COMP90038
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Algorit
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Lecture 13: Priority Oueue Heapsort Add WeChat edu_assist_pro (with thanks to Hara rd)

Andres Munoz-Acosta

munoz.m@unimelb.edu.au

Peter Hall Building G.83

Where to find me?

 My office is at the Peter Hall building (Room G.83)

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- My office is at the Peter Hall building (Room G.83) Assignment Project Exam Help
- Consultation hours: https://eduassistpro.github.io/
 - Wednesdays 10:00am-14:00 aweChat edu_assist_pro
 - By appointment on Monday/Friday (limited slots)

Heaps and Priority Queues

 The heap is a very useful data structure for priority queues, used in many algorithms.

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A priority queue is a set (o

- An element is injected into the priority query r with a **priority** (often the key value itself) and elements dre because edu_assistprocepty.
- We think of the heap as a partially ordered binary tree.
- Since it can easily be maintained as a **complete** tree, the standard implementation uses an array to represent the tree.

The Priority Queue

• As an abstract data type, the priority queue supports the following operations on a "pool" of elements (ordered by some linear order):

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• find an item with maximal priority

- **insert** a new item with ass
- test whether a priority que https://eduassistpro.github.io/
- eject the largest element

- Other operations may be relevant, for example:
 - replace the maximal item with some new item
 - construct a priority queue from a list of items
 - join two priority queues

Some Uses of Priority Queues

- (Discrete event) simul ms (like traffic, or weather). Here prioriti https://eduassistpro.giithuds.io/
- Numerical computations involving at edu_assist pumbers. Here priorities are measures of computat r".

 Many sophisticated algorithms make essential use of priority queues (Huffman encoding and many shortest-path algorithms, for example).

Stacks and Queues as Priority Queues

- Special instances are obtained when we use time for priority:
 - If "large" means "late" https://eduassistpro.github.io/
 - If "large" means "early" we obtain the edu_assist_pro

Possible Implementations of the Priority Queue

Assume priority = key.

Assignment Project Exam Help EJECT()

Unsorted arrhttps://eduassistpro.github.io/

Sorted array Adia WeChat edu assist pro

 $O(\log n)$

 $O(\log n)$

How is this accomplished?

Heap

The Heap

• A heap is a complete binary tree which satisfies the heap condition:

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Each child has a priori eater than its parent's. https://eduassistpro.github.io/

• This guarantees that the root of the edu_assist_pro aximal element.

 (Sometimes we talk about this as a max-heap – one can equally well have min-heaps, in which each child is no smaller than its parent.)

Heaps and Non-Heaps

Which of these are heaps?

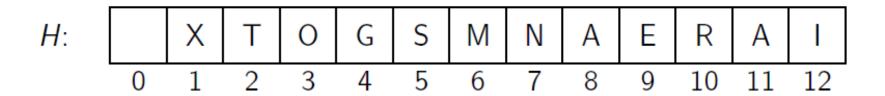
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Heaps as Arrays

 We can utilise the completeness of the tree and place its elements in level-order man Project Exam Help array H. https://eduassistpro.github.io/

• Note that the children of node *i* will be nodes 2i and 2i + 1.

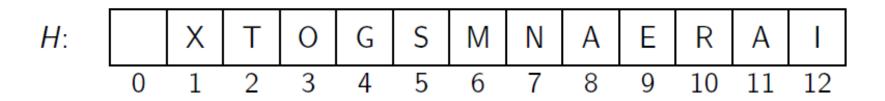


Heaps as Arrays

 This way, the heap condition is very simple:

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• For all $i \subset \{0,1,...,n, \text{ we https://eduassistpro.github.io/have H[i]} \le H[i/2]. Add WeChat edu_assist_pro$



Properties of the Heap

• The root of the tree H[1] holds a maximal item; the cost of EJECT is O(1) plus time to restore the heap.

• The height of the heap is $\lfloor \log n \rfloor$.

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Each subtree is also a heap.

- The children of node i are 2i and 2i+1.
- The nodes which happen to be parents are in array positions 1 to $\lfloor n/2 \rfloor$.
- It is easier to understand the heap operations if we think of the heap as a tree.

Injecting a New Item

 Place the new item at the end; then let it "climb up", repeatedly swapping with parents that are smaller: Assignment Project Exam Help

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Building a Heap Bottom-Up

To construct a heap from an arbitrary set of elements, we can just use the inject operation repeatedly. The construction cost will be n log n. But there is a better way:
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 Start with the last parent and move backwards, in level-order. For each parent node, if the largest child is larger than the parent, swap it with the parent.

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 Start with the last parent and move backwards, in level-order. For each parent node, if the largest child is larger than the parent, swap it with the parent.

