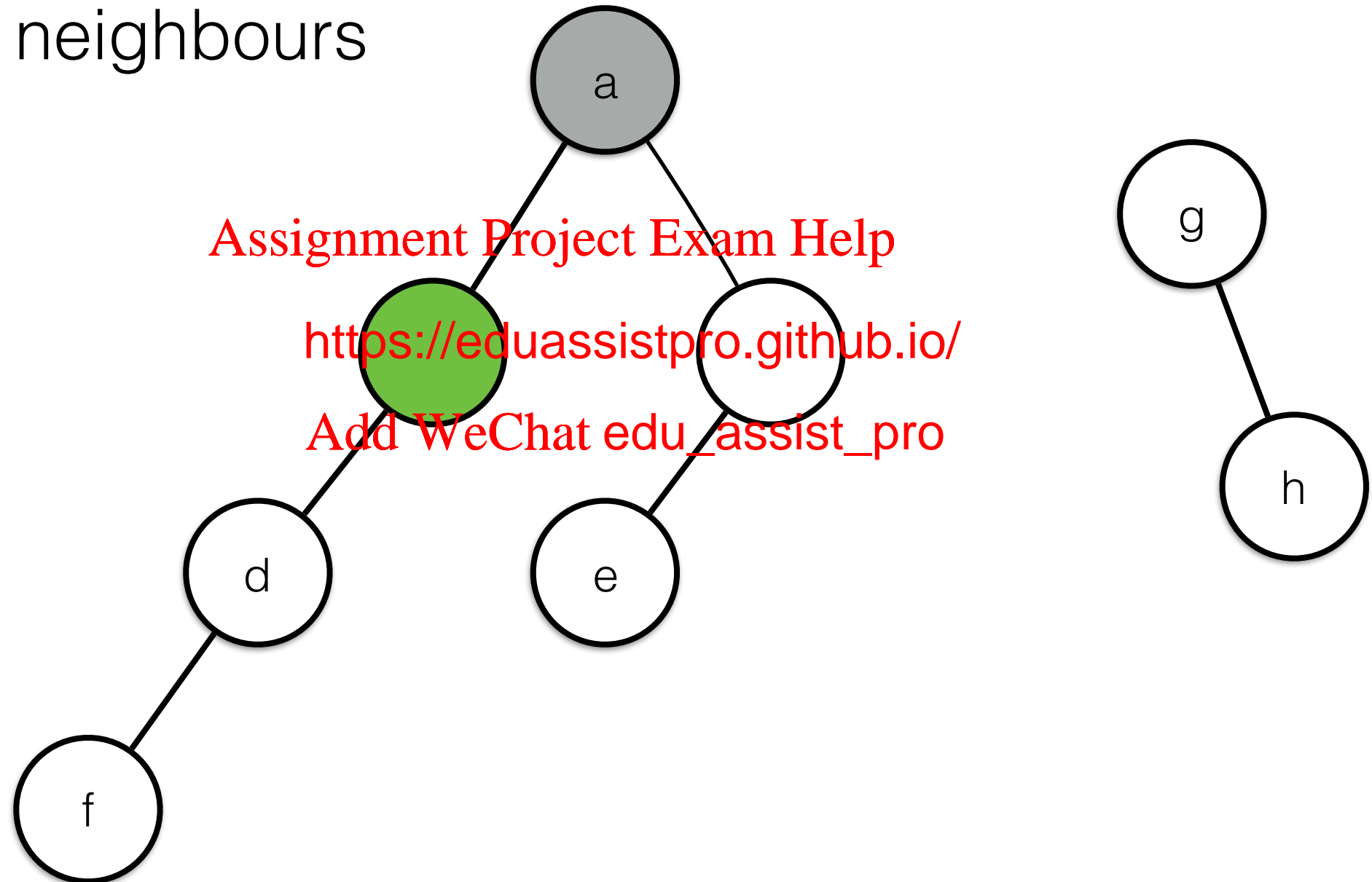


Breadth-First Traversal



Nodes visited in this order: a b

Visit all of its neighbours

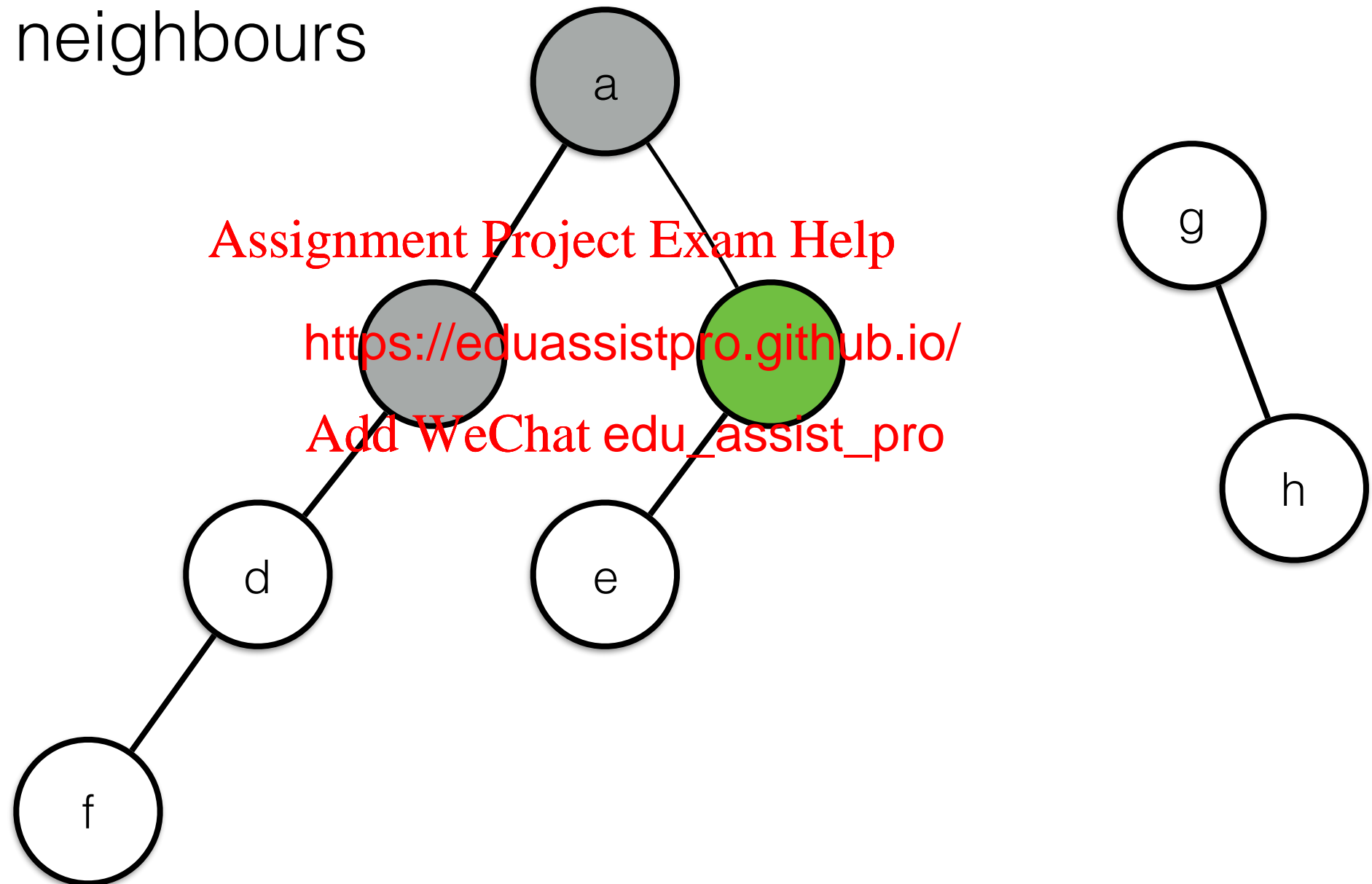


Breadth-First Traversal



Nodes visited in this order: a b c

Visit all of its neighbours

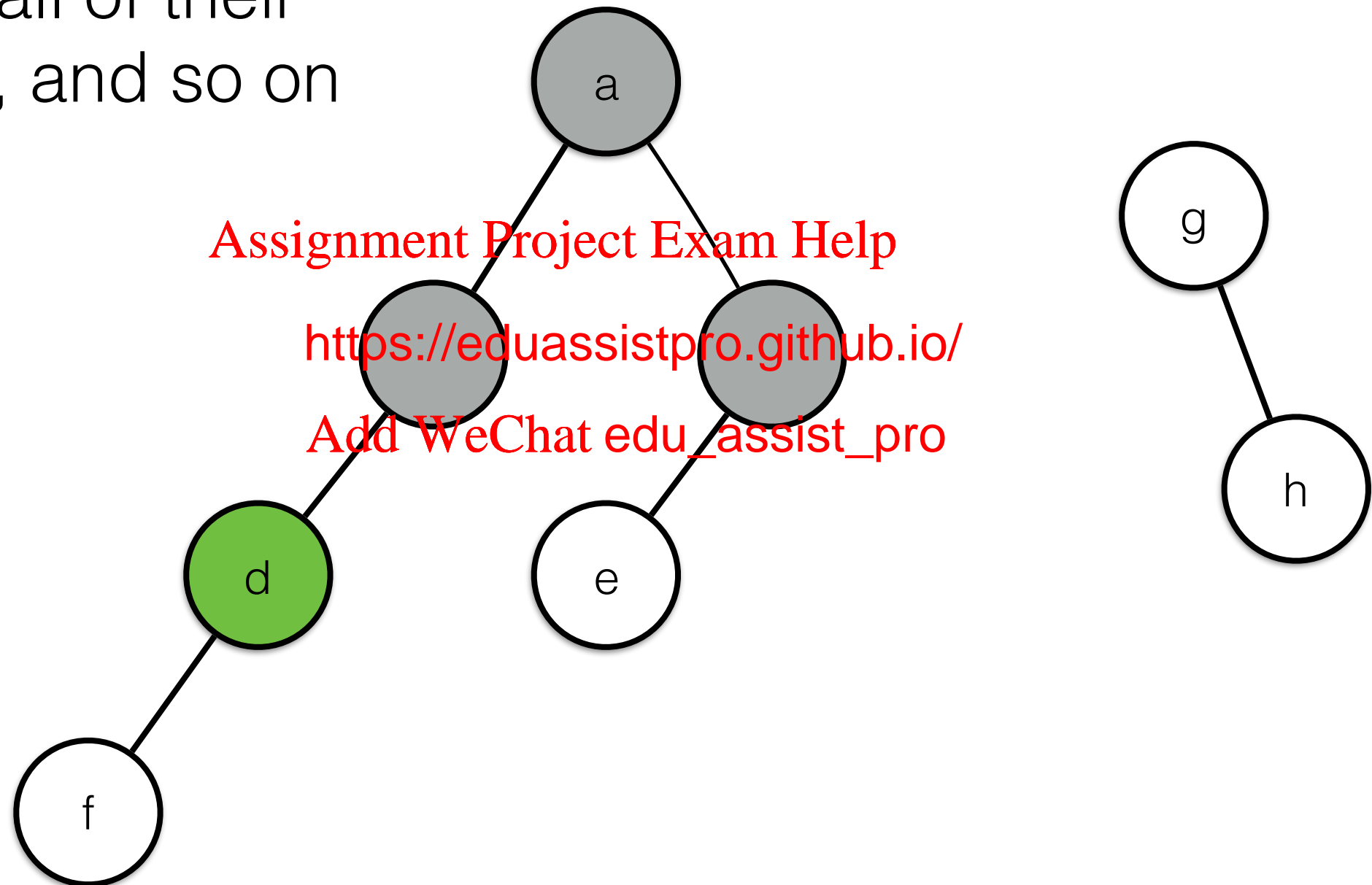


Breadth-First Traversal



Nodes visited in this order: a b c d

Then visit all of their
neighbours, and so on

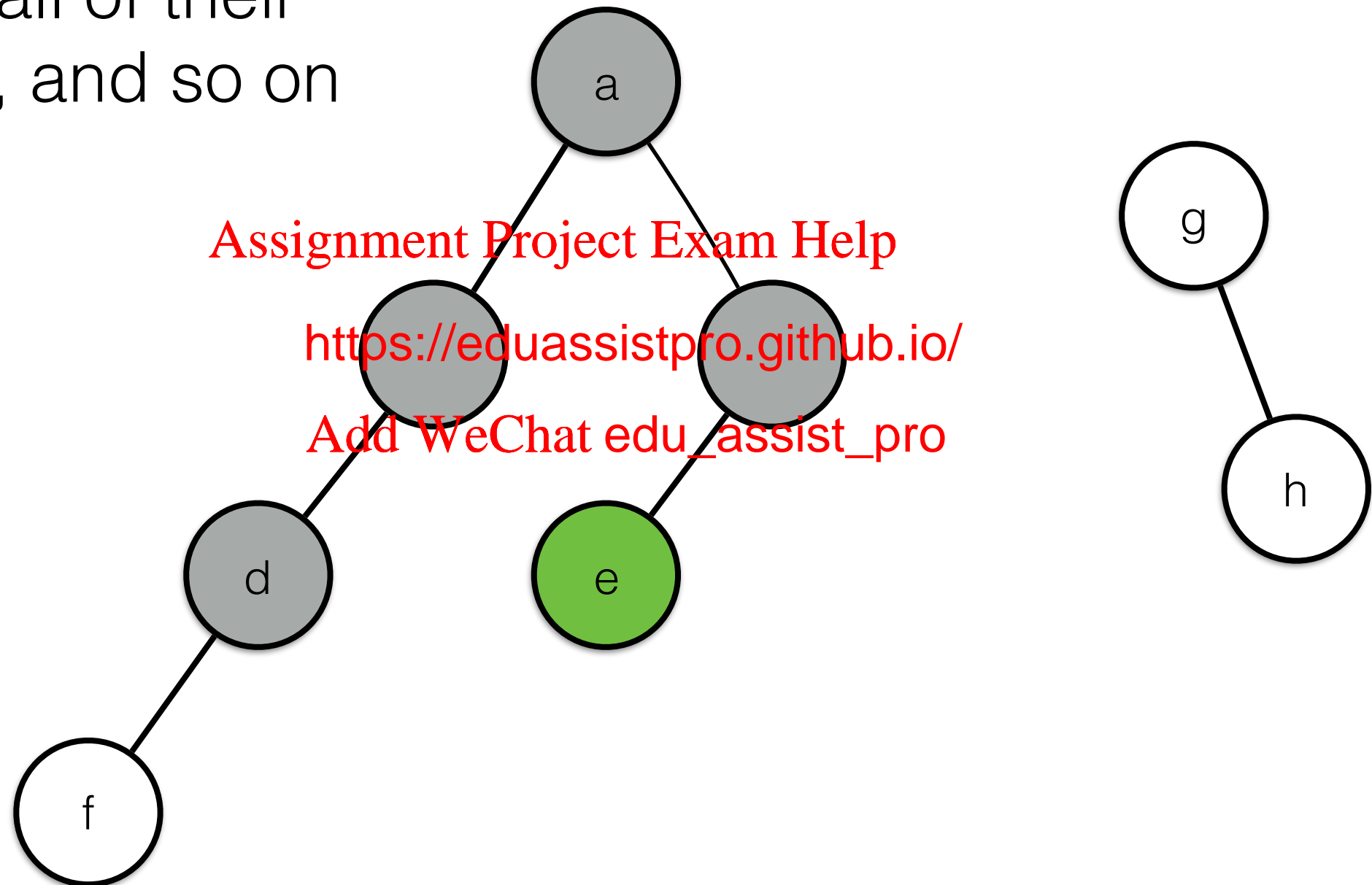


Breadth-First Traversal



