COMP9319 Web Data Compression and Search

Space Efficient Linear Time Construction of Suffix Arrays

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Suffix Array

- Sorted order of suffixes of a string T.
- Represented by the starting position of the suffix.

Text	M	I	S	S	I	S	S	I	P	P	I	$
Index	1	2	3	4	5	6	7	8	9	10	11	12
Suffix Array	12	11	8	5	2	1	10	9	7	4	6	3

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Brief History

- Introduced by Manber and Myers in 1989.
- Takes O(n *log* n) time, and 8n bytes.
- Many other non-linear time algorithms.

Authors	Time	Space (bytes)
String-sorting	$n^2 \log n$	$5n$
Radix-sorting	n^2	$5n$

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Our Result

1. Among the first linear-time direct suffix ion algorithms. Solves an problem.
   ze alphabet, only uses 8n e.
   a space efficient ion algorithm.

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Notation

- String $T = t_1 \ldots t_n$.
- Over the alphabet $\Sigma = \{1 \ldots n\}$.
- t_n = '\$', '\$' is a unique character.
- $T_i = t_i \ldots t_n$, denotes the i-th suffix of T.
- For strings α and β, $\alpha < \beta$ denotes α is lexicographically smaller than β.

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Overview

- Divide all suffixes of T into two types.
  - Type S suffixes = $\{T_i \mid T_i < T_{i+1}\}$
  - Type L suffixes = $\{T_j \mid T_j > T_{j+1}\}$
  - The last suffix is both type S and L.
- Sort all suffixes of one of the types.
- Obtain lexicographical order of all suffixes from the sorted ones.

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Identify Suffix Types

Type	L	S	L	L	S	L	L	S	L	L	L	L/S
Text	M	I	S	S	I	S	S	I	P	P	I	$

The type of each suffix in T can be determined in one scan of the string.

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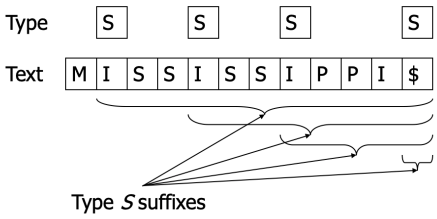
Notation



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Notation

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Notation

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Sorting Type S Suffixes

- Sort all type S substrings.
- Replace each type S substrings by its bucket number.
- New string is the sequence of bucket numbers.
- Sorting all type S suffixes = Sorting all suffixes of the new string.

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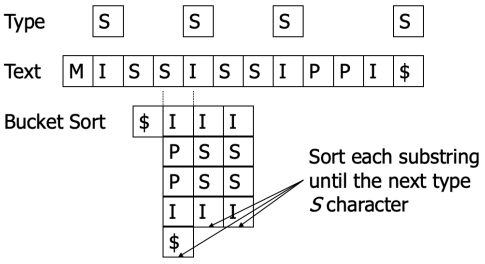
Sorting Type S Substrings



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Sorting Type *S* Substrings

Type

Text | 3 | 3 | 2 | 1 |

Bucket Sort

$	I	I	I
	P	S	S
	P	S	S
	I	I	I
	$		

Substitute the substrings with the bucket numbers to obtain a new string. Apply sorting recursively to the new string.

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Sorting Type *S* Substrings

Type

Text | 3 | 3 | 2 | 1 |

Bucket Sort

$	I	I	I
	P	S	S
	P	S	S
	I	I	I
	$		

Bucket Sort takes potentially $O(n^2)$ time

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Solution

- Observation: Each character participates in the bucket sort at
  - Type *L* characters o bucket sort once.
- Solution:
  - Sort all the characters once.
  - Construct m lists according the distance to the closest type *S* character to the left

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Illustration

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Illustration

Type		S			S			S				S
Index	1	2	3	4	5	6	7	8	9	10	11	12
Text	M	I	S	S	I	S	S	I	P	P	I	$
Distance	0	0	1	2	3	1	2	3	1	2	3	4

The Lists

9	3	6
10	4	7
5	8	11
12		

Sort the type *S* substrings using the lists

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T M I S S I S S I P P I $
Type S S S S
Pos 1 2 3 4 5 6 7 8 9 10 11 12
A 12 2 5 8 11 1 9 10 3 4 6 7

Step 1. Record the S−distances

Pos 1 2 3 4 5 6 7 8 9 10 11 12
Dist 0 0 1 2 3 1 2 3 1 2 3 4

Step 2. Construct S−distance Lists

1 | 9 | 3 | 6 |
2 | 10 | 4 | 7 |
3 | 5 | 8 | 11 |
4 | 12 |

Step 3. Sort all type S substrings

Original | 12 | 2 | 5 | 8 |
Sort according to list 1 | 12 | 8 | 5 | 2 |
Sort according to list 2 | 12 | 8 | 5 | 2 |
Sort according to list 3 | 12 | 8 | 5 | 2 |
Sort according to list 4 | 12 | 8 | 5 | 2 |

Fig. 3. Illustration of the sorting of type *S* substrings of the string MISSISSIPPI$.

