

Solution to COMP9334 Revision Questions Week02A — Part 2

Question 1

- (a) Since the mean arrival rate is 20 requests per second. The mean inter-arrival time is $\frac{1}{20} = 50\text{ms}$.
- (b) The mean number of requests arriving in 1 minute = 20 requests per seconds \times 60 seconds / minute = 1200 requests per minute.
- (c) and (d) Recalling that for Poisson arrivals with mean arrival rate λ and time interval t , the probability of n arrivals is

$$\frac{(\lambda t)^n \exp(-\lambda t)}{n!}. \quad (1)$$

For this question, $\lambda = 20$ and $t = 60$, so $\lambda t = 1200$.

In order to calculate the probability of no arrivals in a minute, we put $n = 0$ to obtain

$$\exp(-\lambda t) = \exp(-1200) \quad (2)$$

In order to calculate

$n = 10$ to obtain

$$\frac{(\lambda t)^n \exp(-\lambda t)}{n!} \quad (3)$$

Question 2

In order to refer to the two Poisson processes in a convenient way, I call P_1 and P_2 . The Poisson processes P_1 and P_2 , have rates r_1 and r_2 , respectively.

Consider a time interval T . Since P_1 is a Poisson process with rate r_1 , we know that the probability that there are k arrivals in time interval T is

$$\frac{e^{-r_1 T} (r_1 T)^k}{k!} \quad (4)$$

Similarly, the probability that there are j arrivals in time interval T from P_2 is

$$\frac{e^{-r_2 T} (r_2 T)^j}{j!} \quad (5)$$

Let us consider the aggregation of the two Poisson processes P_1 and P_2 over the time interval T . The arrivals can come from P_1 or P_2 . Let us find the probability that there are n arrivals in T . If there are n arrivals from P_1 and P_2 together, this can be resulted from

- 0 arrivals from P_1 and n arrivals from P_2
- 1 arrivals from P_1 and $(n - 1)$ arrivals from P_2

- 2 arrivals from P_1 and $(n - 2)$ arrivals from P_2
- ...
- $(n - 1)$ arrivals from P_1 and 1 arrivals from P_2
- n arrivals from P_1 and 0 arrivals from P_2

Therefore

$$\begin{aligned}
 & \text{Probability that there are } n \text{ arrivals over time } T \text{ from } P_1 \text{ and } P_2 \text{ together} \\
 = & \sum_{i=0}^n \text{Probability of } i \text{ arrivals over time } T \text{ from } P_1 \times \text{Probability of } (n - i) \text{ arrivals over time } T \text{ from } P_2 \\
 = & \sum_{i=0}^n \frac{e^{-r_1 T} (r_1 T)^i}{i!} \frac{e^{-r_2 T} (r_2 T)^{n-i}}{(n-i)!} \\
 = & \frac{1}{n!} e^{-(r_1 + r_2)T} \sum_{i=0}^n \frac{n!}{i!(n-i)!} (r_1 T)^i (r_2 T)^{n-i} \\
 = & \frac{1}{n!} e^{-(r_1 + r_2)T} ((r_1 + r_2)T)^n
 \end{aligned}$$

This shows that the a

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