Nodes visited in this order: a b c d e

Then visit all of their
neighbours, and so on

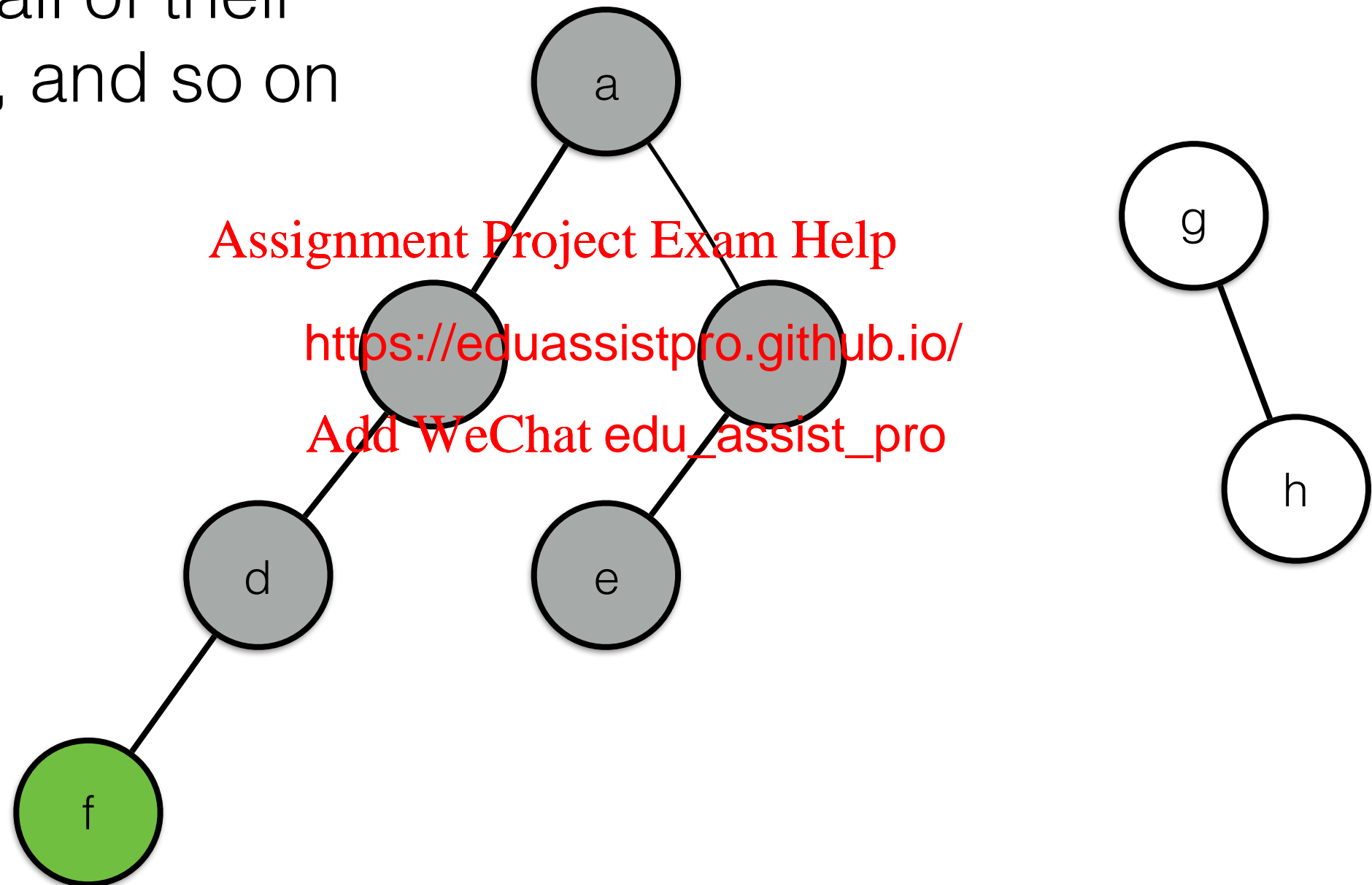


Breadth-First Traversal



Nodes visited in this order: a b c d e f

Then visit all of their
neighbours, and so on

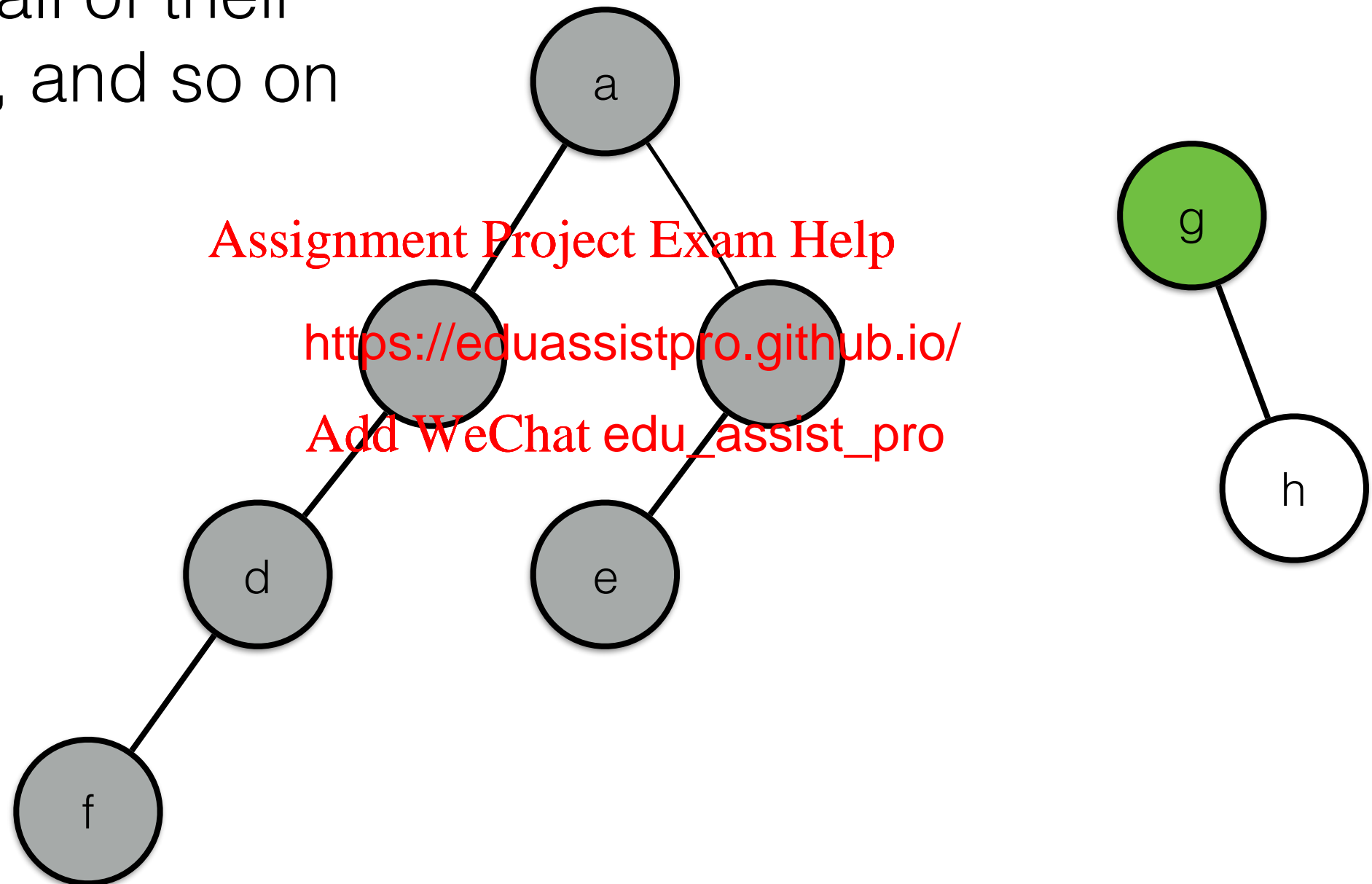


Breadth-First Traversal



Nodes visited in this order: a b c d e f g

Then visit all of their
neighbours, and so on

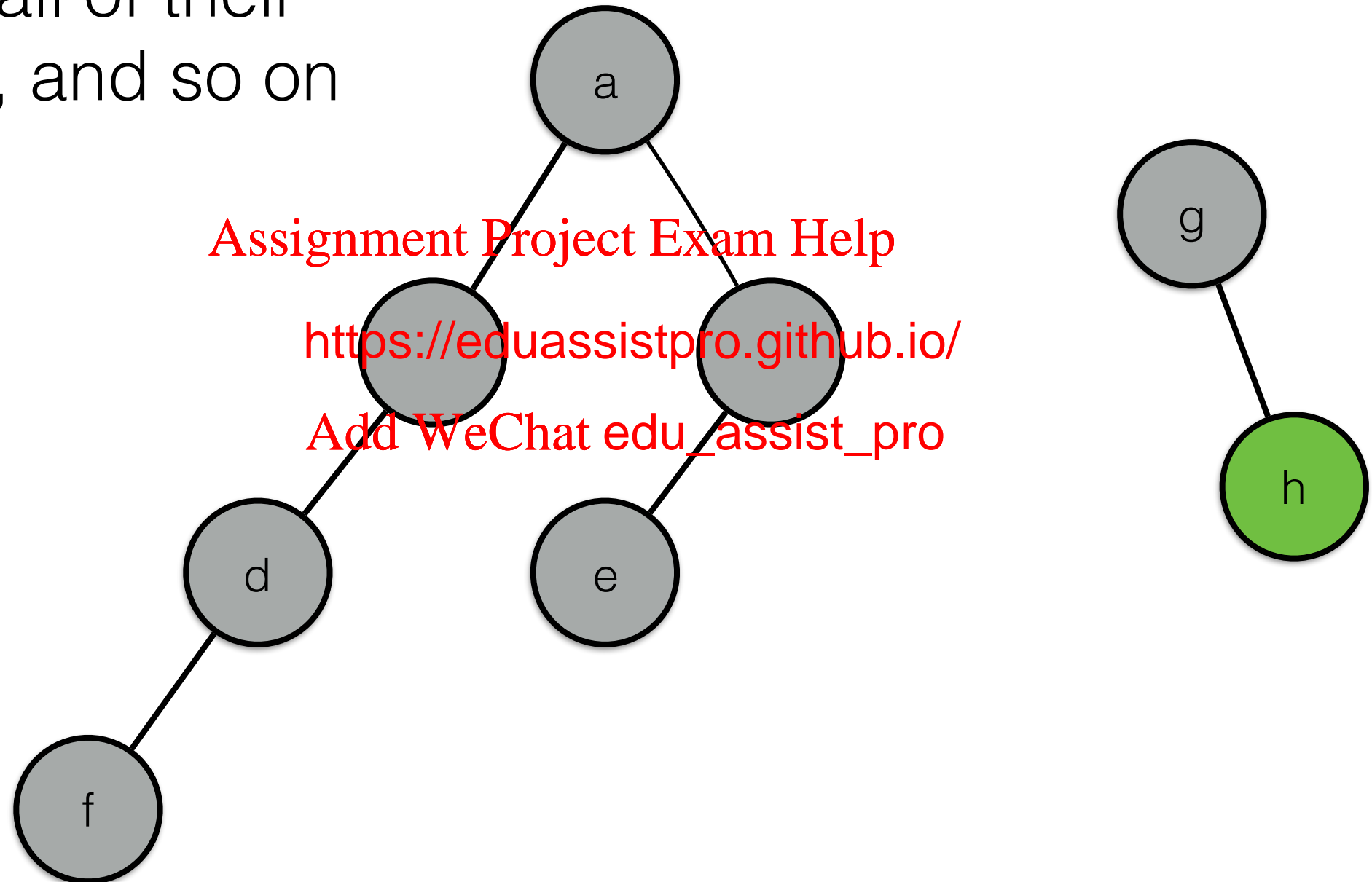


Breadth-First Traversal



Nodes visited in this order: a b c d e f g h