Building a Heap Bottom-Up: Sifting Down

• Whenever a parent is found to be out of order, let it "sift down" until both children are smaller:

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Turning $H[1] \dots H[n]$ into a Heap, Bottom-Up

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Analysis of Bottom-Up Heap Creation

• For simplicity, assume the heap is a full binary tree: $n = 2^{h+1} - 1$. Here is an upper bound on the number of "down-sifts" needed (consider the root to be at level \$h\$, so leaves are at level 0):

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- The last equation is easily proved by matter at isa edu_assist_pro
- Note that $2^{h+1} h 2 < n$, so we perform at most a linear number of down-sift operations. Each down-sift is preceded by two key comparisons, so the number of comparisons is also linear.
- Hence we have a **linear-time** algorithm for heap creation.

• Here the idea is to swap the root with the last item z in the heap, and then let z "sift daws graphent Project Exam Help proper place.

- After this, the last element (here Chat edu_assist_pro shown in green) is no longer considered part of the heap, that is, *n* is decremented.
- Clearly ejection is $O(\log n)$.

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- Clearly ejection is $O(\log n)$.

Exercise: Build and Then Deplete a Heap

• First build a heap from the items S, O, R, T, I, N, G.

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• Then repeatedly eject the end of the heap. https://eduassistpro.github.io/

Exercise: Build and Then Deplete a Heap

• First build a heap from the items S, O, R, T, I, N, G.

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- Then repeatedly eject the end of the heap. https://eduassistpro.github.io/
- Anything interesting to hotice about edu_assist_pro heap?

Heapsort is a Θ(n log n) sorting algorithm, based on the idea from this exercise.

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Given an unsorted arr https://eduassistpro.github.io/

- Step 1: Turn H into a heap.
- **Step 2:** Apply the eject operation *n*-1 times.

Stage 1 (heap construction)

Stage 2 (maximum deletions)

2 9 **7** 6 5 <u>8</u>

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Stage 1 (heap construction)

Stage 2 (maximum deletions)

```
2 9 7 6 5 <u>8</u>
2 9 8 <u>6 5</u> 7
```

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Stage 1 (heap construction)

Stage 2 (maximum deletions)

```
2 9 7 6 5 <u>8</u>
2 9 8 <u>6 5</u> 7
2 <u>9</u> 8 6 5 7
```

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Stage 1 (heap construction)

Stage 2 (maximum deletions)

```
2 9 7 6 5 <u>8</u>
2 9 8 <u>6 5</u> 7
2 <u>9</u> 8 6 5 7
9 2 8 <u>6 5</u> 7
```

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Stage 1 (heap construction)

Stage 2 (maximum deletions)

```
2 9 7 6 5 <u>8</u>
2 9 8 <u>6 5</u> 7
2 9 8 6 5 7
9 2 8 <u>6 5</u> 7
9 6 8 2 5 7
```

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Stage 1 (heap construction)

2 9 **7** 6 5 <u>8</u>

2 9 8 6 5 7

2 **9** 8 6 5 7

- 9 **2** 8 6 5 7
- 9 6 8 2 5 7

Stage 2 (maximum deletions)

9 6 8 2 5 7

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Stage 1 (heap construction)

2 9 7 6 5 <u>8</u> 2 9 8 <u>6 5</u> 7 2 <u>9 8</u> 6 5 7 9 **2** 8 6 5 7

9 6 8 2 5 7

Stage 2 (maximum deletions)

9 6 8 2 5 7
Assignment Project Exam Help² 5 9

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Stage 1 (heap construction)

2 9 7 6 5 <u>8</u> 2 9 8 <u>6 5</u> 7 2 9 8 6 5 7 9 2 8 <u>6 5</u> 7

Stage 2 (maximum deletions)

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Stage 1 (heap construction)

2 9 7 6 5 <u>8</u> 2 9 8 <u>6 5</u> 7 2 <u>9 8</u> 6 5 7 9 2 8 <u>6 5</u> 7

Stage 2 (maximum deletions)

Stage 1 (heap construction)

Stage 2 (maximum deletions)

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$$5 \mid 9$$
 $7 \mid 2 \mid 5 \mid 9$ https://eduassistpro.github2o/ 8 9 Add WeChat edu_assist_po

Stage 1 (heap construction)

2 9 7 6 5 <u>8</u> 2 9 8 <u>6 5</u> 7 2 9 8 6 5 7 9 2 8 <u>6 5</u> 7

Stage 2 (maximum deletions)

Stage 1 (heap construction)

2 9 7 6 5 <u>8</u> 2 9 8 <u>6 5</u> 7 2 9 8 6 5 7 9 2 8 <u>6 5</u> 7

Stage 2 (maximum deletions)

Properties of Heapsort

• On average slower than quicksort, but stronger performance guarantee.

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Truly in-place. https://eduassistpro.github.io/

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Not stable.

Next lecture

- Transform-and-Conquernment Project Exam Help
 - Pre-sorting (Levitin Sechttps://eduassistpro.github.io/