# COMP9334: Capacity Planning of Computer Systems and Networks



W isation (1):

Lin ming

# Three Weeks of Optimisation

- The lectures for next three weeks will focus on optimization methods for network related design and applications
- You will learn:
  - How to formulate opti
  - Tools to solve optimiz
- An introduction only, because optimiza c
  - Emphasis is on applying optimization methods rather than the theory behind

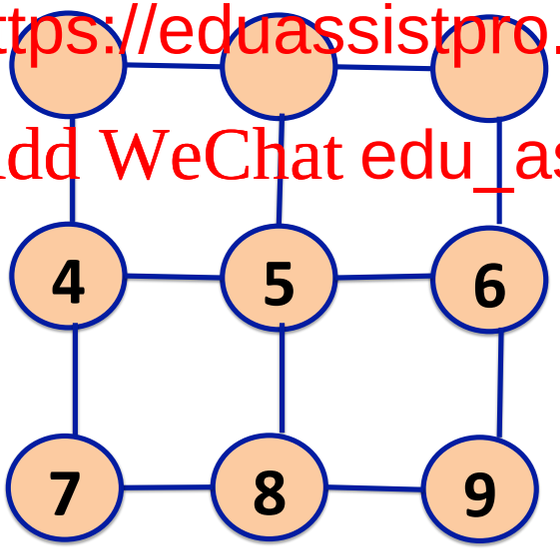
# Motivation (1)

- A modern approach to managing computer networks is based on the concept of *software-defined networking*
- Two types of nodes:
  1. Simple packet switche
  2. Controllers
- A controller can control a number of simple packet switches but they must be placed in a strategic location in the network
- If the delay between the controller and a packet switch is too long, then it can degrade the network performance

# Motivatiation (2)

- Consider the following network where there is a packet switch at each node and the delay on each link is 1 time unit.
- Question: Assuming you want to place one controller in the network, where will you place the controller?






- Question: What if you want to place two controllers?

# Motivation (3)

- How about solving the same problem for a large heterogeneous network?



- Optimisation provides a systematic method to make decisions

# Elements of an optimisation problem

- You want to maximise your WAM and still have a life

Maximise WAM($x_1$,$x_2$,$x_3$,...)

$x_1$ hours/week on COMP9334

$x_2$ hours/week on COMP

$x_3$ hours/week on social

$x_1 \geq 10$

$x_3 \leq$ maxSocialHours

$x_1 + x_2 + x_3 + ... \leq$ totalAwakeHours



- Elements of an optimisation problem
  - Minimise or maximise an objective function
  - Decision variables: $x_1$,$x_2$, ... etc.
  - Constraints

# What is optimization?

- In mathematics, also known as mathematical programming
  - The term programming refers to planning of activities to obtain an optimal result, not computer programming
  - The amount or level of ea as a variable whose valu
- Optimization means solving problems i ek to minimize or maximize the value of an objective function of many decision variables, subject to constraints on the decision variables

# Reference books

- Winston, “Operations Research”, 4th edition
  - Examples from this book tend to come from manufacturing, business, finance, etc
  - The abstraction power of mathematics means many optimization problems have simil on
  - Very often an optimiza y have a similar cousin in other application area thematical formulation are identical
- Ahuja, Magnanti and Orlin, “Network Flows”
- Fourer, Gay and Kernighan, “AMPL: A Modeling Language for Mathematical Programming”, 2nd edition

# Software

- Modeling language: AMPL and Solver CPLEX
  - High-level programming language for describing optimization problems
  - Syntax similar to mathem ion problems
  - Demo version of the software is available fo m: `http://www.ampl.com`. Click , then *Download a Free Demo*

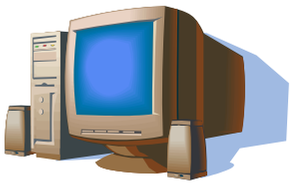


- Note: Demo version of AMPL/CPLEX is full-featured but limited to 500 variables and 500 objectives plus constraints