Then visit all of their
neighbours, and so on



Depth-First vs Breadth-First Search



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Typical Depth-First Search

Depth-First vs Breadth-First Search



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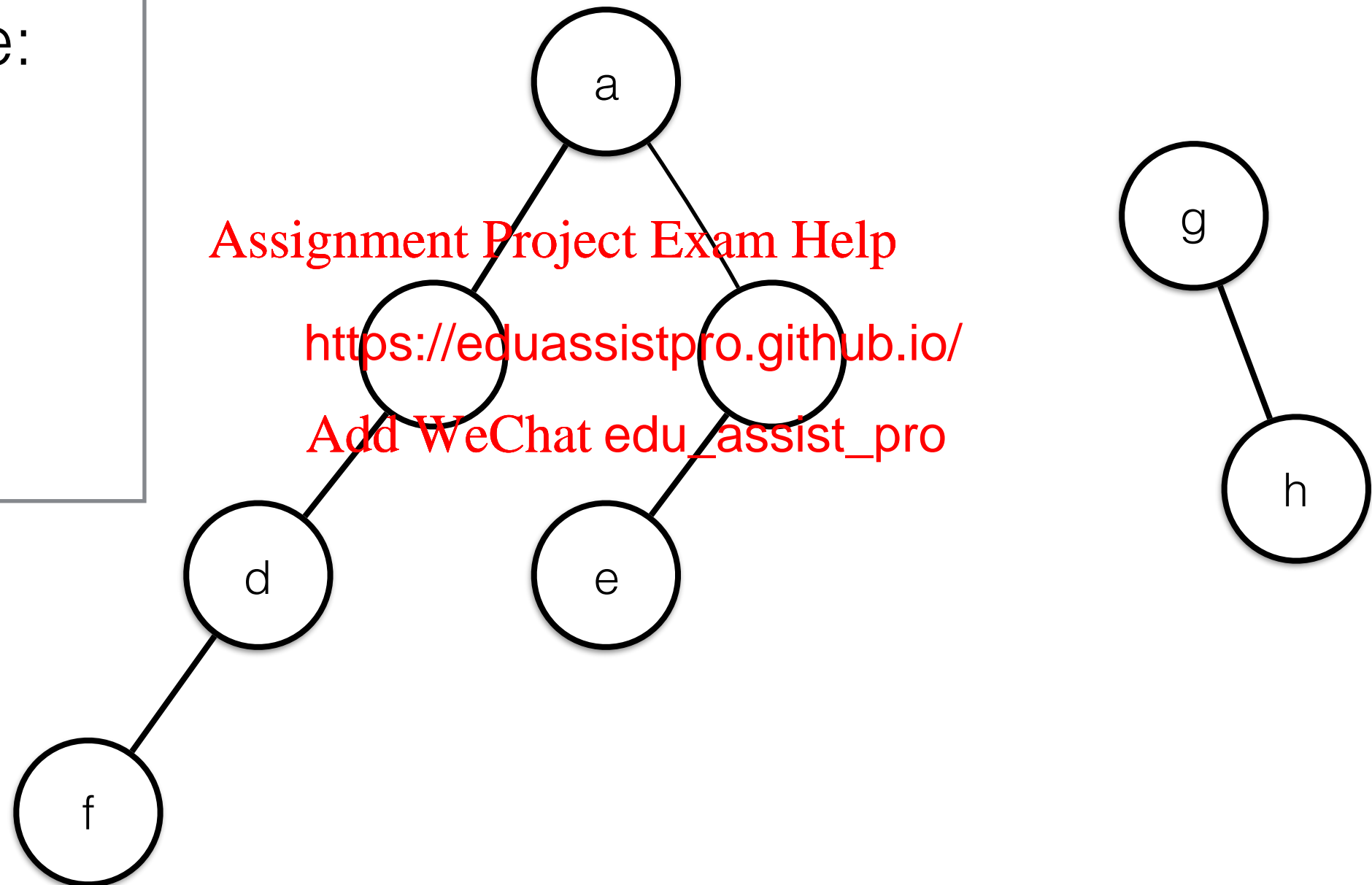
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Typical Breadth-First Search

Breadth-First Search: Queue Discipline



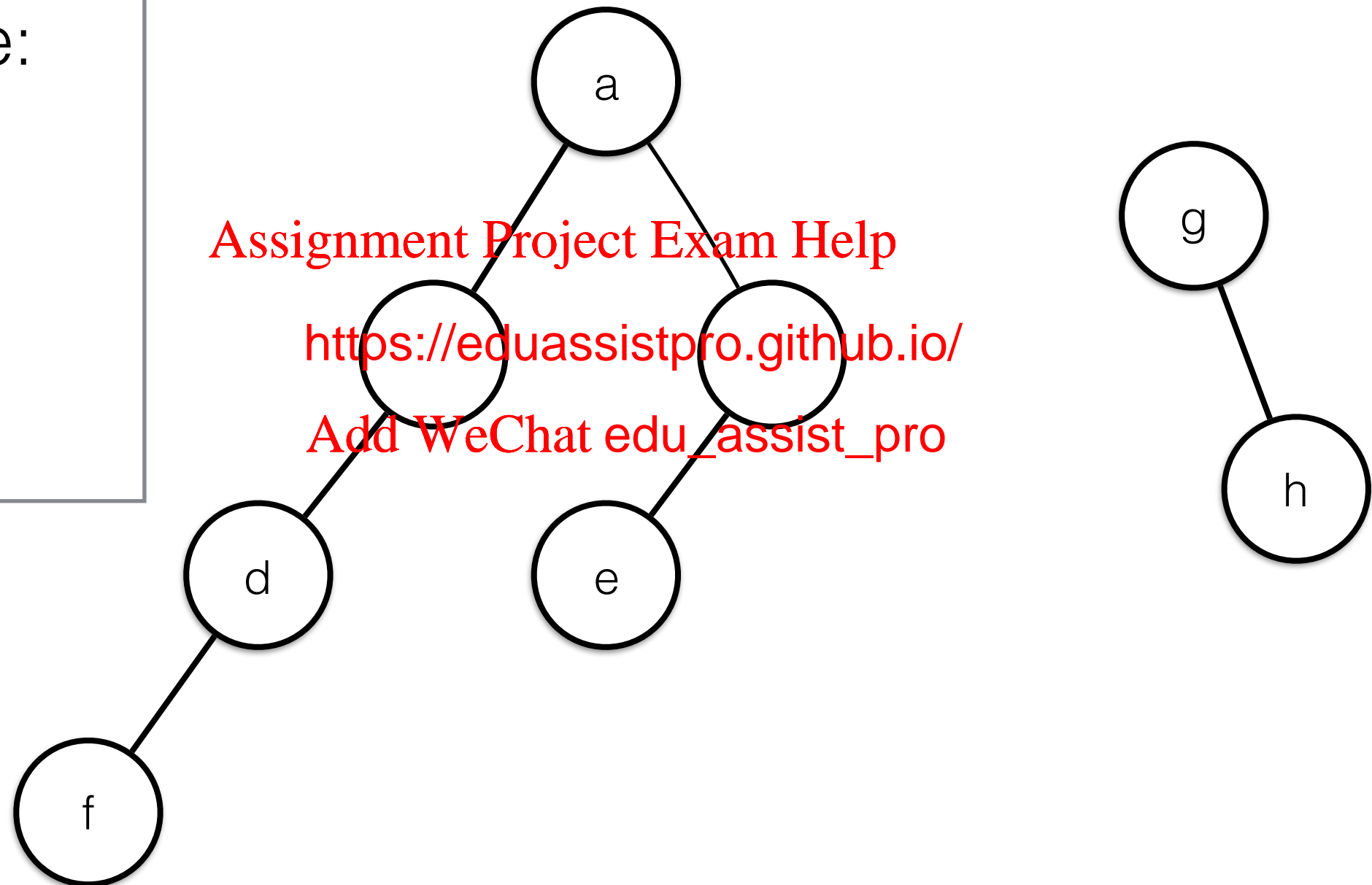
Traversal
Queue:



