COMP9334
Capacity Planning for Computer Systems and Networks

Week 5B_1: Di tion (3).

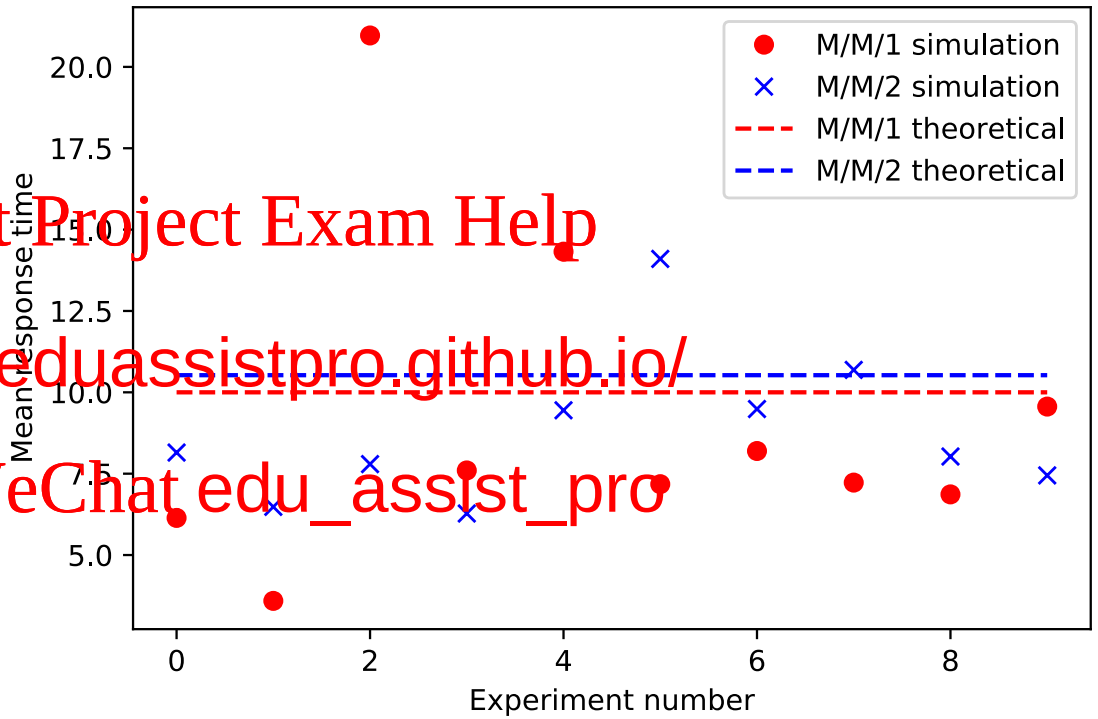
Comparing two systems.

Discrete event simulations so far

- You have learnt:
  - How to write simulation program
  - You know you cannot get exact mean response time from simulation but you can get a confidence interval
  - You can reduce the width of the confidence interval by
    - Simulate for longer
    - Increase the ations
- Today, you will learn how you re two systems in a statistically sound way
- Before that, we show you that comparing systems can be tricky

Problem: How do we compare 2 alternative choices?

- Week 4B's Revision Problem #2. The question asks you to simulate each of the following 2 queues 10 times:
  - M/M/1 queue with λ = 0.9 and μ = 1
  - M/M/2 queue with λ = μ = 0.5



Comparing two systems: motivation

- An application of simulation is to compare two systems

- For example, in Week 4B's revision question, you used simulation to compare the mean response time of
  - System 1: M/M/1 queue with $\lambda = 0.9$ and $\mu = 1$
  - System 2: M/M [illegible] = 0.5 for both server

- If you use analytical method, [illegible] the steady state mean response time of both systems exactly and you compare two numbers

- If you use simulation, you get a confidence interval for each system instead. How do you compare them?

