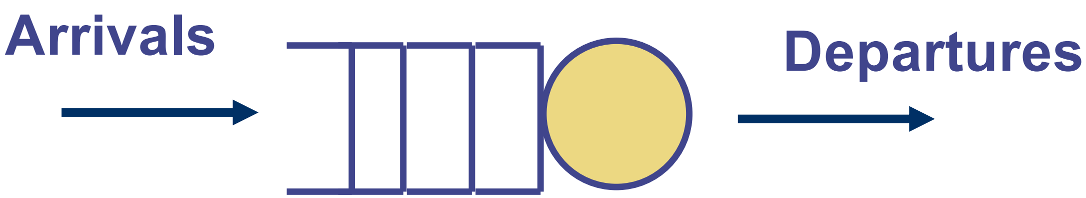
COMP9334
Capacity Planning for Computer Systems and Networks

Week 7A plines

Queuing disciplines



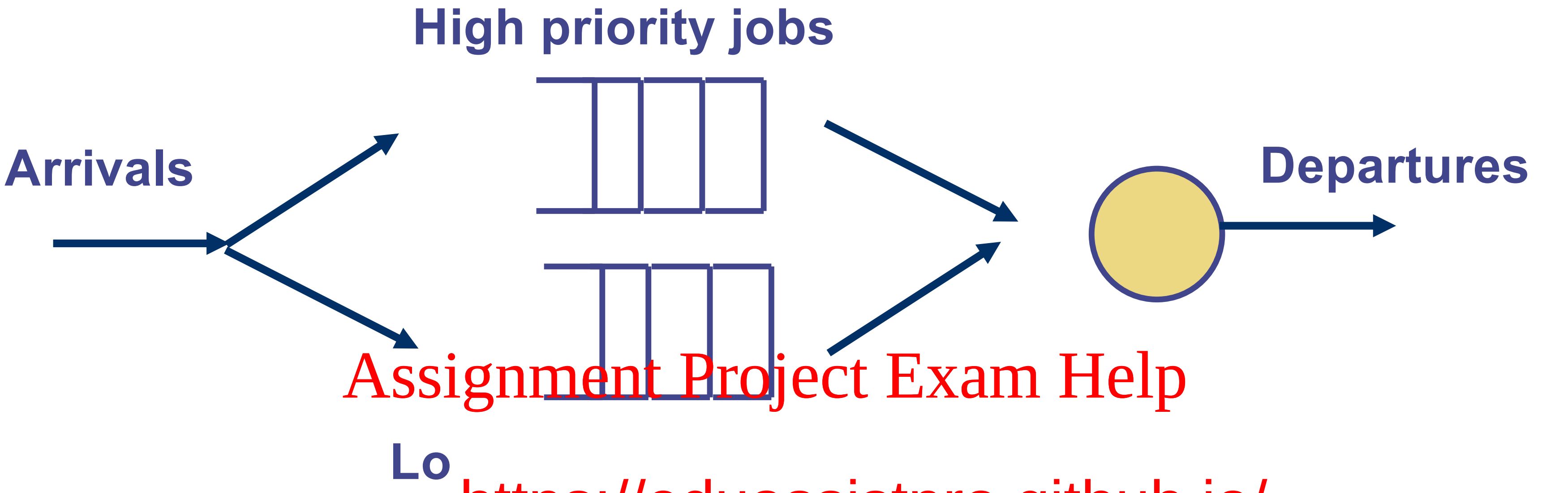
- We have focused on *first-come first-serve* (FCFS) queues so far

- However, somet give some jobs a higher priority th

- Priority queues can be classifi

  - Non-preemptive
  - Preemptive resume

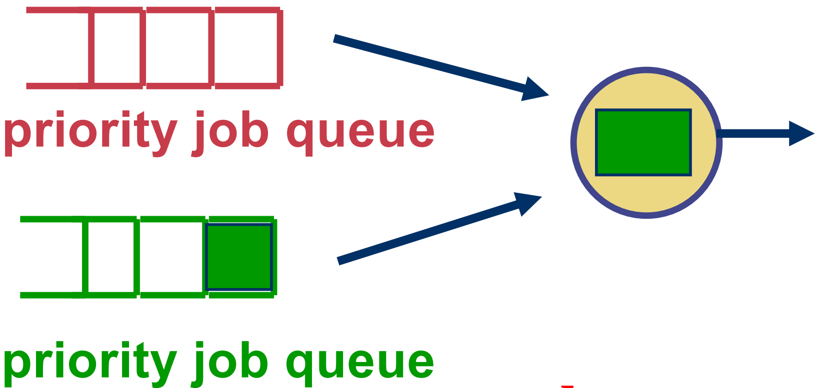
What is priority queueing?