Breadth-First Search: Queue Discipline



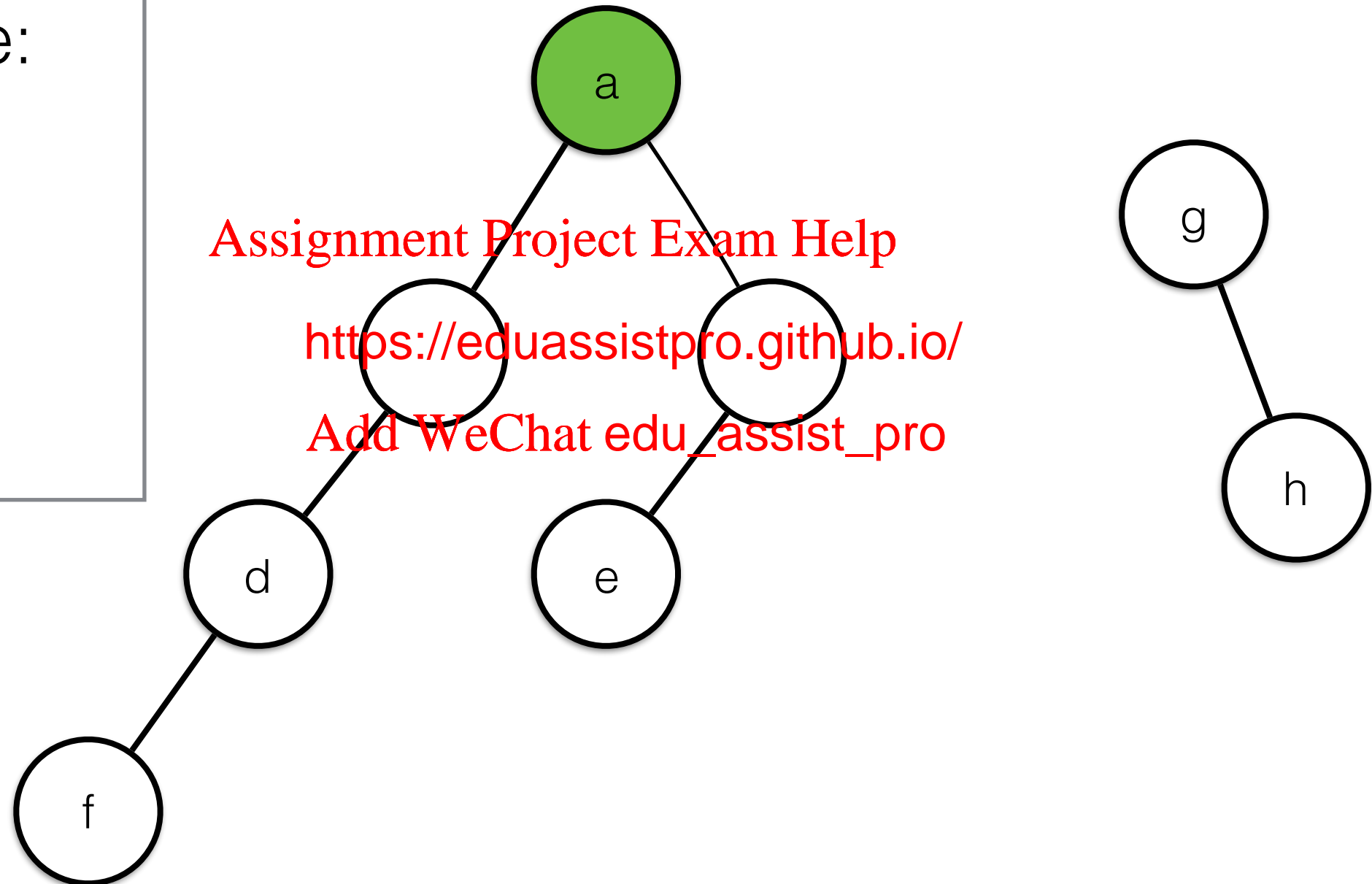
Traversal
Queue:
a



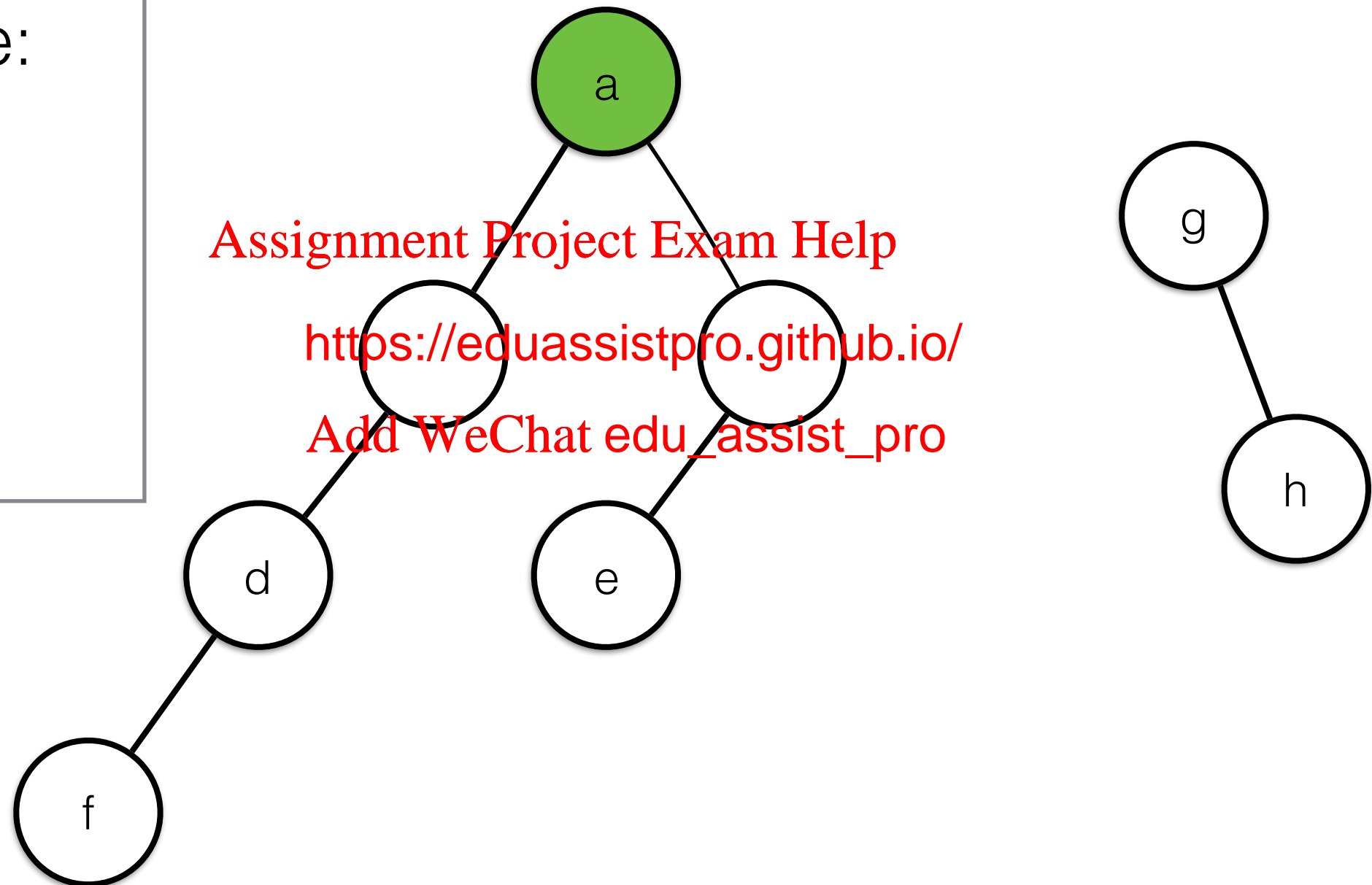
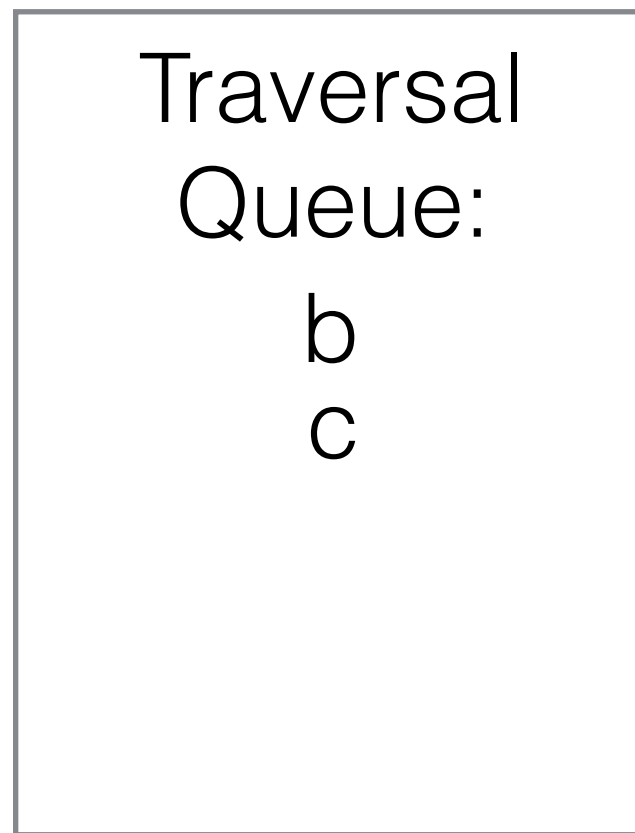
Breadth-First Search: Queue Discipline



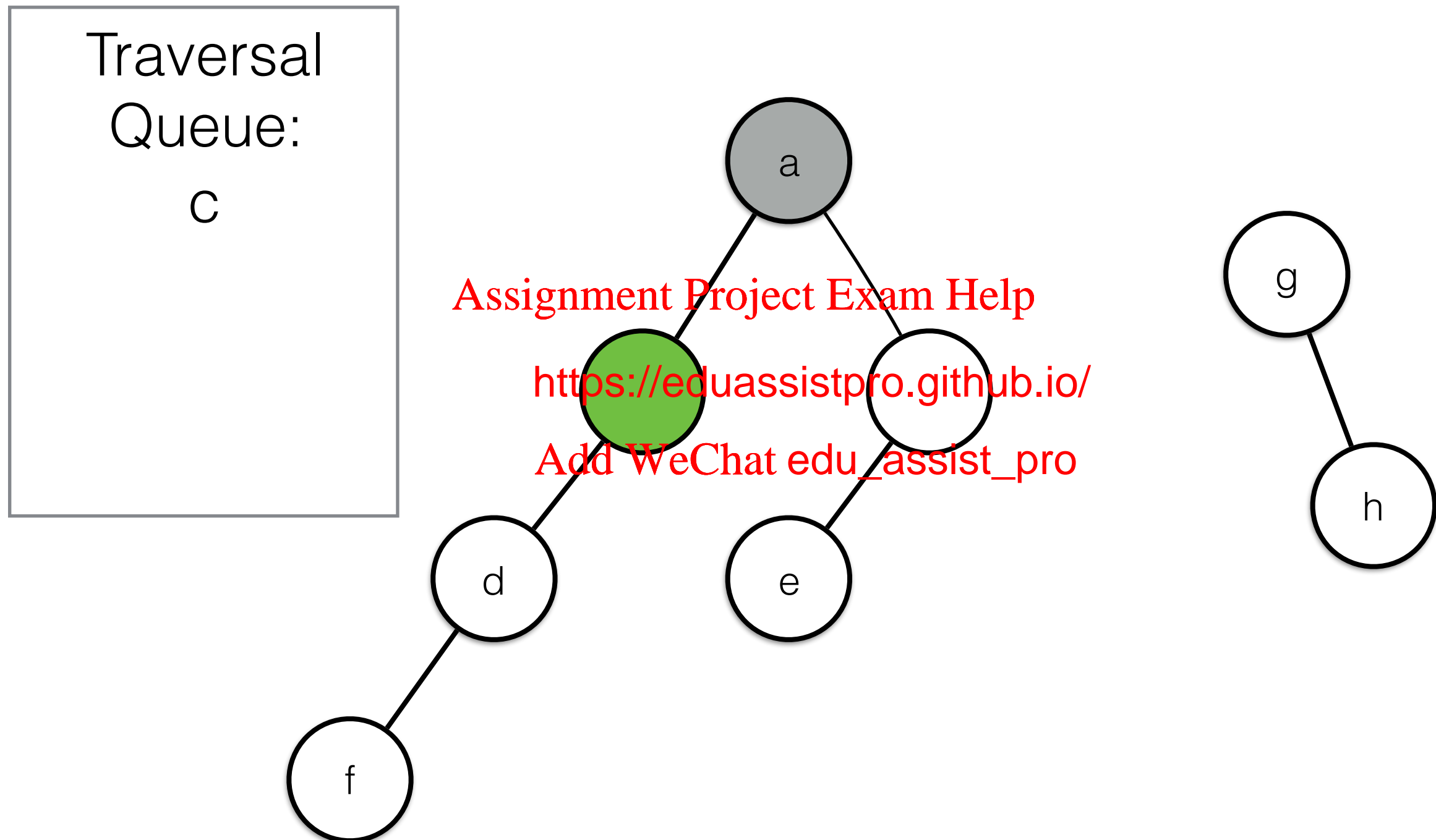
Traversal
Queue:



