COMP9319 Web Data Compression and Search

Regular expressions and basic indexed search

Including some slides modified from comp3402 at cs.ucf.edu, haimk, Tel-Aviv U. and the slides from Andrew Davison.

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Regular Expressions

- Notation to specify a language
  - Declarative
  - Sort of like a programming language.
    - Fundamental in some languages like Perl and applications like grep or lex
  - Capable of describing the same thing as an NFA
    - The two are actually equivalent, so RE = NFA = DFA
  - We can define an algebra for regular expressions

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Definition of a Regular Expression

- R is a regular expression if it is:
  1. **a** for some *a* in the alphabet Σ, standing for the language {a}
  2. ε, standing for the langua
  3. Ø, standing for the empty la
  4. R_1+R_2 where R_1 and R_2 signifies union (sometim
  5. R_1R_2 where R_1 and R_2 are regular expressions and this signifies concatenation
  6. R* where R is a regular expression and signifies closure
  7. (R) where R is a regular expression, then a parenthesized R is also a regular expression

This definition may seem circular, but 1-3 form the basis
Precedence: Parentheses have the highest precedence, followed by *, concatenation, and then union.

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Using Regular Expressions

- Regular expressions are a standard
  l.
  erl, Unix, Python, . . .

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RE Examples

- L(**001**) = {001}
- L(**0+10***) = { 0, 1, 10, 100, 1000, 10000, … }
- L(**0*10***) = {1, 01, 10, 010, 0010, …} i.e. {w | w has exactly a single 1}
- L($\Sigma\Sigma$)* = {w | w is a string of even length}
- L((**0(0+1)**)*) = { ε, 00, 01, 0000, 0001, 0100, 0101, …}
- L(**(0+ε)(1+ ε)**) = {ε, 0, 1, 01}
- L(1Ø) = Ø ; concatenating the empty set to any set yields the empty set.
- Rε = R
- R+Ø = R

- Note that R+ε may or may not equal R (we are adding ε to the language)
- Note that RØ will only equal R if R itself is the empty set.

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Exercise 1

- Let Σ be a finite set of symbols
- Σ = {10, 11}, Σ^* = ?

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Answer

Answer: $\sum^*$ = {ϵ, 10, 11, 1010, 1011, 1110, 1111, …}

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Exercise 2

- L1 = {10, 1}, L2 = {011, 11}, L1L2 = ?

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Answer

- L1L2 = {10011, 1011, 111}

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Exercise 3

- Write RE for

and 1's

and 1's with at least 2

  – All strings of 0's and 1's beginning with 1 and
utive 0's

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Answer

- (0|1)*

All strings of 0's and 1's.

- (0|1)*00(0|1)*

All strings of 0's and 1's with at least 2 consecutive 0's

- (1+10)*

All strings of 0's and 1's beginning with 1 and not having two consecutive 0's

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More Exercises

- 1) (0|1)*011
- 2) 0*1*2*
- 3) 00*11*22*

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More Exercises (Answers)

1) (0|1)*011
Answer: all strings of 0's and 1's ending in 011
2) 0*1*2*
- Answer: any number of 0's followed by any number of 1's followed by any number of 2's
- 3) 00*11*22*
Answer: strings in 0*1*2 with at least one of each symbol

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Deterministic Finite Automata (DFA)

- Simple machine with N states.
- Begin in start state.
- Read first input symbol.
- Move to new state, depending on current state and input symbol.
- Repeat until last input symbol read.
- Accept or reject string depending on label of last state.

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DFA

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Theory of DFAs and REs

- RE. Concise way to describe a set of
recognize whether a
a given set
- **Duality**: for any DFA, there exists a
describe the same
regular expression,
at recognizes the same set.

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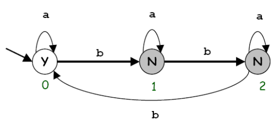
Duality Example

- DFA for multiple of 3 b's:



- RE for multiple of 3 b's:

```
(a*ba*ba*ba*)* a*
```

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Fundamental Questions

- Which languages CANNOT be described by any RE?
- Set of all bit strings with equal number of 0s and 1s.
- Set of all decimal strings that represent prime numbers.
- Many more. . . .