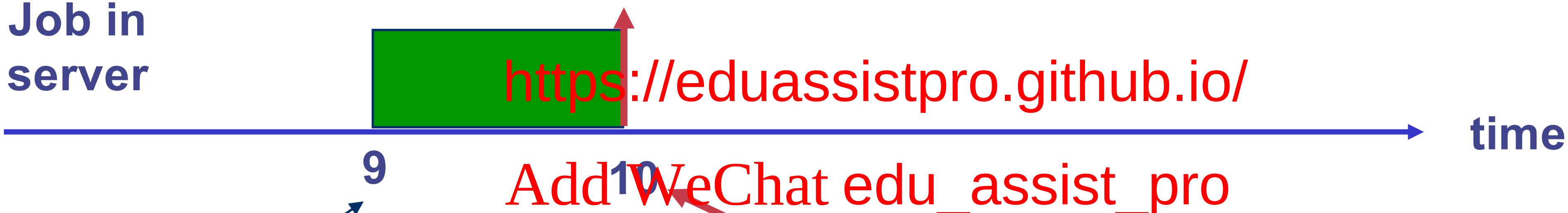


- A job with low priority will only get … high priority queue is empty
- Each priority queue is a FCFS queue
- Exercise: If the server has finished a job and finds 1 job in the high priority queue and 3 jobs in the low priority queue, which job will the server start to work on?
  - Repeat the exercise when the high priority queue is empty and there are 3 jobs in the low priority queue.

Preemptive and non-preemptive priority (1)

- Example:





Time t = 9

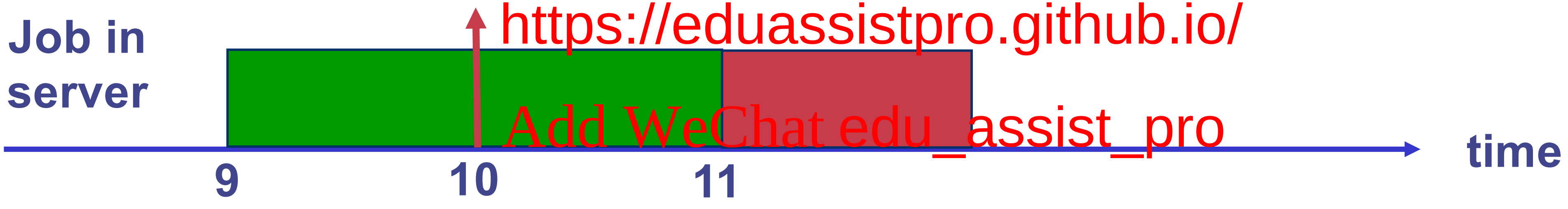
- The high priority job queue is empty
- The server starts serving a low priority job which requires 2s of processing

Time t = 10：A high priority job requiring 1s of processing arrives

Preemptive and non-preemptive priority (2)

- ***Non-preemptive***:
  - A job being served will not be interrupted (even if a higher priority job arrives in the mean time)

- Example: High priority job (red), low priority job (green)



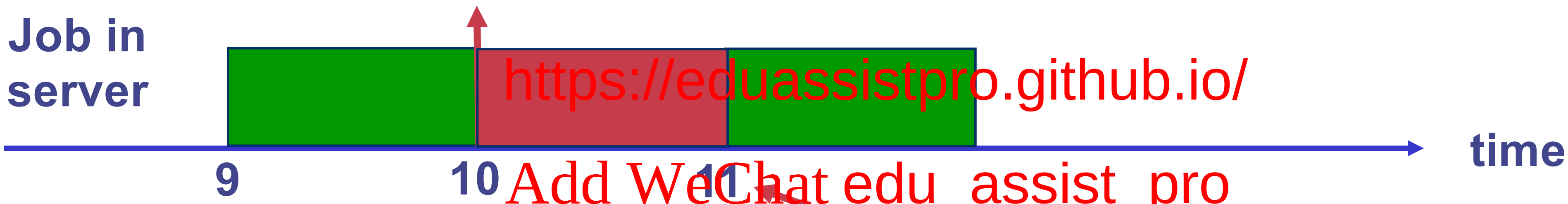
Time t = 10：A high priority job requiring 1s of processing arrives.