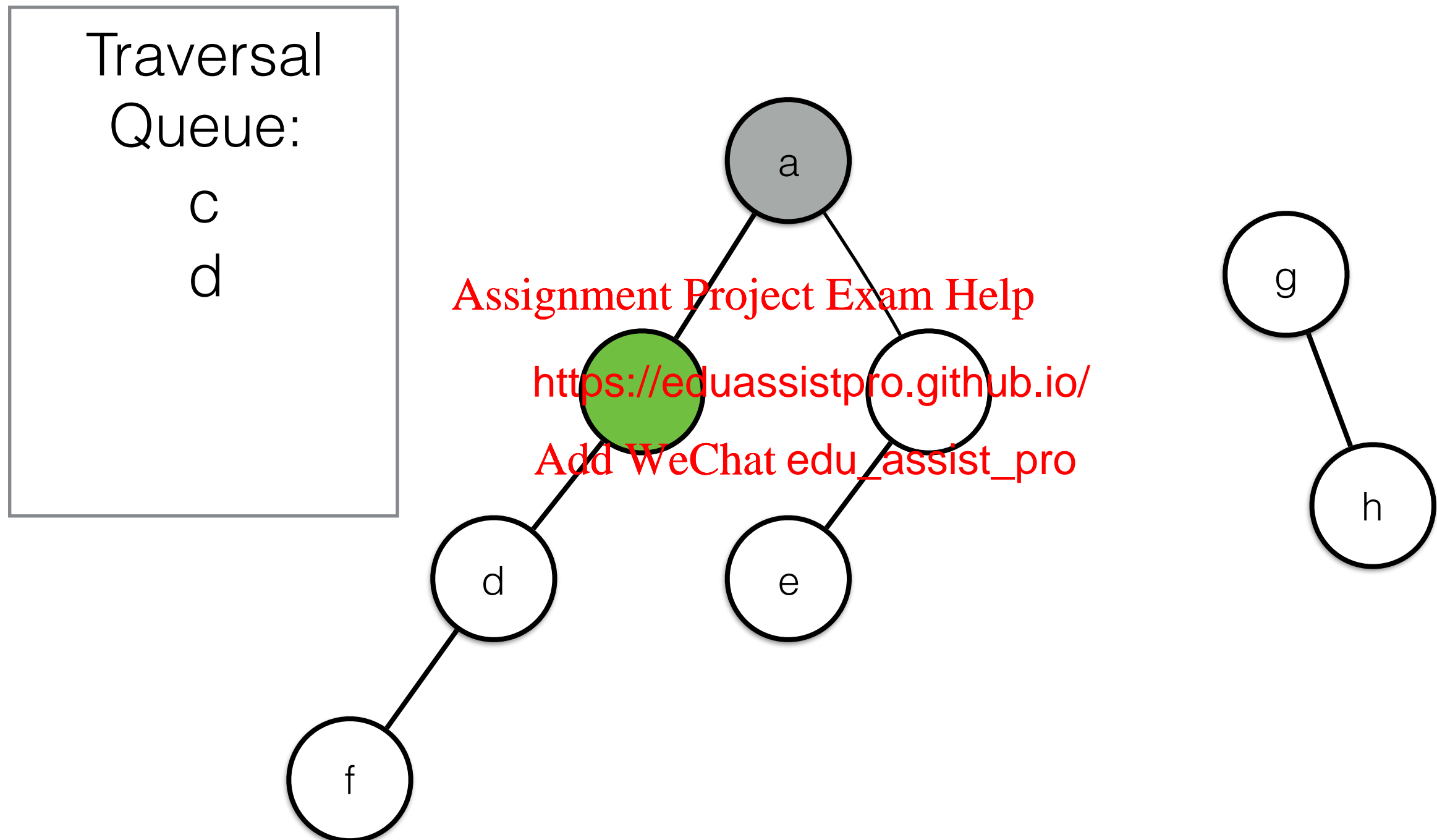
Breadth-First Search: Queue Discipline



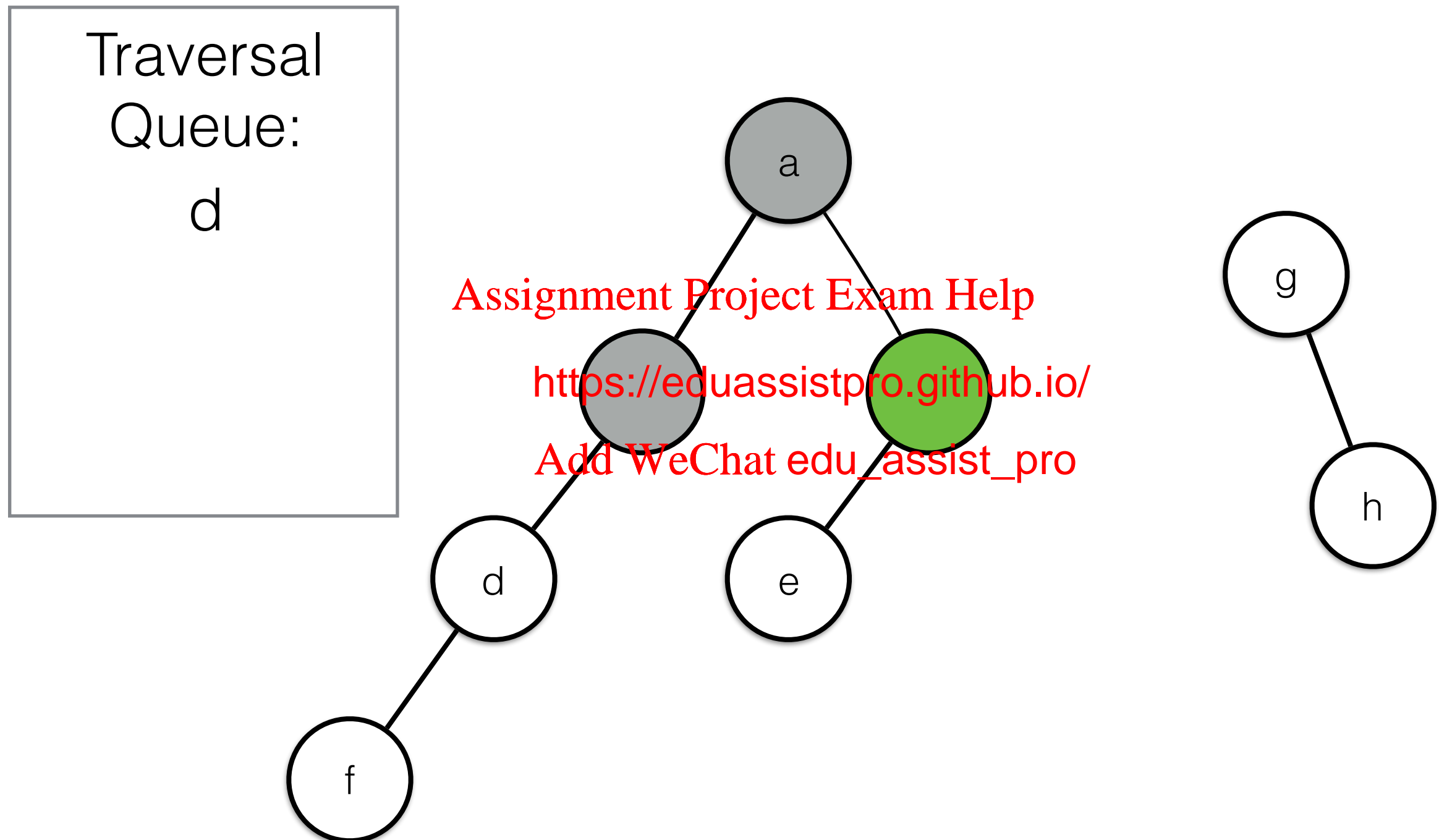
Breadth-First Search: Queue Discipline



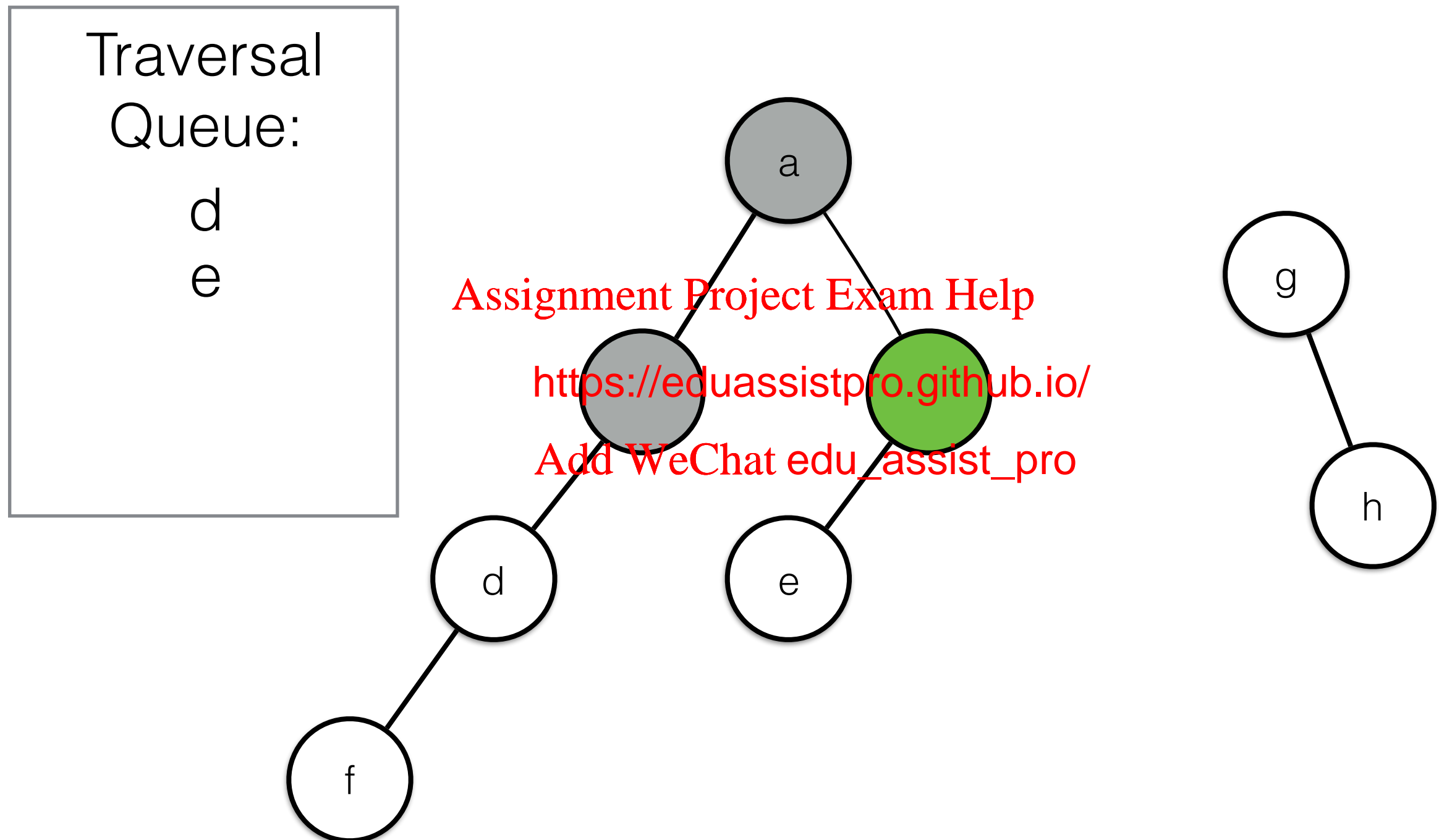
Breadth-First Search: Queue Discipline



Breadth-First Search: Queue Discipline



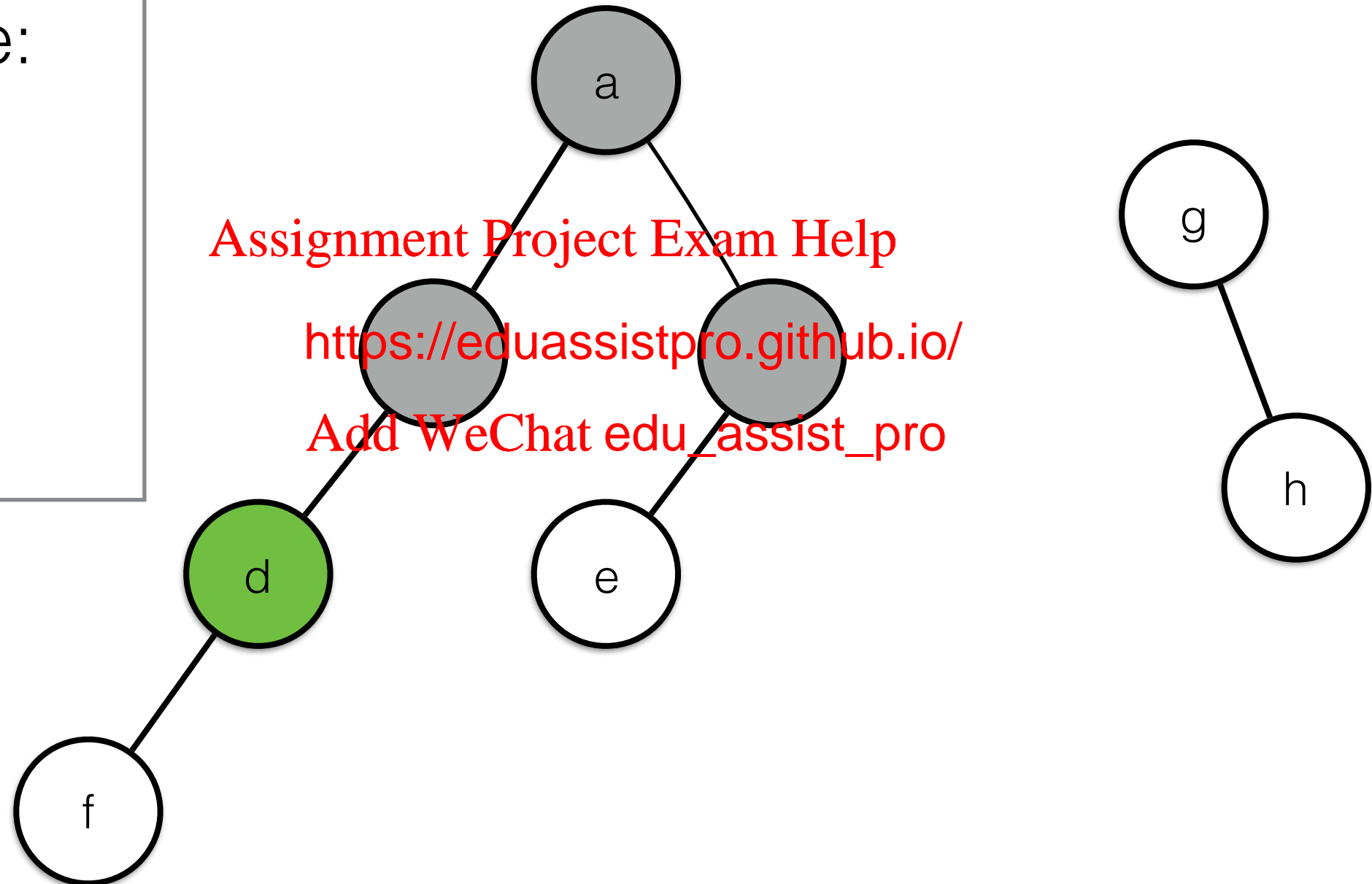
Breadth-First Search: Queue Discipline



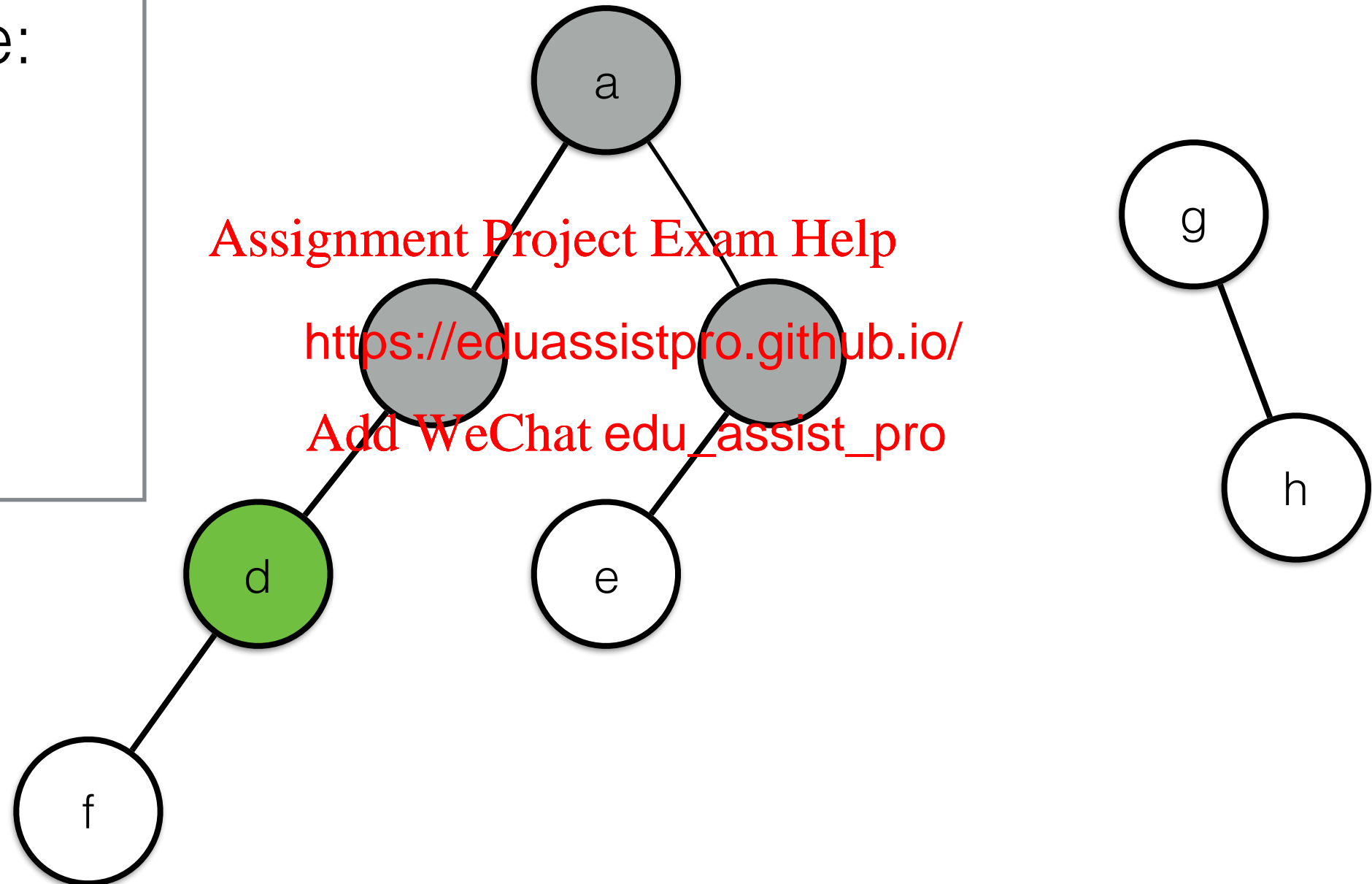
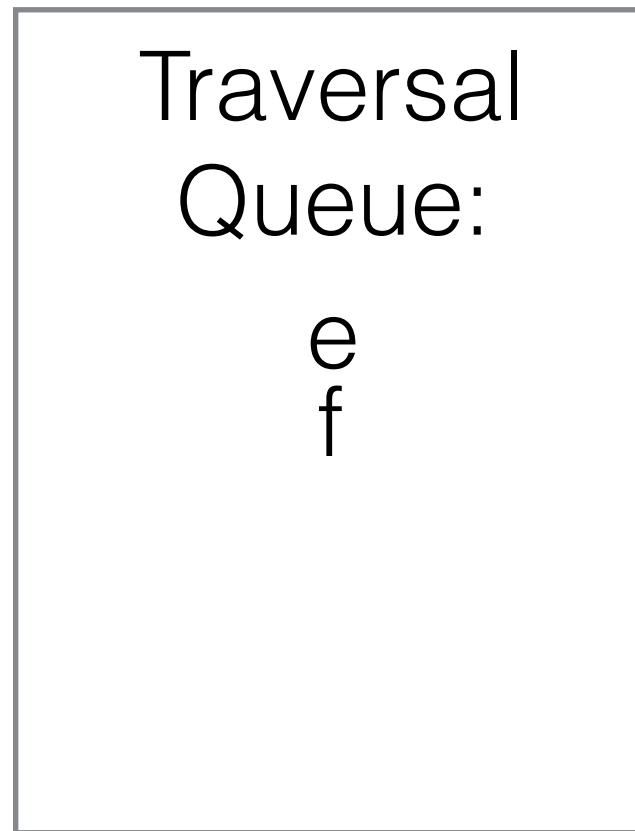
Breadth-First Search: Queue Discipline