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Problem 1

- Make a DFA that accepts the strings in the language denoted by regular expression ab*a

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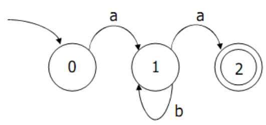
Solution

- ab*a:



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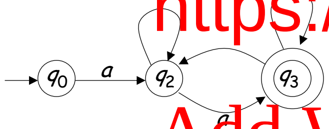
Problem 2

- Write the RE for the following automata:



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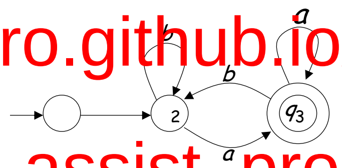
Solution

- a(a|b)*a



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DFA to RE: State Elimination

- Eliminates states of the automaton and replaces the edges with regular expressions that includes the behavior of the eliminated states.
- Eventually we get down to the situation with just a start and final node, and this is easy to express as a RE

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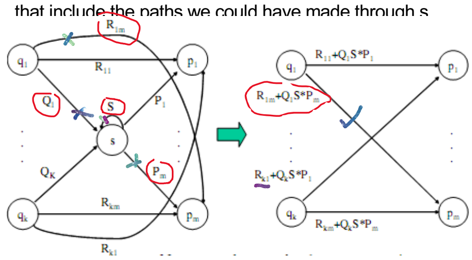
State Elimination

- Consider the figure below, which shows a generic state s about to be eliminated.
- The labels on all edges are regular expressions.
- To remove s, we must make labels from each qi to p1 up to pm that include the paths we could have made through s



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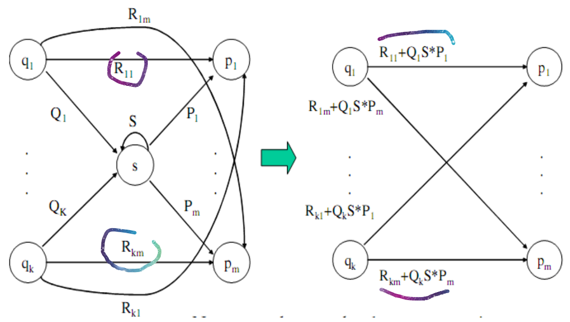

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DFA to RE via State Elimination (1)

- Starting with intermediate states and then moving to accepting states, apply the state elimination process to produce an equivalent automaton with regular expression labels on the edges.
- The result will be a one or two state automaton with a start state and accepting state.

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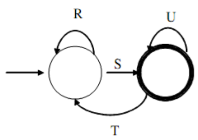
DFA to RE State Elimination (2)

- If the two states are different, we will have an automaton that looks li



- We can describe this automaton as: (R | SU*T)*SU*

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DFA to RE State Elimination (3)

- If the start state is also an accepting state, then rm a state elimination from on that gets rid of every state This leaves the following:

utomaton as simply R*

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DFA to RE State Elimination (4)

- If there are n accepting states, we must repeat the above steps for each accepting states to get n different regular expressions, R1, R2, … Rn.
- For each repeat we turn any other accepting state to non-accepting.
- The desired regular expression for the automaton is then the union of each of the n regular expressions: R1 U R2… U RN

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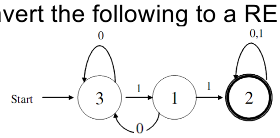
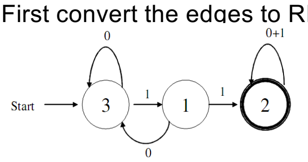
DFA->RE Example

- Convert the following to a RE:



- First convert the edges to RE’s:



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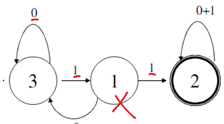
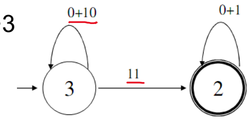
DFA -> RE Example (2)

- Eliminate State 1:



- Note edge from 3->3



- Answer: (0+10)*11(0+1)*

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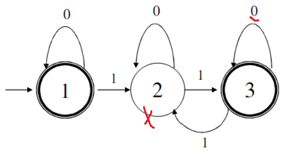
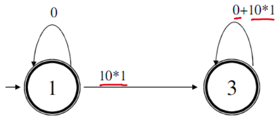
Second Example

- Automata that accepts even number of 1's



- Eliminate state 2:



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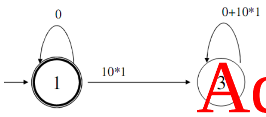
Second Example (2)

- Two accepting states,



- This is just 0*; can ignore going to state 3 since we would "die"

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Second Example (3)

- This is just 0*10*1(0|10*1)*
- Combine from previous slide to get 0* | 0*10*1(0|10*1)*

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Text search

- Pattern matching directly
  - Brute force
  - BM
  - KMP
- Regular expressions
- Indices for pattern matching
  - Inverted files
  - **Signature files**
  - **Suffix trees** and **Suffix arrays**

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Inverted Index

For each term *t*, we store a list of all documents that contain *t*.