The job joins the high priority queue

Time t = 11：Server finishes processing the low priority job. It takes the high priority job in from the queue

Preemptive and non-preemptive priority (3)

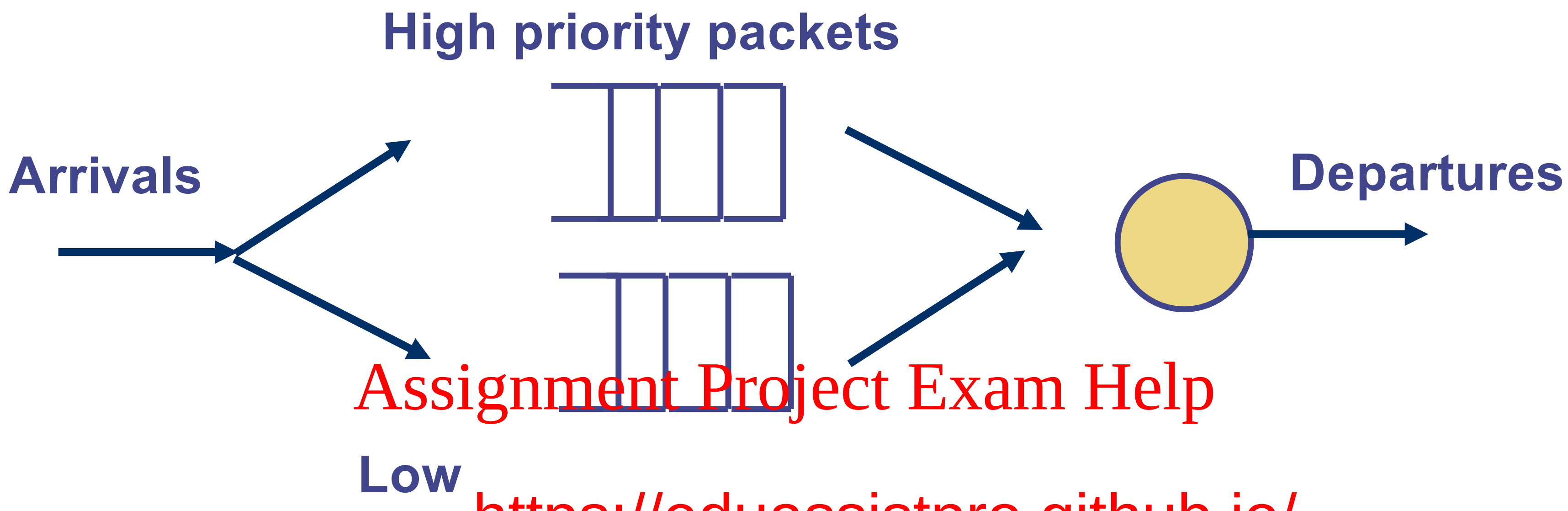
- ***Preemptive resume***:
  - Higher priority job will interrupt a lower priority job under service. Once all higher priorities served, an interrupted lower priority job is resumed.
- Example: High priority job (red), low priority job (green)



Time t = 10：A high priority job requiring 1s of processing arrives.
The server starts processing the high priority job immediately

Time t = 11：Server finishes processing the high priority job. Since no high priority job arrives in (10,11], the high priority job queue is empty, it resumes processing the low priority job that is pre-empted at time t = 10

Example of non-preemptive priority queueing



- Example: In the output port of a router, ive some packets a higher priority
  - In Differentiated Service
    - Real-time voice and video packets are given higher priority because they need a lower end-to-end delay
    - Other packets are given lower priority
- You cannot preempt a packet transmission and resume its transmission later
  - A truncated packet will have a wrong checksum and packet length etc.