Traversal
Queue:
e



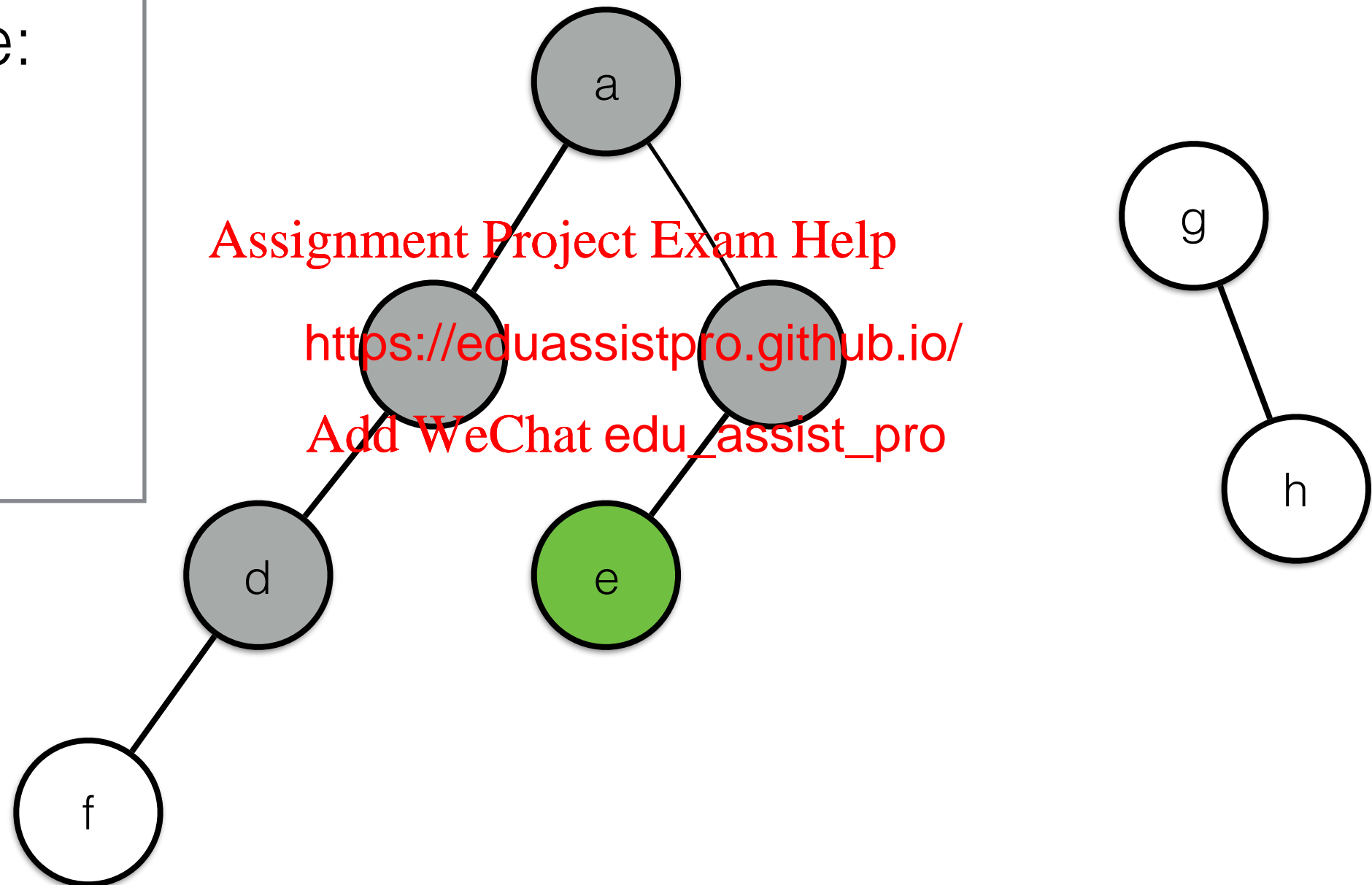
Breadth-First Search: Queue Discipline



Breadth-First Search: Queue Discipline



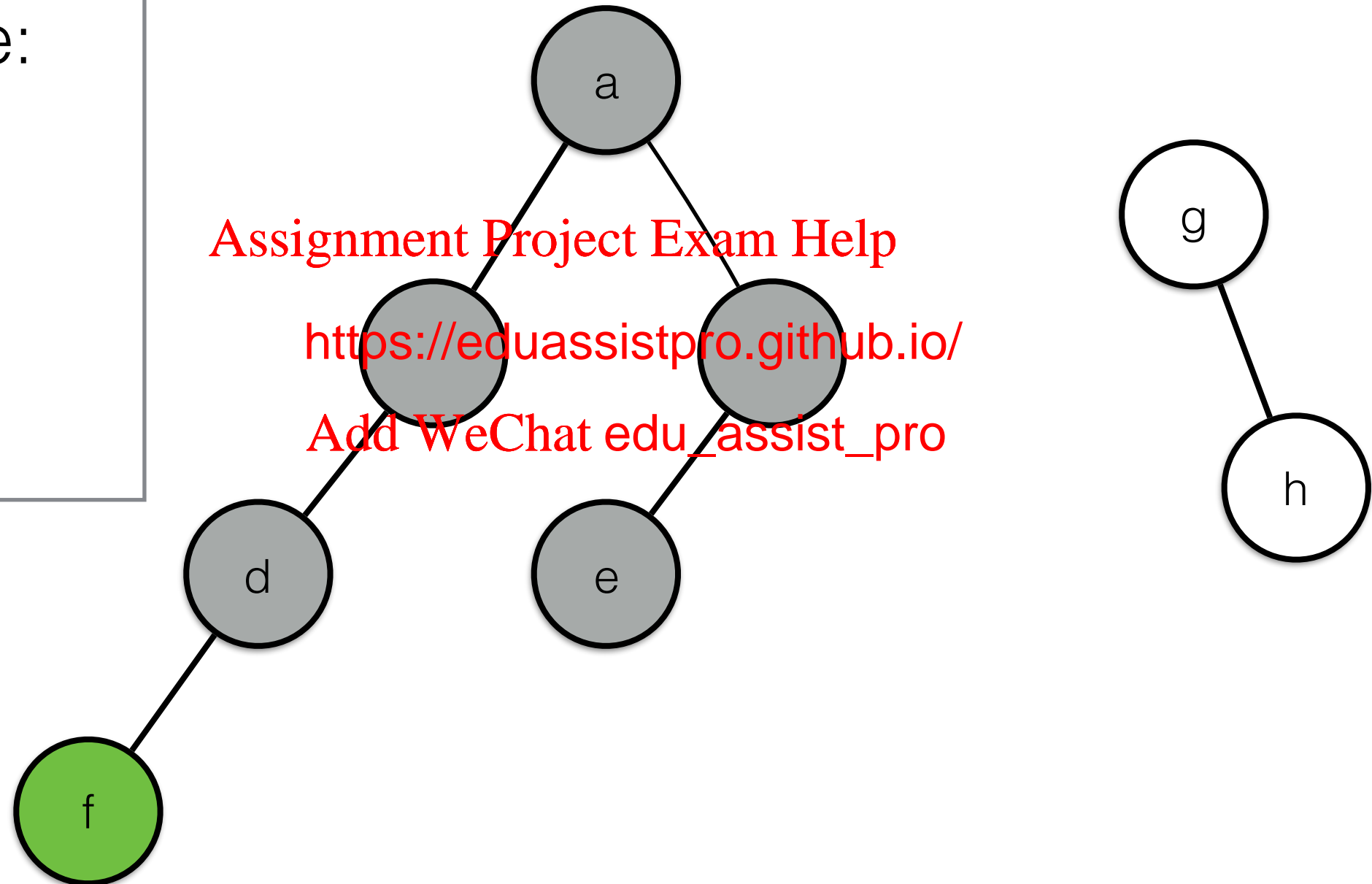
Traversal
Queue:
f



Breadth-First Search: Queue Discipline



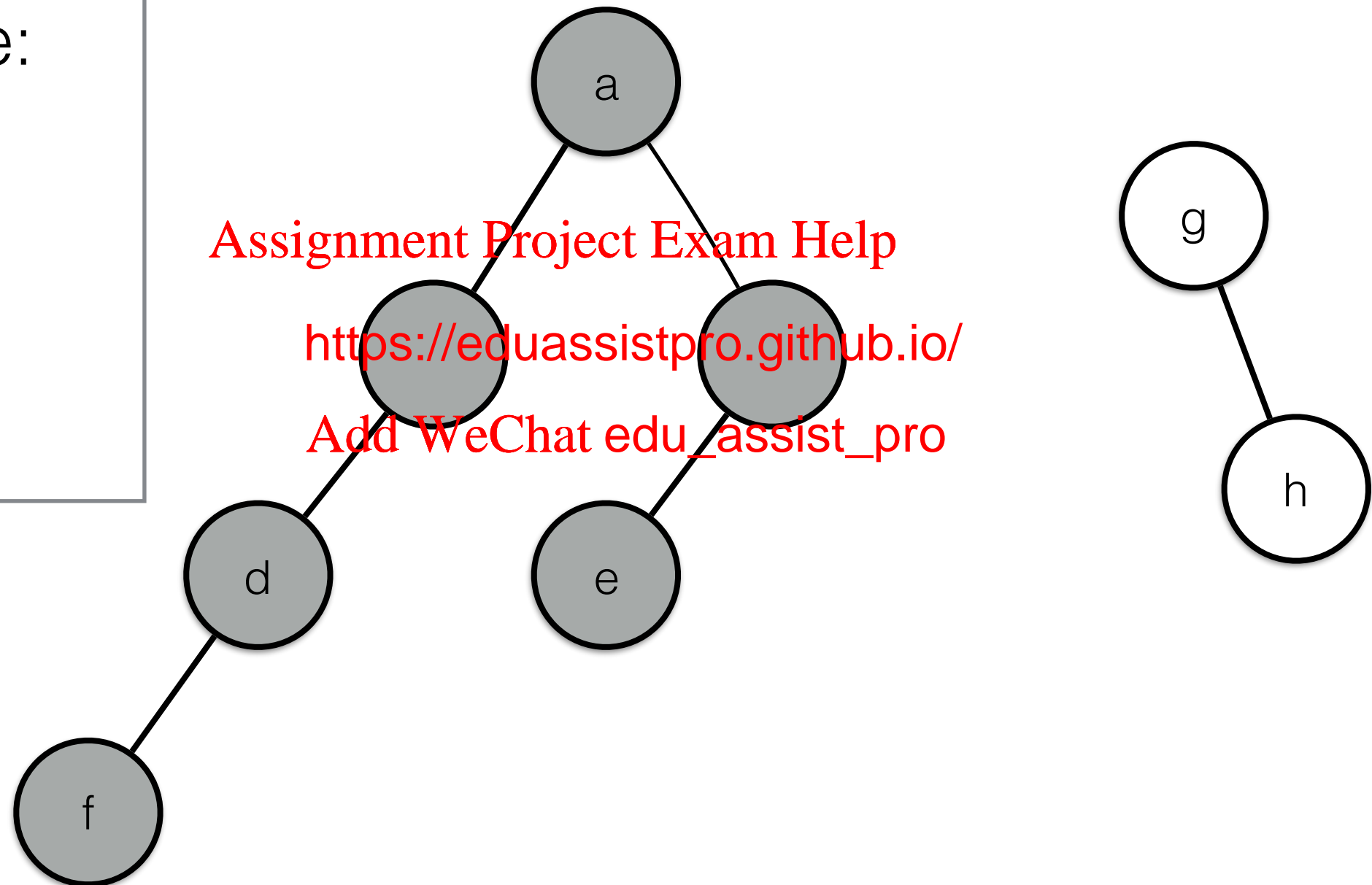
Traversal
Queue:



