COMP9334 Revision Questions for Week 8A - Solution

Question 1

(a) If the service time of the fork join queue is less than x, then it means that the service time at each web service must be less than x.

Therefore, probability that the service time of the fork join queue is less than x is the same as the probability that the service times at the 3 servers are all less than x. Since the probability that the service time at each server is less than x is $1 - \exp(-\mu x)$ (this is the cumulative probability density function of exponential distribution with rate μ) and since we assume that the service time distributions at the servers are independent, we have the probability that the service times at all the 3 servers is less than x is $(1 - \exp(-\mu x))^3$.

(b) Let X be the random variable on the service time of the fork join queue. We have derived in Part (a) that $\text{Prob}[X \leq x] = (1 - \exp(-\mu x))^3$. This is in fact the cumulative probability density of X. The probability density function is the derivative of the cumulative probability density, which is $3\mu \exp(-\mu x)(1 - \exp(-\mu x))^2$

(c) The mean service ti

$$\int_{x=0} dx \quad (1)$$

By doing the integration, we find that

$$E[X] = (1 + \frac{}{2} + \frac{}{3})\frac{}{\mu} \quad (2)$$

Remark: An interesting exercise is to do that for n servers. I leave it to you.