BRUTUS	→	1	2	4	11	31	45	173	174	
CAESAR	→	1	2	4	5	6	16	57	132	...
CALPURNIA	→	2	31	54	101					

⋮

dictionary postings

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Create postings lists, determine document frequency

term	docID
ambitious	2
be	2
brutus	1
brutus	2
capitol	1
caesar	1
caesar	2
caesar	2
did	1
enact	1
hath	1
i	1
i	1
i'	1
it	2
julius	1
killed	1
killed	1
let	2
me	1
noble	2
so	2
the	1
the	2
told	2
you	2
was	1
was	2
with	2

⟹

term	doc. freq.	→	postings lists
ambitious	1	→	2
be	1	→	2
brutus	2	→	1 → 2
capitol	1	→	1
caesar	2	→	1 → 2
did	1	→	1
enact	1	→	1
hath	1	→	2
i	1	→	1
i'	1	→	1
it	1	→	2
julius	1	→	1
killed	1	→	1
let	1	→	2
me	1	→	1
noble	1	→	2
so	1	→	2
the	2	→	1 → 2
told	1	→	2
you	1	→	2
was	2	→	1 → 2
with	1	→	2

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Positional indexes

- Postings lists in a nonpositional index: each posting is just a docID
- Postings lists in a positional index: each posting is a docID and a list of positions

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Positional indexes: Example

Query: "to_1 be_2 or_3 not_4 to_5 be_6"

TO, 993427:

‹ 1: ‹7, 18, 33, 72, 86, 231›;
2: ‹1, 17, 74, 222, 255›;
4: ‹8, 16, 190, 429, 433›;
5: ‹363, 367›;
7: ‹13, 23, 191›; . . . ›

BE, 178239:

‹ 1: ‹17, 25›;
4: ‹17, 191, 291, 430, 434›;
5: ‹14, 19, 101›; . . . › Document 4 is a match!

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Signature files

- Definition
  - Word-oriented index structure based on hashing.

ry large texts.

function that maps words to bit masks.

cks.

ined by bitwise ORing the rds in the text block.

ch between all 1 bits in the ck mask.

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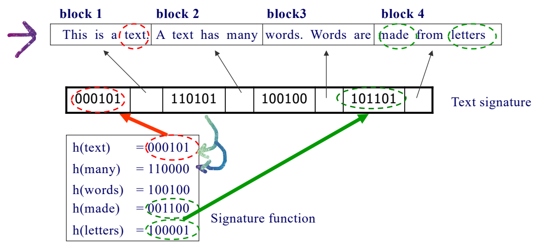
Signature files

- Example:



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Signature files

- False drop Problem
  - The corresponding **bits** are **set** even though the word is **not there**!
  - The **design** should **insure** that the **probability** of false drop is **low**.
    - Also the Signature file should be as short as possible.
  - **Enhance** the **hashing function** to minimize the error probability.

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Signature files

- Searching
  1. For a single word, Hash word to a bit mask W.
  2. For phrases,
     1) Hash words in query to a bit mask.
     2) Bitwise OR of all the query masks to a bit mask W.
  3. Compare W to the bit masks B_i of all the text blocks.
     - If all the bits set in W are also in B_i, then text block may contain the word.
  4. For all candidate text blocks, an online traversal must be performed to verify if the actual matches are there.
- Construction
  1. Cut the text in blocks.
  2. Generate an entry of the signature file for each block.
     - This entry is the bitwise OR of the signatures of all the words in the block.

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Suffix trees and suffix arrays

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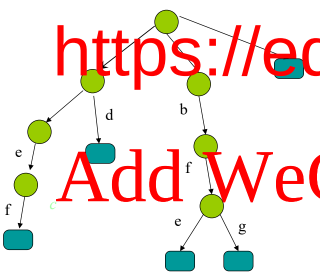
Trie

- A tree representing a set of strings.