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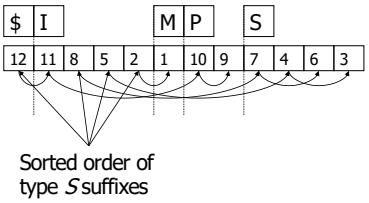
Construct Suffix Array for all Suffixes

- The first suffix in the suffix array is a type S suffix.
- For $1 \le i \le n$, if $T_{SA[i]-1}$ is type L, move it to the current front of its bucket



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Run-Time Analysis (Sketch)

- Identify types of suffixes -- O(n) time.
- Bucket sort type S (or L) substrings -- O(n) time.
- Construct suffix array from sorted type S (or L) suffixes -- O(n) time.

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Conclusion

- Among the first suffix array construction algorithm takes O(
- The algorithm can b
  in 8n bytes (plus a f
- Equal or less space
  time algorithm.

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Exercise

- Consider the popular example string S:

mas$

uffix array of S using the

WT(S)
nip between the
T ?

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Step – Identify the type of each suffix

- **LSLSLSSSLSLSLSL**L/S
- **bananainpajamas$**
- **1**
- **1234567890123456**

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Step – Compute the distance from S

- **LSLSLSSSLSLSLSL**L/S
- **bananainpajamas$**
- **1111111**
- **1234567890123456**
- **0012121112121212**

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Step – Sort order of chars

- LSLSLSSSLSLSLSL$_{L/S}$
- bananainpajamas$
- 1111111
- 1234567890123456
- 0012121112121212
- $a bijmn ps
- 1 111 11 1
- 6246024171335895

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Step – Construct m-Lists

- LSLSLSSSLSLSLSL$_{L/S}$
- bananainpajamas$
- 1111111
- 1234567890123456
- 0012121112121212
- $a bijmn ps
- 1 111 11 1
- 6246024171335895

Scan this once and bucket it according to dist.

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Step – Generate m-Lists

- List 1
- [7],[11],[13],[3,5,8],[9],[15]
- List 2
- [16],[4,6,10,12

- 2022222011111111
- $a bijmn ps
- 1 111 11 1
- 6246024171335895

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Step – Sort S substrings

Bucket the S substrings

[8]

- 1111111
- 90123456
- 21212
- ps
- 1
- 35895

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Step – Sort S substrings

Bucket the S substrings
[16],[2,4,6,10,12,14],[7],[8]
After using List 1:
[16],[6],[10],[12],[2,4],[14],[7],[8]
List 2 useless. Then?

- List 1
- [7],[11],[13],[3,5,8],[9],[15]
- List 2
- [16],[4,6,10,12,14]

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Step – Sort S substrings

Bucket the S substrings
[16],[2,4,6,10,12,14],[7],[8]
After using List 1:
[16],[6],[10],[12],[2,4],[14],[7],[8]
List 2 useless. Consider 6 before 4:
[16],[6],[10],[12],[4],[2],[14],[7],[8]

- List 1
- [7],[11],[13],[3,5,8],[9],[15]
- List 2
- [16],[4,6,10,12,14]

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Step – Generate the Suffix Array

[16],[6],[10],[12],[4],[2],[14],[7],[8]

- $a bijmn ps
- 1 111 11 1
- 6246024171335895

- $a ins
- 1 11 1 1
- 6602424785

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Step – Generate the Suffix Array

- $a bijmn ps
- 1 111 11 1
- 6246024171335895

- $a in s
- 1 11 1 1
- 66024247585

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Step – Generate the Suffix Array

- $a bijmn ps
- 1 111
- 624602417

- $a in
- 1 11 1 1
- 660242475895

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Step – Generate the Suffix Array

- $a bijmn ps
- 1
- 5895

- s
- 1
- 95

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Step – Generate the Suffix Array

- $a bijmn ps
- 1 111 11 1
- 6246024171335895

- $a ijn ps
- 1 11 1 1 1
- 66024247153895

type S

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Step – Generate the Suffix Array

- $a bijmn ps
- 1 111 11 1
- 6246024171335895

- $a bijn ps
- 1 11 1 1 1
- 66024241715389 5

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Step – Generate the Suffix Array

- $a bijmn ps
- 1 111 11 1
- 6246024171335895

- $a bijmn ps
- 1 11 1 11 1
- 6602424171353895

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Final answer

- bananainpajamas$
- 1111111
- 1234567890123456

- Suffix Array:
- 1 11 1 11 1
- 6602424171353895

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Final answer

- bananainpajamas$
-
- 123456789
- Suffix Ar
- 1 11 1 11 1
- 6602424171353895

What is the BWT(S) ?

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BWT is easy!

- bananainpajamas$
- 1111111
- 3456
- 1
- 53895
- 1
- 5591313660242784

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BWT construction in linear time

- bananainpajamas$
- 1111111
- 1234567890123456

- BWT:
- 1 1 11 11 1
- 5591313660242784
- snpjnbm$aaaaaina

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