Example: Comparing two systems

- Let us assume our goal is to use simulation to compare:
  - System 1: M/M/1 queue with $\lambda = 0.9$ and $\mu = 1$
  - System 2: M/M/2 queue with $\lambda = 0.9$ and $\mu = 0.5$ for both server

- For each system we carry out 3 independent replications
  - That is, we use 3 sets of independent random numbers together

- After removing the [illegible] ean response times are:
  - System 1: 6.8769, 8.5769, 10.6340
  - System 2: 8.8087, 7.4616, 9.1565

- In order to compare them, let us pair up these results
  - 1st experiment for System 1 with 1st experiment for System 2
  - 2nd experiment for System 1 with 2nd experiment for System 2 etc.

A paired-*t* confidence interval

- Let us summarise the data in a table
  - EMRT = estimated mean response time

	EMRT System 1	EMRT System 2	EMRT System 2 - EMRT System 1
Rep. 1	6.8769	8.8087	1.9318
Rep. 2	8.5769	7.4616	-1.1154
Rep. 3	10.6340		-1.4775

- We compute the 100 (1-α)% confidence e difference between 2 systems (= last column)

- Let us denote the computed confidence interval by [p,q]
  - Case 1: p,q > 0 ➔ System 1 is better than System 2 with probability (1-α)
  - Case 2: p,q < 0 ➔ System 2 is better than System 1 with probability (1-α)
  - Case 3: q > 0 & p < 0 ➔ Systems 1 and 2 are not different with probability (1-α)

Example: Paired-*t* confidence interval

- We compute the 95% confidence interval of the data showed in the last slide, the confidence interval is:
  - [-4.8721,4.4314]

- Therefore, with 95% probability that the mean response times of the two ent

- Hmmm, we have a problem h ow from queueing theory that System 1 has a better mean response time than System 2, but our simulation does not seem to be able to distinguish them.

- What can we do?

Let us increase the number of replications

- Since increasing the number of replications can reduce the width of the confidence interval, let us try that.
- Let us try 5, 10, 20, 30 replications

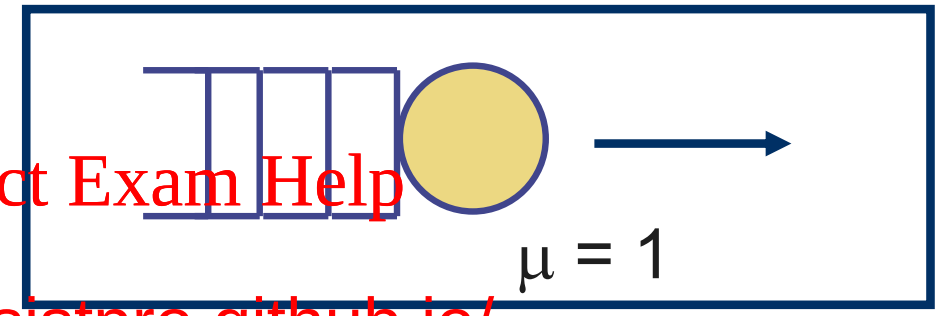
# independent replications	95% Confidence interval of EMRT System 2 - EMRT System 1
5	[-4.9540, 5.0242]
10	[-1.5347, 2.8020]
20	[-1.2724, 1.9870]
30	[-0.6001, 1.8046]

- Increasing the number of replications does reduce the width of the confidence interval
- However, we still cannot conclude which system is better