Example of preemptive resume priority queueing

- E.g. Modelling multi-tasking of processors
- Can interrupt a job but you need to do context switching (i.e. save the registers for the current job so that it can be resumed later)

M/G/1 with priorities

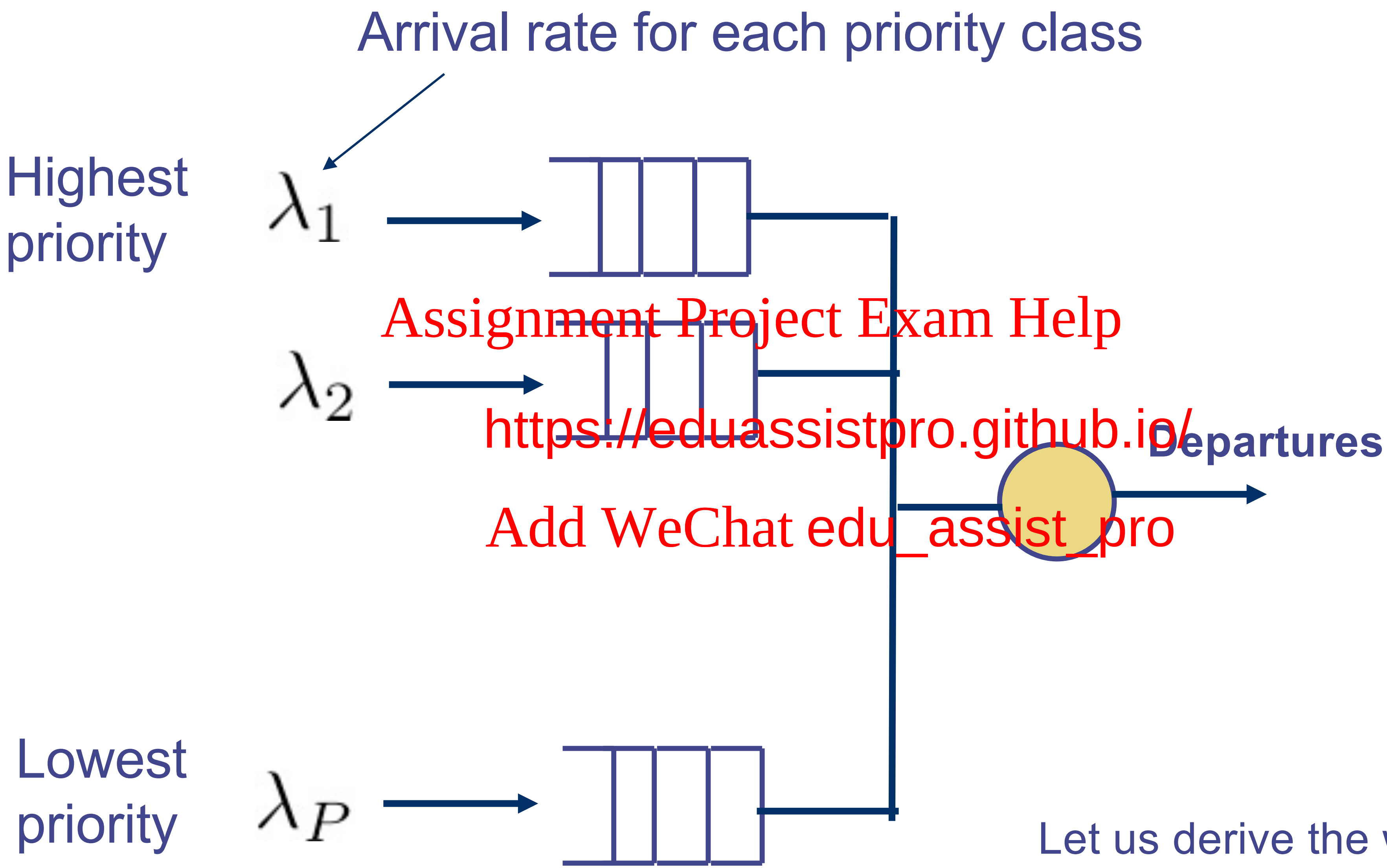
- Separate queue for each priority (see picture next page)
  - Classified into P priorities before entering a queue
  - Priorities numbered 1 to P, Queue 1 being the highest priority
- Arrival rate of priority class p is