Breadth-First Search: Queue Discipline



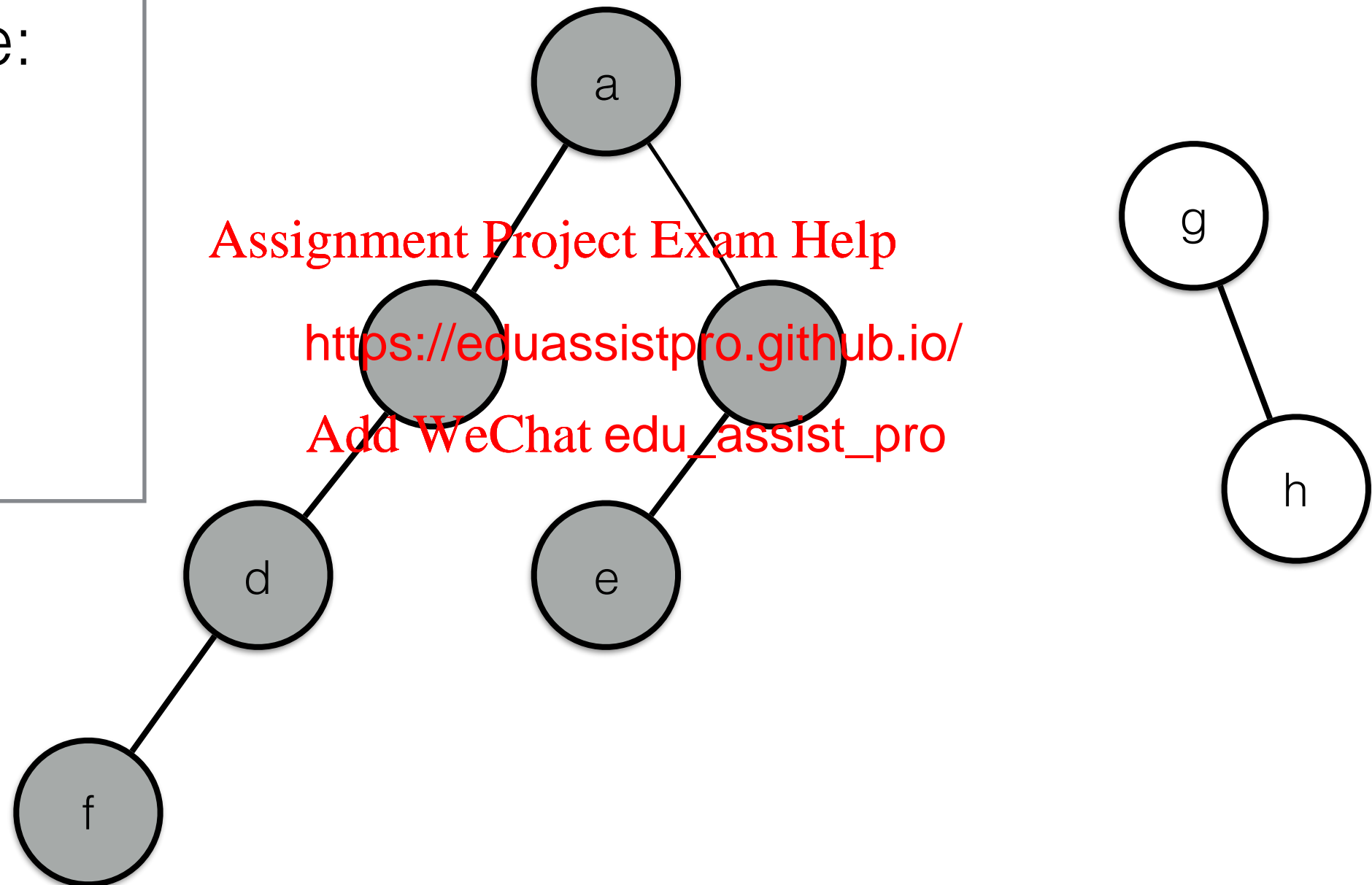
Traversal
Queue:



Breadth-First Search: Queue Discipline



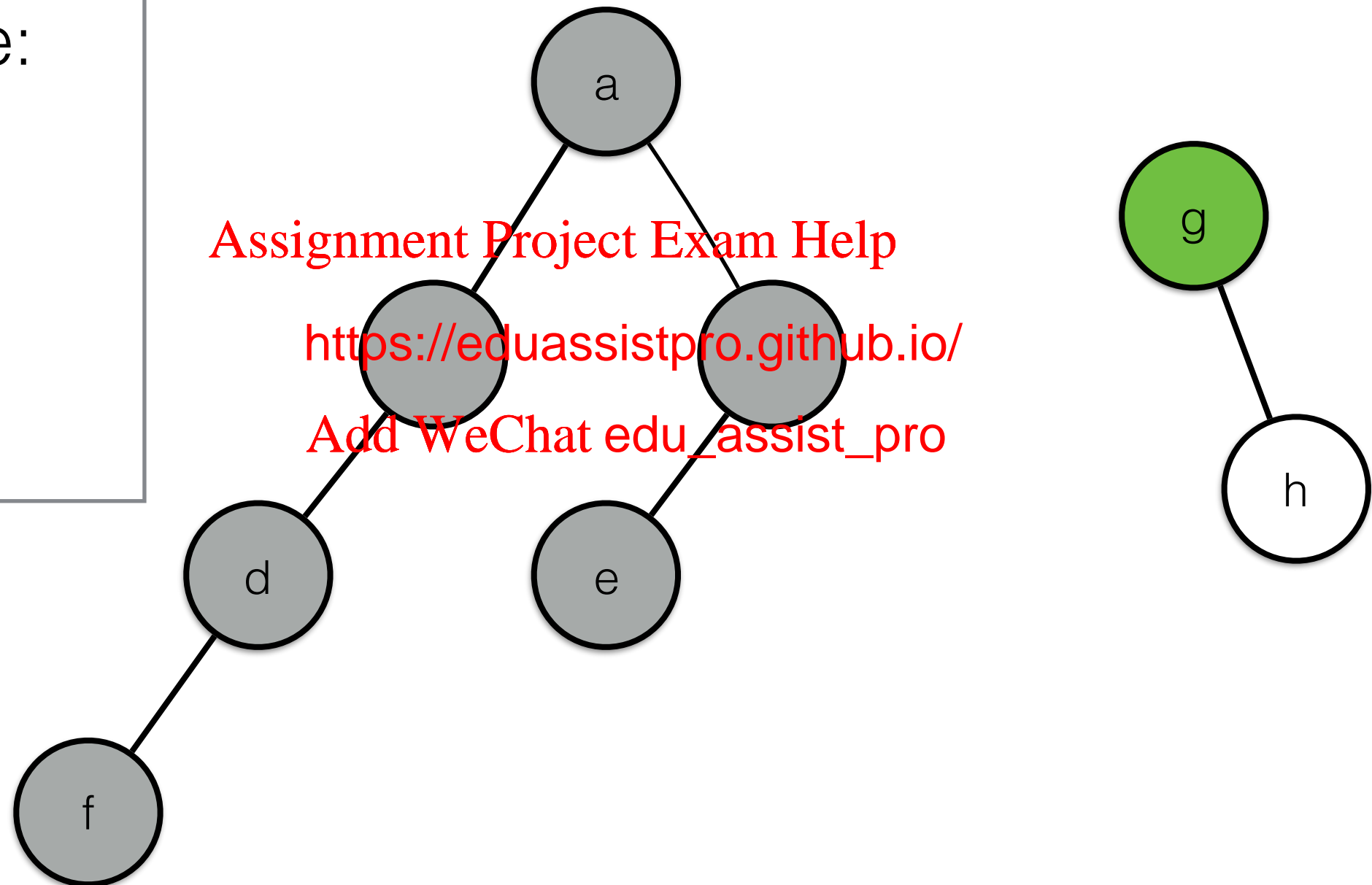
Traversal
Queue:
g



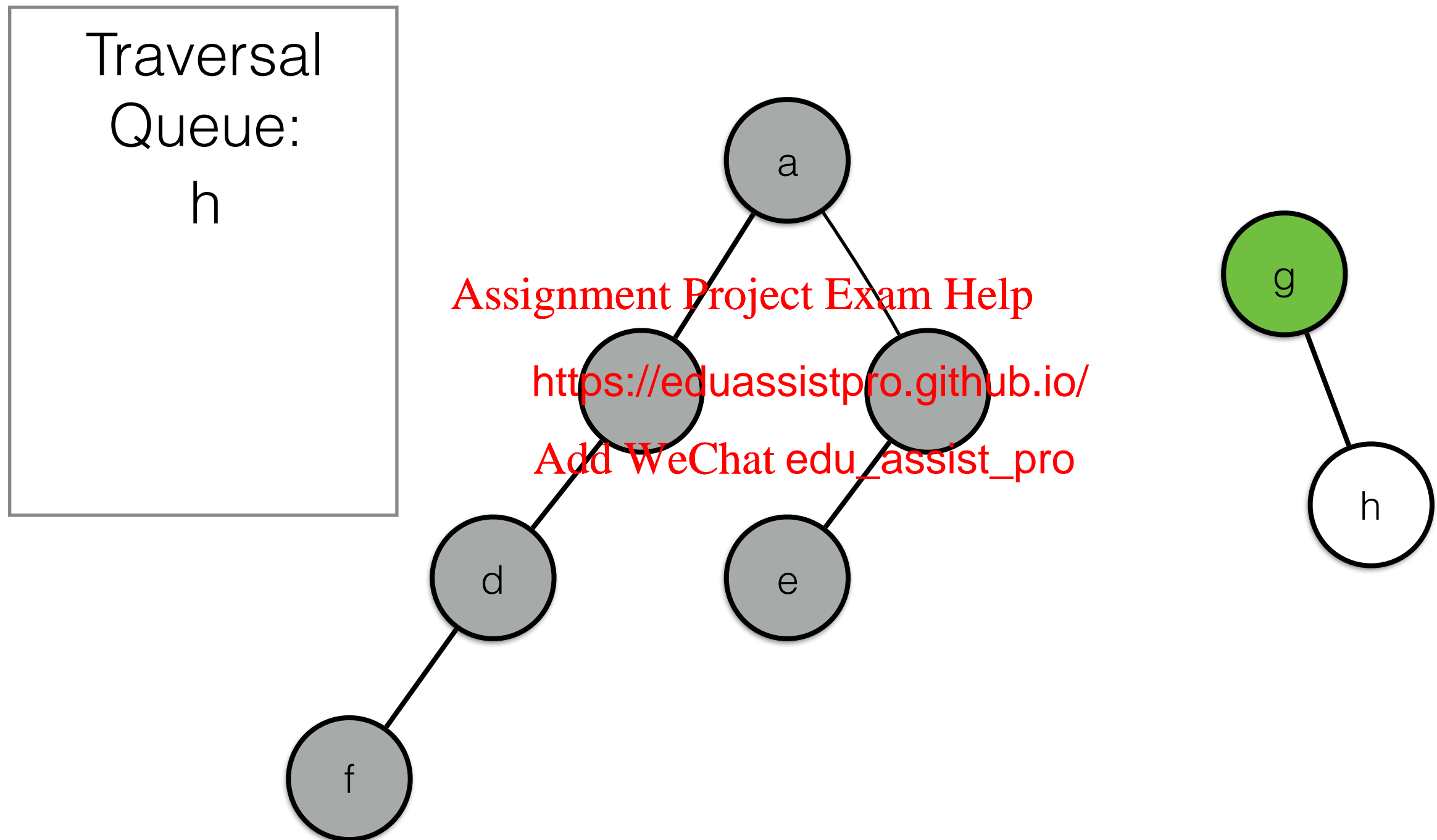
Breadth-First Search: Queue Discipline



Traversal
Queue:



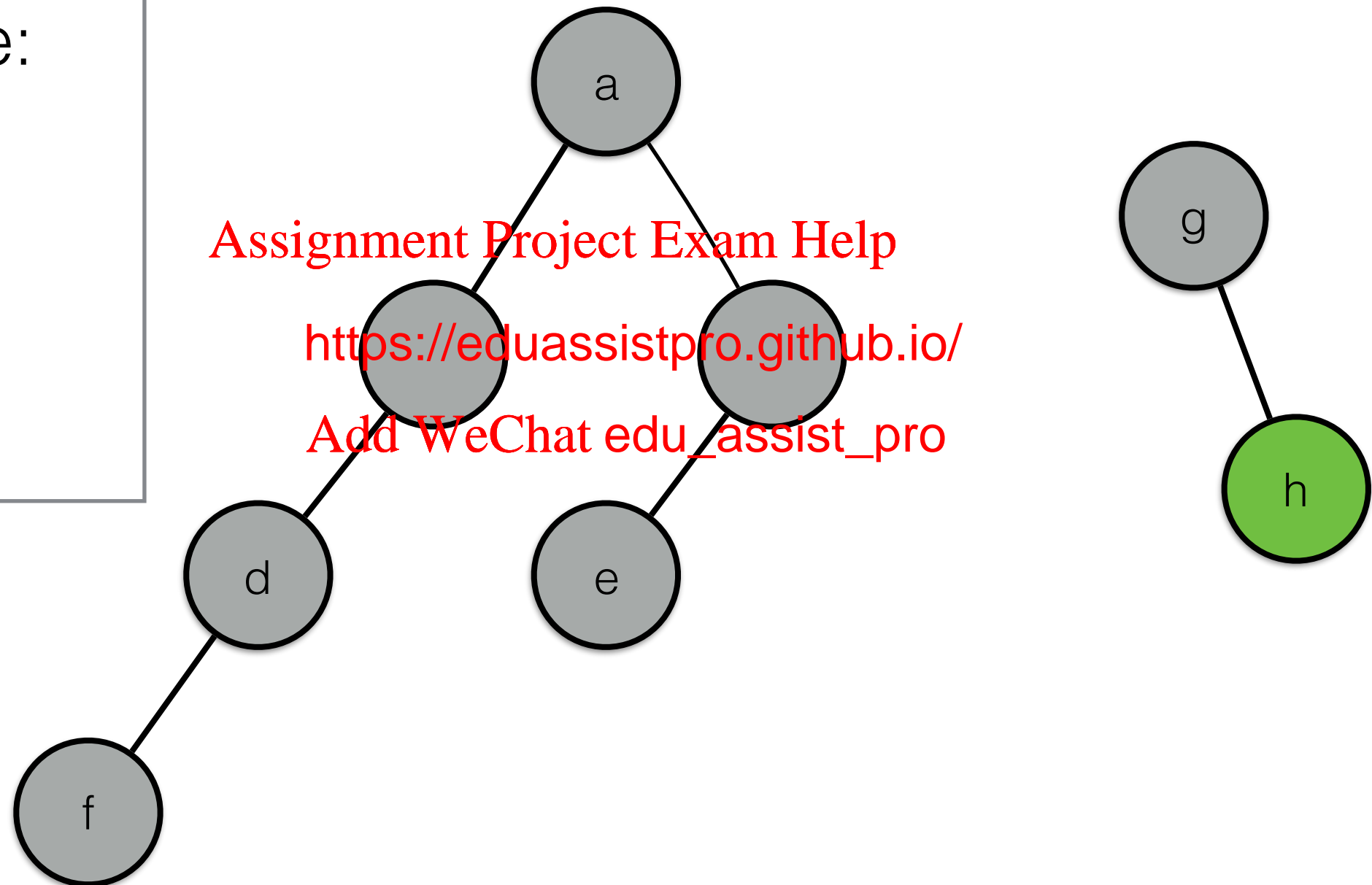
Breadth-First Search: Queue Discipline



Breadth-First Search: Queue Discipline



Traversal
Queue:



Breadth-First Search Algorithm



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BFS Algorithm Notes

- BFS has the same complexity as DFS.
- Again, the same algorithm works for directed graphs as well
- Certain problems adapted by
adapting BFS.
- For example, given a graph and two nodes, a and b in the graph, how would you find the length of the shortest path from a to b?