Let us have a look at how we did our experiments …

- We did our experiment with independent random numbers

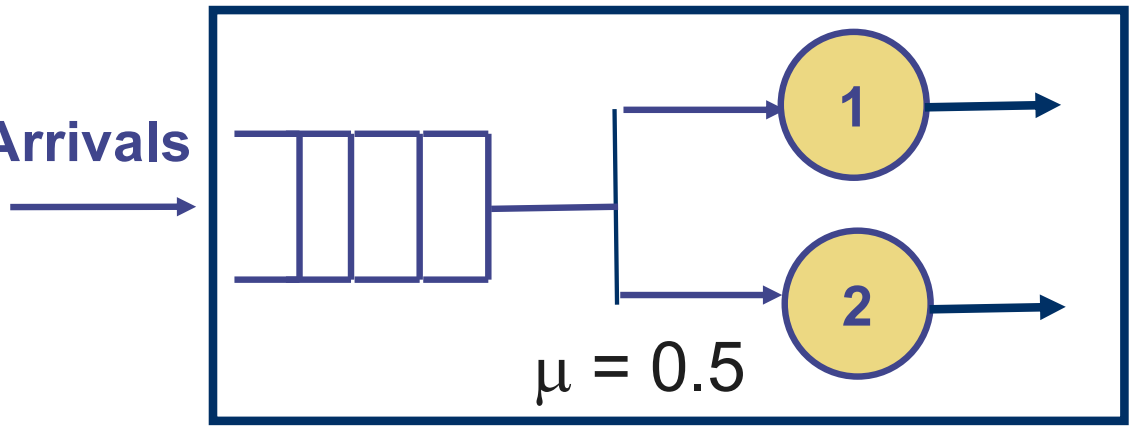
Job #	arrival time	service time
1	0.6329	0.3431
2	3.6392	0.4319
3	4.3919	0.1849
…	…	



Identical probability distribution but different numbers

Different distribution independe d

Job #	arrival time	service time
1	0.2548	0.3228
2	4.1194	0.3877
3	6.4435	0.0728
…	…	….



Common random numbers method

- An alternative is to compare two systems under similar condition

Job #	arrival time	service time
1	0.6329	0.3432
2	3.6392	0.4330
3	4.3919	0.10
…	…	

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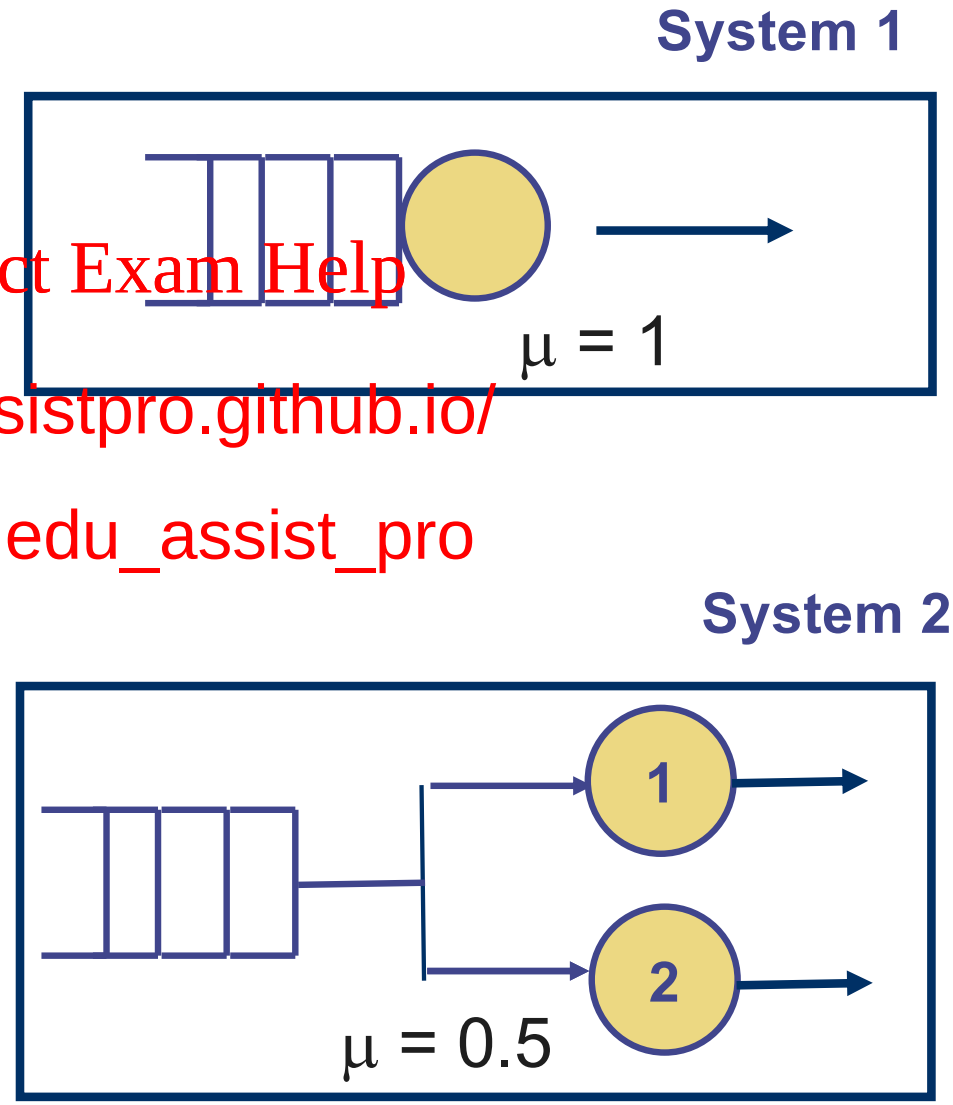
Job #	arrival time	service time
1	0.6329	0.6864
2	3.6392	0.8640
3	4.3919	0.2100
…	…	….