$$\lambda_p \quad \text{W} \qquad P$$

- Average service time and oment of class p requests is given by

$$E\left[S_p\right] \text{ and } E\left[S_p^2\right]$$

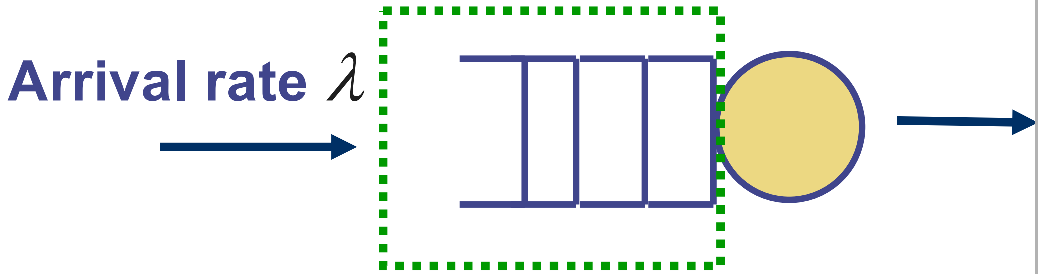
Priority queue



Let us derive the waiting time for P = 2

Lecture 4A: Deriving the P-K formula

- Let
  - W = Mean waiting time
  - N = Mean number of customers in the queue
  - 1/ μ = Mean service time
  - R = Mean residual service time
- We can prove th
  - W = N * (1/ μ) + R



- Applying Little's Law to the queue

= λ W

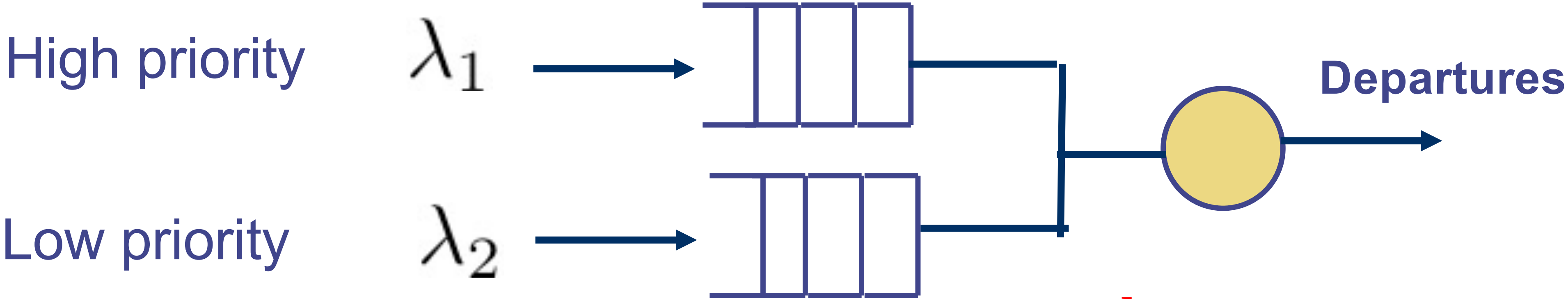
on

$$W = \lambda \times W \times \frac{1}{\mu} + R$$

Mean residual time R

$$R = \frac{1}{2}\lambda E[S^2]$$

Deriving the non-preemptive queue result (1)



- S_1 - service time f E[S_1]

- W_1 = mean waitin omers
- N_1 = number of Class 1 custom queue

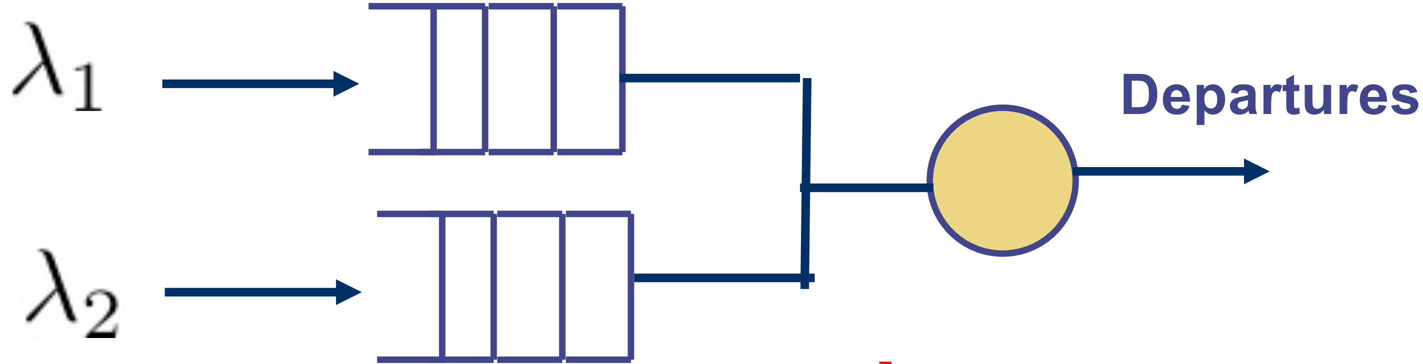
- R = mean residual service time when a customer arrives
- We have for Class 1: $W_1 = N_1 E[S_1] + R$
- Little's Law: $N_1 = \lambda_1 W_1$

$$W_1 = \frac{R}{1 - \rho_1} \quad \text{where} \quad \rho_1 = \lambda_1 E[S_1]$$