{
aceef
ad
bbfe
bbfg
c }



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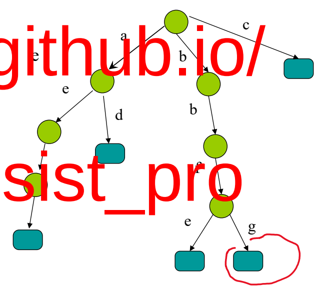
Trie (Cont)

- Assume no string is a prefix of another



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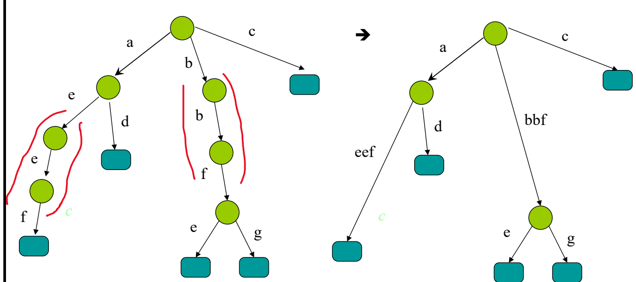
Compressed Trie

- Compress unary nodes, label edges by strings



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Suffix tree

Given a string s a suffix tree of s is a compressed trie of all suffixes of s

To make these suffixes prefix-free we add a special character, say $, at the end of s

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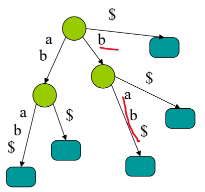
Suffix tree (Example)

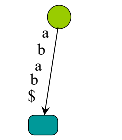
Let s=abab, a suffix tree of s is a compressed trie of all suffixes of s=abab$

{
$
b$
ab$
bab$
abab$
}

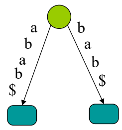


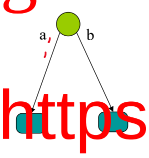
Trivial algorithm to build a Suffix tree

Put the largest suffix in

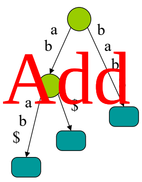


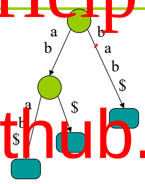
Put the suffix bab$ in

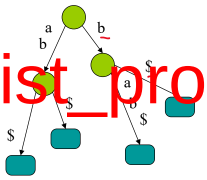


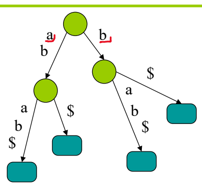


Put the suffix ab$ in

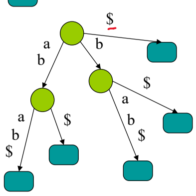


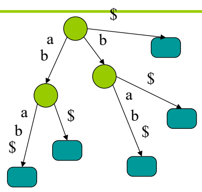






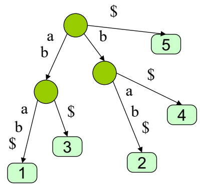
Put the suffix $ in





abab$
1 2 3 4 5

We will also label each leaf with the starting point of the corresponding suffix.



Analysis

Takes O(n^2) time to build.

*It can also be constructed in O(n) time.

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What can we do with it ?

Exact string matching:

Given a Text T, |T| = n, preprocess it such that when a pattern P, |P|=m, arrives you can quickly decide when it occurs in T.

W e may also want to find all occurrences of P in T

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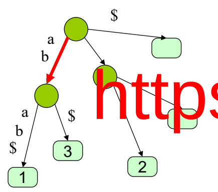
Exact string matching

In preprocessing we just build a suffix tree in O(n) time



Given a pattern P = ab we traverse the tree according to the pattern.

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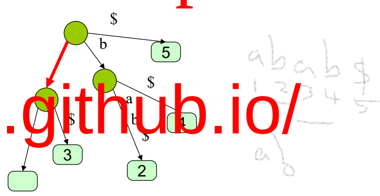

...hen the pattern occurs in the text.

...reach corresponds to an occurrence.

By traversing this subtree we get all k occurrences in O(n+k) time.

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Generalized suffix tree

Given a set of strings S a generalized suffix tree of S is a compressed trie of all suffixes of s ∈ S

To make these suffixes prefix-free we add a special char, say $, at the end of s

To associate each suffix with a unique string in S add a different special char to each s

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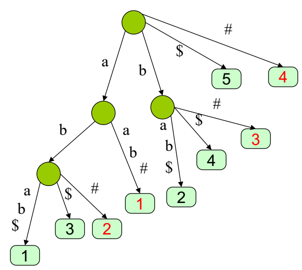
Generalized suffix tree (Example)

Let s_1=abab and s_2=aab here is a generalized suffix tree for s_1 and s_2

{
$ #
b$ b#
ab$ ab#
bab$ aab#
abab$
}



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So what can we do with it ?