Common random numbers method

- A method to reduce the variance when comparing two alternative systems is to subject them to similar experimental condition

- In each replication, generate only one arrival time and one service time sequence
  - Apply this to bot
  - Note: Service tim d according to service rate
- In next replication, generate a new arrival time and a new service time sequence
  - Apply this to both systems

- This method can reduce the variance if the behaviour of the two systems is positively correlated

Applying common random numbers to our problem (1)

- Let us apply the common random numbers method to compare
  - System 1: M/M/1 queue with $\lambda = 0.9$ and $\mu = 1$
  - System 2: M/M/2 queue with $\lambda = 0.9$ and $\mu = 0.5$ for both server
- Let us carry out
- In each replication, we generate inter-arrival time sequence and one service time sequence (adjusted by service rate) and apply to both systems

Applying common random numbers to our problem (2)

- Let us compare the estimated mean response time (EMRT) from the 5 replications:

	EMRT System 1	EMRT System 2	EMRT System 2 - EMRT System 1
Rep. 1	8.3022	8.8087	0.5065
Rep. 2	6.8809	7.4616	0.5807
Rep. 3	8.5769		0.5796
Rep. 4	10.6340		0.7069
Rep. 5	16.2648	16.64	0.3837

- Observation: The EMRT of System 2 is higher than that of System 1 in all 5 replications
- If we compute the 95% confidence interval of the last column, we get [0.4046,0.6983]
- There is a 95% probability that System 1 is better than System 2

Comparing two methods

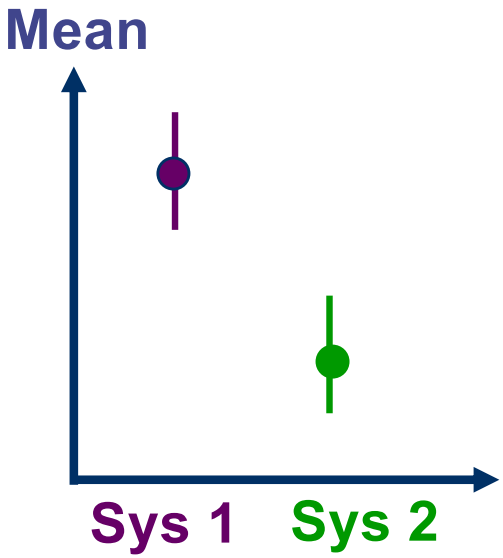
- Let us compare using common random number (CRN) method or not

# independent replications	95% Confidence interval of EMRT System 2 - EMRT System 1	
	Not using CRN	Using CRN
5	[-1.2724, 1.8]	[0.4046, 0.6983]
10	[-0.6001, 1.8046]	[0.4705, 0.6103]
20	[-0.6001, 1.8046]	[0.5127, 0.5942]
30	[-0.6001, 1.8046]	[0.5026, 0.5786]