Deriving the non-preemptive queue result (2)

High priority

Low priority

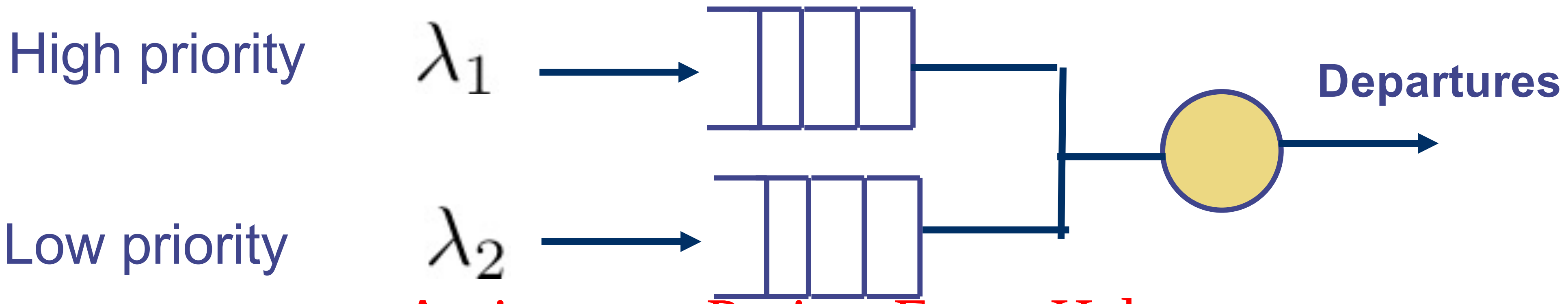


- To find the residu that the customer in the server can be ustomer, we have

- The waiting time is therefore

$$W_1 =$$

Deriving the non-preemptive queue result (3)

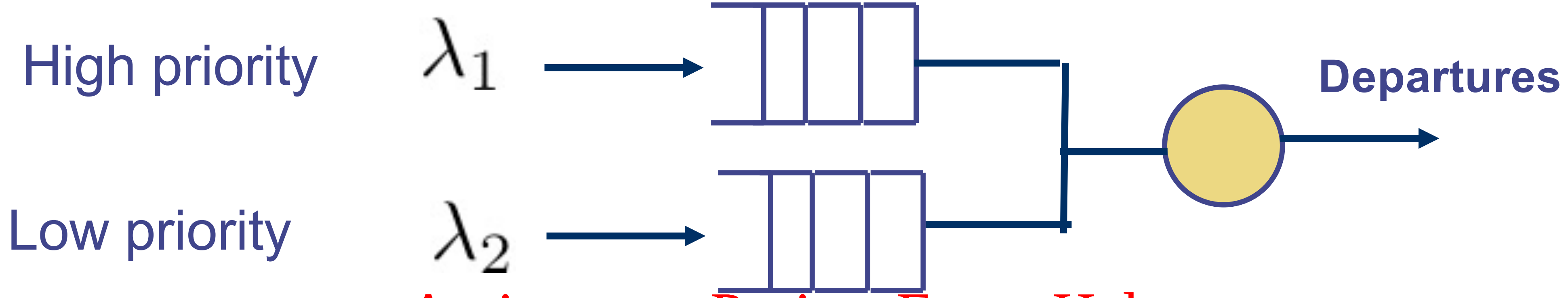


- S_2 - service time f [watermark overlap] $[S_2]$

- W_2 = mean waitin [watermark overlap] omers
- N_2 = number of Class 2 custom [watermark overlap] ueue

- R = mean residual service time when a customer arrives

Deriving the non-preemptive queue result (4)



- For Class 2 custo

Question:

Consider a customer arriving at the low priority queue, when can this customer receive service? This customer has to wait for

- The customer at the server to finish
- The customers who are already in the low priority queue when this customer arrives
- ?????
- ?????