Matching a pattern against a database of strings

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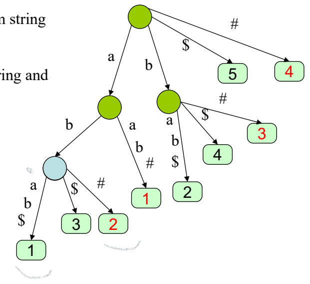
Longest common substring (of two strings)

Every node with a leaf descendant from string S_1 and a leaf descendant from string S_2 represents a maximal common substring and vice versa.

Find such node with largest "string depth"



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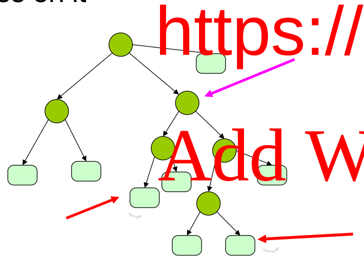
Lowest common ancestor

A lot more can be gained from the suffix tree if we preprocess it so LCA queries on it



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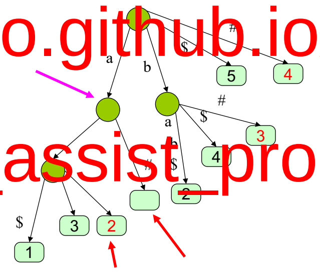
Why?

The LCA of two leaves represents the longest
P) of these 2 suffixes



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Finding maximal palindromes

- A palindrome: caabaac, cbaabc
- Want to find all maximal palindromes in a string s

Let s = cbaaba

The maximal palindrome with center between i-1 and i is the LCP of the suffix at position i of S and the suffix at position m-i+1 of S^r

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Maximal palindromes algorithm

Prepare a generalized suffix tree for
s = cbaaba$ and s^r = abaabc#

For every i find the LCA of suffix i of s and suffix m-i+1 of s^r

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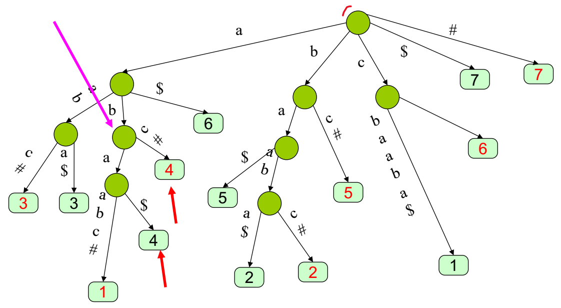
Let s = cbaaba$ then s^r = abaabc#



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Analysis

O(n) time to identify all palindromes

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Drawbacks

- Suffix trees consume a lot of space
- It is O(n) but the co

- Notice that if we indeed want to traverse an edge in O(1) time then we need an array of ptrs. of size |Σ| in each node

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Suffix array

- We loose some of the functionality but we

1234

xicographically:

3 1 4 2

e indices of the

3	1	4	2

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How do we build it ?

- Build a suffix tree
- Traverse the tree in DFS, lexicographically picking edges outgoing from each node and fill the suffix array.
- O(n) time

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How do we search for a pattern ?

- If P occurs in T then all its occurrences are consecutive in the suffix array.
- Do a binary search on the suffix array
- Takes O(mlogn) time

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Example

Let S = mississippi

Let P = issa

P = issi

L	11	i
	8	ippi
M	5	issippi
	2	ississippi
	1	mississippi
M	10	pi
	9	ppi
	7	sippi
	4	sisippi
	6	ssippi
R	3	ssissippi

Supra index

- Structure
  - **Suffix arrays** are **space efficient** implementation of **suffix trees**.
  - Simply an array **containing all the pointers** to the text suffixes listed in lexicographical order.
  - **Supra-indices:**
    - If the suffix array is **large**, this binary search can perform **poorly** because of the number of random disk accesses.
    - Suffix arrays are designed to allow **binary searches** done by comparing the contents of each pointer.
    - To remedy this situation, the use of **supra-indices** over the suffix array has been proposed.

Supra index

- Example