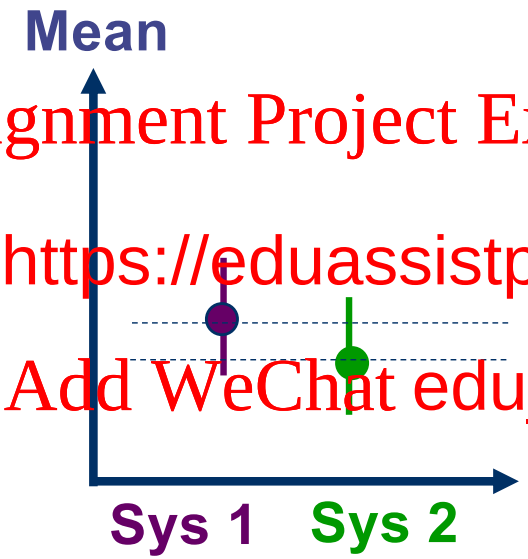
- Observations
  - By using CRN, all 95% confidence interval does not include 0
  - The width of the confidence interval for CRN method is a lot lower!

Approximate visual test

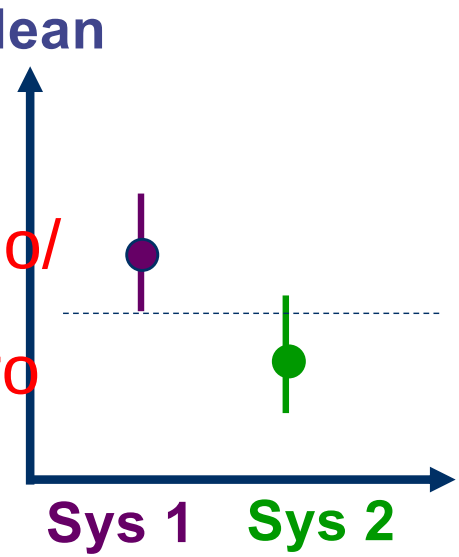
- Let us assume that you know the mean response time and its confidence interval (CI) for 2 systems: **System 1** and **System 2**
- Consider the following 3 possibilities:



CIs do not overlap Mean of System 1 > Mean of Sys. 2



CIs overlap and mean of a system is in the CI of the other: System are *not* different



CIs overlap and mean of any one is not in the CI of the other: do *t*-test

Ex: Multicast protocol design for wireless mesh networks

- Comparing 3 multicast protocols (WCMA, **SPT** and **RCAM**) for wireless mesh networks
- The thin vertical line shows the confidence interval
- What conclusion can you draw?

- Source: Chou et al, “Maximizing Broadcast and Multicast Traffic Load through Link-Rate Diversity in Wireless Mesh Networks”, you can download it from my web site: http://www.cse.unsw.edu.au/~ctchou/

Simulation tools and some applications (1)

- You do not always have to write your own simulation programs from scratch
- There are plenty of simulation tools available
  - Many with GUI

- Simulation tools puter networking research

  - Protocol #1 is the existing protocol. We designed Protocol #2. You want to see whether Protocol #2 is better or not.

  - You have two options (Option #1 and Option #2) to design a network. Which option is better?

Simulation tools and some applications (2)

- Some examples of publicly available simulation tools
  - General purpose: OMNet++
    - http://www.omnetpp.org/
  - For networking research: ns3
    - http://www.isi.edu/nsnam/ns/
- Some commercial tools
  - For network de
    - http://www.o
    - http://web.scalable-networks.c alnet
- **Important note: These tools save you time in writing simulation program but don't forget that you still need to analyse your simulation results using statistically sound methods!**

Summary

- Simulation is not just a computer programming exercise
- You need to make sure that your program is correct
- It is also important to analyse your results using statistically sound methods based on confidence interval
- Unfortunately, a lot of published research papers in computer netwo statistical analysis
  - Optional readin redibility of studies of telecommunication n E Communications Magazine, Pages 132-139, Jan

References

- The primary reference is Law and Kelton, “Simulation Modelling and Analysis”
  - Comparing two alternatives, Section 10.1, 10.2 (10.2.1 only)
  - Common random numbers, Section 11.2
- Raj Jain, “The Art of Computer Systems Performance Analysis” has materials on
  - Comparing two alternatives, Sections 13.3, 13.4 (13.4.1 and 13.4.3 only)
- Note that we have of statistical analysis of simulation data. Th e the specified sections) will provide you with more in depth on the topic.