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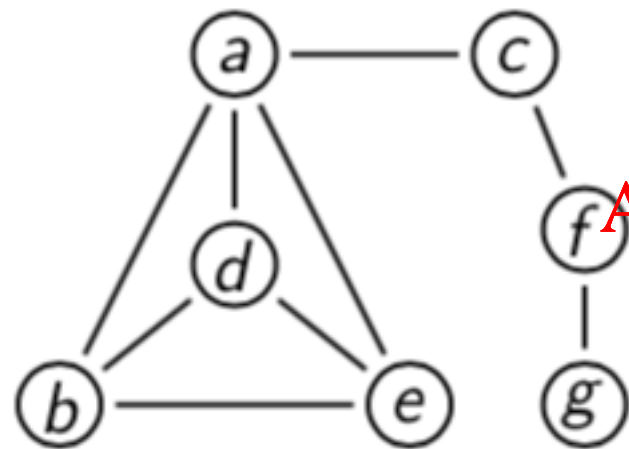
solved by

Breadth-First Search Forest



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BFS **Tree** for this
connected graph:



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In general, we may get a
BFS **Forest**

Topological Sorting

- We mentioned scheduling problems and their representation by directed graphs.
- Assume a directed edge from a to b means that task a must be completed before b can be started.
- The graph must be a DAG. A problem cannot be solved.
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- Assume the tasks are carried out by a single person, unable to multi-task.
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- Then we should try to **linearize** the graph, that is, order the nodes as a sequence v_1, v_2, \dots, v_n such that for each edge $(v_i, v_j) \in E$, we have v_i comes before v_j in the sequence (that is, v_i is scheduled to happen before v_j).

Topological Sorting Example

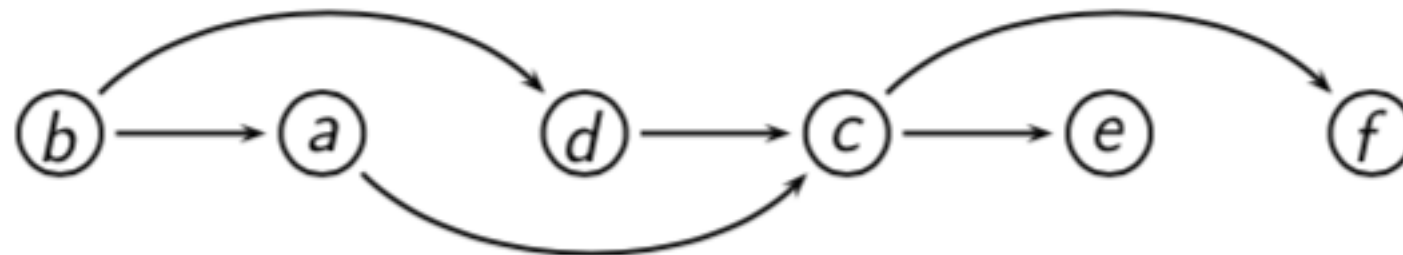
There are 4 ways to linearise the following graph

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Here is one:



Topological Sorting

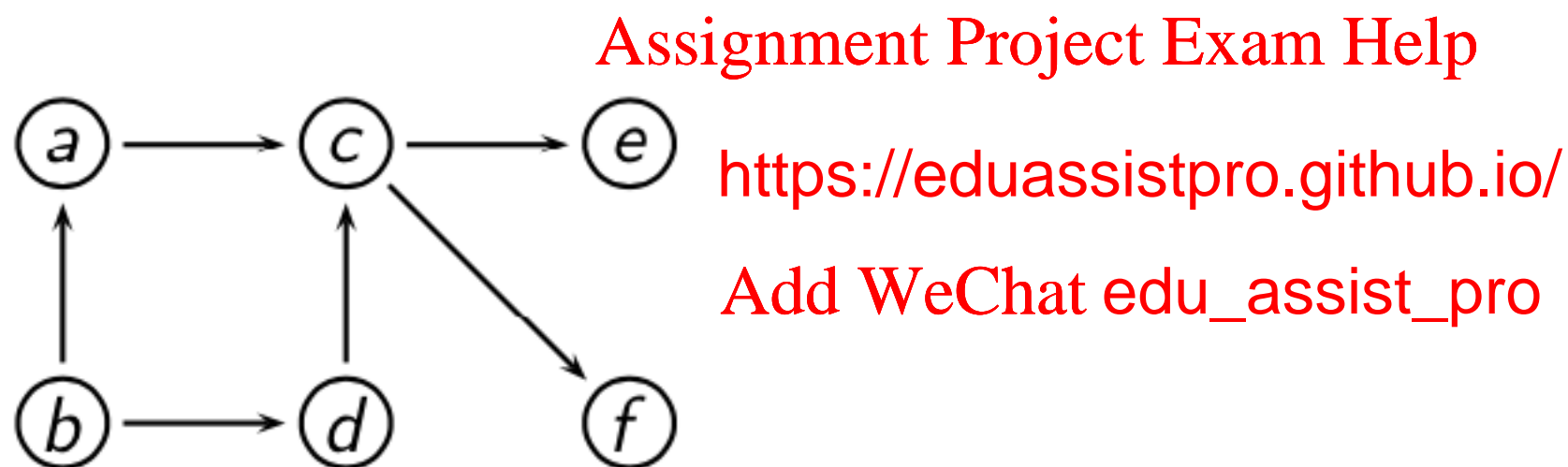
Algorithm 1

- We can solve the top-sort problem with depth-first search:
 1. Perform DFS and note the order in which nodes are popped off the stack.
 2. List the nodes in the reverse of that order.
- This works because <https://eduassistpro.github.io/>
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- If (u,v) is an edge then it is possible (given some way of deciding ties) to arrive at a DFS stack with u sitting below v .
- Taking the “reverse popping order” ensures that u is listed before v .

Topological Sorting

Example Again

Using the DFS method and resolving ties by using alphabetical order, the graph gives rise to the traversal stack shown on the right (the popping order shown in red):



Taking the nodes in reverse popping order yields
b, d, a, c, f, e.

Topological Sorting

Algorithm 2

- An alternative method would be to repeatedly select a random **source** in the graph (that is, a node with no incoming edges), list it, and remove it from the graph (including removing its outgoing edges).

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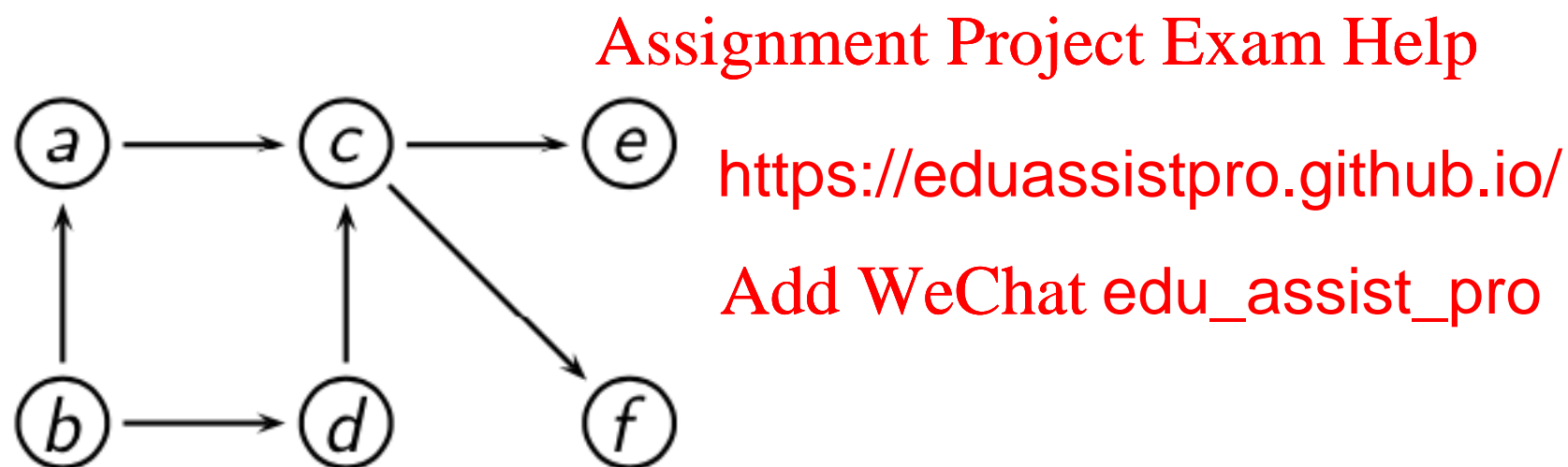
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- This is a very natural approach, but it has the drawback that we repeatedly need to scan the graph for a source.
- However, it exemplifies the general principle of **decrease-and-conquer**.

Topological Sorting

Example Again

Using the source removal method (and resolving ties alphabetically):

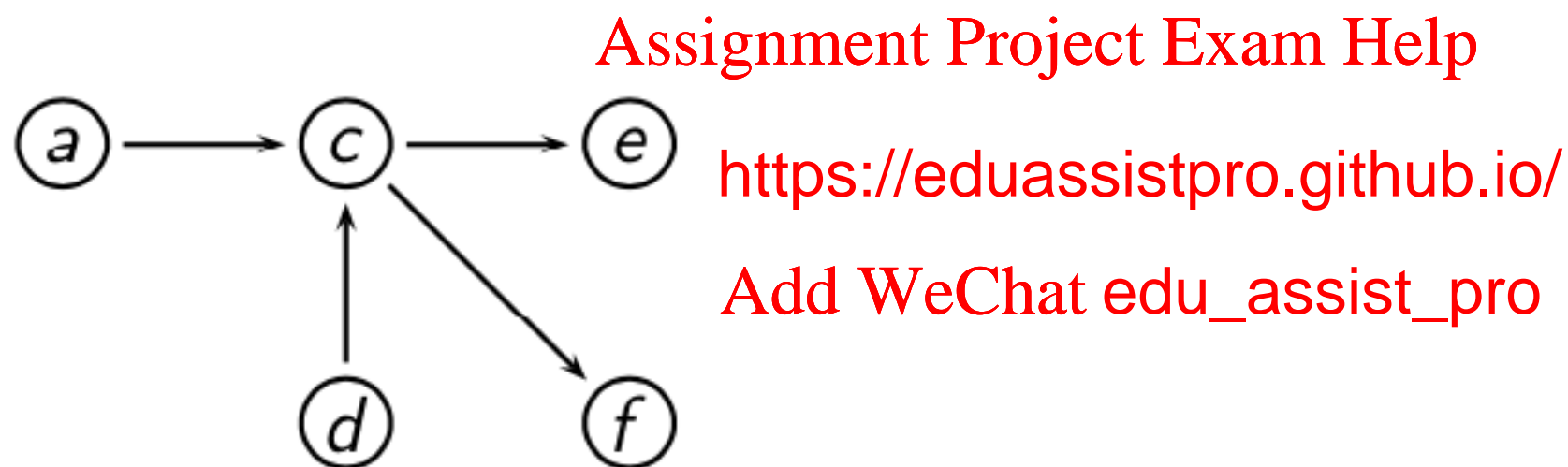


Topological sorted order:

Topological Sorting

Example Again

Using the source removal method (and resolving ties alphabetically):



Topological sorted order:
b

Topological Sorting

Example Again

Using the source removal method (and resolving ties alphabetically):



Topological sorted order:
b, a

Topological Sorting

Example Again

Using the source removal method (and resolving ties alphabetically):



Topological sorted order:
b, a, d

Topological Sorting Example Again

Using the source removal method (and resolving ties alphabetically):

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Topological sorted order:
b, a, d, c

Topological Sorting Example Again

Using the source removal method (and resolving ties alphabetically):

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Topological sorted order:
b, a, d, c, e

Topological Sorting Example Again



Using the source removal method (and resolving ties alphabetically):

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Topological sorted order:
b, a, d, c, e, f

Next time

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- So next we turn on the very useful
“decrease and c <https://eduassistpro.github.io/>
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