<u>Solution to COMP9334 Revision problems - Week 3A</u>

Question 1

This aim of this question is to compare 3 different ways to upgrade an existing system. Let us first calculate the response time of the existing system.

Existing system

The assumptions imply that the queue is an M/M/1 queue with arrival rate $\lambda = 9$ and mean service time $(= 1/\mu) = 0.1$. This gives an utilisation $\rho = \lambda / \mu = 9 * 0.1 = 0.9$. Plugging these numbers into the mean response time T for M/M/1 = $1/(\mu (1- \rho))$, we have T = 1s.

Alternative 1

We upgrade the CPU to one twice as fast and since the service time is inversely proportional to the speed, the new mean service time = 0.05. The system is an M/M/1 queue with $\lambda = 9$ and $1/\mu = 0.05$. The utilisation $\rho = \lambda / \mu = 9 * 0.05 = 0.45$. Using the M/M/1 mean response time formula gives T = 0.0909s.

Alternative 2

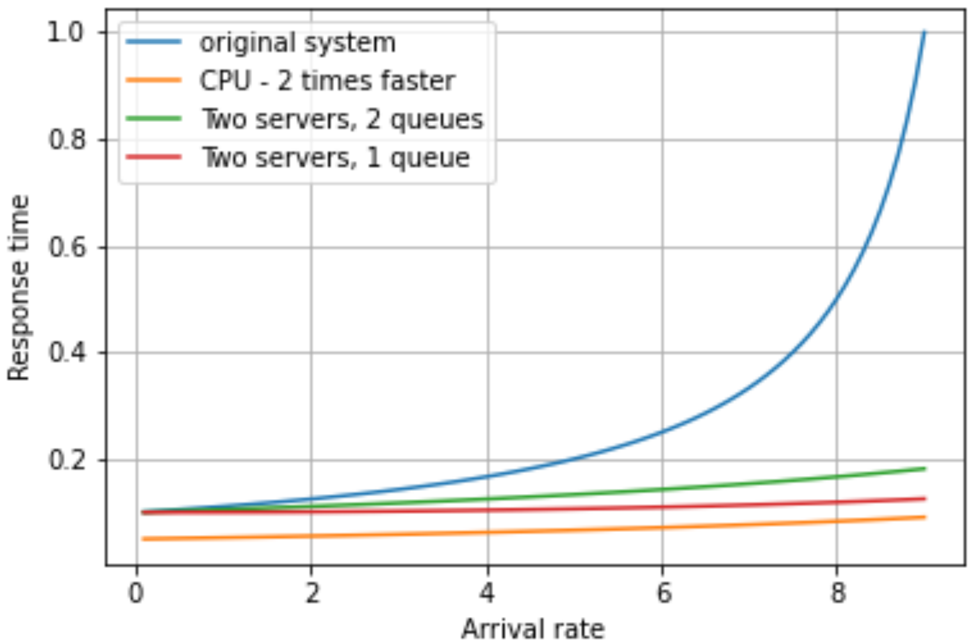
We use 2 existi e. The arrival rate at each queue is oisson splitting, see Section 11. /1 queues with with $\lambda = 4.5$ and $1/\mu = 0.1$. The utilisation ρ = Using the M/M/1 mean response time formula gives T = 0

Alternative 3

This alternative is effectively an M/M/2 queue with $\lambda = 9$ and $1/\mu = 0.1$. The utilisation $\rho = \lambda / \mu / 2 = 9 * 0.1 / 2 = 0.45$. Using the M/M/2 mean response time formula gives = 0.1254s.

Part (b)

We use a number of different arrival rates and we plot a curve of how the response time changes with the arrival rate. See the graph below:



Observations:

Note all the three alternatives have the same utilisation but have different response times. All three nt system but to different extend

Alternative 1 gives the best response time but it he existing hardware and buy an entirely new system. This

Both Alternatives 2 and 3 make use of the exist ative 3 gives a better response time. It may be cheaper to scale up the system by using Alternatives 2 and 3 in the long term because it allows incremental expansion of the system.

Note: I used Python to create the above graph. I've posted the files on the course web site.

Question 2

Part (a)

There is no queueing at all in this case. The mean waiting time is zero second.

Part (b)

The mean arrival rate is 1 customer / s. The mean service rate is 2 customers/s. By the M/M/1 formula, we know the mean response time is 1/(μ (1- ρ)) = 1s. The mean waiting time = mean response time – mean service time = 1 – 0.5 = 0.5s.

You may draw a number of conclusions here:

- Queueing is a result of the randomness in arrival and service patterns. In Part (b), the mean arrival rate is 1 customer/s, which means that the instantaneous arrival rate can sometimes be greater than 1 customer/s, the high instantaneous arrival rate can create a backlog in the system and create queueing.
- Waiting time is not just a function of the mean service time and mean arrival rate. It depends on the arrival and service time distributions too.

Question 3

This is effectively a M/M/4/4 system with arrival rate λ = 3 and mean service rate μ = 1/1.5. The prob me as the probability that the formula

$$P_m = \frac{\frac{\rho^m}{m!}}{\sum_{k=0}^{m} \frac{\rho^k}{k!}} \quad \text{where} \quad \rho = \frac{\lambda}{\mu}$$

With m = 4.