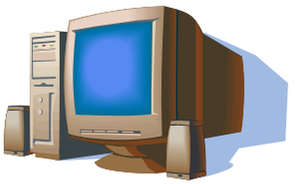
# Motivating example 1: Cloud/Grid computing

- Service providers sell computing power as an utility
  - Computing power measured in CPU cycles
- Target customers
  - Financial company, etc.
- Quality of Service in Cloud computing
  - Different service providers might offer the service at different levels for different costs
  - Optimization problem: How to select service providers (allocate resources) to achieve the best level of service without exceeding budget

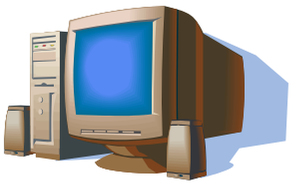
# Cloud computing resource allocation



**Resource 1**
**Speed: 1,000 million cycles/sec**
**Cost: 0.1 dollars/sec**



**Resource 2**
**Speed: 2,000 million cycles/sec**
**Cost: 0.25 dollars/sec**



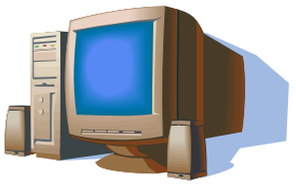
**Resource 3**
**Speed: 3,000 million cycles/sec**
**Cost: 0.6 dollars/sec**



- A computation job:
  - Requires $10^7$ milli
  - Must be completed in at most 4,800 sec
  - Cost must not exceed 1,500 dollars

- Exercises: For the time being, let us ignore the constraint on the completion time and cost.
  - If you use Resource 1 only, what is the completion time and cost?
  - Repeat for Resources 2 and 3.

# Cloud computing resource allocation (cont.)



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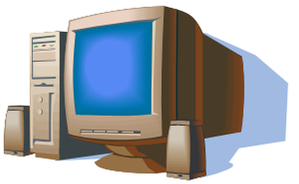

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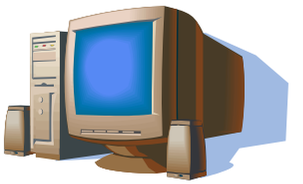

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**Speed: 3,000 million cycles/sec**
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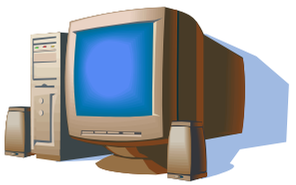
- A computation job:
  - Requires $10^7$ milli
  - Must be completed in at most 4,800 sec
  - Cost must not exceed 1,500 dollars

- Completion time and cost for each resource:
  - Resource 1: Completion time = 10,000 sec, cost = 1,000 dollars
  - Resource 2: Completion time = 5,000 sec, cost = 1,250 dollars
  - Resource 3: Completion time = 3,333 sec, cost = 2,000 dollars

# Cloud computing resource allocation (cont.)



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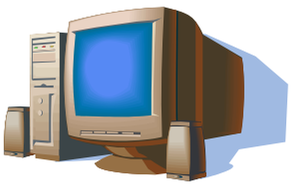

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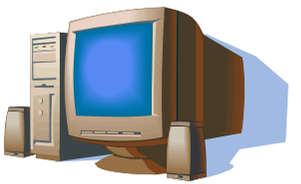


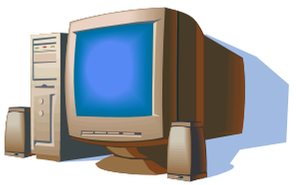
**Resource 3**
**Speed: 3,000 million cycles/sec**
**Cost: 0.6 dollars/sec**

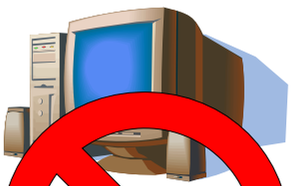


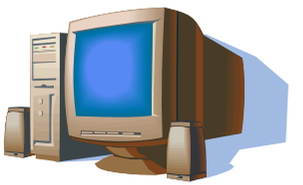
- Assume the computation job can be arbitrarily split to three parallel tasks

- Question: How should the job be split, so that completion time $T$ is minimized subject to two constraints:
  - Completion time constraint: $T \leq$ 4,800 sec
  - Cost constraint: $C \leq$ 1,500 dollars

