Solution to COMP9334 Revision Questions for Week 4B_2

Question 1

We first work out the formula that we should use to generate random numbers with the Weibull distribution using the inverse transform method. Assuming that y and u are related by $y = F^{-1}(u)$, we have

$$\begin{aligned} y &= F^{-1}(u) && (1) \\ \Leftrightarrow u &= F(y) && (2) \\ \Leftrightarrow u &= 1 - \exp(-\alpha y^{\beta}) && (3) \\ \Leftrightarrow y &= \left(-\frac{\log(1-u)}{\alpha}\right)^{\frac{1}{\beta}} && (4) \end{aligned}$$

Therefore we can use the relation

$$\left(-\frac{\log(1-u)}{\alpha}\right)^{\frac{1}{\beta}} \qquad (5)$$

where u is uniformly distributed over $[0,1]$ to generate the Weibull distribution.

By using the above fo

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We have also derived the expected number of random number

it as the red curve in Figure 1. Note that the histogram consists of 50 equally s spanning $[0, y_{\max}]$ where $y_{\max}$ is the largest number gener $\frac{\text{ax}}{50}$, then the k-th bin spans the range $[(k-1)\delta, k\delta]$.

The probability that a number with Weibull distribution falls within the range $[(k-1)\delta, k\delta]$ is $(F(k\delta) - F((k-1)\delta))$. (Note: You can prove this by using the definition of cumulative density function.) If n random numbers are generated, then the expected number of random numbers in the k-th bin is $n(F(k\delta) - F((k-1)\delta))$

Figure 1: Histogram of 10000 random numbers with Weibull distribution. The red curve shows the expected distribution.