Deriving the non-preemptive queue result (5)

$W_2 =$ $\boxed{}$

- Little's Law to Queue 1:
- Little's Law to Queue 2:

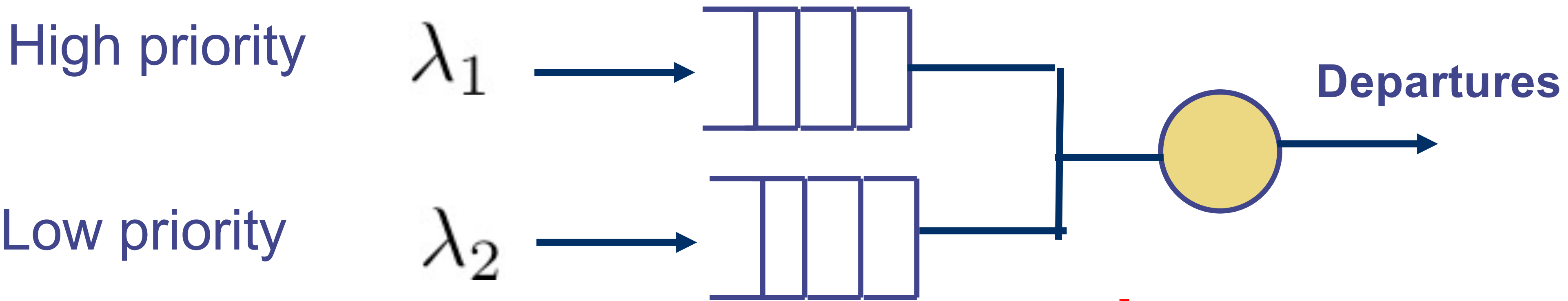
- Combining all of the above

$$W_2 = \frac{R + \rho_1 W_1}{1 - \rho_1 - \rho_2}$$

Where

$$\rho_2 = \lambda_2 E[S_2]$$
$$\rho_1 = \lambda_1 E[S_1]$$

Deriving the non-preemptive queue result (6)



$$W_1 = \frac{R}{1-\rho_1}$$

where

$$\rho_1 = \lambda_1 E[S_1]$$
$$\rho_2 = \lambda_2 E[S_2]$$
$$R = \frac{1}{2}E[S_1^2]\lambda_1 + \frac{1}{2}E[S_2^2]\lambda_2$$

Non-preemptive Priority with *P* classes

Waiting time of priority class k

$$W_k = \frac{R}{(1-\rho_1-\ldots-\rho_{k-1})(1-\rho_1-\ldots-\rho_k)}$$

where

$$\rho_i = \lambda_i E[S_i] \text{ for } i = 1, \ldots, P$$

Example

- Router receives packet at 1.2 packets/ms (Poisson), only one outgoing link
- Assume 50% packet of priority1, 30% of priority 2 and 20% of priority 3. Mean and second moment given in the table below.
- What is the aver
- Solution to be di

Priority	Mean (ms)	2nd Moment (ms^2)
1	0.5	0.375
2	0.4	0.400
3	0.3	0.180

Pre-emptive resume priority (1)

- Can be derived using a similar method to that used for non-preemptive priority
- The key issue to note is that a job with priority k can be interrupted by a job of higher priority even when it is in the server

- For *k* = *1* (highest time T_1 is:

$$T_1 = E[S_1] + \frac{R_1}{(1-\rho_1)}$$

where

$$R_1 = \frac{1}{2} E[S_1^2]\lambda_1$$

$$\rho_1 = E[S_1]\lambda_1$$

A highest priority job only has to wait for the highest priority jobs in front of it.

Preemptive resume priority (2)

- For k ≥ 2, we have response time for a job in Class k:

Question:

Consider a customer arriving in priority class k (≥ 2), what are the components of the w

omer?

Preemptive resume priority (3)

- Solving these equations, we have the response time of Class k jobs is:

$$T_k = T_{k,1} + T_{k,2}$$

where

$$R_k = \frac{1}{2} \sum_{i=1}^{k} E[S_i^2] \lambda_i$$

Other queuing disciplines

- There are many other queueing disciplines, examples include
  - Shortest processing time first
  - Shortest remaining processing time first
  - Shortest expected processing time first
- Optional: For an n queueing disciplines, see ystems Volume 2", Chapter 3.

References

- Recommended reading
  - Bertsekas and Gallager, “Data Networks”
    - Section 3.5.3 for priority queuing
- Optional reading
  - Harchol-Balter,