# Trial and error: Solution 1







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**Speed: 1,000 million cycles/sec**
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**Resource 2**
**Speed: 2,000 million cycles/sec**
**Cost: 0.25 dollars/sec**

**Resource 3**
**Speed: 3,000 million cycles/sec**
**Cost: 0.6 dollars/sec**



- 48% to Resource 1, 52% to Resour

  - Resource 1: Completion time = 4,800 sec, cost = 480 dollars
  - Resource 3: Completion time = 1,733 sec, cost = 1,040 dollars

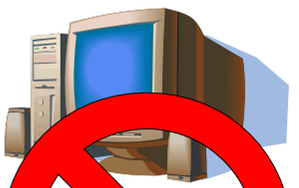
- Job completion time = 4,800 sec (remember jobs run in parallel)

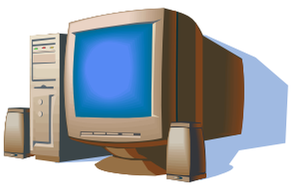
- cost = 1,520 dollars, Infeasible solution

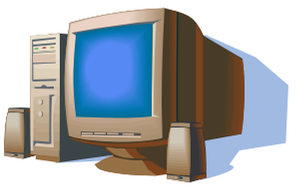
# Terminology

- A solution is feasible if all the constraints are satisfied
- A solution is infeasible if not all the constraints are satisfied

# Trial and error: Solution 2







**Resource 1**
**Speed: 1,000 million cycles/sec**
**Cost: 0.1 dollars/sec**

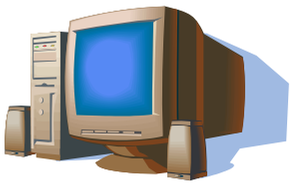
**Resource 2**
**Speed: 2,000 million cycles/sec**
**Cost: 0.25 dollars/sec**

**Resource 3**
**Speed: 3,000 million cycles/sec**
**Cost: 0.6 dollars/sec**

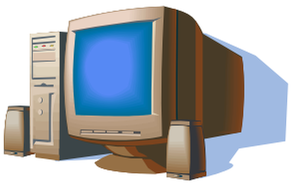


- 70% to Resource 2, 30% to Resour
  - Resource 2: Completion time = 3,500 sec, cost = 875 dollars
  - Resource 3: Completion time = 1,000 sec, cost = 600 dollars
- Job completion time = 3,500 sec, cost = 1,475 dollars
- Feasible solution

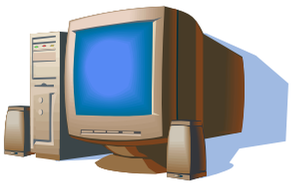
# Trial and error: Solution 3



**Resource 1**
**Speed: 1,000 million cycles/sec**
**Cost: 0.1 dollars/sec**



**Resource 2**
**Speed: 2,000 million cycles/sec**
**Cost: 0.25 dollars/sec**



**Resource 3**
**Speed: 3,000 million cycles/sec**
**Cost: 0.6 dollars/sec**



- 30% to Resource 1, 30% to Resour to Resource 3
  - Resource 1: Completion time = 3,000 s ollars
  - Resource 2: Completion time = 1,500 sec, cost = 375 dollars
  - Resource 3: Completion time = 1,333 sec, cost = 800 dollars
- Job completion time = 3,000 sec, cost = 1,475 dollars
- Feasible solution

# Optimizing resource allocation

- Given:
  - Job requirement = $10^7$ million cycles
  - Completion time $\leq$ 4,800 sec
  - Budget $\leq$ 1,500 dollars
- Let:
  - $x_1$ = fraction of the job to Resource 1
  - $x_2$ = fraction of the job to Resource 2
  - $x_3$ = fraction of the job to Resource 3
- Find $x_1$, $x_2$ and $x_3$ such that
  - All requirements are met
  - Completion time is minimized

# Formulating optimization problem

- Completion time:
  - Resource 1 = $\frac{10^7 \times x_1}{1000} = 10000 \times x_1$
  - Resource 2 = $\frac{10^7 \times x_2}{2000} = 5000 \times x_2$
  - Resource 3 = $\frac{10^7 \times}{300}$ _____
  - Job completion time $000 \times x_2, \frac{10000}{3} \times x_3)$
- Cost:
  - Resource 1 = $0.1 \times 10000 \times x_1 = 1000 \times x_1$
  - Resource 2 = $0.25 \times 5000 \times x_2 = 1250 \times x_2$
  - Resource 3 = $0.6 \times \frac{10000}{3} \times x_3 = 2000 \times x_3$
  - Cost $C = 1000 \times x_1 + 1250 \times x_2 + 2000 \times x_3$

# Formulating optimization problem (cont.)

- Mathematically, the optimization problem can be formulated as

$$\min \quad T$$

subject to



$$\begin{aligned}
T &\geq 10000 \times x_1 \\
T &\geq 5000 \times x_2 \\
T &\geq \frac{10000}{3} \times x_3 \\
T &\leq 4800 \\
1000 \times x_1 + 1250 \times x_2 + 2000 \times x_3 &\leq 1500 \\
x_1 + x_2 + x_3 &= 1 \\
x_1, x_2, x_3 &\geq 0
\end{aligned}$$

# Components of an optimization problem

- Given parameters
- Decision variables
  - In this example, they a
- Objective function
  - Can be minimization or maximization
  - Can be single objective or multi-objective
- Constraints

# Exercise

- Consider the following optimization problem where $x$ is the decision variable:

  $$\min_x \quad 2x - 1$$

  subject to

  $$x \geq 1$$



- What are the feasible solutions?
- What is the optimal solution?

# LP solvers

- Many commercial and free software are available for solving LP problems
- Commercial software
  - Capable of solving lar s of variables
  - A 50,000-variable LP pr standard linux PC
  - You can try out many commercial solve web site
    - `https://neos-server.org/neos/`
- Free software / demo version
  - `http://www.ampl.com`
  - `http://ampl.com/try-ampl/download-a-demo-version/`

# LP solvers (cont.)

- LP solvers require the user to write the problem in fixed format
- Can be embedded in C, C++ or Java, e.g.



```
model.add(IloMi                    ]+x[1]+4*x[2]));
model.add(-x[0]
model.add( x[0]-x[1] <= 1);
```



- Can be used with some modeling languages
  - AMPL
  - MPS
  - GAMS

# AMPL/CPLEX for solving example 1

- In AMPL, the grid computing problem formulated earlier becomes

```
var T;
var x_1 >= 0;
var x_2 >= 0;
var x_3 >= 0;
minimize time: T;
subject to T_1: T >= 10000*x_1;
subject to T_2: T >= 5000*x_2;
subject to T_3: T >= 10000/3*x_3;
subject to T_max: T <= 4800;
subject to C_max: 1000*x_1+1250*x_2+2000*x_3 <= 1500;
subject to x_sum: x_1+x_2+x_3 = 1;
```



- This is saved in the file `grid_lp.mod`

# AMPL/CPLEX for solving example 1 (cont.)

- The problem can be solved by CPLEX with the batch file `grid_lp_batch`

```
model grid_lp.mod;
option solver cplex;
solve;
display x_1;
display x_2;
display x_3;
display T;
```

# AMPL/CPLEX for solving example 1 (Command line version)

- At the ampl command prompt, type `commands grid_lp_batch;`, it returns

```
commands grid_lp_batch;
CPLEX 12.6.0.0: optimal solution; objective 2000
4 dual simplex iteration
x_1 = 0.2

x_2 = 0.4

x_3 = 0.4

T = 2000

1000*x_1 + 1250*x_2 + 2000*x_3 = 1500
```



- Note: All these files can be downloaded from the course web site

# AMPL/CPLEX for solving example 1 (IDE version)

- At the AMPL prompt, type `commands grid_lp_batch;`
- Need to `reset;` before working on a new problem

# Acknowledgment


- Grid computing example based on Menascé and Casalicchio, “QoS in computing”, IEEE Internet Computing, pp. 85–87, Jul./Aug. 2004.